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MASSIVE ϕ^4 MODEL AT FINITE TEMPERATURE – RESUMMATION PROCEDURE *A LA* RG IMPROVEMENT –

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In this paper the phase structure of the massive $\lambda\phi^4$ model at finite temperature ($T \neq 0$) is investigated by applying a resummation method inspired by the renormalization-group (RG) improvement to the one-loop effective potential. The resummation method *a la* RG-improvement is shown to work quite successfully by resumming up systematically large correction-terms of $O(\lambda T/\mu)$ and of $O(\lambda(T/\mu)^2)$. The temperature-dependent phase transition of the model is shown to proceed through the second order transition. The critical exponents are determined analytically and are compared with those in other analyses.

1 Introduction

Understanding the phase structure and the mechanism of phase transition of quantum field theories at finite temperature/density is important to understand the evolution of Universe and the physics to be searched by the ultrarelativistic heavy-ion experiments planned at the BNL-RHIC and CERN-LHC [1].

To investigate analytically the phase structure of relativistic quantum field theory, the effective potential (EP) is used as a powerful tool. Perturbative calculation of the EP at finite temperature, however, suffers from various troubles: the poor convergence or the breakdown of the loop expansion [2], and the strong dependence on the renormalization-scheme (RS). These troubles have essentially the same origin, i.e., they come about with the emergence of large perturbative corrections depending explicitly on the RS. Thus to break a way out of these troubles we must carry out the systematic resummation [3,4,5,6] of dominant large correction terms, and at the same time we must also solve the problem of the RS-dependence.

Recently simple but very efficient renormalization group (RG) improvement procedures for resumming dominant large correction terms are proposed in vacuum [7] and in thermal [8] field theories. This procedure was originally proposed to solve the problem of the strong RS-dependence of the EP calculated through the loop-expansion method.

It is worth noticing here that in the massive scalar $\lambda\phi^4$ model at high temperature the large correction terms appearing in the L-loop EP have the structures as follows; i) terms proportional to powers of the temperature T:

i-a) $(\lambda(T/\mu)^2)^L$, i-b) $(\lambda(T/\mu))^L$, and ii) terms proportional to powers of the *logs*: ii-a) $(\lambda \ln(T/\mu))^L$, ii-b) $(\lambda \ln(M/\mu))^L$, where M is the large mass scale appearing in the theory.

In this paper we present the result of application of the resummation procedure *a la* RG [8,9] to the massive scalar ϕ^4 model renormalized at the temperature of the environment T. We have found that the proposed resummation procedure *a la* RG works efficiently, not only by resolving the problem of the RS-dependence, but also by properly as well as systematically resumming terms having the structures i-a) and i-b) above. As for the details of the analyses, see Ref. 9 and the paper to appear [10].

2 Improving the effective potential through resummation and the phase structure of massive ϕ^4 model at $T \neq 0$

Let us consider the massive scalar ϕ^4 model at finite temperature renormalized at an arbitrary mass-scale μ and at the temperature of the environment T (hereafter we call this scheme as the T-renomalization). The key idea to resolve the RS-ambiguity is to use correctly and efficiently the fact that the exact EP satisfies a homogeneous renormalization group equation (RGE) with respect to change of the arbitrary parameter $\mu \to \bar{\mu} = \mu e^t$.

In the scalar ϕ^4 model the dominant large corrections appear as a power function of the effective variable τ (for more details, see Refs. 9 and 10)

$$\begin{aligned} \tau/\lambda &\equiv \Delta_1 \\ &= \frac{T^2}{2\pi^2 M^2}\left\{ L_1\left(\frac{T^2}{M^2}\right) - \frac{\pi^2}{12}\right\} - \frac{1}{2\pi^2} L_2\left(\frac{T^2}{\mu^2}\right) + \frac{b}{2}\left(\ln\frac{M^2}{\mu^2} - 1\right), \quad (1) \end{aligned}$$

where $b = 1/16\pi^2$, $M^2 = m^2 + \lambda\phi^2/2$ and $M^2\Delta_1$ is nothing but (a part of) the renormalized one-loop self-energy correction,

$$\begin{aligned} M^2\Delta_1 &\equiv \frac{1}{2}\sum\!\!\!\!\!\!\int \frac{1}{k^2 - M^2} + (\text{one-loop counter term}), \quad (2) \\ L_i\left(\frac{1}{a^2}\right) &\equiv \frac{\partial^i}{\partial (a^2)^i} L_0\left(\frac{1}{a^2}\right) \ , \quad (i \geq 1), \\ L_0\left(\frac{1}{a^2}\right) &\equiv \int_0^\infty k^2\, dk\, \ln[1 - \exp\{-\sqrt{k^2 + a^2}\}] \ . \end{aligned}$$

The resummation of dominant $O(\lambda(T/\mu)^2)$ terms in the T-renomalization can be automatically performed through renormalization, giving the remormalized mass-squared $m^2 \simeq m_0^2 + \frac{1}{24}\lambda T^2$, appearing as a mass-term in the

propagator with which the perturbative calculation is performed, where m_0 denotes the renormalized mass in the vacuum theory.

At the one-loop level the RGE's satisfied by the renormalized coupling and mass-squared can be solved exactly, giving solutions to the running parameters $\bar{m}^2$ and $\bar{\lambda}$ as

$$\bar{M}^2 = \bar{m}^2 + \frac{1}{2}\bar{\lambda}\phi^2, \quad \bar{m}^2 = m^2 f^{-1/3}, \quad \bar{\lambda} = \lambda f^{-1}, \tag{3}$$
$$f = 1 - 3\lambda\left[bt + \frac{1}{2\pi^2}\left\{L_2\left(\frac{T^2}{\bar{\mu}^2}\right) - L_2\left(\frac{T^2}{\mu^2}\right)\right\}\right].$$

Up to now $\bar{\mu}$ in the above eqations (3) can be arbitrary, with μ being fixed at the initial value of renormalization. *Our RG-improvement procedure, i.e., the resummation procedure a la RG, can be carried out by choosing the RS-fixing parameter $\bar{\mu}$ so as to satisfy $\bar{\tau}(t) = 0$, namely to make the one-loop radiative correction to the mass fully vanish.*

The RS-fixing equation $\bar{\tau}(t) = 0$ gives the mass-gap equation [6,8,9]

$$M^2 = m^2 + f(\bar{M}^2)\bar{M}^2 - f(\bar{M}^2)^{2/3}m^2 \ , \tag{4}$$

which determines, in the HT regime where $T/\mu \gg 1$, the RS-parameter $\bar{\mu}$ being exact up to T-independent constant as

$$\bar{\mu} = \frac{\bar{M}}{2} \ . \tag{5}$$

Now we can study the consequences of the RG-improvement in the T-renormalization, with solutions $\bar{\lambda}$, $\bar{m}^2$, and $\bar{\mu}$, Eqs. (3) and (5). The RG improvement can then be performed analytically [8,9,10], obtaining the improved EP as

$$\bar{V}_1 = \frac{1}{2}\bar{m}^2\phi^2 + \frac{1}{4!}\bar{\lambda}\phi^4 + \bar{h}\bar{m}^4$$
$$+\frac{\bar{M}^4}{2}\left[-\frac{b}{4} + \frac{T^4}{\pi^2\bar{M}^4}L_0\left(\frac{T^2}{\bar{M}^2}\right) - \frac{T^2}{2\pi^2\bar{M}^2}L_1\left(\frac{T^2}{\bar{M}^2}\right) - \frac{T^2}{24\bar{M}^2}\right] \tag{6}$$
$$= \frac{1}{2}\bar{m}^2\phi^2 + \frac{1}{4!}\bar{\lambda}\phi^4 - \frac{\bar{m}^4}{2\bar{\lambda}} - \frac{T\bar{M}^3}{48\pi} + \cdots. \tag{7}$$

With the RG-improved EP, $\bar{V}_1$, Eqs. (6) and (7), we can see the nature of the temperaure-dependent phase-transition of the model; i) At low temperature below $T_c \sim \sqrt{24|m^2|/\lambda}$ the EP has twofold structure showing the existence of two phases, Fig. 1a, the ordinary mass phase and the

Figure 1: The RG improved effective potential of the massive ϕ^4 model at two temperatures in the T-renormalization: a) $\tilde{T}_1 = 21.6$ and b) $\tilde{T}_2 = \tilde{T}_c = 21.689$ with $\tilde{T} \equiv T/|m_0|$. $\tilde{V} \equiv [\bar{V}_1(\tilde{\phi}) - \bar{V}_1(\tilde{\phi}_{min})]/|m_0|^4$, $\tilde{\phi} \equiv \phi/|m_0|$ and $\lambda = 1/20$.

small mass phase. The ordinary mass phase, with its counterpart in the tree-level potential, is the symmetry-broken phase which develops its minimum at $\phi = \phi_0 \sim \{T_c^4|m^2|^3/\lambda\}^{1/10}$. The small mass phase is a new "symmetric" phase, without having any counterpart in the tree-level potential, with a linearly decreasing potential unbounded from below, indicating the simple ϕ^4 model becoming an unstable theory at low temperature. As the temperature becomes higher the the minimum of the ordinary mass phase eventually diminishes, and ii) at the critical temperature T_c the minimum of the potential at non-zero ϕ completely disappears. The EP shows a symmetric structure in ϕ with the minimum at $\phi = 0$, $V(\phi) - V(0) \propto \phi^{\delta+1}$, $\delta \sim 5.0$, Fig. 1b, and iii) at high temperature above T_c the EP remains symmetric in ϕ with its minimum at $\phi = 0$. Transition between the symmetry-broken phase at low temperature and the symmetric phase at high temperature proceeds through the second order transition.

3 Critical exponents

Here we present the critical exponents determined from the RG-improved one-loop EP in the T-renormalization, Eqs. (6) and (7). In this case we can calculate the critical exponents through analytic manipulation.

The definition of the critical exponents are as follows; 1) On the behavior at $\phi = \phi_0$ around $T \simeq T_c$: $\phi_0 \propto (T_c - T)^\beta$, $d^2V/d\phi^2|_{\phi=\phi_0} \propto |T_c - T|^\gamma$, $V(\phi_0) - V(0) \propto |T_c - T|^{2-\alpha}$. 2) On the behavior at $T = T_c$: $V(\phi) - V(\phi_0 = 0) \propto \phi^{\delta+1}$ or $dV/d\phi \propto \phi^\delta$. Here ϕ_0 denotes the position of the true min-

Table 1: Critical exponents obtained from various methods.

	β	γ	δ	α
Our result	0.3	1.2	5.0	0.2
mean-field	0.5	1.0	3.0	0.0
lattice [12]	0.324	1.24	4.83	0.113
experimental [11]	0.325	1.24		0.112

imum, and T_C denotes the critical temperature, which are determined as $\phi_0 \simeq \{54^3 T_c^4 |m^2|^3/\lambda(8\pi)^4\}^{1/10}$, $T_c \simeq \sqrt{24|m_0^2|/\lambda} - 3|m_0^2|/4\pi\mu$.

Results are summarized in Table 1, showing that our result deviates significantly from the mean-field values and agrees reasonably with the experimental data [11,12].

4 Summary and discussion

In this paper we proposed a new resummation method inspired by the renormalization-group improvement. Applying this resummation procedure *a la* RG-improvement to the one-loop effective potential in the massive scalar ϕ^4 model renormalized at the temperature of the environment T, we found important observations; The $O(\lambda(T/\mu)^2)$-term resummation, thus the so-called hard-thermal-loop resummation [3] in this model, can be simply done through the T-renormalization itself. With the lack of freedom we can set only one condition to choose the RS-fixing parameter, which actually ensures to absorb the large terms of $O(\lambda T/\mu)$, thus only the partial resummation of these terms can be carried out.

It is to be noted that all the results obtained are essentially the same as those in the T_0-renormalization case [9,10]: the second order phase transition between the ordinary mass broken phase at low temperature and the symmetric phase at high temperature, and the existence of the unstable small mass phase at low temperature. In this sense our resummation method gives stable results so long as the terms of $O(\lambda T/\mu)$ are systematically resummed. The critical exponents are determined by analytic manipulation, showing the significant deviation from the mean-field values and the reasonable agreement with the experimental data [11,12]. For details, see Refs. 9 and 10.

As noticed above, the RG-improved EP in the simple massive ϕ^4 model has,

below the critical temperature T_c, an unstable small-mass phase in addition to the ordinary symmetry-broken phase. This unstable phase also appears in the same model at exact zero-temperature, indicating its appearence being not the artifact due to the crudeness of approximation on the temperature-dependent correction terms. Though the origin of its appearence is not fully understood, it may have a relation with the triviality of the model, which is an interesting problem for further studies. The $O(N)$ symmetric model in the large-N limit exists as a stable theory without having such an unstable phase [8].

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