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What one can learn about the QCD parton cascades studying the multiplicity distributions at HERA ?

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ABSTRACT

New properties of the multiplicity distributions predicted by higher order QCD and their physical origin are discussed briefly. Several studies which can be performed at HERA are proposed.

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1 Definitions

Multiplicity distributions (MD) of particles produced in high energy collisions are the most typical and widely discussed characteristic of the interaction dynamics. In a condensed form MD provide information about the fluctuations of energy spent for multiple particle production during a collision.

The goal of the present paper is to review briefly the new features of the multiplicity distributions predicted by higher order QCD.

There are two complementary ways of dealing with multiplicity fluctuations :

– studying the distribution $P_n = \sigma_n/\sigma$ which is the number of produced particles per event, or

– measuring the inclusive multiplicity correlators.

In practice, one uses often the normalized factorial moments F_q and cumulants K_q (for review see [2]) defined as

$$F_q = \sum_{n=0}^{\infty} n(n-1)...(n-q+1)P_n/\langle n\rangle^q = \frac{\langle n(n-1)...(n-q+1)\rangle}{\langle n\rangle^q}, \qquad (1)$$

$$K_q = F_q - \sum_{m=1}^{q-1} C_{q-1}^m K_{q-m} F_m . \qquad (2)$$

Here $C_q^m = \frac{q!}{m!(q-m)!}$ are the binomial coefficients and $F_0 = F_1 = K_1 = 1$.

These moments have an important advantage over the original moments [3]. The average shown in (1) implies mean value of the corresponding expressions over the available set of experimental events. In experiment this averaging takes into account both statistical and dynamical effects. If one assumes that random fluctuations due to limited number of detected particles are described by the Poissonian distribution, then the total average of the factorial moments is equivalent to the dynamical average of usual moments [3].

In the Feynman diagram's language, F_q corresponds to the set of all graphs while the cumulants K_q describe the connected graphs only. The cumulants provide the knowledge about the "true" correlations , non-reducible to the product of the correlations of lower orders. At asymptotic energies the normalized factorial moments (as well as the ordinary ones) do not depend on energy and are the functions of their rank only. The higher the rank of the moment is the more sensitive F_q and K_q are to the "tail" of MD at large n. The steeper decrease of the distribution at large n leads to smaller values of the high rank factorial moments .

In a theoretical analysis instead of studying of the numerical series P_n it is more convenient to analyse the function "generating" it, namely the generating function (GF). F_q and K_q are easily calculated if the generating function $G(u)$ is known [2]

$$G(u) = \sum_{n=0}^{\infty} P_n(1+u)^n . \qquad (3)$$

Then

$$P_n = \frac{1}{n!}\frac{d^n G(u)}{du^n}\Big|_{u=-1}, \qquad (4)$$

$$F_q \;=\; \frac{1}{\langle n\rangle^q}\frac{d^q G(u)}{du^q}\Big|_{u=0}, \tag{5}$$

$$K_q \;=\; \frac{1}{\langle n\rangle^q}\frac{d^q \ln G(u)}{du^q}\Big|_{u=0}. \tag{6}$$

Thus, the knowledge of GF gives us a possibility to calculate both the multiplicity distribution and cumulant and factorial moments i.e. (3)-(6) demonstrate mathematical equivalence the description of MD by functions P_n, F_q and K_q . In [4] it has been proposed to use the ratio of cumulant to factorial moments $H_q \equiv K_q/F_q$ which behaves in a qualitatively different way for various distributions and is more sensitive to specific features of P_n which are invisible when just plotted P_n or even F_q (see Sec. 3).

2 Some properties of the multiplicity distributions

In pre-QCD time Koba, Nielsen and Olesen published the paper [5] with a hypothesis about the scaling properties of the multiplicity distributions at asymptotic energies (the KNO scaling). If z is the scaled multiplicity $z = n/\langle n\rangle$, then the KNO scaling implies a universal form

$$\psi(z) \;=\; \langle n\rangle\, P_n$$

for the multiplicity distribution. During last 30 years the KNO-like behavior of MD was experimentally confirmed in various types of high energy particle production processes except the data on proton-antiproton interactions at the highest energies $\sqrt{s}$ =546 and 900 GeV obtained by UA5 collaboration [6] in CERN.

The negative binomial distribution (NBD)

$$G(u) \;=\; \left(1 \;-\; \frac{u\langle n\rangle}{k}\right)^{-k},$$

$$P_n \;=\; \frac{(n+k-1)!}{n!(k-1)!}\left(\frac{\langle n\rangle/k}{1+\langle n\rangle/k}\right)^n\left(1 \;+\; \frac{\langle n\rangle}{k}\right)^{-k} \tag{7}$$

$$F_q \;=\; \frac{(k+1)\cdots(k+q-1)}{k^{q-1}}, \qquad K_q \;=\; \frac{(q-1)!}{k^{q-1}}$$

$$H_q \;=\; \frac{(q-1)!}{(k+1)\cdots(k+q-1)} \tag{8}$$

is another example of the distribution which is in good agreement with experimental data in full phase space and in smaller phase space domains. NBD depends on two parameters, the average multiplicity $\langle n\rangle$ and a positive parameter k describing the shape of the distribution. Here we will mention only two classes of mechanisms proposed to generate NBD, (partial)stimulated emission [7, 8] and cascading [8].

One feature of H_q for NBD is that it always positive and tends to zero with a q^{-k} behaviour at high ranks. For the Poisson distribution H_q is identically equal to zero (except for $H_1 = 1$)

3 What does QCD tell us about the multiplicity distributions ?

The KNO hypothesis was strongly supported by QCD when the equations for generating function were solved in the so-called double logarithmic approximation (DLA). DLA happens to be too crude however for making reasonable predictions even for asymptotic energies: the predicted KNO shape of the distribution appeared to be much wider than the experimental one. On the qualitative level, DLA can be thought to overestimate cascading processes, ignoring completely energy-momentum conservation since the energy of the radiating particles remains unchanged after a soft gluon emission. Therefore DLA apparently overestimates the gluon multiplicity because the parton characteristic energy is higher and the parton multiplicate more actively. Taking into account higher order perturbative corrections leads to a more accurate control over the parton splitting processes and energy conservation.

Such an approach has been realized (see [9], [10]) in the framework of the modified leading logarithmic approximation (MLLA) by a generalization of the standard LLA scheme following the logic of the famous Dokshitzer-Gribov-Lipatov-Altarelli-Parisi approach and including the exact angular ordering (AO) (instead of the strong AO within DLA). Thus the system of the MLLA integro-differential equations for the quark and gluon GF has been derived.

A recent series of publications [4], [11]–[14] was devoted to solving of these equation in the case of e^+e^--collisions with account of different next-to-next-to leading (NNL) effects. Corresponding corrections can be looked upon [15] as being due to a more accurate account of energy conservation in the course of parton splitting. For example, the approximation used in [11] allowed in the framework of gluodynamics the derivation of analytical expressions for the asymptotic behaviour of factorial moments and the KNO function, which are in better agreement with the data, by reducing substantially the width of the theoretical distribution. Cumulant and factorial moments of the multiplicity distributions in the perturbative gluodynamics have been calculated in [4], [12]. Accounting for the degrees of freedom associated with quarks [13] does not change the essential qualitative features of F_q , K_q and influence only weakly H_q . The exact solutions of the QCD equations for quark and gluon GF are obtained for the case of fixed coupling in [14].

The ratio H_q is more sensitive to the form of P_n at large n than F_q (see Fig. 1). It was shown in [13] that the predictions of F_q shown in Fig. 1a, have qualitatively the same behaviour and are very close to each other for $q \leq 10$. However H_q (Fig. 1b) demonstrate much stronger sensitivity to the assumptions used. The most typical feature of the ratio H_q predicted by QCD [12] is its quasi-oscillating form with a changing sign (Fig. 2). Such an oscillating behavior of H_q is a specific property of higher order QCD. Less complete account of nonlinearities in the equation for GF leads [4], [13] only to one minimum with a very small value of H_q (the solid line in Fig. 1b).

The results of [4] have initiated a search for the peculiarities of H_q from the experimental data. According to the H_q measurements from multiplicity distributions in e^+e^-–annihilation in the energy range from 22 to 91 GeV, and in hh–collisions, in the energy range from 24 to 900 GeV, made in [16], its behaviour corresponds to the predic-

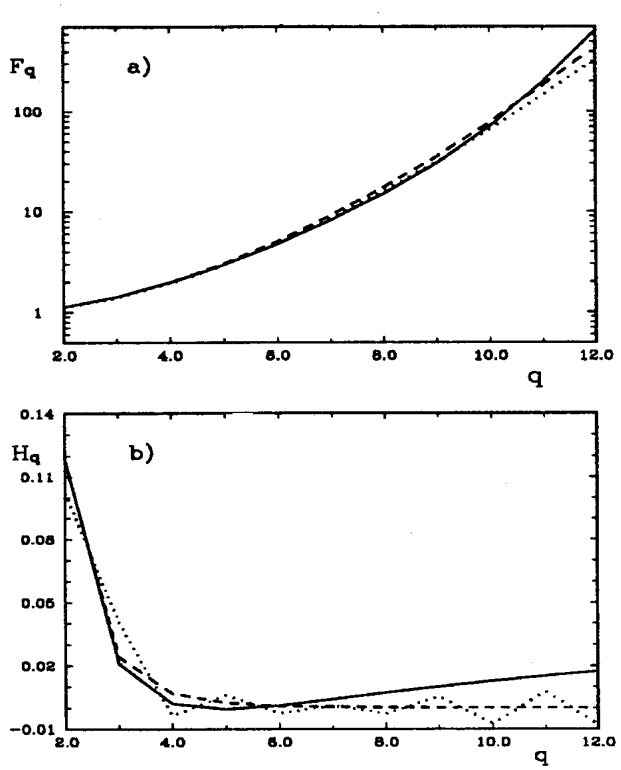

Fig.1. a) The factorial moments for different QCD distributions ([4] - solid line; [11] – dotted line) and for NBD with k=7.6 (dashed line). b) The ratio H_q for the same distributions as in a).

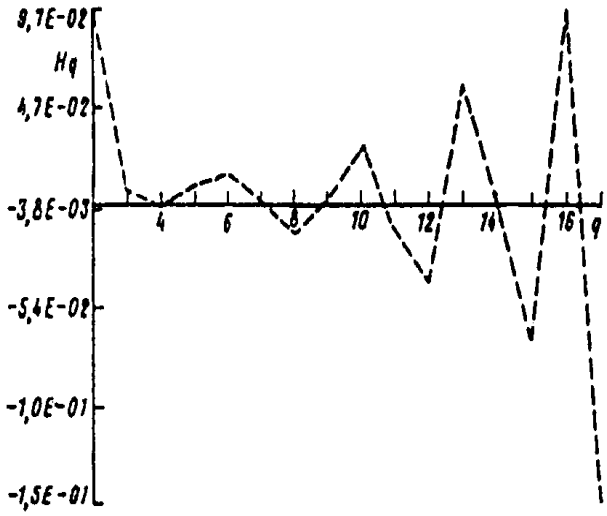

Fig. 2. The ratio H_q predicted by QCD [12].

tions of higher order QCD. A few examples are presented in Fig. 3. It is a surprise for us that the theoretical results obtained for hard processes at asymptotic energies, are in a qualitative agreement with experimental data at low and high energies both for e^+e^- processes and soft hadronic collisions.

The behaviour of H_q for NBD shown in Fig 1, where H_q falls monotone but always positive, tending to zero at large ranks q, is not compatible to results shown in Fig 3. Therefore, despite the fact that NBD fits experimental MD very well it is not appropriate for the complete of description MD in particle production processes as claimed in [13] and [16]. However, as will be seen from Sec.5, after modifications NBD is able to generate the oscillating H_q as well.

4 Monte Carlo Generators

All Monte Carlo (MC) generators for high energy physics [17] and, in particular, those which simulate deep inelastic scattering (DIS) [18] are based on the leading logarihm (LL) picture with two body parton splitting $a \to d + c$. However, as one mentioned in the previous section, higher orders in the perturbative QCD are necessary for a proper description of multiproduction at high energies.

At present this can only be achieved in the generators through approximate methods implemented in different QCD cascades e.g. the Lund parton shower (PS) [19], the color

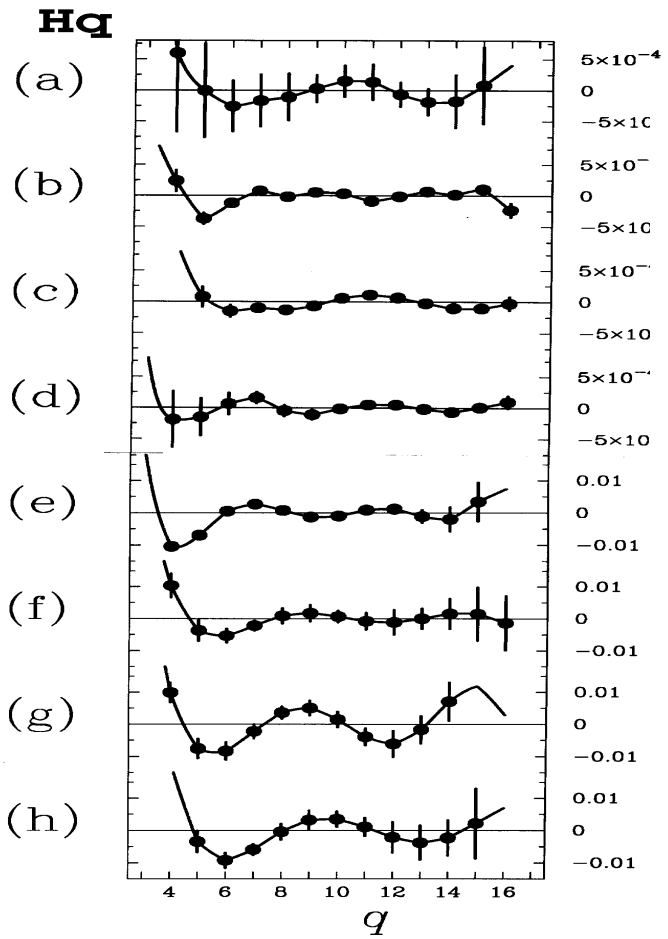

Fig.3. Experimental data [16] on H_q for a)-d) e^+e^- ($\sqrt{s}$=29, 34.8, 43.8, 91 GeV) and e)-h) hh ($\sqrt{s}$=62.2, 200, 546, 900 GeV) collisions. Lines are to guide the eye.

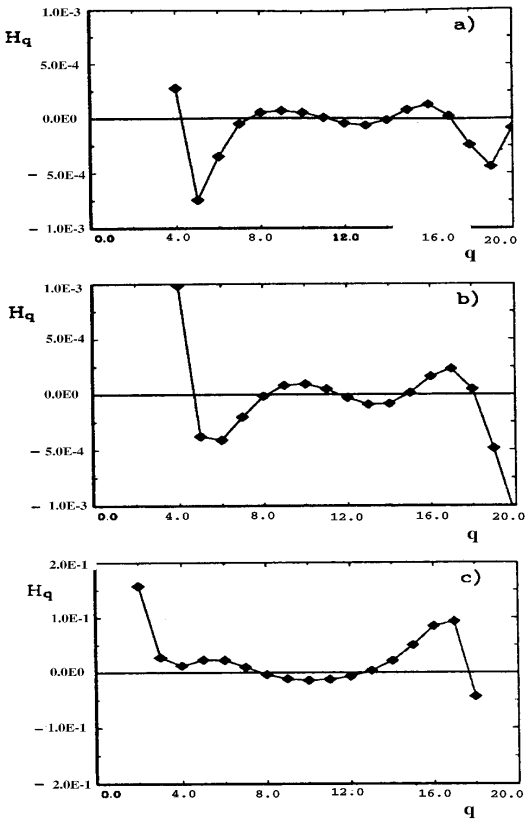

Fig. 4. The ratio H_q due to the QCD MC codes: a) JETSET 7.3, e^+e^-, $\sqrt{s}$= 91 GeV; b) ARIADNE 4.4, e^+e^-, $\sqrt{s}$= 91 GeV; c) PYTHIA 5.5, e^-p, $\sqrt{s}$= 314 GeV. Lines are to guide the eye.

dipole model (CDM)[20]. The LLA used in PS and CDM does not give a proper treatment of hard emissions. A method was developed to let a single hard emission to be controlled by the exact $O(\alpha_s)$ or $O(\alpha_s^2)$ QCD matrix elements and then modelling subsequent radiation using the PS technique.

One can ask a question: are the above-mentioned improvements of the MC models enough for a proper description of H_q ? The answer seems to us obvious: since LLA is the base of PS one should not expect an oscillatory behavior of H_q . However, according to our calculations of the correlators with the MC generators JETSET 7.3 [21], ARIADNE 4.4 [18](e^+e^-, $\sqrt{s}$ =91 GeV) and PYTHIA 5.5 [18](e^-p, $\sqrt{s}$ =314 GeV) H_q has, nevertheless, an oscillating form as shown in Fig. 4. An explanation of such a phenomenon can be found immediately if one recall two facts: 1) each MC generator takes a special care about both the local (in the course of parton splitting) and global energy-momentum conservation in the collision; 2) the finite energy of collisions is the physical origin of large $O(\alpha_s)$ corrections [15]. Thus, the LLA in conjunction with the energy-momentum conservation in the MC models imitate in some part the higher order corrections leading to the oscillation of H_q . The question arises, though, how much of the higher order corrections are accounted for?

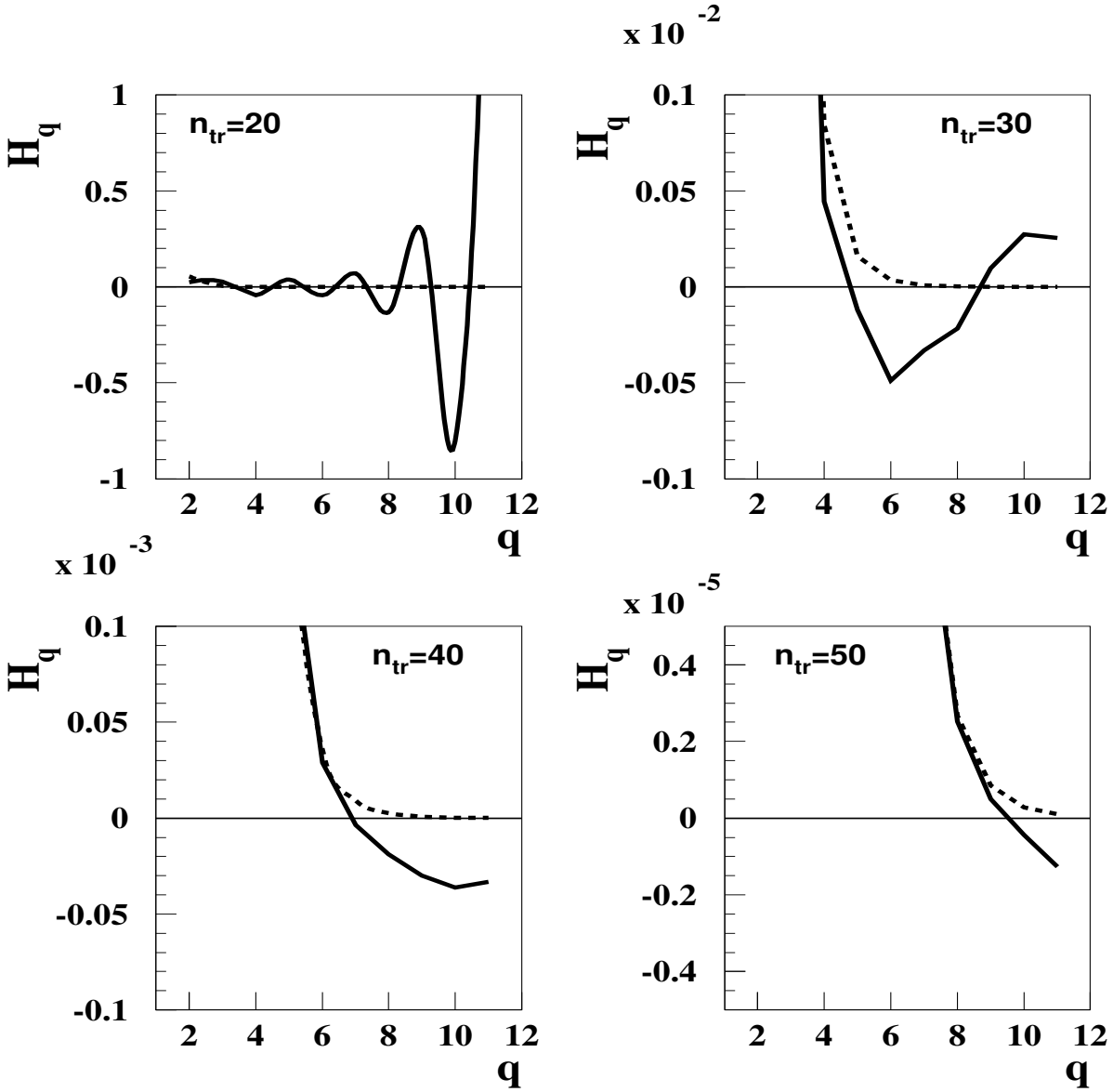

Fig.5. The ratio H_q calculated for NBD with $\langle n\rangle$=9.22, k=17.24. The solid line are for NBD truncated at n_{tr} and the dished lines are for NBD without truncation.

5 Phenomenological examples

The conclusions from the previous section can be confirmed by the following arguments [22]. Formally, according to (7) NBD has an infinite "tail" at finite collision energy (finite $\langle n\rangle$). This results in positive K_q and monotone declining H_q (8). On the other hand, an infinite "tail" of MD is possible only for production of massless particles or through neglecting energy conservation during the reaction. Taking into account these factors leads to a truncation of the MD "tail" at some finite multiplicity $n_{tr}(s)$. As a result, H_q calculated for the truncated NBD oscillates around the curve q^{-k} with alternating sign. The amplitude of the oscillation tends to zero quickly as $n_{tr} \to \infty$ and $H_q^{(tr)}$ tends to $H_q^{(NBD)}$ (Fig. 5). The same behaviour of H_q has been found for the truncated Poisson

distribution (PD).

Another example of the behaviour of H_q is in soft $p\bar{p}$ collisions at $Sp\bar{p}S$ and Tevatron energies calculated in [23] in the framework of the Dual Parton Model [24]. It was found [23] the properties of H_q (amplitude of the oscillation, positions of minima and maxima) are very sensitive to the number of cutted Pomerons accounted for in the calculation.

6 What can be done at HERA ?

In high energy reactions used in the study of the oscillations of H_q [16] only DIS data are missing. New data from the ep collider HERA will be able to rectify this situation. The invariant mass W of the hadronic final state in DIS at HERA extends, with significant cross sections, to the phase space limit ($\sqrt{s}$ = 314 GeV). This circumstance allows us to formulate several problems related to properties of MD which can be studied with the H1 and ZEUS detectors:

1. Detailed study of MD as a function of $z = n/\langle n \rangle$ over the whole kinematical region of W. Does the KNO scaling violated at large W ?

2. High precision measurement of the ration $H_q = K_q/F_q$ of cumulant and factorial moments both for the full phase space and restricted rapidity windows, for events with 1+1, 1+2, ... jets, etc. Does H_q as a function of the order q shows an oscillations around $H_q = 0$? If so, confronting the data with predictions of the MC models we would learn more about the higher order effects implemented in these MC models.

3. Measurements of H_q at different W will shed more light on the problem how the finite energy effects influence the H_q shape.

To conclude, energy-momentum conservation plays a very important role in the correct description of the multiplicity distributions , F_q , K_q and H_q in the framework of QCD and different phenomenological models. The ratio H_q is extremely sensitive to the length of the MD "tail". In perturbative QCD the behaviour of the MD "tail" is controlled by higher order corrections while for phenomenological approaches (NBD, PD etc.) the finite energy effects have to be accounted for by truncating the MD "tail".

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