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THE QCD-ORIENTED VECTOR DOMINANCE MODEL

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Abstract

The total photoabsorption cross section $\sigma_{\gamma N}$ is studied using nondiagonal GVDM. Vector meson-nucleon scattering amplitudes are calculated in two-gluon exchange approximation of QCD. The off-diagonal transitions of diffraction dissociation type between different vector mesons and their contributions to the expression for $\sigma_{\gamma N}$ are also calculated. It is shown that destructive interference of diagonal and off-diagonal terms is not effective. The main conclusion is that a p -cut-off in $\gamma \to q\bar{q}$ transition is necessary for obtaining the convergence in the summation over all vector meson contributions.

1. Introduction

The nondiagonal GVDM [1-3] is based on an assumption about existence of a double mass dispersion relation for the forward Compton scattering amplitude [4]. In particular, in case of transversely polarized virtual photons one has

$$\sigma_T(s,q^2) = \int dm^2 \int dm'^2 \, \frac{\rho_T(s,m^2,m'^2)m^2m'^2}{(m^2-q^2)(m'^2-q^2)} \qquad (1)$$

The spectral function ρ_T is expressed in GVDM through imaginary parts of vector meson-nucleon scattering $V_nN \to V_{n'}N$ amplitudes

$$\rho_T \sim \frac{1}{m_n^2 m_{n'}^2} \, \mathrm{Im}\, T_{nn'}(m_n, m_{n'}, s)$$

(2)

It is well known, that GVDM approach, using hadron basis describing, e.g., total photoabsorption cross section is dual to the approach, based on covariant parton model (CPM). In CPM the forward Compton amplitude is determined by a ?handbag? diagram [5] which, in turn, is expressed through off-mass shell parton-hadron amplitudes. Further, the hypothesis is used that parton-nucleon amplitudes are strongly damped when any parton line is far from mass shell. As a result, CPM leads to Bjorken scaling feature for $\sigma_T(s,q^2)$, while having a nondiagonal spectral function ρ_T (the correct behavior is reached after cancellations of leading diagonal contributions to ρ_T with negative off-diagonal ones). Analogously, in nondiagonal GVDM, the scaling requires destructive interference between the diagonal ($V \to V$) and off-diagonal ($V \to V'$) transitions.

Nowadays, with advent of QCD, it became clear that the use of hadron basis in VDM-type approaches has some limitations. The process of photoabsorption can be divided on two stages: firstly, the production of $q\bar{q}$-pair and, then, the interaction of this pair with target nucleon. One can easily show that the average transverse distance between particles of the pair is inversely proportional to the intrinsic transverse momentum of the pair, $\langle r_\perp \rangle \sim 1/p_\perp$. Evidently, the condition of vector-meson dominating is

$$\langle r_\perp \rangle \geq R_\perp, \text{ or } \quad p_{\perp \leq} p_{\perp \max} \sim \frac{1}{R_\perp}. \qquad (3)$$

Here, $R_\perp$ is the average transverse radius of a vector meson. So, for consistent use of GVDM concept the cut-off in the $P_\perp$ must be introduced. Correspondingly this leads to appearing of non-GVDM contributions which can be taken into account by the QCD perturbation theory.

2. Outline of the model

For a calculation of the spectral function one needs hadronic forward scattering amplitudes. We use the Low-Nussinov [6] QCD model of Pomeron in which this amplitudes are calculated in two-gluon exchange approximation. This is a

perturbation approach: the non perturbative effects are simulated only by given an effective mass to the gluons and using an effective value of the cuark-gluon coupling constant.

At high energies, due to large lifetime of $q\bar{q}$-fluctuations of the vector meson, the transverse size of virtual $q\bar{q}$-pair does not change (i.e., is ?frozen?) in the scattering process. Owing to this, the diagonal amplitude can be expressed as an average of the scattering amplitude of ?eigenstate? having definite size of the pair, over $q\bar{q}$-wave function of the vector meson. Further one can easily calculate the off-diagonal amplitudes assuming that $V \to V'$ transition is generated by different absorption of various eigenstates [7].

The wave functions and mass spectra of vector mesons which are necessary for the calculation of the spectral function and σ_T are obtained from the solution of the reduced Bethe-Salpeter equation describing $V_{q\bar{q}}$-vertex.

As is discussed above, the transverse momentum cut-off must be introduced leading to decrease of high-mass contributions. The present model shows straightforwardly that there is no strong destructive interference, so that without of p -cut-off the integral in (1) would diverge.

As for nonGVDM contribution to σ_T, it is important to note that due to ?color transparency? property, the cross section of scattering of small-size $q\bar{q}$-pair is small, $\sigma \sim \left\langle r_{q\bar{q}}^2 \right\rangle$ [8]. Correspondingly, the direct (perturbative) contribution to σ_T is also not large and GVDM remains to be useful approach for a prediction of the photoabsorption cross section and related quantities.

3. Meson-nucleon scattering amplitudes

The straightforward calculation of the elastic meson-nucleon scattering amplitude using two-gluon exchange diagram leads to the expression:

$$F(s,t) = \frac{i}{(2\pi)} s \frac{2}{9} 2q^4 \iint d^2\mathbf{r}_\perp \, dy \times$$

$$\times \left\{ \int \frac{d^2k_\perp \, V(k_\perp, Q)}{\left[\left(\frac{\vec{Q}}{2} - \vec{k}\right)^2 + \mu^2\right]\left[\left(\frac{\vec{Q}}{2} + \vec{k}\right)^2 + \mu^2\right]} \left(e^{-i\frac{\vec{Q}}{2}\vec{r}_\perp} - e^{-i\vec{k}_\perp \vec{r}_\perp} \right) \right\} \Psi_V^2(\vec{r}, y)$$

(4)

Here, $\Psi_V(\vec{r},y)$ is the two-particle ($q\bar{q}$) wave function of the meson; $r_\perp$ is the transverse distance between q and $\bar{q}$ of the meson; the variable y is equal to ½ - x, here x is a fraction of longitudinal momentum of the meson, carried by one the quarks. The variable y is conjugate to a longitudinal size between q and $\bar{q}$. Further, μ is the gluon effective mass (parameter of the model), $\vec{Q}$ is a transverse momentum transfer in the scattering, $t = -\vec{Q}^2$. Transverse moments of two exchange gluons are equal to ($\vec{Q}/2 \pm \vec{k}_\perp$). At least, the quantity $V(k_\perp, Q)$ is the factor, containing nucleon wave function. Using the simplest assumptions about these wave functions [9] one obtains for the $V(k_\perp, Q)$ the following expression:

$$V(k_\perp, Q) = 3\exp\left(\frac{Q^2\left\langle r_N^2\right\rangle}{6}\right)\left\{1 - \exp\left(-\frac{Q^2\left\langle r_N^2\right\rangle}{8}\right)\exp\left(-\frac{k_\perp^2\left\langle r_N^2\right\rangle}{2}\right)\right\} \quad (5)$$

Here, $\left\langle r_N^2\right\rangle$ is the mean square radius of the nucleon.

One can rewrite Eq.(4) for the elastic scattering of V_n meson on nucleon in the form

$$F_{nn}(s,t) = \langle n|F_{r_\perp}(s,t)|n\rangle, \quad (6)$$

where $F_{nn}(s,t)$ is the scattering amplitude for the eigenstate, i.e. for the $q\bar{q}$-pair with the transverse size $r_\perp$. The brackets in Eq. (6) mean averaging over the wave function.

It is convenient to work in the plane of impact parameters:

$$\Omega_{nn}(s,b) = \langle n|\Omega_{r_\perp}(s,t)|n\rangle, \quad (7)$$

where

$$\Omega_{nn}(s,b) = \frac{1}{2\pi i}\int \frac{1}{4\pi s} F(s,t) e^{i\vec{Q}\vec{b}} d^2Q. \quad (8)$$

Calculating $\Omega_{nn}(s,b)$ for each vector meson one can parameterize it in the Regge-type form:

$$\Omega_{nn}(s,b) = \frac{\sigma_{2g}}{4\pi}\frac{1}{B_{2G}} e^{-\frac{b^2}{2B_{2G}}}. \quad (9)$$

Here, index 2G means ?two-gluon exchange?; σ_{2G} is the total cross section of V_nN-interaction and B_{2G} is the slope of diffraction cone in elastic V_nN-scattering.

To take into account the energy dependence of scattering amplitudes the expression (9) must be modified. We use for this aim the two-pole parameterization [10]:

$$\Omega_{nn}(s,b)=\frac{\sigma_{2g}}{4\pi}\left(\frac{1}{B_{2G}}e^{-\frac{b^2}{2B_{2G}}}+\frac{1}{R}\frac{e^{\Delta_F\xi}}{B_{2G}+2\alpha'_F\xi}e^{-\frac{b^2}{B_{2G}+2\alpha'_F\xi}}\right). \quad (10)$$

Here $\xi=\ln s/s_0-i\pi/2$, Δ_F,α_F and R are constants ($\Delta_F=\alpha_F-1$, where α_F is intercept of the F-trajectory, α'_F is the trajectory slope). It is assumed here, for simplicity, that the slopes of diffraction cones are the same for 2G- and F-terms. If R>>1, the relation contribution of the F-term at low energies is small and Eq.(9) is valid.

Returning to the calculation of forward scattering amplitudes, one has

$$F_{nn}(s,0)=2is\int\Omega(s,b)d^2b. \quad (11)$$

Unitarized amplitude accounting the rescatterings of $q\bar{q}$-pair is given by the formula

$$T_{nn}(s)=2is\int\langle n\left|\left(1-e^{-\Omega_{r_\perp}(s,b)}\right)\right|n\rangle d^2b=$$
$$=2is\int\left(1-\langle n|e^{-\Omega_{r_\perp}(s,b)}|n\rangle\right)d^2b. \quad (12)$$

Finally, for off-diagonal amplitudes one has, evidently ($n\neq n'$):

$$T_{nn'}(s)=-2is\int\langle n|e^{-\Omega_{r_\perp}(s,b)}|n\rangle d^2b. \quad (13)$$

The imaginary parts of just these T-amplitudes are to be used in the expression (2) for the spectral function.

4. Mass spectrum and wave functions of vector mesons

In the initial form, the formulas for hadron scattering matrix elements contain $h_{q\bar{q}}$-vertex functions expressed through Bete-Salpeter $q\bar{q}$-wave functions of the hadrons (we use the constituent quark model of hadrons). These wave functions can be found by approximate solution of Bete-Salpeter equation using the null-plane formalism [11] and additional angular condition [12]. The resulting wave functions depend on one variable $\vec{q}^2$ and are solutions of simple equation ($\vec{q}^2 \equiv (q_\perp^2 + (My)^2)$:

$$\left(\vec{q}^2 - \frac{M^2}{4} + m_q^2\right)\Psi(\vec{q}) = \frac{\varpi_{q\bar{q}}^2}{2M}(\nabla_{\vec{q}}^2 + \varpi_0^2)\Psi(\vec{q}) \qquad (14)$$

Here, M and m_q are masses of the meson and quark, respectively, $\varpi_{q\bar{q}}^2$ and ϖ_0^2 are parameters. From formal point of view this is the equation for a wave function of a particle moving in three-dimensional oscillator potential (we assumed here that the confining $q-\bar{q}$ -interaction is of harmonic oscillator type).

The solutions of Eq.(14) are well known. The mass spectrum of the mesons with L=0 (e.g., mesons of ρ -family) is

$$\frac{1}{2\beta^2}\left(\frac{M^2}{4} - m_q^2 + \beta^4\varpi_0^2\right) = N + \frac{3}{2}; \quad N = 0,2... \qquad (15)$$

Here, β is given by the definition

$$\varpi_{q\bar{q}}^2/(2M) \equiv \beta^4 \qquad (16)$$

One can see from Eq.(15) that, if β and ϖ_0^2 are constants, the mass spectrum has the form

$$m_n^2 = a + bn, \quad (17)$$

i.e., the spectrum is equidistant for square masses.

5. Cut-off factor

One can show that the invariant mass of produced $q\bar{q}$-pair is connected with $p_\perp^2$ and x by simple relation,

$$M_{q\bar{q}}^2 = p_\perp^2 /[x(1-x)], \quad (18)$$

We will assume, for simplicity, that a meson is formed only if $p_\perp < p_{\perp max}$ and this value $p_{\perp max}$ is the same for all vector mesons (irrespectively on their mass). In this case the relation of phase volume part of produced pair with $p_\perp < p_{\perp max}$ to total phase volume is equal to (for large $M_{q\bar{q}}$)

$$\eta(M_{q\bar{q}}^2) \approx 3p_{\perp max}^2 / M_{q\bar{q}}^2. \quad (19)$$

All hadronic amplitudes contained in the expression (2) for the spectral function must be multiplied on these cutting factors (assuming that $M_{q\bar{q}} \sim M_V$). The value of $p_{\perp max}$ is a parameter of model.

6. Results of the calculations and conclusions

We calculated the contribution to $\sigma_T(s, q^2 = 0)$ from mesons of the ρ-family (ρ, ρ',

ρ'' etc). Mass spectrum of the mesons is obtained from Eq.(15) if masses of any two family members are known. For $m_\rho = 0.77$ and $m_{\rho'} = 1.45$ one obtains from the system of equations

$$\frac{1}{4}m_\rho^2 - m_q^2 + \beta^4 \varpi_0^{-2} = \frac{3}{2}2\beta^2$$
$$\frac{1}{4}m_{\rho'}^2 - m_q^2 + \beta^4 \varpi_0^{-2} = \left(2 + \frac{3}{2}\right)2\beta^2 \qquad (20)$$

the following values of the parameters ($m_q = 0.3$):

$$\beta^2 = 0.094\text{GeV}^2,\ \omega_0^2 = 0.04\text{GeV}^2$$

From here one obtains the mass spectrum

$$m_n^2 \cong m_\rho^2(1 + 2.55n) \qquad (21)$$

and the value $m_{\rho''} = 1.9$ which does not contradict with the data.

For normalization of the scattering amplitudes we use the relation from the additive quark model,

$$\sigma_{\rho p} = \frac{1}{2}\left(\sigma_{\pi^+ p} + \sigma_{\pi^- p}\right) \qquad (22)$$

And two-pole parameterization of Eq.(10) for the πp-amplitudes. Corresponding parameters for ρp-amplitude, found from comparison with the data, are:

$$\sigma_{\rho p} = 27\,\text{mbn}\ ;\ R=10;\ \Delta_F = 0.2\ ;\ \alpha'_F = 0.13$$

Further, knowing $\Omega_{00}(s,b)$ we perform in Eqs.(12,13) the changes:

$$\Omega r_{\perp}(s,b) \to \frac{\Omega r_{\perp}(s,b)}{\langle 0|\Omega r_{\perp}(s,b)|0\rangle}\Omega_{00}(s,b)$$

For the calculation of T_{nn} , $T_{nn'}$ we use the value $\mu = m_{\pi}$; this choice leads to more or less acceptable predictions of effective slopes for known mesons.

The calculation showed that, at least in the present model, the cancellations of diagonal and off-diagonal amplitudes are not effective (in particular, amplitudes T_{01}, T_{10} have the same sign as T_{00}, T_{11} and the same order of magnitude). It is clear that, in this case, the use of the $p_{\perp}$ -cutting procedures is essential for the convergence of the summation over vector mesons. From the comparison with data on $\sigma_T(s,0) = \sigma_{\gamma N}$ we obtained (adding ϖ, φ -contributions without their families) that

$$p_{\perp max} \approx 0.5 \text{ GeV}$$

One should note, in conclusion, that probably this physical result depends rather strongly on the model of hadrons used here (constituent quark picture, confining potential of harmonic oscillator type). In principle al required information about hadrons can be taken from experiment (this is an advantage of using hadron basis in VDM approaches) but this information must be very detailed because the off-diagonal transitions can, in principle, be rather large.

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