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Searching for Extra Dimensions and New String-Inspired Forces in the Casimir Regime*

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Abstract

The appearance of new fundamental forces and extra-dimensional modifications to gravity in extensions of the Standard Model has motivated considerable interest in testing Newtonian gravity at short distances ($\lesssim 10^{-3}$ m). Presently a number of new gravity experiments are searching for non-Newtonian effects in the ranges $\sim 10^{-4}$–10^{-3} m. However, as challenging as these experiments are, formidable new obstacles await the next generation of experiments which will probe gravity at distances $\lesssim 10^{-4}$ m where Casimir/van der Waals forces become dominant. Here we will review the motivation for conducting such very short distance gravity experiments, and discuss some of the new problems that may arise in future experiments. Finally, we suggest schematic designs for null experiments which would address some of these problems using the "iso-electronic" and "finite-size" effects.

1 Introduction

When Isaac Newton formulated his law of universal gravity over 300 years ago, he provided the first mathematical description of one of the fundamental forces of nature. Yet, physicists have realized only relatively recently that tests of Newtonian gravity can still provide a unique window into new physics [1–7]. Within the past 20 years, experimentalists have put Newtonian gravity to the test for distance scales 10^{-3}–10^{15} m by searching for violations of the weak equivalence principle (WEP) and inverse square law (ISL). The fact that no such violations have been observed places stringent constraints on extensions of the Standard Model that would naturally lead to such effects [1]. Despite this effort, a number of authors have pointed out that very little is known of the validity of Newtonian gravity at distances $\lesssim 10^{-3}$ m [8–12]. Several experimental groups are currently attempting to extend these limits down to 10^{-4} m [9,13,14], which is near the point where Casimir/van der Waals forces overcome gravity to become the dominant force between neutral, non-magnetic bodies. This strong intermolecular force background will become a major challenge for experimentalists who attempt to probe gravity at much smaller distances. The purpose of this paper is show that such experiments are worth the effort despite the new difficulties, and to suggest ideas which may be useful in detecting new forces of gravitational strength against a strong intermolecular force background.

We begin by examining the theoretical motivation for studying Newtonian gravity and the phenomenology used to characterize non-Newtonian effects which would be the signal of

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new physics in a gravity experiment. After briefly reviewing the current constraints on new forces achieved by longer distance gravity experiments ($\gtrsim 10^{-3}$ m), we will see that new problems arise when one attempts to set comparable limits in the Casimir/van der Waals regime ($\lesssim 10^{-4}$ m) where intermolecular forces become large. We then investigate these problems quantitatively by computing the forces between two parallel plates over a range of separations. Finally, we propose two schematic designs for null experiments designed to subtract out the unwanted intermolecular and gravitational backgrounds using the "isoelectronic" and "finite-size" effects. These will hopefully allow one to search for signals of new physics at very short distances in the presence of Casimir/van der Waals forces.

2 Theoretical Motivation and Phenomenology

2.1 Overview

The Standard Model currently provides an adequate description of the electromagnetic, weak, and strong interactions within the framework of quantum field theory. However, a consistent description of quantum gravity has yet to be formulated despite intense work over the past fifty years. The lack of a quantum theory of gravity currently provides much of the motivation for studying extensions of the Standard Model which would bring all the fundamental forces into the quantum realm. In fact, many believe that the present Standard Model is really only an effective theory which would be superseded at much higher energies by a more fundamental theory, such as string or M-theory [15, 16]. One of the main problems with these more fundamental theories is that, despite their purported mathematical beauty, many of their principal consequences lay far beyond the reach of most foreseeable experiments. It is therefore vital to investigate the low-energy limits of these fundamental theories to allow experimentalists the opportunity to constrain the proliferation of models which would otherwise go unchecked.

It is against this backdrop that one should view recent and future experiments testing Newtonian gravity. Many extensions of the Standard Model, including string theory, contain new light bosons which would manifest themselves as new fundamental forces [1, 6, 7, 12]. These new forces would compete with the other known forces, but they would most likely be revealed in a gravity experiment for several reasons. First, in many ways gravity remains the least understood of the fundamental forces and is relatively untested over a wide range of distance scales. Second, any new forces probably couple very weakly with matter—otherwise they would have been seen already. Since gravity is by far the weakest fundamental force, it sets a natural scale from which to measure new weak forces. Third, there are two signatures of gravity which help one extract a signal from the background of other forces: 1) Since the gravitational force couples to mass, it obeys the weak equivalence principle (WEP), so violations of the WEP would indicate the presence of a non-gravitational force. 2) The Newtonian gravitational force between point particles obeys an inverse square law (ISL), hence any departures from the ISL might be attributed to new forces. Finally, Newtonian gravity is the weak-field, non-relativistic limit of General Relativity, a theory in which gravity is seen as a manifestation of spacetime that has been curved by mass-energy. Therefore, any theory that impacts our understanding of space and time must involve gravity. This is important because string theory requires that there exist more than three spatial dimensions, the new extra dimensions being rendered invisible to current experiments by some yet-to-be-understood mechanism. As will be discussed below, recent models suggest that these new dimensions may modify Newtonian gravity at short, but macroscopic, distances.

2.2 Yukawa Potentials

The form of the violations of Newton's law of gravity arising from new physics will be to some extent model dependent, but one finds in practice that most theories yield modifications that have similar generic features [1]. For example, suppose there exists a new vector field $A^\mu(x)$ which couples to fermions via the Lagrangian density

$$\mathcal{L}(x) = if\overline{\psi}(x)\gamma_\mu\psi(x)A^\mu(x). \tag{1}$$

Here f is the dimensionless vector-fermion coupling constant ($\hbar = c = 1$) and $\psi(x)$ is the fermion field operator. If two fermions 1 and 2 exchange a single vector boson with mass m via this coupling, the lowest order interaction in the non-relativistic limit yields a Yukawa potential

$$V_v(r) = \pm\frac{f_1 f_2}{4\pi}\frac{e^{-mr}}{r}, \tag{2}$$

where "+" ("−") indicates that the force is repulsive (attractive) between like charges. If this was electromagnetism, massless photons give $m = 0$, and for electrons $f = -e$. In the units we use, the range of the interaction is $\lambda \equiv 1/m$, so that $m \sim 10^{-5}$ eV gives $\lambda \sim 1$ cm, for example. If the exchanged bosons were scalars instead of vectors, one arrives at an attractive Yukawa potential between identical fermions:

$$V_s(r) = -\frac{f_1 f_2}{4\pi}\frac{e^{-mr}}{r}, \tag{3}$$

where f_i is now the scalar coupling constant.

If the fermions have masses m_1 and m_2, the total interaction potential including gravity and scalar/vector interactions can be written in the general form

$$V(r) = -\frac{Gm_1m_2}{r}\left(1 + \alpha_{12}\, e^{-r/\lambda}\right), \tag{4}$$

where

$$\alpha_{12} \equiv \mp\frac{f_1 f_2}{4\pi G m_1 m_2}. \tag{5}$$

The dimensionless constant α_{12} then characterizes the strength of the interaction relative to gravity, and its sign depends on type of boson exchanged. When $r \ll \lambda$, $|\alpha_{12}| = 1$ indicates a force of gravitational strength.

Yukawa potentials also arise naturally in models where new gravitational forces appear from extra spatial dimensions. String theory requires there to be more than 3 spatial dimensions, but until recently it was thought that all the extra dimensions were compactified on the Planck scale and thus invisible to any conceivable experiment. However, much attention recently has focused on a number of string-inspired models in which all Standard Model particles are confined to the usual 3 spatial dimensions (a 3-brane) while gravity can "see" all dimensions [17–21]. In such models, one would never expect to see the effects of extra dimensions in Standard Model physics, but their effects would appear in gravitational physics. Since so much of the parameter space of gravity remains unexplored, these effects could have easily escaped detection. For example, in models in which the extra dimensions are compact, it is possible that the compactification radius r_c could be as large as 10^{-3} m and thus would have not been seen in any experiment to date [17, 18]. These models would produce dramatic deviations from Newtonian gravity at short distances since they imply that the r-dependence of the gravitational potential between point masses changes when

the particle separation approaches r_c:

$$V_{\rm grav}(r) = \begin{cases} -\dfrac{G_4 m_1 m_2}{r} & \text{for } r \gg r_c \\ -\dfrac{G_{4+n} m_1 m_2}{r^{1+n}} & \text{for } r \ll r_c. \end{cases} \tag{6}$$

Here n is the number of extra spatial dimensions, $G_4 = G$ is the usual macroscopic Newtonian gravitational constant, and G_{4+n} is the more fundamental gravitational constant for the total $4+n$ dimensional spacetime. Thus, in this model, Newton's law of gravity is merely a projection of a more fundamental law of gravity onto 3 spatial dimensions, and the unusual weakness of gravity relative to the other fundamental forces is attributed to this projection. One of the striking features of some recent string models is that compactification can occur over scales much larger than the Planck scale ($1/M_{\rm Planck} \sim 10^{-35}$ m) [17, 18, 21]. For example, if in a $4+n$ dimensional spacetime the fundamental mass scale $M_{\rm fund} \sim M_{\rm EW}$, where $M_{\rm EW} \sim 1$ TeV is the electroweak scale, then one expects the compactification scale r_c to be given by [17, 18, 21]

$$r_c \sim \frac{1}{M_{\rm fund}} \left(\frac{M_{\rm Planck}}{M_{\rm fund}} \right)^{2/n} \sim (10^{-19}\ {\rm m})(10^{16})^{2/n}, \tag{7}$$

which yields,

$$r_c \sim \begin{cases} 10^{13}\ {\rm m}, & n = 1, \\ 10^{-3}\ {\rm m}, & n = 2, \\ 10^{-9}\ {\rm m}, & n = 3. \end{cases} \tag{8}$$

Since no deviations from Newtonian gravity have been observed for $r \gtrsim 10^{-3}$ m, theories which suggest that $r_c \lesssim 10^{-3}$ m (e.g., $n \geq 2$) are compatible with current experimental limits. As one tests gravity over smaller distance scales, the effects of new extra dimensions would first appear as corrections to the usual gravitational potential. It has been shown that these corrections for $r \gg r_c$ have a Yukawa form [18, 22, 23]:

$$V_{\rm grav}(r) \sim -\frac{G_4 m_1 m_2}{r} \left(1 + \alpha_n e^{-r/\lambda} \right). \tag{9}$$

Here α_n is a composition-independent universal constant that depends on n and the nature of the compactification, and the range of the interaction is $\lambda \sim r_c$, the compactification scale. For example, for n extra dimensions compactified on an n-torus, $\alpha_n = 2n$ [18, 22, 23]. It is thus conceivable that the first evidence supporting the existence of extra spatial dimensions (and string theory) could come from the detection of a composition-independent Yukawa modification of Newton's law of gravity.

2.3 Current Constraints on New Yukawa Forces

Let us now turn to the current laboratory constraints on new Yukawa forces which arise from a generic potential of the form,

$$V_Y(r) = -\alpha \frac{G m_1 m_2}{r} e^{-r/\lambda}. \tag{10}$$

(Here we assume that the interactions are attractive for positive α.) This potential will lead to a violation of the WEP in a gravity experiment if α is composition-dependent, as in the case of a vector or scalar interaction. In addition, even if α is independent of the composition of the test masses, as in the case of the extra dimension theories described

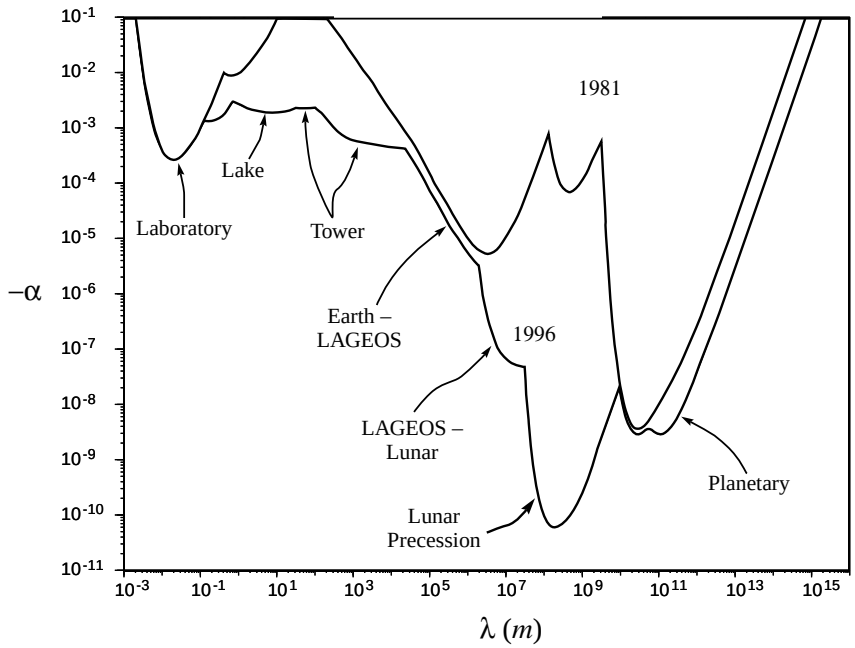

Figure 1: 2σ constraints on the coupling constant α as a function of the range λ from composition-independent experiments [1]. The dark shaded area indicates the region excluded as of 1981, and the light hatched region gives the 1996 limits which remain current.

earlier, small violations of the WEP will still be present in an experiment using different materials due to the "finite-size" effect [1]. This effect arises because a non-uniform Yukawa field will "capture" different fractions of two finite-sized objects having the same mass, but different densities, as will be the case in the null experiments considered below. Finally, in addition to violating the WEP, $V_Y(r)$ will also violate the ISL, and so constraints on α can be inferred from tests of the gravitational ISL.

As shown in Fig. 1, the current experimental constraints on the Yukawa coupling constant α as a function of range λ are quite stringent (allowed $|\alpha| \ll 1$) for 10^{-3} m $\lesssim \lambda \lesssim 10^{15}$ m, but they fall exponentially outside this region. Composition-dependent experiments have set strong limits for specific couplings (e.g., to baryon number) when $\lambda \gtrsim 10^{15}$ m [1], but also fall off exponentially for $\lambda \lesssim 10^{-3}$ m.

As discussed in more detail in Ref. [9], current constraints allow $\alpha \gtrsim 1$ for $\lambda \lesssim 10^{-3}$ m. For 10^{-4} m $\lesssim \lambda \lesssim 10^{-3}$ m, these limits were obtained from a test of the gravitational ISL by Mitrofanov and Ponomareva [24], but they still permit a new force with $\alpha \sim 10^4$ for $\lambda \sim 10^{-4}$ m. However, a new round of gravity experiments [9,13,14] should fill in much of this region of parameter space within the next few years. At shorter distances, Casimir/van der Waals forces dominate gravity so the current limits are set by Casimir force experiments [9,25–27] and are much less restrictive than those obtained from the longer ranged gravity experiments. [See also Refs. [28,29] for detailed discussions on extracting limits on new forces from Casimir force experiments.] The region $\lambda \lesssim 10^{-4}$ m will remain essentially unexplored until the next generation of experiments specifically dedicated to search for new forces is designed and carried out. We turn next to a discussion of some of the difficulties likely to be encountered in developing this next round of experiments.

3 Problems in Testing Gravity at Very Short Distances

3.1 General Problems

As noted in the Introduction, a number of authors [8–12] have called attention to the huge gap in our understanding of gravity at very short distances, and to its potential to reveal new physics. The fact that short-distance gravity experiments can potentially expose the presence of extra spatial dimensions is particularly tantalizing. However, since $|\alpha| \sim 1$ in these string models, the ultimate experimental goal is quite challenging, namely to set limits $|\alpha| \lesssim 1$ for $\lambda \lesssim 10^{-3}$ m. To accomplish this, one has to be able to sense and distinguish a force of gravitational strength at these distance scales. Since the current laboratory limits in this region are orders of magnitude less sensitive than this goal, we discuss briefly some of the difficulties in studying gravity at short distances.

An obvious problem is scaling [9]. Suppose we have two identical spheres of density ρ and radius R. If we wish to test the law of gravity between these spheres at short distances, the dimensions of the spheres have to be made comparable to the small distance scales that we are interested in probing. Since the minimum separation distance between centers is $2R$, this leads to a maximum force

$$F_{max} = \frac{G(4\pi\rho R^3/3)^2}{(2R)^2} \propto R^4. \tag{11}$$

This example illustrates that the gravitational force between macroscopic objects, which is already quite small, decreases rapidly with size and separation of the test masses.

A second problem in searching for new short-ranged forces is that their short range limits the effective mass of a body that can participate in the interaction. Suppose we have a sphere of uniform density and radius R. Since gravity is a long-range force, another identical sphere close by will interact with all of the mass of the first sphere. However, if there exists a new force of range $\lambda \ll R$, then only the layer of material of thickness $\sim \lambda$ on the surface of the sphere will interact with external objects, and hence only a fraction $\sim \lambda/R$ of the total mass participates in interactions. But this problem is actually much worse in general. If we have two spheres nearly touching, it is only the mass within a range λ of the contact point that interacts, which is much less than the fraction λ/R, while the gravitational force is still felt by all the mass. Therefore, even if the new force is intrinsically of gravitational strength ($\alpha = 1$) between point masses when $r \ll \lambda$, this new force between macroscopic bodies will usually be much smaller than the corresponding gravitational force. This situation was not encountered in previous longer range gravity experiments since in those cases, $\lambda \gtrsim L$, where L was the characteristic size of the test bodies used.

A third problem occurs when the separation distance is $\lesssim 10^{-4}$ m, where intermolecular forces become significant. Since these forces have a power-law form $1/r^n$ [30], where n depends on the geometry of the macroscopic bodies, they grow very rapidly as r decreases and overwhelm gravity at very short distances. Distinguishing a force of gravitational strength from this background will be a major challenge.

3.2 Quantitative Example: Parallel Plate Gravity Experiment

Idealized Setup

To better appreciate how these problems might arise in actual experiments, let us now consider a simple experimental setup. Our goal here is to estimate the size of the various effects which might appear, and not to propose the optimal experimental design, and hence we will ignore practical problems which might be encountered when one actually attempts to realize such a design. Since we are searching for forces of very short range, the discussion

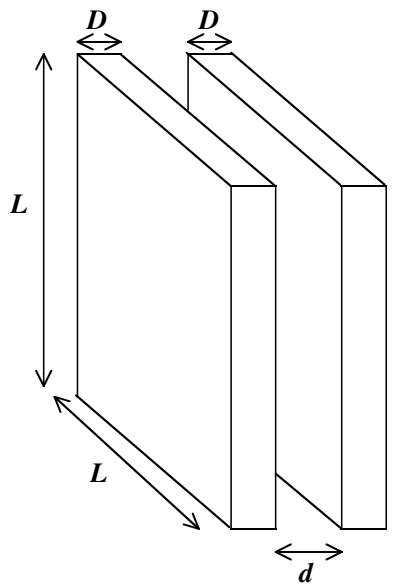

Figure 2: The idealized parallel plate setup used to quantitatively estimate the relative magnitudes of the gravitational, Casimir, and Yukawa forces. Numerical results were obtained by letting $L = 1$ cm, $D = 1$ mm, $\rho_{\text{copper}} = 8.96 \times 10^3$ m^3, $T = 300$ K, and $d \ll L$.

of the previous section suggests we should have most, if not all, of the mass of the two test bodies in an experiment contributing in order to realize the largest possible force. This means that we need to have all the mass in one body as close as possible to all the mass of the second body. The simplest way to accomplish this is to use parallel plates as our test bodies [29] which maximizes the "effective mass" for any short-range range force. It then follows that the most appropriate configuration for searching for new forces between macroscopic bodies is a parallel-plate experiment analogous to those used to study the Casimir effect [31].

Let us now consider two identical plates of density ρ, thickness D, area $A = L^2$, separated by a distance d (Figure 2). If we assume $d \ll L$, we can then safely neglect edge effects and calculate the pressures between the plates as if $L = \infty$. In addition, we assume that the plates are perfectly smooth, perfectly conducting, and at temperature $T = 300$ K.

Force Formulas

We begin our investigation of the forces between these plates with the known forces, starting with gravity. For this particular configuration, the gravitational force acting on the plates is given by

$$F_{\text{Gravity}}(d) = -2\pi G \rho^2 L^2 D^2, \tag{12}$$

where the minus sign indicates an attractive force. We see that when the plates are sufficiently close, the gravitational attraction is constant, independent of the separation d.

If there are no stray charges, etc., gravity is the dominant force at large plate separations, but as d decreases, the Casimir force grows rapidly and quickly overwhelms gravity.

Calculating the Casimir force for this geometry for real metals can become quite complicated, involving corrections for finite conductivity and surface roughness [32–34]. However, for present purposes we will ignore these difficulties by assuming the plates to be smooth and perfectly conducting over all frequencies, which should be a good approximation as long as the plates are not too close. However, we will include thermal effects which become large when the plate separation is large. The Casimir force between our plates at temperature T can be written as [35, 36]

$$\begin{aligned} F_{\rm Casimir}(d) &= -\frac{\pi^2}{240}\frac{\hbar c L^2}{d^4} - \frac{\pi k T L^2}{d^3}\sum_{n=1}^{\infty} n^2 \ln\left[1-\exp\left(-\frac{n\pi\hbar c}{kTd}\right)\right] \\ &\quad - \frac{\pi^2 (kT)^4 L^2}{45(\hbar c)^3}, \end{aligned} \tag{13}$$

where k is Boltzmann's constant and factors of $\hbar$ and c have been included for convenience. Eq. (13) simplifies in the two limiting cases [35, 36]:

$$F_{\rm Casimir}(d) = \begin{cases} -1.202\left(\dfrac{kTL^2}{4\pi d^3}\right), & d \gg \dfrac{\pi\hbar c}{kT}, \\ -\dfrac{\pi^2\hbar c L^2}{240 d^4}, & d \ll \dfrac{\pi\hbar c}{kT}. \end{cases} \tag{14}$$

Having obtained expressions for the known forces acting between the plates in this idealized setup, let us now determine the forces arising from possible new interactions. The attractive Yukawa potential between point masses as given by Eq. (10) leads to a force between the plates (with $d \ll L$) given by

$$F_{\rm Yukawa}(d) = -2\pi\alpha\lambda^2 G\rho^2 L^2 \left(1-e^{-D/\lambda}\right)^2 e^{-d/\lambda}. \tag{15}$$

We then notice that the ratio of this Yukawa force to the gravitational force in Eq. (12) is

$$\frac{F_{\rm Yukawa}(d)}{F_{\rm Gravity}(d)} = \alpha\left(\frac{\lambda}{D}\right)^2 \left(1-e^{-D/\lambda}\right)^2 e^{-d/\lambda}. \tag{16}$$

If $\lambda \ll D$, then

$$\frac{F_{\rm Yukawa}(d)}{F_{\rm Gravity}(d)} \simeq \alpha\left(\frac{\lambda}{D}\right)^2 e^{-d/\lambda}. \tag{17}$$

Thus, even if the Yukawa coupling is intrinsically of gravitational strength ($\alpha = 1$), the actual Yukawa force is suppressed relative to gravity not only by the usual exponential factor $e^{-d/\lambda}$, but also by $(\lambda/D)^2$ which arises because only a fraction λ/D of the total mass of each plate contributes to the Yukawa force. This effect was discussed earlier and clearly illustrates how a short-ranged force intrinsically of gravitational strength is strongly suppressed in an experiment using macroscopic bodies.

Numerical Results

Having found the general formulas for all the forces that we will be considering, let us now obtain numerical values for the following setup. We assume that the plates have dimensions $L \times L \times D$, where $L = 1$ cm and $D = 1$ mm, which are roughly comparable to the values used in some of the current short distance gravity experiments [9, 13]. Since our previous calculations assumed $d \ll L$, we focus our attention on the region 10^{-8} m $\lesssim d \lesssim 10^{-3}$ m.

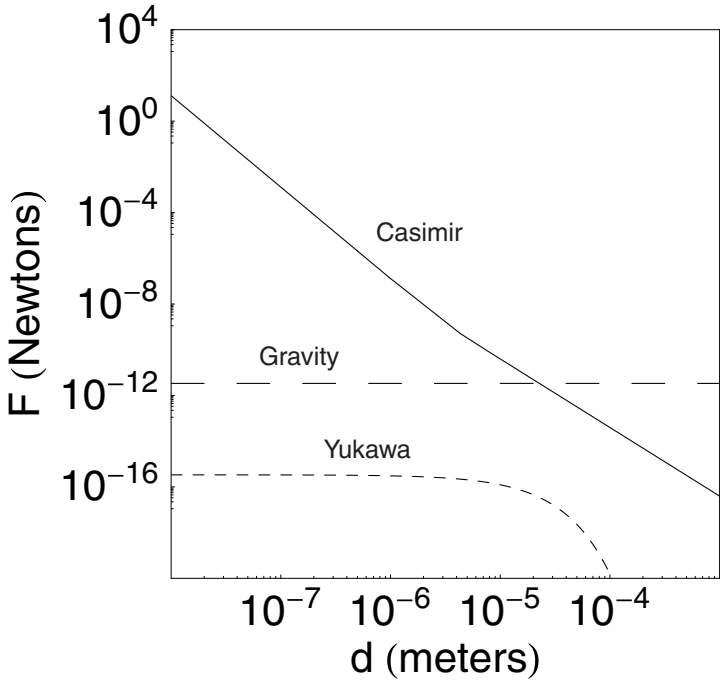

Figure 3: The Casimir, gravitational, and Yukawa ($\alpha = 1$, $\lambda = 10^{-5}$ m) forces between the parallel plates shown in Fig. 2.

Next we will assume that the plates are made of pure copper which has a density $\rho = 8.96 \times 10^3$ kg/m^3. Except for the new force parameters α and λ, our problem is now completely specified.

Using these numbers, we first calculate the known forces, gravity and Casimir, for the plates. As discussed earlier, the gravitational force under the conditions assumed here is constant and given by Eq. (12). Substituting the parameters given above yields

$$F_{\rm Gravity} = 3.37 \times 10^{-12} \text{ Newtons}. \tag{18}$$

To determine the Casimir force for this configuration, we use Eq. (13). The cross-over distance d_c, where temperature-dependent effects become important at $T = 300$ K, is

$$d_c = \frac{\pi\hbar c}{kT} = 2.4 \times 10^{-5} \text{ m} = 24\ \mu\text{m}. \tag{19}$$

Graphs of the Casimir force using Eq. (14), and the gravitational force between the plates, are shown together in Figure 3, and numerical values of these forces at various distances can be found in Table 1. The gravitational and Casimir forces are equal to each other when $T = 300$ K at $d = 2.3 \times 10^{-5}$ m $= 23\ \mu$m, which just happens to coincide with d_c here. Thus, for $d \lesssim 23\ \mu$m, the Casimir force will dominate gravity in this setup.

Now let us turn to new Yukawa forces, which are characterized by two free parameters, the relative strength α and the range λ. If for illustrative purposes we consider a force of gravitational strength ($\alpha = 1$) and set $\lambda = 10^{-5}$ m, then Eq. (15) yields $F_{\rm Yukawa}(d)$ exhibited

Table 1: The magnitudes of the gravitational, Casimir, and Yukawa (using $\alpha = 1$ and $\lambda = 10^{-5}$ m) forces arising in the idealized parallel plate experiment discussed in the text. Here $F_{\rm Back} = F_{\rm Gravity} + F_{\rm Casimir}$ is the total background force against which the signal of $F_{\rm Yukawa}$ must be seen.

d (m)	$F_{\rm gravity}$ (N)	$F_{\rm Casimir}$ (N)	$F_{\rm Yukawa}$ (N)	$F_{\rm Yukawa}/F_{\rm Back}$
10^{-3}	3.4×10^{-12}	4.2×10^{-17}	1.3×10^{-59}	4×10^{-48}
10^{-4}	3.4×10^{-12}	4.0×10^{-14}	1.5×10^{-20}	4×10^{-9}
10^{-5}	3.4×10^{-12}	4.0×10^{-11}	1.2×10^{-16}	3×10^{-6}
10^{-6}	3.4×10^{-12}	1.3×10^{-7}	3.0×10^{-16}	2×10^{-9}
10^{-7}	3.4×10^{-12}	1.3×10^{-3}	3.3×10^{-16}	3×10^{-13}
10^{-8}	3.4×10^{-12}	1.3×10^{1}	3.4×10^{-16}	3×10^{-17}

in Figure 3 and Table 1. We see that when $d \ll \lambda$, the force becomes constant:

$$F_{\rm Yukawa}(d \ll \lambda) = 3.37 \times 10^{-16} \text{ Newtons}, \tag{20}$$

which is $(\lambda/D)^2 = 10^{-4}$ times smaller than the corresponding gravitational force given by Eq. (18). Thus, as explained earlier, even though the Yukawa force between *point* particles is of gravitational strength at short distances, $F_{\rm Yukawa}$ is much smaller than gravity for these macroscopic plates. We also clearly see that $F_{\rm Yukawa}/F_{\rm Back}$ is maximized when $d \sim \lambda$ and falls off rapidly from this plate separation (Fig. 4). This is because $F_{\rm Yukawa}$ levels off when $d \lesssim \lambda$ while $F_{\rm Casimir}$ continues to increase via a power-law ($1/d^4$ if $d \lesssim d_c$).

Constraining New Short-Ranged Yukawa Forces

This analysis using an obviously idealized setup reveals the two critical problems that will be encountered in devising experiments using macroscopic bodies to search for very short-ranged Yukawa interactions of gravitational strength ($\alpha \sim 1$). The first is that the absolute magnitude of such a force will be very small, possibly even smaller than the gravitational force if α is small. Thus, an experiment must be sensitive to the smallest possible forces. Second, since the Casimir background force grows rapidly as the separation decreases, one must be able to extract the signal of a very weak force from a background of very strong intermolecular forces. A direct attack on this problem would be to attempt to calculate as accurately as possible the background forces in a gravity experiment, and to then subtract these from the observed force to set limits using what remains [9, 26, 28, 29]. However, recent experiments studying the Casimir force reveal the difficulty of accurately calculating the background forces to better than 1% [25, 27, 32–34, 37, 38]. While this approach is still possible, we will describe in the next section a new method of performing a null short-distance gravity experiment specifically designed to directly subtract out the unwanted background effects. Calculating intermolecular forces precisely then becomes much less important.

4 Very Short Distance Null Gravity Experiments

Some of the best constraints on Yukawa interactions come from tests of the WEP [1]. In these experiments, one compares the accelerations of compositionally-different test bodies toward a common source body. Any differences in these accelerations can then be attributed to non-gravitational forces. We wish to utilize the same principle in a short-distance gravity experiment, and thus avail ourselves to the extreme sensitivity of such experiments.

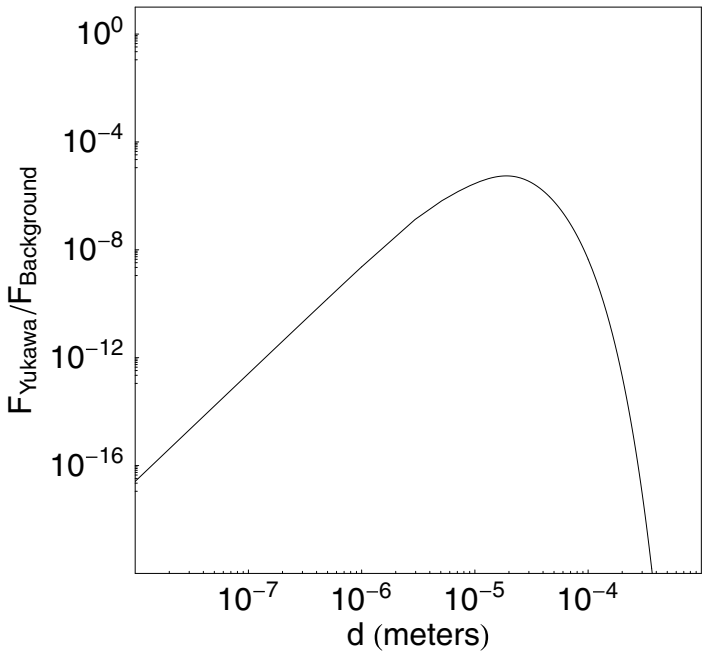

Figure 4: The ratio of the Yukawa ($\alpha = 1$, $\lambda = 10^{-5}$ m) force to the background force $F_{\rm Back} = F_{\rm Gravity} + F_{\rm Casimir}$ for the parallel plate setup shown in Fig. 2.

4.1 Null Experiment #1

Inspired by two ongoing short distance experiments [9, 13], a possible design for one such experiment is shown in Figure 5. It consists of two parallel plates, a source plate 1 and a detector plate made of two smaller plates 2 and $2'$. The source plate is driven sinusoidally with angular frequency ω such that the separation distance d is given by

$$d(t) = d_0 + d_1 \cos\omega t. \tag{21}$$

Instead of detecting a force, this experiment would be sensitive to a torque, modulated by the frequency ω, about an axis passing along the boundary where plates 2 and $2'$ are joined, as shown in Fig. 5. If we assume that the plates are conducting, the net torque $\tau_{\rm net}$ on the detector plate will arise from contributions from the gravitational, Casimir, and possibly, Yukawa forces:

$$\begin{aligned} \tau_{\rm net} &= \tau_{\rm Gravity} + \tau_{\rm Casimir} + \tau_{\rm Yukawa} \\ &= \frac{L}{2}\left[(F_2^{\rm Gravity} - F_{2'}^{\rm Gravity}) + (F_2^{\rm Casimir} - F_{2'}^{\rm Casimir}) \right. \\ &\quad \left. + (F_2^{\rm Yukawa} - F_{2'}^{\rm Yukawa})\right], \end{aligned} \tag{22}$$

where $F_i^{\rm Gravity}$, $F_i^{\rm Casimir}$, and $F_i^{\rm Yukawa}$ are the gravitational, Casimir, and Yukawa forces on plates $i = 2$ and $2'$ respectively. One then selects plates 2 and $2'$ such that the torque $\tau_{\rm Gravity} + \tau_{\rm Casimir}$ due to background forces vanishes while $\tau_{\rm Yukawa} \neq 0$ if $\alpha_{1i} \neq 0$. [Here

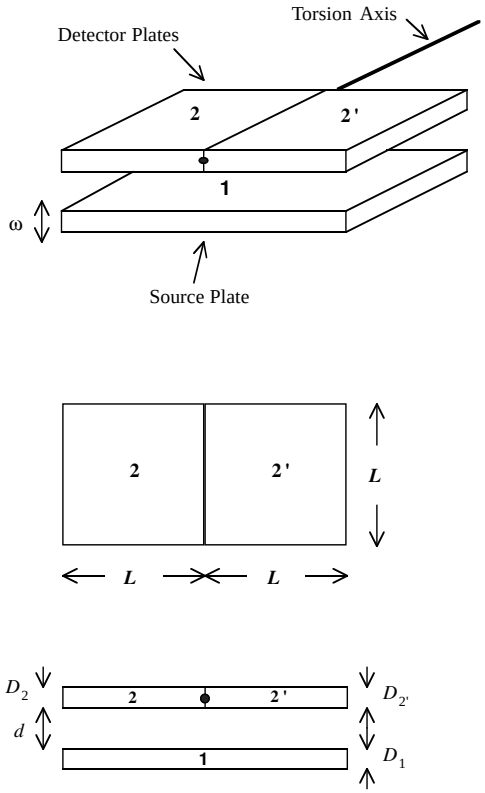

Figure 5: Schematic design for Very Short Distance Null Experiment #1. See text for details.

we allow for the possibility that α_{12} and $\alpha_{12'}$, the Yukawa couplings between the materials comprising plates 1 and 2, and 1 and $2'$ respectively, are different.]

At very small separations, $F_i^{\rm Gravity}$ will be negligible (and independent of ω to first approximation) , but it is still easy to make $\tau_{\rm Gravity}$ vanish anyway. Using Eq. (12), we see that

$$|F_2^{\rm Gravity} - F_{2'}^{\rm Gravity}| = 2\pi G L^2 \rho_1 D_1 \left|\rho_2 D_2 - \rho_{2'} D_{2'}\right| , \tag{23}$$

where ρ_i and D_i are the density and plate thickness of the ith plate. Then

$$(\rho_2 D_2 = \rho_{2'} D_{2'}) \Rightarrow (M_2 = M_{2'}) \Rightarrow \tau_{\rm Gravity} = 0, \tag{24}$$

so if the detector plates 2 and $2'$ have the same mass, the gravitational torque will vanish.

Of course, the much bigger challenge is choose materials for plates 2 and $2'$ such that the Casimir torque $\tau_{\rm Casimir}$ also vanishes. If the plates were perfectly conducting, this would be the case since Eq. (13) would be identical for all such plates with the same surface area. However, the finite conductivity of real metallic plates becomes very important when the plate separation $d \sim \lambda_P$, where $\lambda_P = 2\pi c/\omega_P$, and ω_P is the plasma frequency of the metal. Still, it was shown recently [32] that the Casimir force between pairs of copper and gold plates are equal to a good approximation for separations $d \gtrsim 10^{-6}$ m at $T = 0$.

Such calculations are difficult for real materials, but this certainly raises the hope that it is possible to choose appropriate plates 2 and $2'$ such that

$$F_2^{\rm Casimir} - F_{2'}^{\rm Casimir} \simeq 0 \quad \Rightarrow \quad \tau_{\rm Casimir} \simeq 0. \tag{25}$$

At the very least, one should be able to fabricate the plates using two different isotopes of the same element (e.g., ^{24}Mg and ^{26}Mg) such that Eq. (25) is satisfied. The underlying premise of the "iso-electronic" effect (IE) is that to a good approximation the Casimir effect depends on the *electronic* properties of the materials, and hence is largely independent of their *nuclear* properties. By contrast, the gravitational interaction, and virtually all proposed new Yukawa interactions, involve couplings to both electrons and nucleons. Hence, subtracting out the electronic contributions by choosing two isotopes of some material, or by choosing materials with similar electronic properties (such as Cu and Au), we can enhance the signal from a new Yukawa force while simultaneously reducing the Casimir background.

The remaining torque after the gravitational and Casimir torques have been suppressed might be due to a new Yukawa force. Using Eq. (15), the net torque due to a putative Yukawa force would be

$$\begin{aligned} \tau_{\rm Yukawa} &= 2\pi G\lambda^2 (L/2) L^2 \rho_1 \left(1 - e^{-D_1/\lambda}\right) e^{-d/\lambda} \\ &\quad \times \left[\alpha_{12}\rho_2 \left(1 - e^{-D_2/\lambda}\right) - \alpha_{12'}\rho_{2'} \left(1 - e^{-D_{2'}/\lambda}\right)\right]. \end{aligned} \tag{26}$$

If $\lambda \ll D_i$, then Eq. (26) simplifies to

$$\tau_{\rm Yukawa} \simeq 2\pi G\lambda^2 (L/2) L^2 \rho_1 e^{-d/\lambda} \left[\alpha_{12}\rho_2 - \alpha_{12'}\rho_{2'}\right]. \tag{27}$$

If the Yukawa force arises from an extra-dimensional modification of Newtonian gravity such that $\alpha_{12} = \alpha_{12'} = \alpha_n$, Eq. (27) reduces to

$$\tau_{\rm Yukawa} \simeq 2\pi \alpha_n G\lambda^2 (L/2) L^2 \rho_1 \rho_2 e^{-d/\lambda} \left(1 - \frac{\rho_{2'}}{\rho_2}\right). \tag{28}$$

This result depends on the difference in the mass densities $\rho_2 - \rho_{2'}$ for a simple reason: When $\lambda \ll D_i$, the force only sees the mass within a distance λ of the surface. Thus, if plates 2 and $2'$ have different densities, the effective mass seen by the Yukawa force will be different and a net torque arises due to the "finite-size" effect discussed earlier. It is then clear that one should choose materials 2 and $2'$ such that ρ_2 and $\rho_{2'}$ differ as much as possible while still ensuring that the Casimir torque vanishes. For the gold/copper and ^{24}Mg/^{26}Mg combinations suggested above,

$$1 - \frac{\rho_{2'}}{\rho_2} \simeq \begin{cases} 0.32 & 2 = {\rm Au},\ 2' = {\rm Cu}, \\ 0.077 & 2 = {}^{26}{\rm Mg},\ 2' = {}^{24}{\rm Mg}. \end{cases} \tag{29}$$

The hope is that the suppression factor Eq. (29) is more than compensated by the reduction of the unwanted background torques.

4.2 Null Experiment #2

We conclude this section by briefly describing another possible design for a short distance null experiment motivated by another set of gravity experiments [14,39]. As shown schematically in Figure 6, this experiment consists of two disks, one serving as the source mass, while the other (detector mass) is the pendant of a torsion pendulum. Each disk is divided into alternating wedges made of two different materials 1 and 2. The wedges and materials 1

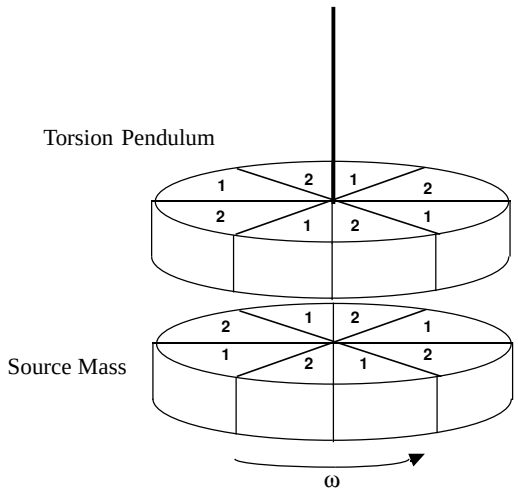

Figure 6: Schematic design for Very Short Distance Null Experiment #2. See text for details.

and 2 are designed such that no Casimir torque would arise when the source disk rotates below the pendulum. For example, as indicated above, 1 and 2 might be gold/copper or ^{24}Mg/^{26}Mg which would significantly reduce the Casimir torque. Then, if the separation between the disks is small, the remaining torque on the pendulum would arise from a putative Yukawa force because the effective mass within a distance of λ will be different for alternating wedges.

5 Discussion

It is clear that there is significant motivation for testing Newtonian gravity at very short distances. However, as we have seen, new problems will face experimentalists who wish to extend current constraints on the Yukawa coupling constant α down to ranges $\lambda \lesssim 10^{-4}$ m. Our aim in this paper has been to point out some of the most obvious difficulties, but there may be others that have passed unnoticed. While we have not addressed the significant issue of improving the sensitivity of experiments to very small forces, we have taken the first steps towards dealing with the problem of detecting small forces against a large intermolecular force background. Our schematic designs for null experiments are meant as illustrations of the principles involved in canceling background forces (the "iso-electronic" and "finite-size" effects), and hence are not intended to suggest optimal designs. It is hoped that experimentalists, who face the realities of imperfect materials and incomplete theories, can extract some useful ideas from these schematic designs or, perhaps, will be able to point to flaws which preclude them from working as actual experiments. Finally, we conclude with the encouraging note that since so little is known about gravity at separations $\lesssim 10^{-4}$ m, virtually *any* good experiment in this region will tell us something new.

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