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Light-Cone Kaluza-Klein Geometry: Confined Propagation as a Particle Model

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Abstract

A modified Kaluza-Klein theory is proposed in which propagation takes place only at the speed
of light. The propagation can be confined to a small volume, forming a particle with rest mass.
The usual four space-time coordinates locate the confinement volume, and Kaluza's fifth coordinate
is replaced by an internal degree of freedom. Electromagnetism corresponds to a gauge field on
the phase of the internal motion. Self-gravity might create the confinement, as in a geon, but the particle
would have a Planck mass. This large mass could be made closer to the masses of observed particles if
gravity were allowed to increase in strength within the confinement volume.

Introduction. Kaluza-Klein theory provides a promising basis for unifying the physical forces. The idea of extending the dimensionality of space to combine electromagnetism with gravity [1] has proven to be far reaching, ultimately offering the possibility of placing all particle interactions within a single space-time framework [2]. Although there are aspects of higher dimensionality that have not been completely satisfactory, the success of the Kaluza-Klein approach is so impressive that we can expect a correct theory of particle dynamics to contain some version of its formalism.

This paper describes a modification of the original Kaluza-Klein theory that changes its conceptual basis while preserving its compelling features. Setting the Kaluza-Klein five-dimensional (5-D) space-time interval [3] equal to zero produces the relation

$$dx_m\, dx^m = a^2(dx_o + bA_m\, dx^m)^2 , \qquad (1)$$

where $dx_m = g_{mn}\, dx^n$, m and n = 0, 1, 2, 3, and the summation convention is adopted. This light-cone geometry retains the unification of gravity and electromagnetism. The left side of (1) is the conventional 4-D space-time invariant metric. Kaluza introduced the term on the right, with a fifth dimension x_o gauge-coupled to the electromagnetic field A_m. Both sides are just the square of the 4-D proper displacement, $ds^2 = c^2dt^2$ (t is the 4-D proper time). If x^m locates a particle in 4-D space-time, then $a(dx_o + bA_m\, dx^m) = ds$ is a scalar displacement that remains present when the particle is at rest (i.e., when dx^i

= 0, where i = 1, 2, 3). The right side of (1) therefore describes an extra portion of a particle?s motion that is not included in its observed motion in 3-D space. The extra degree of freedom x_o parameterizes the internal motion of the particle. In this way, physical propagation can be modeled as taking place on the light-cone in 5-D space-time, with the fifth dimension representing an internal variable.

These ideas can be illustrated more easily when there are no gravitational or electromagnetic fields present (and when the factor a = 1). Equation (1) then has the familiar Galilean form

$$ds^2 = dx_o^{\,2} = c^2dt^2 - dx^2 - dy^2 - dz^2 \,. \qquad (2)$$

The scalar displacement dx_o is both the 4-D invariant interval ds and also an internal spatial interval. Equation (2) describes a particle that has internal motion at the speed of light in its rest frame. This is demonstrated by defining the rest frame of the particle as that in which the average external position $x_R^{\,i} = x_R, y_R, z_R$ of the confined internal motion does not change (i.e., $dx_R^{\,i} = 0$), and in which the internal speed is $dx_o / dt = c$. Then by going to another frame, with new x^i, in which the external velocity of the particle is $v^i = dx^i / dt$ and the internal speed is $dx_o / dt = (c^2 - v^i v^i)^{1/2}$, the internal and external motions are described by (2).

Although the scalar dx_o represents an internal portion of the motion, the form of (2) is a result of purely external considerations, since the external coordinates x^i and t must obey the Lorentz transformations. In its role as an internal displacement in the rest frame, dx_o is not an extra dimension in the usual sense. Although Eq. (2) gives c as the orthogonal sum of internal and external component speeds, and the picture of motion on a 5-D light-cone is thereby justified, the concept of an extra dimension is not required. The importance of the 5-D light-cone model is that the fifth Kaluza-Klein coordinate becomes identified with the conventional proper displacement.

The conventional 4-D theory of relativity for observed velocities v £ c becomes a theory of motion in which the total velocity is c, but a portion is hidden in an unobserved component. When the internal speed is non-zero the external 3-D speed of the particle is less than c and the particle has rest mass. At the extremes, a particle that is at rest in 3-D space has an internal speed c, and a particle with no internal motion moves at c in 3-D space.

In the presence of fields, described by (1), the overall speed varies from the vacuum value c. However, there is still only one speed of propagation at each space-time location, with internal and external parts. The electromagnetic field bA_m and the scalar factor a allow the internal speed in the rest frame to be different from c. With gravity and electromagnetism, restricting motion to the light-cone in the Kaluza-Klein geometry still describes the movement of a particle traveling at v£c in conventional, curved 4-D space-time.

Conjectures. To show how (1) arises from the concept of confined propagation, begin by stating the assumptions: 1) *all physical propagation occurs at the speed of light*; and 2) *physical propagation can be confined to small volumes of space*. The first conjecture states that, if ds is the combined external and internal spatial displacement, the relativistic invariant $c^2dt^2 - ds^2$ is zero for any propagation. The second conjecture says that a propagation path can be bounded, i.e., made to curl back on itself and form an orbit. We presume that some kind of variation in the speed of propagation, in the manner of refraction,

produces the tightly curled-up path needed to confine the motion. The possibility of gravitational confinement is discussed later in this paper. For now the mechanism of confinement does not need to be specified, but we can expect that the second conjecture will eventually become unnecessary when the cause is understood.

Internal motion. Beginning in the rest frame of the confinement volume where the motion is entirely internal, define external rest coordinates x_R^m of the confinement volume. Note that $x_R^o = ct$, where t is the proper time for the confinement volume. Let the confined, internal motion have a single circular degree of freedom, with r the radius of the circle and j the angular position. The first conjecture says that the proper displacement is $c\,dt = r\,dj$. Now allow the internal angle q to differ from j by a phase $\int f(x_R)\,dx^m$, where $f(x_R)$ is a function of the external coordinates. In terms of the internal displacement, the proper displacement is then $c\,dt = r(dq - f\,c\,dt)$. The field f introduces a variance of the speed $r\,dq/dt$ from c.

Next go to a frame in which the confinement volume is moving, accomplished by transforming to an arbitrary external coordinate system x^m. If the external space-time is flat, this will be a Lorentz transformation. For arbitrary curvature, the square of the external proper displacement, $c^2dt^2 = dx_m\,dx^m$, is invariant. Equating the internal and external expressions for the proper displacement, we have

$$r^2(dq - f_m\,dx^m)^2 = dx_m\,dx^m\,. \qquad (3)$$

The terms r, dq and $f_m\,dx^m$ are scalars under transformations of x^m. The field $f_m(x)$ is therefore a four-vector field in external space-time. Equation (3) has the form of the Kaluza-Klein light-cone metric equation (1), with a absorbed into r. In addition to its invariance under transformations of x^m, the proper displacement is also invariant under the gauge transformations

$$\begin{aligned} \theta &\to \theta + f \\ \phi_\mu &\to \phi_\mu + \partial_\mu f \end{aligned} \qquad (4)$$

where f is any continuous function of x^m.

Mass, charge and gravity. It is straightforward to demonstrate that this theory encompasses electromagnetism. For the lowest quantum state, with one wavelength around the circumference of the circle, the internal angular momentum is Planck's reduced constant h (excited states would have nh , where n is an integer). Using the internal angular velocity $w = dq/dt$, define the Lagrangian L that gives $dL/dw = h$. From Eq. (3) this also gives the correct Lagrangian for a charged particle in an electromagnetic field. Writing $u^m = dx^m/dt$,

$$\begin{aligned} L &= \hbar\omega \\ &= \frac{\hbar}{r}\sqrt{u_\mu u^\mu} + \hbar\phi_\mu u^\mu \\ &= mc\sqrt{u_\mu u^\mu} + \frac{e}{c}A_\mu u^\mu \end{aligned}\,. \qquad (5)$$

The physical meanings of the coefficients has been established here by identifying: *i*) the quantum relation $mc = \hbar / r$, where m is the rest mass; and *ii*) the electromagnetic momentum $eA_m/c = \hbar f_m$, where e is the quantum of charge. The correspondence of (3) with (1) then gives $ab = -e/mc^2$. The gauge invariance of the electromagnetic field is assured by (4). All external canonical momenta $P_\mu = \partial L / \partial u^\mu$ are properly given by this Lagrangian. The action is proportional to the integrated internal angle, $\int L\, d\tau = \hbar\theta$.

Although this theory is consistent with reality if we are free to set the mass and charge equal to their experimental values, we do not have this freedom when when gravity is included. The original derivation of the equations of gravity and electromagnetism from the stationary 5-D Hilbert action remains valid here, since the geometry described by (1) and (3) is the light-cone manifold imbedded in the Kaluza-Klein space-time. However, by imposing the 5-D light-cone constraint we have a more direct coupling between charge and mass than was present in the original Kaluza-Klein theory. When 5-D general relativity is applied to our case we have the result

$$\frac{e}{m} = \pm\sqrt{16\pi G}, \qquad (6)$$

where G is the gravitational constant. This predicts a value $e/m = 2\times10^{-3}$ gm$^{-1/2}$ cm$^{3/2}$ sec^{-1}. But this is in wide disagreement with actual particles. For the proton and electron the values of e/m are 3×10^{14} and 5×10^{17} gm$^{-1/2}$ cm$^{3/2}$ sec^{-1}, respectively. Klein [4] avoided this problem by disregarding the large mass implied by (6). Without mentioning mass, he adopted the experimental value of e and interpreted the derived Planck-like radius, $r\sim10^{-31}$ cm, to be the size of his extra compact dimension. More recently [2], the problem of the large mass has been eliminated by using only the massless states of the particle spectrum, giving up on the idea of letting (6) determine the quantum of charge. We do not wish to do that here because it ignores the formal advantages of the theory. We therefore must deal with the unrealistic consequence of (6) that the particle has a Planck-like mass. If we want to pursue this theory as a possible description of nature we are forced to take the "incorrect" value for e/m quite seriously, because it implies that the observed values of charge, mass and gravitational constant are mutually incompatible. The root of this disparity may lie in the theory of gravity, as the following section demonstrates.

Gravitational confinement. What causes the internal motion to be captured? Without introducing a new force, gravity seems to be the best candidate for the required confinement field. From general relativity we know that a strong gravitational field can curve speed-of-light propagation into a closed path. Furthermore, a sufficient amount of energy propagating in an orbit can be held in that configuration by its own gravity. Wheeler calls such a self-maintained object a "geon" [5]. This seems to be a tenable model of internal confinement, i.e., *internal propagation takes place in a volume of tightly curved space-time formed gravitationally by the trapped energy itself*. The particle forms as a solution to the gravitational equations, with the short-range field acting to confine the propagating energy and the long-range field acting as the external Newtonian gravity of the particle. For gravitational confinement the factor $a$relates the propagation speeds inside and outside the confinement region, and is greater than unity because the speed of light decreases inside a gravitational field.

A quantum object formed in this way has very specific features; in particular, its mass and size are determined uniquely. In a Schwarzchild-type geon the orbital radius is given by $r = kGm/c^2$(the constant

k is likely to be of order 10). But, in addition, we can assume that the configuration of the object will be stable when the quantum relation $r = \hbar /mc$ is satisfied. If stable orbits form where the two radii are equal, we have the constrained values for the mass

$$m = \sqrt{\frac{\hbar c}{kG}} \sim 10^{-5} \text{ gm} \qquad (7)$$

and the orbital radius

$$r = \sqrt{\frac{k\hbar G}{c^3}} \sim 10^{-33} \text{ cm} \quad . \qquad (8)$$

Again this is a Planck particle, with a mass vastly larger than that of real particles and a radius that is much too small. In contrast, the quantum of electromagnetic charge is given by Eqs. (6) and (7) as

$$e = \pm\sqrt{\frac{16\pi\hbar c}{k}} \sim 10^{-8} \text{ gm}^{1/2}\,\text{cm}^{3/2}\,\text{sec}^{-1} \quad , \qquad (9)$$

which, in turn, gives the fine-structure constant

$$\frac{e^2}{\hbar c} = \frac{16\pi}{k} \sim 10 \quad . \qquad (10)$$

These are larger than observed, but not by the huge factor associated with the mass.

Thus gravity can produce an object with internal orbital speed-of-light motion in the rest frame. It has attributes of a real particle; it is extremely small and quantized, with uniquely determined rest mass, radius and charge. Of course, the confined orbit in a real particle is likely to be more complicated than a circle; indeed, we can expect a variety of confinement geometries and many internal degrees of freedom. However, it seems to be a fundamental prediction of this theory that a particle formed by gravitational confinement will have a mass and radius on the Planck scale. This is clearly a shortcoming.

Nevertheless, gravity appears to be able to produce a particle-like object with a minimum of assumptions. This is a characteristic that we would expect of a successful theory of matter. Therefore, gravitational confinement deserves further consideration, and we are faced with the possibility that the theory of gravity needs to be altered in some way to make the predicted mass more realistic.

The role of *G*. It is suggestive that *m* and *r*, which depend on *G*, have theoretical values that are far from their measured values, while *e*, which is independent of *G*, is close to measurement. This indicates that *G* might be varied in an attempt to reduce the mass. Consider the consequences of making *G* larger than its observed value by a factor of 10^{38}. If we allow them to follow the dependencies described by (7), (8), and (9), the mass and size become close to values observed for real particles, $m \sim 10^{-24}$ gm and $r \sim 10^{-14}$ cm, while the charge *e* does not change. In this way, by adjusting the value of *G*, we are able to make the mass, size and charge of the Planck particle similar to those of real particles. In this scenario, *G* would be near Newton?s value outside the particle, but would become large within the confinement volume. Such a behavior would be conceivable, for instance, if gravity were a strong function of energy density. For example we might suppose that $G(T) = G_N \exp(r/r_c)$, where G_N is Newton?s constant, r is

the radial-dependent energy density, and $r \sim 8\pi_c$ at the radii of real particles.

Note that a 10^{38}-fold increase of G would make the gravitational interaction approximately equal in strength to the other three fundamental interactions. This could suggest a commonality among the forces within the confinement volume. This discussion is only meant to be suggestive, but the implication is clear; a modification of gravity at high densities, besides solving the problem of the Planck mass, might also be a key to unification of the physical forces.

Conclusions. The concept of confined motion allows two changes in the foundation of Kaluza-Klein theory. First, a universal propagation speed can be used to describe all physical motion. Second, a new dimension does not need to be introduced, only an internal degree of freedom, so that a particle with rest mass is described as having internal dynamics. The important breakthrough of the original theory, combining electromagnetism and gravity, is preserved. Electromagnetism corresponds to the dependence of internal phase on external displacement. This clearly implies that additional internal degrees of freedom might be considered as a basis for the strong and weak forces, as is done in higher dimensional Kaluza-Klein theories. (Along these lines, the internal propagation loops may prove to have string properties.)

The possibility that a modified theory of gravity can explain confinement on the scale of real particles should be investigated. Such an approach would retain the prediction of the quantum of charge while solving the problem of the large mass. The gravitational equation would have to be modified in some way to make G depend on the energy density.

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