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Worldsheet and Spacetime Properties of $p-p'$ System with B Field and Noncommutative Geometry *

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Abstract

We study worldsheet and spacetime properties of the p-p' ($p < p'$) open string system with constant B_{ij} field viewed from the Dp'-brane. The description of this system in terms of the CFT with spin and twist fields leads us to consider the renormal ordering procedure from the $SL(2,\mathbf{R})$ invariant vacuum to the oscillator vacuum. We compute the attendant two distinct superspace two-point functions as well as their difference (the subtracted two-point function). These bring us an integral (Koba-Nielsen) representation for the multiparticle tree scattering amplitudes consisting of $N-2$ vectors and two tachyons. We evaluate them explicitly for the $N = 3, 4$ cases. Several novel features are observed which include a momentum dependent multiplicative factor to each external vector leg and the emergence of a symplectic tensor multiplying the polarization vectors. In the zero slope limit, the principal parts of the amplitudes translate into a noncommutative field theory in $p'+1$ dimensions in which a scalar field decaying exponentially in $(p'-p)$ dimensions and a noncommutative $U(1)$ gauge field interact via the minimal coupling and a new interaction. A large number of nearly massless states noted before are shown to propagate in the t-channel.

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I. Introduction

After decades of investigations, string perturbation theory has now become a well-established old subject. It is yet a nontrivial task to uncover spacetime properties, given a first quantized worldsheet theory. Some of our previous endeavors on strings are regarded as a search for a formulation in which these spacetime properties come out in a more transparent way. String theory with constant B_{ij} background offers an intriguing situation in which the emerging spacetime picture is given in terms of noncommutative geometry and is a focus of the recent intensive studies [1][2][3][4][5].

Several important steps have been taken in [6]. In particular, the proper spacetime metric on the Dp-brane (the open string metric) has been extracted from the worldsheet theory of an open string with its both ends on a Dp-brane, (the p-p open string system): the distances measured with respect to this metric are kept finite at all scales. The attendant noncommutative field theory in the zero slope limit lives on this metric together with the parameter representing noncommutativity of spacetime.

In the previous paper [7], we have examined the more complex p-p' ($p < p'$) open string system where the both ends of the open string are on a Dp-brane and on a Dp'-brane respectively. We have obtained the open string metric and the noncommutativity parameter on the Dp'-brane from the worldsheet two-point function. They in fact agree with those of the p-p system. We have computed the spectrum in each case of (p, p'), uncovered the emergence of a large number of nearly massless states in some cases and clarified the connections among the GSO projection, branes at angles and supersymmetry. Yet a number of other properties, in particular, spacetime properties on the Dp'-brane with Dp-brane inside have remained elusive. Elucidating upon these is a major goal of the present paper.

It should be mentioned that, in the vanishing B_{ij} background, several properties of the p-p' open string system have been studied. For example, amplitudes of some scattering processes taking place on the Dp-brane worldvolume (i.e. that of the lower dimensional D-brane) has been evaluated in [8] and the conformal field theory correlation functions have been studied in [9]. In the presence of B_{ij} background the four point tachyon amplitudes have been given in [10] for the $p' = p + 2$ case.

In [6] several properties of the 0-4 system have been derived. The description of the system from the D0-brane has offered a new perspective to the moduli space of noncommutative instantons [11] where the noncommutativity of the system is measured by the presence of the Fayet-Iliopoulos D term. In contrast to this moduli space point of view, the thrust of the present paper is to uncover the spacetime properties of the system viewed from the higher dimensional D-brane, namely the Dp'-brane. This can be accomplished by placing vertex

operators on the worldvolume of the Dp'-brane and by considering the scattering processes. This is an extension of the computation of scattering amplitudes in the p-p open string system with and without B_{ij} background [12][13][5][14]. This line of reasonings has led us to carry out a systematic study which begins with evaluating superspace two-point functions in the relevant CFT with the spin and twist fields, proceeds to the computation of scattering amplitudes on the Dp'-brane and ends with identifying a proper low energy noncommutative field theory in the zero slope limit.

In the next section, we begin with quantizing an open string ending on Dp and Dp' ($p < p'$) branes. We exploit superspace formulation, which we find extremely efficient in the calculation pursued in the subsequent sections. We evaluate two distinct two-point functions on superspace and observe the importance of the renormal ordering procedure from the $SL(2, \mathbf{R})$ invariant vacuum to the oscillator vacuum. This procedure leads us to consider the difference of these two two-point functions as well. This third quantity plays an important role in section 4 and will be referred to as subtracted two-point function. The conformal weights of the twist and spin fields are readily computed from the renormal ordering procedure.

In section 3, we examine the tachyon vertex operator of the p-p' open string and the vector vertex operator of the p'-p' open string. We derive the on-shell conditions in terms of the open string metric, $p + 1$ dimensional momenta of the tachyon, $p' + 1$ dimensional momenta and polarizations of the massless vector and mass of the tachyon.

In section 4, we consider the multiparticle tree scattering amplitudes consisting of external states of $N - 2$ vectors and two tachyons. We are able to derive an integral (Koba-Nielsen) representation of these quantities as integrals over $N - 3$ locations of the vertex operators as well as the $N - 2$ Grassmann counterparts and the $N - 2$ Grassmann sources conjugate to the polarization vectors. (See [15] for the case of the p-p string with vanishing B field.) Several striking properties emerge from this representation. Among other things, we find a momentum dependent exponential factor to each external vector leg, which the subtracted two-point function is responsible for, as well as a new symplectic tensor multiplying the polarizations of the massless vector. We observe that some parts of the amplitudes are expressible in terms of the inner products of polarizations, momenta and the symplectic tensor with respect to the open string metric, while there are a host of other parts which do not permit such generic description by the inner product. We evaluate the amplitudes for the $N = 3, 4$ cases explicitly.

In section 5, we examine the zero slope limit of the system. We find that the parts of the amplitudes expressible in terms of the inner product in this limit (the principal parts) can be summarized as a noncommutative field theory of a scalar field and a noncommutative

$U(1)$ gauge field in $p'+1$ dimensions in which the scalar field decays exponentially in the $x^{p+1},\ldots,x^{p'}$-directions. They interact via the minimal coupling and a new interaction which consists of the field strength and the scalar bilinear contracted with the symplectic tensor. The contributions from the residual parts are consistent with the propagations in the t-channel of a large number of nearly massless states found in [7].

II. Basic Properties of p-p' System with B_{ij} Field

In this section, we will provide the two-point functions, and the twist and the spin fields for a p-p' open string in constant B field background. This will also help us to establish our notations. We introduce two types of normal ordering: the one is taken with respect to the $SL(2,\mathbf{R})$ invariant vacuum and the other is with respect to the oscillator vacuum. We will establish the relationship between these two, which will be important for our calculation in the subsequent sections.

A. Action and boundary condition

The action of the NSR superstring in the constant B background takes the form of

$$S=\frac{1}{2\pi}\int d^2\xi\int d\theta d\overline{\theta}\,(g_{\mu\nu}+2\pi\alpha' B_{\mu\nu})\,\overline{D}\mathbf{X}^{\mu}(\mathbf{z},\overline{\mathbf{z}})D\mathbf{X}^{\nu}(\mathbf{z},\overline{\mathbf{z}})\ , \tag{2.1}$$

where $\mathbf{z}=(z,\theta)$ and $\overline{\mathbf{z}}=(\overline{z},\overline{\theta})$ are the superspace coordinates on the worldsheet, $D=\frac{\partial}{\partial\theta}+\theta\frac{\partial}{\partial z}$ and $\overline{D}=\frac{\partial}{\partial\overline{\theta}}+\overline{\theta}\frac{\partial}{\partial\overline{z}}$ are the superspace covariant derivatives and $g_{\mu\nu}$ denotes the space-time metric which is taken to be flat. $z=\xi^1+i\xi^2$ and $\overline{z}=\xi^1-i\xi^2$ are complex coordinates on the plane which are related to the strip coordinates (τ,σ) by $z=e^{\tau+i\sigma}$ and $\overline{z}=e^{\tau-i\sigma}$ respectively. The superfield $\mathbf{X}^{\mu}(\mathbf{z},\overline{\mathbf{z}})$ is the string coordinate which is expressed in terms of component fields as

$$\mathbf{X}^{\mu}(\mathbf{z},\overline{\mathbf{z}})=\sqrt{\frac{2}{\alpha'}}X^{\mu}(z,\overline{z})+i\theta\psi^{\mu}(z,\overline{z})+i\overline{\theta}\widetilde{\psi}^{\mu}(z,\overline{z})+i\theta\overline{\theta}F^{\mu}(z,\overline{z})\ . \tag{2.2}$$

In terms of the component fields the action (2.1) is given by

$$S=\frac{1}{2\pi}\int d^2\xi\,(g_{\mu\nu}+2\pi\alpha' B_{\mu\nu})\left(\frac{2}{\alpha'}\overline{\partial}X^{\mu}\partial X^{\nu}-\overline{\partial}\psi^{\mu}\,\psi^{\nu}+\widetilde{\psi}^{\mu}\partial\widetilde{\psi}^{\nu}\right)\ . \tag{2.3}$$

Here we have eliminated the auxiliary field $F^{\mu}(z,\overline{z})$ by using the equation of motion $F^{\mu}=0$, and the operators ∂ and $\overline{\partial}$ denote $\frac{\partial}{\partial z}$ and $\frac{\partial}{\partial\overline{z}}$ respectively. Since the B dependent terms do not couple to the worldsheet metric, the energy-momentum tensor of this system has the

same form as that of the string without B field background:

$$\mathrm{T}(\mathbf{z}) \equiv T_F(z) + \theta T_B(z) \equiv -\frac{1}{2} g_{\mu\nu} D\mathbf{X}^\mu D^2 \mathbf{X}^\nu \ , \tag{2.4}$$

$$\text{with} \begin{cases} T_F(z) = -\dfrac{i}{2}\sqrt{\dfrac{2}{\alpha'}} g_{\mu\nu} \psi^\mu \partial X^\nu \\ T_B(z) = -\dfrac{1}{\alpha'} g_{\mu\nu} \partial X^\mu \partial X^\nu - \dfrac{1}{2} g_{\mu\nu} \psi^\mu \partial \psi^\nu \end{cases} . \tag{2.5}$$

Let us consider a p-p' open string in the type IIA theory with $p < p'$ and p and p' being even integers. We concentrate on the situation in which a Dp-brane extends in the $(x^0, x^1, \ldots, x^p)$-directions and a Dp'-brane extends in the $(x^0, x^1, \ldots, x^{p'})$-directions with the D$p$-brane inside. The worldsheet of the open string corresponds to the upper half-plane: $\mathrm{Im} z \geq 0$ ($\Leftrightarrow 0 \leq \sigma \leq \pi$). The D$p$-brane worldvolume contains the boundary $\sigma = 0$ while the Dp'-brane worldvolume contains the boundary $\sigma = \pi$. As the space-time is flat with the metric

$$g_{\mu\nu} = \begin{pmatrix} -1 & & & & \\ & g_{ij} & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{pmatrix} , \qquad g_{ij} = \varepsilon \delta_{ij} \quad (i,j = 1, \ldots, p') \ , \tag{2.6}$$

we can bring $B_{\mu\nu}$ into a canonical form

$$B_{ij} = \frac{\varepsilon}{2\pi\alpha'} \begin{pmatrix} 0 & b_1 & & & \\ -b_1 & 0 & & & \\ & & 0 & b_2 & \\ & & -b_2 & 0 & \\ & & & & \ddots \end{pmatrix} \quad (i,j = 1, \ldots, p') \ , \quad \text{otherwise } B_{\mu\nu} = 0 \ . \tag{2.7}$$

In what follows we will investigate the system on this background.

The boundary conditions for the string coordinates in the NS sector are [7][6]

$$\begin{aligned} & D\mathbf{X}^0 - \overline{D}\mathbf{X}^0\Big|_{\sigma=0,\pi \ \theta=\overline{\theta}} = 0 \ , \quad D\mathbf{X}^{p'+1,\ldots,9} + \overline{D}\mathbf{X}^{p'+1,\ldots,9}\Big|_{\sigma=0,\pi \ \theta=\overline{\theta}} = 0 \ , \\ & g_{kl}(D\mathbf{X}^l - \overline{D}\mathbf{X}^l) + 2\pi\alpha' B_{kl}(D\mathbf{X}^l + \overline{D}\mathbf{X}^l)\Big|_{\sigma=0,\pi \ \theta=\overline{\theta}} = 0 \quad (k,l = 1, \ldots, p) \\ & D\mathbf{X}^i + \overline{D}\mathbf{X}^i\Big|_{\sigma=0 \ \theta=\overline{\theta}} = g_{ij}(D\mathbf{X}^j - \overline{D}\mathbf{X}^j) + 2\pi\alpha' B_{ij}(D\mathbf{X}^j + \overline{D}\mathbf{X}^j)\Big|_{\sigma=\pi \ \theta=\overline{\theta}} = 0 \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (i,j = p+1, \ldots, p') \ . \end{aligned} \tag{2.8}$$

For the bosonic components these conditions read

$$
\begin{aligned}
&(\partial-\overline{\partial})X^0\Big|_{\sigma=0,\pi}=0\ , \qquad (\partial+\overline{\partial})X^{p'+1,\ldots,9}\Big|_{\sigma=0,\pi}=0\ ,\\
&g_{kl}(\partial-\overline{\partial})X^l+2\pi\alpha' B_{kl}(\partial+\overline{\partial})X^l\Big|_{\sigma=0,\pi}=0 \quad (k,l=1,\ldots,p)\ ,\\
&(\partial+\overline{\partial})X^i\Big|_{\sigma=0}=\ g_{ij}(\partial-\overline{\partial})X^j+2\pi\alpha' B_{ij}(\partial+\overline{\partial})X^j\Big|_{\sigma=\pi}=0 \quad (i,j=p+1,\ldots,p')
\end{aligned}
\tag{2.9}
$$

and for the fermionic components

$$
\begin{aligned}
&\psi^0-\widetilde{\psi}^0\Big|_{\sigma=0,\pi}=0\ , \qquad \psi^{p'+1,\ldots,9}+\widetilde{\psi}^{p'+1,\ldots,9}\Big|_{\sigma=0,\pi}=0\ ,\\
&g_{kl}(\psi^l-\widetilde{\psi}^l)+2\pi\alpha' B_{kl}(\psi^l+\widetilde{\psi}^l)\Big|_{\sigma=0,\pi}=0 \quad (k,l=1,\ldots,p)\\
&\psi^i+\widetilde{\psi}^i\Big|_{\sigma=0}=\ g_{ij}(\psi^j-\widetilde{\psi}^j)+2\pi\alpha' B_{ij}(\psi^j+\widetilde{\psi}^j)\Big|_{\sigma=\pi}=0 \quad (i,j=p+1,\ldots,p')
\end{aligned}
\tag{2.10}
$$

The boundary conditions for the R fermions are

$$
\begin{aligned}
&\psi^0-\widetilde{\psi}^0\Big|_{\sigma=0}=\ \psi^0+\widetilde{\psi}^0\Big|_{\sigma=\pi}=0\ , \quad \psi^{p'+1,\ldots,9}+\widetilde{\psi}^{p'+1,\ldots,9}\Big|_{\sigma=0}=\ \psi^{p'+1,\ldots,9}-\widetilde{\psi}^{p'+1,\ldots,9}\Big|_{\sigma=\pi}=0\ ,\\
&g_{kl}(\psi^l-\widetilde{\psi}^l)+2\pi\alpha' B_{kl}(\psi^l+\widetilde{\psi}^l)\Big|_{\sigma=0}=\ g_{kl}(\psi^l+\widetilde{\psi}^l)+2\pi\alpha' B_{kl}(\psi^l-\widetilde{\psi}^l)\Big|_{\sigma=\pi}=0\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (k,l=1,\ldots,p)\ ,\\
&\psi^i+\widetilde{\psi}^i\Big|_{\sigma=0}=\ g_{ij}(\psi^j+\widetilde{\psi}^j)+2\pi\alpha' B_{ij}(\psi^j-\widetilde{\psi}^j)\Big|_{\sigma=\pi}=0 \quad (i,j=p+1,\ldots,p')\ .
\end{aligned}
\tag{2.11}
$$

It should be noted that these boundary conditions are written on the complex plane, while the boundary conditions in [7] are written on the strip.

B. Quantization of a p-p' string

In this subsection we will give the mode expansions of the string coordinates of a p-p' open string and the commutation relations among their modes.

Let us first consider the x^0-direction. In this direction the string coordinate obeys the Neumann boundary condition on both ends. The coordinate $X^0(z,\overline{z})$ is expanded as

$$
X^0(z,\overline{z})=x^0-i\alpha' p^0\ln(z\overline{z})+i\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}\frac{\alpha^0_m}{m}\left(z^{-m}+\overline{z}^{-m}\right)\ . \tag{2.12}
$$

The modes satisfy the following commutation relations,

$$
[x^0,p^0]=ig^{00}\ , \quad [x^0,x^0]=[p^0,p^0]=0\ ; \quad [\alpha^0_m,\alpha^0_n]=g^{00}m\delta_{m+n}\ . \tag{2.13}
$$

The mode expansions of the NS fermions ψ^0 and $\widetilde{\psi}^0$ become

$$\psi^0(z) = \sum_{r\in\mathbf{Z}+1/2} b^0_r z^{-r-\frac{1}{2}} \ , \quad \widetilde{\psi}^0(\overline{z}) = \sum_{r\in\mathbf{Z}+1/2} b^0_r \overline{z}^{-r-\frac{1}{2}} \ . \tag{2.14}$$

The oscillators satisfy

$$\{b^0_r, b^0_s\} = g^{00}\delta_{r+s} \ . \tag{2.15}$$

In the R sector, we have

$$\psi^0(z) = \sum_{m\in\mathbf{Z}} d^0_m z^{-m-\frac{1}{2}} \ , \quad \widetilde{\psi}^0(\overline{z}) = \sum_{m\in\mathbf{Z}} d^0_m \overline{z}^{-m-\frac{1}{2}} \ , \tag{2.16}$$

and

$$\{d^0_m, d^0_n\} = g^{00}\delta_{m+n} \ . \tag{2.17}$$

Next we consider the x^i-directions, $i = 1, \ldots, p$. In these directions, the boundary conditions on the string coordinates are the same as those on the string coordinates along the Dp-branes in the Dp-Dp system with B field. This implies that the mode expansions and the commutation relations among the oscillators take the same form as those in the Dp-Dp system in the B field background. The mode expansions of the bosonic coordinates $X^i(z,\overline{z})$ are

$$\begin{aligned} X^i(z,\overline{z}) = x^i &- i\alpha'\Big[g^{-1}(g-2\pi\alpha' B)\Big]^i{}_j\, p^j \ln z - i\alpha'\Big[g^{-1}(g+2\pi\alpha' B)\Big]^i{}_j\, p^j \ln \overline{z} \\ &+ i\sqrt{\frac{\alpha'}{2}} \sum_{m\neq 0} \left[\Big(g^{-1}(g-2\pi\alpha' B)\Big)^i{}_j z^{-m} + \Big(g^{-1}(g+2\pi\alpha' B)\Big)^i{}_j \overline{z}^{-m}\right] \frac{\alpha^j_m}{m} \ . \end{aligned} \tag{2.18}$$

Under this expansion, the commutation relations among the oscillators are given [3][4] by

$$[x^i, x^j] = i\theta^{ij} \ , \quad [p^i, p^j] = 0 \ , \quad [x^i, p^j] = iG^{ij} \ ; \quad [\alpha^i_m, \alpha^j_n] = G^{ij} m\delta_{m+n} \ , \tag{2.19}$$

where θ^{ij} is the noncommutativity parameter [6] and G^{ij} is the inverse of the open string metric G_{ij} [6] defined respectively as

$$\theta^{ij} = -(2\pi\alpha')^2 \left(\frac{1}{g+2\pi\alpha' B} B \frac{1}{g-2\pi\alpha' B}\right)^{ij} \ , \quad G^{ij} = \left(\frac{1}{g+2\pi\alpha' B} g \frac{1}{g-2\pi\alpha' B}\right)^{ij} \ . \tag{2.20}$$

We will further generalize G^{ij} to include the time direction. This is denoted by $G^{\mu\nu}$ and will simplify our formulas in the subsequent sections. Taking eqs. (2.6) and (2.7) into account, we obtain

$$G^{\sigma\rho} = \left(\begin{array}{c|c} g^{00} & \\ \hline & G^{ij} \end{array}\right) = \left(\begin{array}{c|cccccc} -1 & & & & & & \\ \hline & \frac{1}{\varepsilon(1+b_1^2)} & 0 & & & & \\ & 0 & \frac{1}{\varepsilon(1+b_1^2)} & & & & \\ & & & \frac{1}{\varepsilon(1+b_2^2)} & 0 & & \\ & & & 0 & \frac{1}{\varepsilon(1+b_2^2)} & & \\ & & & & & \ddots & \end{array}\right) . \tag{2.21}$$

We obtain for the NS fermions

$$\begin{cases} \psi^i(z) = \sum_{r\in\mathbf{Z}+1/2} \left[g^{-1}(g-2\pi\alpha' B)\right]^i{}_j b^j_r z^{-r-\frac{1}{2}} \\ \widetilde{\psi}^i(\overline{z}) = \sum_{r\in\mathbf{Z}+1/2} \left[g^{-1}(g+2\pi\alpha' B)\right]^i{}_j b^j_r \overline{z}^{-r-\frac{1}{2}} \end{cases} , \quad \text{with} \quad \{b^i_r, b^j_s\} = G^{ij}\delta_{r+s} \ , \tag{2.22}$$

and for the R fermions

$$\begin{cases} \psi^i(z) = \sum_{m\in\mathbf{Z}} \left[g^{-1}(g-2\pi\alpha' B)\right]^i{}_j d^j_m z^{-m-\frac{1}{2}} \\ \widetilde{\psi}^i(\overline{z}) = \sum_{m\in\mathbf{Z}} \left[g^{-1}(g+2\pi\alpha' B)\right]^i{}_j d^j_m \overline{z}^{-m-\frac{1}{2}} \end{cases} , \quad \text{with} \quad \{d^i_m, d^j_n\} = G^{ij}\delta_{m+n} \ . \tag{2.23}$$

Finally we investigate the x^i-directions ($i = p+1, \ldots, p'$). We complexify the string coordinates $\mathbf{X}^i(\mathbf{z}, \overline{\mathbf{z}})$ in these directions as

$$\begin{aligned} \mathbf{Z}^I(\mathbf{z}, \overline{\mathbf{z}}) &= \mathbf{X}^{2I-1}(\mathbf{z}, \overline{\mathbf{z}}) + i\mathbf{X}^{2I}(\mathbf{z}, \overline{\mathbf{z}}) = \sqrt{\frac{2}{\alpha'}} Z^I(z, \bar{z}) + i\theta\Psi^I(z) + i\overline{\theta}\widetilde{\Psi}^I(\overline{z}) \ , \\ \overline{\mathbf{Z}}^{\overline{I}}(\mathbf{z}, \overline{\mathbf{z}}) &= \mathbf{X}^{2I-1}(\mathbf{z}, \overline{\mathbf{z}}) - i\mathbf{X}^{2I}(\mathbf{z}, \overline{\mathbf{z}}) = \sqrt{\frac{2}{\alpha'}} \overline{Z}^{\overline{I}}(z, \overline{z}) + i\theta\overline{\Psi}^{\overline{I}}(z) + i\overline{\theta}\widetilde{\overline{\Psi}}^{\overline{I}}(\overline{z}) \ , \end{aligned} \tag{2.24}$$

where $I, \overline{I} = \frac{p+2}{2}, \ldots, \frac{p'}{2}$ and we have eliminated the auxiliary field F^i. From the boundary conditions eq. (2.9) and equations of motion, we find that the mode expansions of Z^I and $\overline{Z}^{\overline{I}}$ are given by

$$\begin{aligned} Z^I(z, \overline{z}) &= i\sqrt{\frac{\alpha'}{2}} \sum_{n\in\mathbf{Z}} \frac{\alpha^I_{n-\nu_I}}{n-\nu_I} \left(z^{-(n-\nu_I)} - \overline{z}^{-(n-\nu_I)}\right) \ , \\ \overline{Z}^{\overline{I}}(z, \overline{z}) &= i\sqrt{\frac{\alpha'}{2}} \sum_{m\in\mathbf{Z}} \frac{\overline{\alpha}^{\overline{I}}_{m+\nu_I}}{m+\nu_I} \left(z^{-(m+\nu_I)} - \overline{z}^{-(m+\nu_I)}\right) \ , \end{aligned} \tag{2.25}$$

where ν_I are defined by

$$e^{2\pi i\nu_I} = -\frac{1+ib_I}{1-ib_I} \ , \quad 0 < \nu_I < 1 \ . \tag{2.26}$$

Now we can introduce the open string metric G^{IJ}, $G^{\overline{I}\overline{J}}$, $G^{I\overline{J}}$ and $G^{\overline{I}J}$ concerning the $x^{p+1}, \ldots, x^{p'}$ directions,

$$G^{IJ} = G^{\overline{I}\overline{J}} = 0 \ , \quad G^{I\overline{J}} = G^{\overline{J}I} = \frac{2}{\varepsilon(1+b_I^2)}\delta^{I\overline{J}} \ . \tag{2.27}$$

Similarly, the boundary conditions eq. (2.10), eq. (2.11) and equations of motion lead us to the mode expansions of the NS-fermions,

$$\begin{aligned} \Psi^I(z) &= \sum_{r\in\mathbf{Z}+1/2} b^I_{r-\nu_I} z^{-(r-\nu_I)-\frac{1}{2}} \ , \qquad \widetilde{\Psi}^I(\overline{z}) = -\sum_{r\in\mathbf{Z}+1/2} b^I_{r-\nu_I} \overline{z}^{-(r-\nu_I)-\frac{1}{2}} \ , \\ \overline{\Psi}^{\overline{I}}(z) &= \sum_{s\in\mathbf{Z}+1/2} \overline{b}^{\overline{I}}_{s+\nu_I} z^{-(s+\nu_I)-\frac{1}{2}} \ , \qquad \widetilde{\overline{\Psi}}^{\overline{I}}(\overline{z}) = -\sum_{s\in\mathbf{Z}+1/2} \overline{b}^{\overline{I}}_{s+\nu_I} \overline{z}^{-(s+\nu_I)-\frac{1}{2}} \ , \end{aligned} \tag{2.28}$$

and that of R-fermions,

$$\Psi^I(z) = \sum_{n\in\mathbf{Z}} d^I_{n-\nu_I} z^{-(n-\nu_I)-\frac{1}{2}} \ , \qquad \widetilde{\Psi}^I(\overline{z}) = -\sum_{n\in\mathbf{Z}} d^I_{n-\nu_I} \overline{z}^{-(n-\nu_I)-\frac{1}{2}} \ ,$$

$$\overline{\Psi}^{\overline{I}}(z) = \sum_{m\in\mathbf{Z}} \overline{d}^{\overline{I}}_{m+\nu_I} z^{-(m+\nu_I)-\frac{1}{2}} \ , \qquad \widetilde{\overline{\Psi}}^{\overline{I}}(\overline{z}) = -\sum_{m\in\mathbf{Z}} \overline{d}^{\overline{I}}_{m+\nu_I} \overline{z}^{-(m+\nu_I)-\frac{1}{2}} \ . \tag{2.29}$$

The commutation relations are

$$[\alpha^I_{n-\nu_I} \, , \, \overline{\alpha}^{\overline{J}}_{m+\nu_J}] = \frac{2}{\varepsilon}\delta^{I\overline{J}}(n-\nu_I)\delta_{n+m} \ ,$$

$$\{b^I_{r-\nu_I} \, , \, \overline{b}^{\overline{J}}_{s+\nu_J}\} = \frac{2}{\varepsilon}\,\delta^{I\overline{J}}\delta_{r+s} \ , \qquad \{d^I_{n-\nu_I} \, , \, \overline{d}^{\overline{J}}_{m+\nu_J}\} = \frac{2}{\varepsilon}\,\delta^{I\overline{J}}\delta_{n+m} \ . \tag{2.30}$$

As for the $x^{p'+1}, \ldots, x^9$-directions, the string coordinates obey the Dirichlet boundary condition. Our analysis in the remaining part of this paper does not involve these directions.

C. Two-point functions on superspace

In this subsection we construct two-point functions of the p-p' open string coordinates on superspace. For this purpose, we begin by defining the oscillator vacuum of the system.

As shown in the last subsection, the mode expansions in the x^0 and the x^i-directions ($i = 1, \ldots, p$) are similar to those of the usual open strings obeying Neumann boundary conditions in the sense that the bosons and the R fermions have integral moding oscillators and the NS fermions have half-integral moding ones. Therefore we can define the vacuum in these directions in the same way as the usual open string. In the NS sector the vacuum $|0\rangle$ is defined by

$$\begin{cases} \alpha^0_m|0\rangle = 0 \ , \quad \alpha^i_m|0\rangle = 0 \ , & \text{for } m \geq 0 \\ b^0_r|0\rangle = 0 \ , \quad b^i_r|0\rangle = 0 \ , & \text{for } r \geq \frac{1}{2} \end{cases} \ , \tag{2.31}$$

where $\alpha^\mu_0 = \sqrt{2\alpha'}p^\mu$ ($\mu = 0, 1, \ldots, p$). In the R sector the vacuum $|S^\alpha\rangle$ belongs to the spinor representation of the $SO(p,1)$ group. Now that the commutation relations and the vacuum $|0\rangle$ are determined, we can evaluate the two-point functions of the string coordinates in these directions [6][16][12],

$$\begin{aligned} \mathbf{G}^{00}(\mathbf{z}_1, \overline{\mathbf{z}}_1|\mathbf{z}_2, \overline{\mathbf{z}}_2) &\equiv \langle 0|\mathcal{R}\mathbf{X}^0(\mathbf{z}_1, \overline{\mathbf{z}}_1)\mathbf{X}^0(\mathbf{z}_2, \overline{\mathbf{z}}_2)|0\rangle \\ &= -g^{00}\left[\ln(z_1 - z_2 - \theta_1\theta_2)(\overline{z}_1 - \overline{z}_2 - \overline{\theta}_1\overline{\theta}_2) + \ln(z_1 - \overline{z}_2 - \theta_1\overline{\theta}_2)(\overline{z}_1 - z_2 - \overline{\theta}_1\theta_2)\right] \ , \\ \mathbf{G}^{ij}(\mathbf{z}_1, \overline{\mathbf{z}}_1|\mathbf{z}_2, \overline{\mathbf{z}}_2) &\equiv \langle 0|\mathcal{R}\mathbf{X}^i(\mathbf{z}_1, \overline{\mathbf{z}}_1)\mathbf{X}^j(\mathbf{z}_2, \overline{\mathbf{z}}_2)|0\rangle \\ &= -g^{ij}\ln(z_1 - z_2 - \theta_1\theta_2)(\overline{z}_1 - \overline{z}_2 - \overline{\theta}_1\overline{\theta}_2) + (g^{ij} - 2G^{ij})\ln(z_1 - \overline{z}_2 - \theta_1\overline{\theta}_2)(\overline{z}_1 - z_2 - \overline{\theta}_1\theta_2) \\ &\quad -2\frac{\theta^{ij}}{2\pi\alpha'}\ln\frac{z_1 - \overline{z}_2 - \theta_1\overline{\theta}_2}{\overline{z}_1 - z_2 - \overline{\theta}_1\theta_2} - 2D^{ij} \ , \end{aligned} \tag{2.32}$$

where $\mathcal{R}$ stands for the radial ordering. D^{ij} are the contributions from the zero modes x^i and these will be fixed conveniently as is done in [6]. When we restrict these two-point functions onto the Dp'-brane worldvolume, i.e. the worldsheet boundary characterized by $z = e^{\tau+i\pi} = -e^{\tau}$ and $\theta = \overline{\theta}$, they become

$$\begin{aligned}
\mathbf{G}^{00}\left(-e^{\tau_1},\theta_1|-e^{\tau_2},\theta_2\right) &\equiv \mathbf{G}^{00}\left(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2\right)\Big|_{\sigma=\pi,\theta=\overline{\theta}} = -2g^{00}\ln(e^{\tau_1}-e^{\tau_2}+\theta_1\theta_2)^2 \ , \\
\mathbf{G}^{ij}\left(-e^{\tau_1},\theta_1|-e^{\tau_2},\theta_2\right) &\equiv \mathbf{G}^{ij}\left(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2\right)\Big|_{\sigma=\pi,\theta=\overline{\theta}} \\
&= -2G^{ij}\ln(e^{\tau_1}-e^{\tau_2}+\theta_1\theta_2)^2 - \frac{i}{\alpha'}\theta^{ij}\epsilon(\tau_1-\tau_2) \ , \qquad (2.33)
\end{aligned}$$

where $\epsilon(x)$ is the sign function.

In the x^i-directions ($i = p+1,\ldots,p'$), the situation is more complex as the string coordinates are expanded in non-integer power of z and $\overline{z}$. We define the oscillator vacuum $|\sigma\rangle$ for the bosonic sector so that this should be annihilated by the negative energy modes:

$$|\sigma\rangle = \bigotimes_I |\sigma_I\rangle \qquad \text{with} \qquad \begin{cases} \alpha^I_{n-\nu_I}|\sigma_I\rangle = 0 & n > \nu_I \\ \overline{\alpha}^{\overline{I}}_{m+\nu_I}|\sigma_I\rangle = 0 & m > -\nu_I \end{cases} . \qquad (2.34)$$

For the fermions in the NS sector we define the oscillator vacuum $|s\rangle$ by[1]

$$|s\rangle = \bigotimes_I |s_I\rangle \qquad \text{with} \qquad \begin{cases} b^I_{r-\nu_I}|s_I\rangle = 0 & r \geq \frac{1}{2} \\ \overline{b}^{\overline{I}}_{s+\nu_I}|s_I\rangle = 0 & s \geq \frac{1}{2} \end{cases} , \qquad (2.35)$$

and for the R sector we define the oscillator vacuum $|S\rangle$ by

$$|S\rangle = \bigotimes_I |S_I\rangle \qquad \text{with} \qquad \begin{cases} d^I_{n-\nu_I}|S_I\rangle = 0 & n > \nu_I \\ \overline{d}^{\overline{I}}_{m+\nu_I}|S_I\rangle = 0 & m > -\nu_I \end{cases} . \qquad (2.36)$$

By using the commutation relations and the defining relations of the vacua, we can calculate the two-point functions[2]:

$$\mathcal{G}^{I\overline{J}}\left(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2\right) \equiv \langle\sigma,s|\,\mathcal{R}\mathbf{Z}^I(\mathbf{z}_1,\overline{\mathbf{z}}_1)\overline{\mathbf{Z}}^{\overline{J}}(\mathbf{z}_2,\overline{\mathbf{z}}_2)\,|\sigma,s\rangle$$

[1] The vacuum $|s\rangle$ is defined in order that the negative energy and the positive energy modes for $0 < \nu_I < 1/2$ should be the annihilation and the creation modes respectively. The energy carried by the lowest creation mode $\overline{b}^{\overline{I}}_{-\frac{1}{2}+\nu_I}$ becomes negative when ν_I becomes greater than $1/2$. We could define another oscillator vacuum $|\widetilde{s}\rangle$ by

$$|\widetilde{s}\rangle = \bigotimes_I |\widetilde{s}_I\rangle \qquad \text{with} \qquad \begin{cases} b^I_{r-\nu_I}|\widetilde{s}_I\rangle = 0 & r \geq \frac{3}{2} \\ \overline{b}^{\overline{I}}_{s+\nu_I}|\widetilde{s}_I\rangle = 0 & s \geq -\frac{1}{2} \end{cases} ,$$

which makes the negative energy and the positive energy modes for $1/2 < \nu_I < 1$ to be the annihilation and the creation modes respectively.

[2] If we adopted $|\widetilde{s}\rangle$ as the vacuum of the NS fermions instead of $|s\rangle$, the two-point function of the super-

$$= \Theta(|z_1|-|z_2|)\frac{2\delta^{I\overline{J}}}{\varepsilon}\left[\mathcal{F}\left(1-\nu_I\,;\,\frac{z_2+\theta_1\theta_2}{z_1}\right)+\mathcal{F}\left(1-\nu_I\,;\,\frac{\overline{z}_2+\overline{\theta}_1\overline{\theta_2}}{\overline{z_1}}\right)\right.$$
$$\left.-\mathcal{F}\left(1-\nu_I\,;\,\frac{\overline{z}_2+\theta_1\overline{\theta_2}}{z_1}\right)-\mathcal{F}\left(1-\nu_I;\,\frac{z_2+\overline{\theta}_1\theta_2}{\overline{z_1}}\right)\right]$$
$$+\Theta(|z_2|-|z_1|)\frac{2\delta^{I\overline{J}}}{\varepsilon}\left[\mathcal{F}\left(\nu_I\,;\,\frac{z_1}{z_2+\theta_1\theta_2}\right)+\mathcal{F}\left(\nu_I\,;\,\frac{\overline{z_1}}{\overline{z}_2+\overline{\theta}_1\overline{\theta_2}}\right)\right.$$
$$\left.-\mathcal{F}\left(\nu_I\,;\,\frac{z_1}{\overline{z}_2+\theta_1\overline{\theta_2}}\right)-\mathcal{F}\left(\nu_I;\,\frac{\overline{z_1}}{z_2+\overline{\theta}_1\theta_2}\right)\right]\ , \tag{2.37}$$

where $\Theta(x)$ is the step function, $\mathcal{F}(\nu\,;\,z)$ is defined as

$$\mathcal{F}(\nu\,;\,z)=\frac{z^\nu}{\nu}\,F(1,\nu;1+\nu;z)=\sum_{n=0}^{\infty}\frac{1}{n+\nu}z^{n+\nu}\ , \tag{2.38}$$

and $F(a,b;c;z)$ is the hypergeometric function. When we restrict this two-point function onto the worldsheet boundary on the Dp'-brane worldvolume, this becomes

$$\boldsymbol{\mathcal{G}}^{I\overline{J}}\left(-e^{\tau_1},\theta_1|-e^{\tau_2},\theta_2\right)\equiv\boldsymbol{\mathcal{G}}^{I\overline{J}}\left(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2\right)\Big|_{\sigma=\pi,\theta=\overline{\theta}}$$
$$=4G^{I\overline{J}}\left[\Theta(\tau_1-\tau_2)\mathcal{F}\left(1-\nu_I\,;\,\frac{e^{\tau_2}-\theta_1\theta_2}{e^{\tau_1}}\right)+\Theta(\tau_2-\tau_1)\mathcal{F}\left(\nu_I\,;\,\frac{e^{\tau_1}}{e^{\tau_2}-\theta_1\theta_2}\right)\right] \tag{2.39}$$

Now we would like to study the two-point function eq. (2.39) more closely. Let us recast the right hand side of eq. (2.39) into

$$4G^{I\overline{J}}\left[\frac{1}{2}\left\{\mathcal{F}\left(1-\nu_I;\frac{e^{\tau_2}}{e^{\tau_1}}\right)+\mathcal{F}\left(\nu_I;\frac{e^{\tau_1}}{e^{\tau_2}}\right)\right\}-\theta_1\theta_2\frac{\left(\frac{e^{\tau_1}}{e^{\tau_2}}\right)^{\nu_I}}{e^{\tau_1}-e^{\tau_2}}\right]$$
$$+\epsilon(\tau_1-\tau_2)\,\frac{4}{\varepsilon}\frac{\delta^{I\overline{J}}}{1+b_I^2}\left\{\mathcal{F}\left(1-\nu_I;\frac{e^{\tau_2}}{e^{\tau_1}}\right)-\mathcal{F}\left(\nu_I;\frac{e^{\tau_1}}{e^{\tau_2}}\right)\right\}\ . \tag{2.40}$$

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coordinates $\mathbf{Z}^I$ and $\overline{\mathbf{Z}}^{\overline{I}}$ would be

$$\widetilde{\boldsymbol{\mathcal{G}}}^{I\overline{J}}\left(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2\right)\equiv\langle\sigma,\widetilde{s}|\,\mathcal{R}\mathbf{Z}^I(\mathbf{z}_1,\overline{\mathbf{z}}_1)\overline{\mathbf{Z}}^{\overline{J}}(\mathbf{z}_2,\overline{\mathbf{z}}_2)\,|\sigma,\widetilde{s}\rangle$$
$$=\Theta(|z_1|-|z_2|)\frac{2\delta^{I\overline{J}}}{\varepsilon}\left[\mathcal{F}\left(1-\nu_I\,;\,\frac{z_2}{z_1-\theta_1\theta_2}\right)+\mathcal{F}\left(1-\nu_I\,;\,\frac{\overline{z}_2}{\overline{z}_1-\overline{\theta}_1\overline{\theta}_2}\right)\right.$$
$$\left.-\mathcal{F}\left(1-\nu_I\,;\,\frac{\overline{z}_2}{z_1-\theta_1\overline{\theta}_2}\right)-\mathcal{F}\left(1-\nu_I\,;\,\frac{z_2}{\overline{z}_1-\overline{\theta}_1\theta_2}\right)\right]$$
$$+\Theta(|z_2|-|z_1|)\frac{2\delta^{I\overline{J}}}{\varepsilon}\left[\mathcal{F}\left(\nu_I\,;\,\frac{z_1-\theta_1\theta_2}{z_2}\right)+\mathcal{F}\left(\nu_I\,;\,\frac{\overline{z}_1-\overline{\theta}_1\overline{\theta}_2}{\overline{z}_2}\right)-\mathcal{F}\left(\nu_I\,;\,\frac{z_1-\theta_1\overline{\theta}_2}{\overline{z}_2}\right)-\mathcal{F}\left(\nu_I\,;\,\frac{\overline{z}_1-\overline{\theta}_1\theta_2}{z_2}\right)\right]\ .$$

As noncommutativity of the Dp'-brane worldvolume originates from the term proportional to the sign function $\epsilon(\tau_1 - \tau_2)$ in the above equation [7][6], we will also refer to this term as noncommutativity term in what follows. Here it should be noted that by using the relations,

$$\begin{aligned} \frac{d}{dz}\left[z^{c-1}F(a,b;c;z)\right] &= (c-1)z^{c-2}F(a,b;c-1;z)\ , \\ F(a,b;b;z) &= (1-z)^{-a}\ . \end{aligned} \tag{2.41}$$

we can obtain

$$\frac{d}{dz}\left[\mathcal{F}\left(1-\nu_I;\frac{1}{z}\right) - \mathcal{F}\left(\nu_I;z\right)\right] = 0. \tag{2.42}$$

This implies that the noncommutativity term in eq. (2.40) is constant. The value of this constant can be fixed by evaluating the noncommutativity term at a certain point on the real axis, such as $\frac{e^{\tau_1}}{e^{\tau_2}} = 1$. By using the hypergeometric series, we find that

$$\mathcal{F}(1-\nu_I;1) - \mathcal{F}(\nu_I;1) = -\sum_{n=-\infty}^{\infty}\frac{1}{n+\nu_I} = -\pi\cot\left(\pi\nu_I\right) = \pi b_I\ . \tag{2.43}$$

Thus we find that the noncommutativity term becomes

$$\frac{4}{\varepsilon}\frac{\delta^{I\overline{J}}}{1+b_I^2}\left\{\mathcal{F}\left(1-\nu_I;\frac{e^{\tau_2}}{e^{\tau_1}}\right) - \mathcal{F}\left(\nu_I;\frac{e^{\tau_1}}{e^{\tau_2}}\right)\right\} = \frac{4}{\varepsilon}\frac{\delta^{I\overline{J}}}{1+b_I^2}\pi b_I\ . \tag{2.44}$$

When we rewrite the complex string coordinates $\mathbf{Z}^I$ and $\overline{\mathbf{Z}}^{\overline{I}}$ into the real one $\mathbf{X}^i$ ($i = p+1,\ldots,p'$), this noncommutativity term takes the same form as that in eq. (2.33). This means that, as is pointed out in [7], the noncommutativity on the D-brane worldvolume in the p-p' system is the same as that in the p-p system [6][3][4]. From eq. (2.44), we conclude that

$$\mathcal{G}^{I\overline{J}}\left(-e^{\tau_1},\theta_1|-e^{\tau_2},\theta_2\right) = 4G^{I\overline{J}}\,\mathcal{H}\left(\nu_I;\frac{e^{\tau_1}}{e^{\tau_2}-\theta_1\theta_2}\right) + \epsilon(\tau_1-\tau_2)\frac{4}{\varepsilon}\frac{\delta^{I\overline{J}}}{1+b_I^2}\pi b_I\ , \tag{2.45}$$

where $\mathcal{H}(\nu;z)$ is defined by using the hypergeometric series as

$$\mathcal{H}(\nu;z) = \begin{cases} \mathcal{F}\left(1-\nu_I;\dfrac{1}{z}\right) - \dfrac{\pi}{2}b_I = \displaystyle\sum_{n=0}^{\infty}\frac{z^{-n-1+\nu_I}}{n+1-\nu_I} - \frac{\pi}{2}b_I & \text{for } |z|>1 \\ \mathcal{F}\left(\nu_I;z\right) + \dfrac{\pi}{2}b_I = \displaystyle\sum_{n=0}^{\infty}\frac{z^{n+\nu_I}}{n+\nu_I} + \frac{\pi}{2}b_I & \text{for } |z|<1 \end{cases}\ . \tag{2.46}$$

The two infinite series in the above defining relation should be analytically continued to each other.

In Appendix we give another derivation of the two-point function (2.45) from eq. (2.39).

D. Twist field and spin field

Here we would like to make further consideration on the oscillator vacuum consisting of the bosonic sector $|\sigma\rangle$ and the fermionic sector $|s\rangle$. As is explained in the last subsection, the primary fields $D\mathbf{Z}^I$, $\overline{D}\mathbf{Z}^I$, $D\overline{\mathbf{Z}}^{\overline{I}}$ and $\overline{D\mathbf{Z}}^{\overline{I}}$ defined on the upper half plane are expanded in non-integer powers of z and $\overline{z}$. It follows that when we extend the defining region of these fields to the whole complex plane through the doubling trick these fields become multi-valued functions on the whole plane. For example, when the primary fields $\partial Z^I(z)$, $\overline{\partial} Z^I(\overline{z})$, $\partial\overline{Z}^{\overline{I}}(z)$ and $\overline{\partial Z}^{\overline{I}}(\overline{z})$ on the whole plane are transported once around the origin, they gain phase factors:

$$\begin{aligned} \partial Z^I(e^{2\pi i}z) &= e^{2\pi i\nu_I}\partial Z^I(z) \ , \qquad \overline{\partial} Z^I(e^{2\pi i}\overline{z}) = e^{2\pi i\nu_I}\overline{\partial} Z^I(\overline{z}) \\ \partial\overline{Z}^{\overline{I}}(e^{2\pi i}z) &= e^{-2\pi i\nu_I}\partial\overline{Z}^{\overline{I}}(z) \ , \qquad \overline{\partial Z}^{\overline{I}}(e^{2\pi i}\overline{z}) = e^{-2\pi i\nu_I}\overline{\partial Z}^{\overline{I}}(\overline{z}) \ . \end{aligned} \tag{2.47}$$

This implies that a twist field $\sigma_I^+(\xi^1)$ and an anti-twist field $\sigma_I^-(\xi^1)$, both of which are mutually non-local with respect to Z^I and $\overline{Z}^I$, are located at the origin and at infinity on the plane respectively. They create a branch cut between themselves. The twist field σ^+ serves as a boundary changing operator from the p'-brane to the p-brane and the anti-twist field σ^- acts in the opposite way [8][10]. The incoming vacuum $|\sigma_I\rangle$ defined in eq. (2.34) should be interpreted as being excited from the $SL(2,\mathbf{R})$-invariant vacuum $|0\rangle$ by the twist field σ_I^+:

$$|\sigma_I\rangle = \lim_{\xi^1\to 0}\sigma_I^+(\xi^1)\,|0\rangle \ . \tag{2.48}$$

In the same way, the outgoing vacuum $\langle\sigma_I|$ should be regarded as

$$\langle\sigma_I| = \lim_{\widetilde{\xi}^1\to 0}\left(\frac{1}{\widetilde{\xi}^1}\right)^{2h_{\sigma_I}}\langle 0|\ \sigma_I^-\left(-\frac{1}{\widetilde{\xi}^1}\right) \ , \tag{2.49}$$

where h_{σ_I} denotes the weight of the (anti-) twist field. We will later explain that $h_{\sigma_I} = \frac{1}{2}\nu_I(1-\nu_I)$ [17]. We can read off the OPE's of Z^I and $\overline{Z}^I$ with $\sigma_I^\pm$ from eq. (2.34):

$$\begin{cases} \partial Z^I(z)\sigma_J^+(0) \sim \delta_J^I z^{-(1-\nu_I)}\tau_I^+(0) \ , & \overline{\partial} Z^I(\overline{z})\sigma_J^+(0) \sim \delta_J^I\overline{z}^{-(1-\nu_I)}\widetilde{\tau}_I^+(0) \ , \\ \partial\overline{Z}^{\overline{I}}(z)\sigma_J^+(0) \sim \delta_J^{\overline{I}} z^{-\nu_I}\tau_I'^+(0) \ , & \overline{\partial Z}^{\overline{I}}(\overline{z})\sigma_J^+(0) \sim \delta_J^{\overline{I}}\overline{z}^{-\nu_I}\widetilde{\tau}_I'^+(0) \ , \end{cases} \tag{2.50}$$

$$\begin{cases} \partial Z^I(z)\sigma_J^-(0) \sim \delta_J^I z^{-\nu_I}\tau_I^-(0) \ , & \overline{\partial} Z^I(\overline{z})\sigma_J^-(0) \sim \delta_J^I\overline{z}^{-\nu_I}\widetilde{\tau}_I^-(0) \ , \\ \partial\overline{Z}^{\overline{I}}(z)\sigma_J^-(0) \sim \delta_J^{\overline{I}} z^{-(1-\nu_I)}\tau_I'^-(0) \ , & \overline{\partial Z}^{\overline{I}}(\overline{z})\sigma_J^-(0) \sim \delta_{\overline{J}}^{\overline{I}}\overline{z}^{-(1-\nu_I)}\widetilde{\tau}_I'^-(0) \ , \end{cases} \tag{2.51}$$

where τ 's are excited twist fields.

Similar argument holds for the fermionic coordinates. In the NS sector, the spin fields s_I^+ and s_I^- are mutually non-local with respect to the fermions. They are located at the origin

and at the infinity on the worldsheet respectively. They exchange the boundary conditions corresponding to the p-brane and those to the p'-brane by generating a branch cut between themselves. The incoming vacuum $|s_I\rangle$ and the outgoing vacuum $\langle s_I|$ should be regarded as being excited from the $SL(2,\mathbf{R})$-invariant vacuum by spin fields:

$$|s_I\rangle = \lim_{\xi^1\to 0} s_I^+(\xi^1)|0\rangle \ , \quad \langle s_I| = \lim_{\widetilde{\xi}^1\to 0} \left(\frac{1}{\widetilde{\xi}^1}\right)^{2h_{s_I}} \langle 0|\ s_I^-\left(-\frac{1}{\widetilde{\xi}^1}\right) \ , \tag{2.52}$$

where h_{s_I} is the weight of the spin fields which will be found to be $h_{s_I} = \frac{1}{2}{\nu_I}^2$. The defining relation eq. (2.35) yields the OPE's,

$$\begin{cases} \Psi^I(z)s_J^+(0) \sim \delta_J^I z^{+\nu_I} t_I'^+(0) \ , & \widetilde{\Psi}^I(\overline{z})s_J^+(0) \sim \delta_J^I \overline{z}^{+\nu_I} \widetilde{t}_I'^+(0) \ , \\ \overline{\Psi}^{\overline{I}}(z)s_J^+(0) \sim \delta_J^{\overline{I}} z^{-\nu_I} t_I^+(0) \ , & \widetilde{\overline{\Psi}}^{\overline{I}}(\overline{z})s_J^+(0) \sim \delta_J^{\overline{I}} \overline{z}^{-\nu_I} \widetilde{t}^+(0) \ , \end{cases} \tag{2.53}$$

$$\begin{cases} \Psi^I(z)s_J^-(0) \sim \delta_J^I z^{-\nu_I} t_I'^-(0) \ , & \widetilde{\Psi}^I(\overline{z})s_J^-(0) \sim \delta_J^I \overline{z}^{-\nu_I} \widetilde{t}_I'^-(0) \\ \overline{\Psi}^{\overline{I}}(z)s_J^-(0) \sim \delta_J^{\overline{I}} z^{+\nu_I} t_I^-(0) \ , & \widetilde{\overline{\Psi}}^{\overline{I}}(\overline{z})s_J^-(0) \sim \delta_J^{\overline{I}} \overline{z}^{+\nu_I} \widetilde{t}_I^-(0) \ , \end{cases} \ , \tag{2.54}$$

In the fermionic sector, the bosonization simplifies the treatment of the spin fields. While we will not invoke this treatment here, we include it here for the sake of completeness. We write

$$\Psi^I(z) \cong \sqrt{\frac{2}{\varepsilon}} e^{iH^I(z)} \ , \quad \overline{\Psi}^{\overline{I}}(z) \cong \sqrt{\frac{2}{\varepsilon}} e^{-iH^I(z)} \ , \tag{2.55}$$

where $H^I(z)$ are free bosons normalized as $H^I(z)H^J(w) \sim -\delta^{IJ}\ln(z-w)$. Then the OPE (2.54) tells us that the spin fields $s_I^\pm(z)$ should be bosonized as [3]

$$s_I^+(z) \cong e^{+i\nu_I H^I(z)} \ , \quad s_I^-(z) \cong e^{-i\nu_I H^I(z)} \ . \tag{2.56}$$

We can repeat the same analysis in the R sector. We find that the incoming vacuum eq. (2.36) and the outgoing vacuum are excited from the $SL(2,\mathbf{R})$-invariant vacuum by the spin fields $S_I^+(z)$ and $S^-(z)$ respectively. They are bosonized as

$$S_I^+(z) \cong e^{i\left(-\frac{1}{2}+\nu_I\right)H^I(z)} \ , \quad S_I^-(z) \cong e^{-i\left(-\frac{1}{2}+\nu_I\right)H^I(z)} \ . \tag{2.57}$$

[3] We are also able to perform the same analysis on the other incoming and the outgoing vacua, $|\widetilde{s}_I\rangle$ and $\langle\widetilde{s}|$, in the NS sector defined in the footnote 1. These states are excited from the $SL(2,\mathbf{R})$-invariant vacuum by $\widetilde{s}_I^+(z)$ and $\widetilde{s}_I^-(z)$ respectively which are bosonized as

$$\widetilde{s}_I^+(z) \cong e^{+i(-1+\nu_I)H^I(z)} \ , \quad \widetilde{s}_I^-(z) \cong e^{-i(-1+\nu_I)H^I(z)} \ .$$

E. Subtracted two-point functions and weights of twist and spin fields

In the x^i-directions ($i = p+1, \ldots, p'$), we have two types of vacuum: the one is the $SL(2,\mathbf{R})$-invariant vacuum and the other is the oscillator vacuum. We can define the normal ordering corresponding to each one. We will use the symbols : : and :: : to denote the normal orderings with respect to the $SL(2,\mathbf{R})$-invariant vacuum and the oscillator vacuum respectively.

In the other directions we have a single type of vacuum, namely $SL(2,\mathbf{R})$-invariant vacuum. We have : :-normal ordered product only. For free bosons and free fermions in these directions, it is defined by a subtraction:

$$: \mathbf{X}^\mu(\mathbf{z}_1,\overline{\mathbf{z}}_1)\mathbf{X}^\nu(\mathbf{z}_2,\overline{\mathbf{z}}_2) := \mathcal{R}\mathbf{X}^\mu(\mathbf{z}_1,\overline{\mathbf{z}}_1)\mathbf{X}^\nu(\mathbf{z}_2,\overline{\mathbf{z}}_2) - \mathbf{G}^{\mu\nu}\left(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2\right) \ , \tag{2.58}$$

for $\mu,\nu = 0,1,\ldots,p$. Here $\mathbf{G}^{\mu\nu}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2)$ is the two-point function defined in eq. (2.32).

In the same way we can define :: :-normal ordered product for the free fields in the x^i-directions ($i = p+1,\ldots,p'$) as

$$:: \mathbf{Z}^I(\mathbf{z}_1,\overline{\mathbf{z}}_1)\overline{\mathbf{Z}}^{\overline{J}}(\mathbf{z}_2,\overline{\mathbf{z}}_2) :: = \mathcal{R}\mathbf{Z}^I(\mathbf{z}_1,\overline{\mathbf{z}}_1)\overline{\mathbf{Z}}^{\overline{J}}(\mathbf{z}_2,\overline{\mathbf{z}}_2) - \boldsymbol{\mathcal{G}}^{I\overline{J}}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2) \ , \tag{2.59}$$

for $I,\overline{J} = \frac{p+2}{2},\ldots,\frac{p'}{2}$. Here $\boldsymbol{\mathcal{G}}^{I\overline{J}}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2)$ is the two-point function defined in eq. (2.37). In addition to the :: :-normal ordered product, we would like to define the : :-normal ordered product for these free fields $\mathbf{Z}^I$ and $\overline{\mathbf{Z}}^{\overline{I}}$. In order to apply the definition (2.58) directly to these fields, we have to evaluate their two-point functions on the $SL(2,\mathbf{R})$-invariant vacuum. As is pointed out, the twist and the spin fields play the role of boundary changing operators. This implies that if we delete the twist and the spin fields and thus remove the cut generated by them, the boundary conditions imposed on the positive real axis of the z-plane is expected to be identical to those imposed on the negative real axis. This leads us to conclude that the two point function of $\mathbf{Z}^I$ and $\overline{\mathbf{Z}}^{\overline{I}}$ evaluated on the $SL(2,\mathbf{R})$-invariant vacuum takes the same form as that of the p'-p' system with B field:

$$\begin{aligned}
&\mathbf{G}^{I\overline{J}}\left(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2\right) \equiv \langle 0|\mathcal{R}\mathbf{Z}^I(\mathbf{z}_1,\overline{\mathbf{z}}_1)\overline{\mathbf{Z}}^{\overline{J}}(\mathbf{z}_2,\overline{\mathbf{z}}_2)|0\rangle \qquad (2.60)\\
&= \frac{2\delta^{I\overline{J}}}{\varepsilon}\Big[-\ln(z_1-z_2-\theta_1\theta_2)(\bar{z}_1-\bar{z}_2-\bar{\theta}_1\bar{\theta}_2)+\ln(z_1-\bar{z}_2-\theta_1\bar{\theta}_2)(\bar{z}_1-z_2-\bar{\theta}_1\theta_2)\\
&-\frac{2}{1+b_I^2}\ln(z_1-\bar{z}_2-\theta_1\bar{\theta}_2)(\bar{z}_1-z_2-\bar{\theta}_1\theta_2)-2i\frac{b_I}{1+b_I^2}\ln\frac{z_1-\bar{z}_2-\theta_1\bar{\theta}_2}{\bar{z}_1-z_2-\bar{\theta}_1\theta_2}\Big]+\text{D-term}
\end{aligned}$$

Computing this two point function at the boundary $\sigma=\pi$ and $\theta=\overline{\theta}$, we obtain

$$\begin{aligned}
&\mathbf{G}^{I\overline{J}}\left(-e^{\tau_1},\theta_1|-e^{\tau_2},\theta_2\right) \equiv \mathbf{G}^{I\overline{J}}\left(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2\right)\Big|_{\sigma=\pi,\theta=\overline{\theta}}\\
&= -\frac{4\delta^{I\overline{J}}}{\varepsilon(1+b_I^2)}\ln\left(-e^{\tau_1}+e^{\tau_2}-\theta_1\theta_2\right)^2+\frac{4\pi\delta^{I\overline{J}}b_I}{\varepsilon(1+b_I^2)}\epsilon(\tau_1-\tau_2) \ . \qquad (2.61)
\end{aligned}$$

From the fact that the normal ordered product is defined by a subtraction, we can readily find that for an arbitrary functional $\mathcal{O}$ of the free fields $\mathbf{Z}^I$ and $\overline{\mathbf{Z}}^{\overline{I}}$ the normal ordering is formally expressed as (see e.g. [18])

$$
\begin{aligned}
:\mathcal{O}: &= \exp\left(-\int d^2\mathbf{z}_1 d^2\mathbf{z}_2\, \mathbf{G}^{I\overline{J}}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2)\frac{\delta}{\delta\mathbf{Z}^I(\mathbf{z}_1,\overline{\mathbf{z}}_1)}\frac{\delta}{\delta\overline{\mathbf{Z}}^{\overline{J}}(\mathbf{z}_2,\overline{\mathbf{z}}_2)}\right)\mathcal{O}\ , \\
\vdots\,\mathcal{O}\,\vdots &= \exp\left(-\int d^2\mathbf{z}_1 d^2\mathbf{z}_2\, \boldsymbol{\mathcal{G}}^{I\overline{J}}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2)\frac{\delta}{\delta\mathbf{Z}^I(\mathbf{z}_1,\overline{\mathbf{z}}_1)}\frac{\delta}{\delta\overline{\mathbf{Z}}^{\overline{J}}(\mathbf{z}_2,\overline{\mathbf{z}}_2)}\right)\mathcal{O}\ , \quad (2.62)
\end{aligned}
$$

where $d^2\mathbf{z}$ is defined as $d^2\mathbf{z} = d^2\xi d\theta d\overline{\theta}$. From these general definitions of the normal orderings, we can read off the formula of the reordering between them:

$$
:\mathcal{O}: = \exp\left(\int d^2\mathbf{z}_1 d^2\mathbf{z}_2\, {\boldsymbol{\mathcal{G}}_{\rm sub}}^{I\overline{J}}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2)\frac{\delta}{\delta\mathbf{Z}^I(\mathbf{z}_1,\overline{\mathbf{z}}_1)}\frac{\delta}{\delta\overline{\mathbf{Z}}^{\overline{J}}(\mathbf{z}_2,\overline{\mathbf{z}}_2)}\right)\vdots\,\mathcal{O}\,\vdots\ . \qquad (2.63)
$$

Here ${\boldsymbol{\mathcal{G}}_{\rm sub}}^{I\overline{J}}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2)$ is a subtracted two-point function defined as

$$
\begin{aligned}
{\boldsymbol{\mathcal{G}}_{\rm sub}}^{I\overline{J}}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2) &\equiv \langle\sigma,s|:\mathbf{Z}^I(\mathbf{z}_1,\overline{\mathbf{z}}_1)\overline{\mathbf{Z}}^{\overline{J}}(\mathbf{z}_2,\overline{\mathbf{z}}_2):|\sigma,s\rangle \\
&= \boldsymbol{\mathcal{G}}^{I\overline{J}}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2) - \mathbf{G}^{I\overline{J}}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2)\ . \qquad (2.64)
\end{aligned}
$$

Let us compute the subtracted two-point function ${\boldsymbol{\mathcal{G}}_{\rm sub}}^{I\overline{J}}$ at the worldsheet boundary $\sigma=\pi$ and $\theta=\overline{\theta}$. From eqs. (2.39) and (2.61), we find that

$$
\begin{aligned}
&{\boldsymbol{\mathcal{G}}_{\rm sub}}^{I\overline{J}}(-e^{\tau_1},\theta_1|-e^{\tau_2},\theta_2) \equiv \langle\sigma,s|:\mathbf{Z}^I(-e^{\tau_1},\theta_1)\overline{\mathbf{Z}}^{\overline{J}}(-e^{\tau_2},\theta_2):|\sigma,s\rangle \\
&= \frac{8\delta^{I\overline{J}}}{\varepsilon(1+b_I^2)}\left[-\gamma - \frac{\boldsymbol{\psi}(\nu_I)+\boldsymbol{\psi}(1-\nu_I)}{2} + \frac{1}{2}\ln e^{\tau_1+\tau_2}\right. \\
&\quad +\Theta(\tau_1-\tau_2)\left\{\frac{1}{2}\ln\frac{e^{\tau_1}}{e^{\tau_2}} - \left(\frac{e^{\tau_2}}{e^{\tau_1}}\right)^{1-\nu_I}\sum_{n=1}^{\infty}\frac{(1-\nu_I)_n}{n!}\sum_{m=0}^{n-1}\frac{\nu_I}{(m+1)(m+1-\nu_I)}\left(1-\frac{e^{\tau_2}}{e^{\tau_1}}\right)^n\right\} \\
&\quad \left.+\Theta(\tau_2-\tau_1)\left\{\frac{1}{2}\ln\frac{e^{\tau_2}}{e^{\tau_1}} - \left(\frac{e^{\tau_1}}{e^{\tau_2}}\right)^{\nu_I}\sum_{n=1}^{\infty}\frac{(\nu_I)_n}{n!}\sum_{m=0}^{n-1}\frac{1-\nu_I}{(m+1)(m+\nu_I)}\left(1-\frac{e^{\tau_1}}{e^{\tau_2}}\right)^n\right\}\right] \\
&+\theta_1\theta_2\frac{8\delta^{I\overline{J}}}{\varepsilon(1+b_I^2)}\left\{\left(\frac{e^{\tau_1}}{e^{\tau_2}}\right)^{\nu_I}-1\right\}\frac{1}{-e^{\tau_1}+e^{\tau_2}}\ , \qquad (2.65)
\end{aligned}
$$

where γ is Euler's constant, $\boldsymbol{\psi}(w)$ denotes the digamma function defined as $\boldsymbol{\psi}(w)=\frac{d}{dw}\ln\Gamma(w)$ and $(a)_n \equiv \frac{\Gamma(a+n)}{\Gamma(a)}$. Here we have used the formulas (A.1) and (A.5).

Let us compute the conformal weights of the twist and spin fields. This helps us appreciate these subtracted two-point functions better. The relevant part of the energy-momentum

tensor in this computation consists of the terms depending on the string coordinates in the x^i-directions, $i = p+1, \ldots, p'$: $T_B^{(Z,\Psi)}(z) = T_B^{(Z,\overline{Z})}(z) + T_B^{(\Psi,\overline{\Psi})}(z)$ with

$$T_B^{(Z,\overline{Z})}(z) = -\frac{\varepsilon\,\delta_{I\overline{J}}}{\alpha'} : \partial Z^I \partial \overline{Z}^{\overline{J}}(z) : \ , \quad T_B^{(\Psi,\overline{\Psi})}(z) = -\frac{\varepsilon\,\delta_{I\overline{J}}}{4}\left[: \Psi^I \partial \overline{\Psi}^{\overline{J}}(z) : - : \partial \Psi^I\, \overline{\Psi}^{\overline{J}}(z) : \right] . \tag{2.66}$$

Using the subtracted two point function, we obtain

$$\begin{aligned} \langle \sigma_I | T_B(z) | \sigma_I \rangle &= -\frac{\varepsilon}{\alpha'} \lim_{w\to z} \langle \sigma_I | : \partial Z^I(w) \partial \overline{Z}^{\overline{I}}(z) : | \sigma_I \rangle \\ &= \lim_{w\to z} \left[\left(\frac{w}{z}\right)^{\nu_I - 1} \frac{1}{w-z} \left(\frac{1-\nu_I}{z} + \frac{1}{w-z} \right) - \frac{1}{(w-z)^2} \right] = \frac{1}{z^2} \frac{\nu_I(1-\nu_I)}{2} \ . \end{aligned} \tag{2.67}$$

This implies that the twist fields $\sigma_I^{\pm}$ have the weights $h_{\sigma_I} = \frac{1}{2}\nu_I(1-\nu_I)$. In the same way, we obtain

$$\begin{aligned} \langle s_I | T_B(z) | s_I \rangle &= -\frac{\varepsilon}{4} \lim_{w\to z} \langle s_I | : \left(\Psi^I(w) \partial \overline{\Psi}^{\overline{I}}(z) - \partial \Psi^I(w)\, \overline{\Psi}^{\overline{I}}(z) \right) : | s_I \rangle \\ &= \lim_{w\to z} \left[\left(\frac{w}{z}\right)^{\nu_I} \frac{1}{w-z} \left\{ \frac{\nu_I}{2} \left(\frac{1}{w} + \frac{1}{z} \right) - \frac{1}{w-z} \right\} + \frac{1}{(w-z)^2} \right] = \frac{1}{z^2} \frac{{\nu_I}^2}{2} \ . \end{aligned} \tag{2.68}$$

From this equation we can read [4] that the spin fields $s_I^{\pm}$ have the weights $h_{s_I} = \frac{1}{2}{\nu_I}^2$.

III. Vertex operators

Let us pay some attention to vertex operators of our system before we start calculating scattering amplitudes. We focus on two types of vertex operators. The one is the tachyon vertex operators, and the other is the massless vector vertex operators. These are the relevant ones in order for us to find out the spacetime processes occurring on the Dp'-brane worldvolume.

Here we make a comment on the GSO projection. In this paper we take the GSO projection in the NS sector so that the oscillator vacuum corresponding to the tachyon vertex operator survives. It follows that the GSO projection adopted in this paper is not always the same as that in our previous work [7]: in the cases of $p' = p+2,\ p+6$ they are opposite to each other, while in the cases of $p' = p+4,\ p+8$ they are the same. This is attributed to the sign convention of the Dp-brane charge: in the cases of $p' = p+2,\ p+6$ the Dp-branes in this paper should be referred to as anti-Dp-branes in the convention of [7].

[4] This result can also be obtained from the fact that $T_B^{(\Psi,\overline{\Psi})}(z)$ is bosonized as $T_B^{(\Psi,\overline{\Psi})}(z) \cong -\frac{1}{2}\delta_{IJ} : \partial H^I \partial H^J(z) : .$

A. Tachyon vertex operator of p-p' string

First let us investigate vertex operators of the p-p' open string which contain the twist and the spin fields. We will focus on the vertex operator that corresponds to the ground state in the NS sector of this open string. This vertex operator is seen for instance in [10][17]:

$$\mathbf{V}_T^{\pm}(\xi^1,\theta) = V_T^{\pm\,(-1)}(\xi^1) + \theta V_T^{\pm\,(0)}(\xi^1) = \boldsymbol{\mathcal{T}}^{\pm}(\xi^1,\theta)\ :\exp\left(i\sqrt{\frac{\alpha'}{2}}\sum_{\mu=0}^{p} k_\mu \mathbf{X}^\mu(\xi^1,\theta)\right):\ , \quad (3.1)$$

where $\mathbf{X}^\mu(\xi^1,\theta)$ is the boundary value of the superfield $\mathbf{X}^\mu(\mathbf{z},\overline{\mathbf{z}})$: $\mathbf{X}^\mu(\xi^1,\theta) \equiv \mathbf{X}^\mu(\mathbf{z},\overline{\mathbf{z}})|_{z=\overline{z}=\xi^1,\theta=\overline{\theta}}$, and $\boldsymbol{\mathcal{T}}^{\pm}(\xi^1,\theta)$ is the superfield whose lowest component $\mathcal{T}_0^{\pm}(\xi^1)$ consists of the twist and the spin fields,

$$\mathcal{T}_0^{\pm}(\xi^1) = \prod_I \sigma_I^{\pm}(\xi^1) s_I^{\pm}(\xi^1)\ . \quad (3.2)$$

The upper component is obtained by applying the supercurrent $T_F(z)$ to $\mathcal{T}_0^{\pm}(\xi^1)$ [17]. In eq. (3.1), $V_T^{\pm\,(0)}(\xi^1)$ denotes the 0-picture vertex and $V_T^{\pm\,(-1)}(\xi^1)$ is the matter field contribution to the (-1)-picture vertex

$$\mathcal{V}_T^{\pm\,(-1)}(\xi^1) = e^{-\phi}(\xi^1)\, V_T^{\pm\,(-1)}(\xi^1) = e^{-\phi}\prod_I \sigma_I^{\pm} s_I^{\pm}\ :\exp\left(i\sum_{\mu=0}^{p} k_\mu X^\mu\right):\ , \quad (3.3)$$

where $e^{-\phi}$ comes from the $\beta\gamma$-ghost sector.

It is worth noting that space-time momentum k_μ of the tachyon vertex operator eq. (3.1) is not $(p'+1)$ dimensional but $(p+1)$ dimensional. This is because the p-p' string coordinates in $x^{p+1},\ldots,x^{p'}$-directions do not possess zero-modes and thus the momenta in these directions are not defined. This implies that an initial/final tachyon field corresponding to this vertex operator is frozen in these space-time directions.

Let us study the physical state conditions for this vertex operator. If the (-1)-picture vertex satisfies the physical state condition, the 0-picture vertex is automatically physical because of the worldsheet supersymmetry. We will therefore concentrate on the (-1)-picture vertex. Through the operator-state mapping this vertex operator corresponds to the state[5],

$$\left|V_T^{(-1)}\right\rangle \equiv \lim_{\xi^1\to 0} V_T^{(-1)}(\xi^1)|0\rangle = |0;k_\mu\rangle \otimes |\sigma,s\rangle\ , \quad (3.4)$$

where the state $|0;k_\mu\rangle$ is defined as $|0;k_\mu\rangle = \exp\left(i\sum_{\mu=0}^{p} k_\mu x^\mu\right)|0\rangle$. Here x^μ are the zero modes of the bosonic coordinates. It is evident that $\left|V_{T\,(-1)}\right\rangle$ is a primary state. We just

[5] Here we ignore the ghost sector.

require that this state should have weight $\frac{1}{2}$. From the calculation in the last subsection, we find that the state $|\sigma, s\rangle$ has a weight

$$h\Big[\,|\sigma,s\rangle\,\Big] = \sum_I \left(\frac{\nu_I(1-\nu_I)}{2} + \frac{{\nu_I}^2}{2}\right) = \sum_I \frac{\nu_I}{2}\ . \tag{3.5}$$

Substituting the mode expansions eqs. (2.18) and (2.22) into the defining relation eq. (2.5), we see that the terms in $T_B(z)$ which depend on the string coordinates in x^μ-directions ($\mu = 0, 1, \ldots, p$) are

$$T_B^{(X,\psi)}(z) \equiv\ -\sum_{\mu=0}^{p}\left(\frac{1}{\alpha'}g_{\mu\nu}\partial X^\mu \partial X^\nu(z) + \frac{1}{2}g_{\mu\nu}\psi^\mu\partial\psi^\nu(z)\right) \equiv \sum_{m\in\mathbf{Z}} L_m z^{-m-2}\ ,$$

$$\text{with}\quad L_m = \frac{1}{2}\sum_{n\in\mathbf{Z}}\sum_{\sigma,\rho=0}^{p} G_{\sigma\rho}\,:\alpha^\sigma_{m-n}\alpha^\rho_n: + \frac{1}{4}\sum_{r\in\mathbf{Z}+1/2}(2r-m)\sum_{\sigma,\rho=0}^{p} G_{\sigma\rho}\,:b^\sigma_{m-r}b^\rho_r:\ , \tag{3.6}$$

where $\alpha_0^\mu = \sqrt{2\alpha'}p^\mu$. Here $G_{\sigma\rho}$ is the open string metric including time direction, its inverse is given in eq. (2.21). This yields

$$L_0|0;k_\mu\rangle = \alpha'\sum_{\sigma,\rho=0}^{p} G^{\sigma\rho}k_\sigma k_\rho|0;k_\mu\rangle\ . \tag{3.7}$$

Gathering all results obtained above, we conclude that the weight of the state $|V_T^{(-1)}\rangle$ is $L_0 = \sum_I \frac{1}{2}\nu_I + \alpha'\sum_{\sigma,\rho=0}^{p} G^{\sigma\rho}k_\sigma k_\rho$. Thus the on-shell condition $L_0 = \frac{1}{2}$ requires that the mass squared of this state should be

$$\alpha' m_T^2 \equiv -\alpha'\sum_{\sigma,\rho=0}^{p} G^{\sigma\rho}k_\sigma k_\rho = -\frac{1}{2}\left(1 - \sum_I \nu_I\right)\ . \tag{3.8}$$

B. Massless vector vertex operator of p'-p' string

The vector emission vertex operator takes the form of

$$\mathbf{V}_{\rm vec}(\xi^1,\theta) \equiv V^{(-1)}_{\rm vec}(\xi^1) + \theta V^{(0)}_{\rm vec}(\xi^1) \equiv \frac{i}{2}\sum_{\mu=0}^{p'} :\zeta_\mu(k)\dot{\mathbf{X}}^\mu(\xi^1,\theta)\,\exp\left(i\sqrt{\frac{\alpha'}{2}}\sum_{\rho=0}^{p'}k_\rho\mathbf{X}^\rho(\xi^1,\theta)\right):\ , \tag{3.9}$$

where $\zeta_\mu(k)$ denotes the polarization vector and $\dot{\mathbf{X}}^\mu(\xi^1,\theta) \equiv (D+\overline{D})\mathbf{X}^\mu(\mathbf{z},\overline{\mathbf{z}})\Big|_{z=\overline{z}=\xi^1,\theta=\overline{\theta}}$. The operator $V^{(0)}_{\rm vec}(\xi^1)$ is the 0-picture vertex and $V^{(-1)}_{\rm vec}(\xi^1)$ is the matter field contribution to the (-1)-picture vertex $\mathcal{V}^{(-1)}_{\rm vec}(\xi^1) = e^{-\phi}V^{(-1)}_{\rm vec}(\xi^1)$. Their explicit forms are

$$V^{(-1)}_{\rm vec}(\xi^1) = -\frac{1}{2}\sum_{\mu=0}^{p'} :\zeta_\mu(k)\left(\psi^\mu + \widetilde{\psi}^\mu\right)\exp\left(i\sum_{\rho=0}^{p'}k_\rho X^\rho\right):\ , \tag{3.10}$$

$$V^{(0)}_{\rm vec}(\xi^1) = \frac{1}{\sqrt{2\alpha'}}\sum_{\mu=0}^{p'} :\zeta_\mu(k)\left[i\dot{X}^\mu + \frac{\alpha'}{2}\left(\sum_{\rho=0}^{p'}k_\rho\left(\psi^\rho + \widetilde{\psi}^\rho\right)\right)\left(\psi^\mu + \widetilde{\psi}^\mu\right)\right]\exp\left(i\sum_{\lambda=0}^{p'}k_\lambda X^\lambda\right):\ ,$$

where $\dot{X}^\mu(\xi^1)$ is defined as $\dot{X}^\mu(\xi^1) = (\partial + \overline{\partial})X^\mu(z,\overline{z})\Big|_{z=\overline{z}=\xi^1}$.

The string coordinates of a p'-p' open string in the x^i-directions ($i = 1, \ldots, p'$) obey the same boundary conditions as the x^i-directions with $i = 1, \ldots, p$ of the p-p' string. They have the same mode expansions and the commutators among the oscillating modes. Therefore the vector emission vertex operator in each picture corresponds to the respective state

$$\begin{aligned}
\left|V_{\rm vec}^{(-1)}\right\rangle &\equiv \lim_{\xi^1\to 0} V_{\rm vec}^{(-1)}(\xi^1)|0\rangle = -\sum_{\mu=0}^{p'} \zeta_\mu(k)\, b^\mu_{-\frac{1}{2}}|0;k_\rho\rangle' \ , \\
\left|V_{\rm vec}^{(0)}\right\rangle &\equiv \lim_{\xi^1\to 0} V_{\rm vec}^{(0)}(\xi^1)|0\rangle = \sum_{\mu=0}^{p'} \zeta_\mu(k) \left(\alpha^\mu_{-1} + \sqrt{2\alpha'}\left(\sum_{\lambda=0}^{p'} k_\lambda b^\lambda_{-\frac{1}{2}}\right) b^\mu_{-\frac{1}{2}}\right)|0;k_\rho\rangle' \ ,
\end{aligned} \tag{3.11}$$

where $|0;k_\rho\rangle'$ is defined as $|0;k_\rho\rangle' = \exp\left(\sum_{\rho=0}^{p'} k_\rho x^\rho\right)|0\rangle$. Let us consider the physical state conditions on these states. The relevant part of the energy-momentum tensor for this analysis is eq. (3.6) with p being replaced by p'. This yields

$$\begin{aligned}
L_0\left|V_{\rm vec}^{(0)}\right\rangle &= \left[\alpha' \sum_{\sigma,\rho=0}^{p'} G^{\sigma\rho}k_\sigma k_\rho + 1\right]\left|V_{\rm vec}^{(0)}\right\rangle \ , \\
L_1\left|V_{\rm vec}^{(0)}\right\rangle &= \sqrt{2\alpha'}\sum_{\sigma,\rho=0}^{p'} G^{\sigma\rho}k_\sigma\zeta_\rho(k)|0;k_\rho\rangle' \ .
\end{aligned} \tag{3.12}$$

From these relations we find that the physical state conditions, $L_0 = 1$ and $L_1 = 0$, require that

$$\alpha' m^2_{\rm vec} \equiv -\alpha' \sum_{\sigma,\rho=0}^{p'} G^{\sigma\rho}k_\sigma k_\rho = 0 \ , \qquad \sum_{\sigma,\rho=0}^{p'} G^{\sigma\rho}k_\sigma\zeta_\rho(k) = 0 \ . \tag{3.13}$$

We will write the vector emission vertex operator $\mathbf{V}_{\rm vec}(\xi^1,\theta)$ in an exponential form [15],

$$\mathbf{V}_{\rm vec}(\xi^1,\theta) = \int d\eta : \exp\left[i\sum_{\mu=0}^{p'}\left\{\sqrt{\frac{\alpha'}{2}}k_\mu + \eta\zeta_\mu(k)\frac{1}{2}\left(D+\overline{D}\right)\right\}\mathbf{X}^\mu(\mathbf{z},\overline{\mathbf{z}})\right] : \Bigg|_{\substack{z=\overline{z}=\xi^1 \\ \theta=\overline{\theta}}} \ , \tag{3.14}$$

by introducing a Grassmann parameter η.

We will write the part of vertex operator eq. (3.14) which depends on the string coordinates in the x^i-directions ($i = p+1, \ldots, p'$) as

$$: \exp\left[\sum_{I=\frac{p+2}{2}}^{p'/2}\mathbf{E}_I(\zeta,k,\eta)\mathbf{Z}^I(\mathbf{z},\overline{\mathbf{z}}) + \sum_{\overline{J}=\frac{p+2}{2}}^{p'/2}\overline{\mathbf{E}}_{\overline{J}}(\zeta,k,\eta)\overline{\mathbf{Z}}^{\overline{J}}(\mathbf{z},\overline{\mathbf{z}})\right] : \Bigg|_{z=\overline{z}=\xi^1,\theta=\overline{\theta}} \ , \tag{3.15}$$

using the complex variables. Here $\mathbf{E}_I(\zeta,k,\eta)$ and $\overline{\mathbf{E}}_{\overline{I}}(\zeta,k,\eta)$ are differential operators on the superspace defined as

$$\mathbf{E}_I(\zeta,k,\eta) = i\sqrt{\frac{\alpha'}{2}}\kappa_I + i\eta e_I(k)\frac{1}{2}\left(D+\overline{D}\right) \ ,$$
$$\overline{\mathbf{E}}_{\overline{I}}(\zeta,k,\eta) = i\sqrt{\frac{\alpha'}{2}}\overline{\kappa}_{\overline{I}} + i\eta \overline{e}_{\overline{I}}(k)\frac{1}{2}\left(D+\overline{D}\right) \ , \tag{3.16}$$

with

$$\kappa_I = \frac{1}{2}\left(k_{2I-1} - ik_{2I}\right) \ , \qquad \overline{\kappa}_{\overline{I}} = \frac{1}{2}\left(k_{2I-1} + ik_{2I}\right) \ ;$$
$$e_I(k) = \frac{1}{2}\Big(\zeta_{2I-1}(k) - i\zeta_{2I}(k)\Big) \ , \qquad \overline{e}_{\overline{I}}(k) = \frac{1}{2}\Big(\zeta_{2I-1}(k) + i\zeta_{2I}(k)\Big) \ . \tag{3.17}$$

IV. Scattering Amplitudes

We would like to find out a proper description of the physical processes taking place on the worldvolume of the Dp'-brane with the Dp-brane inside ($p < p'$) in the case where the B field is nonvanishing. In this section, we will consider multiparticle tree scattering amplitudes consisting of the external states of $N-2$ vectors obtained from the mode of the p'-p' open string and two tachyons from the mode of the p-p' open string. The spacetime picture of this string scattering process is depicted in Fig.1. That this process is possible is easy to see once we draw a spacetime diagram and map the end points of the open strings onto a circle. See Fig.2.

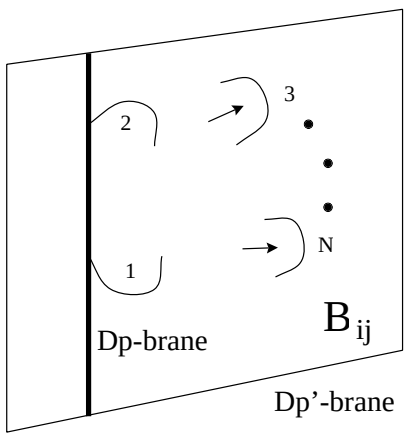

Figure 1: The space-time picture of the process.

Open string tree amplitudes in general are obtained by placing vertex operators on the boundary of the upper half plane, namely, the real axis, integrating over the positions of

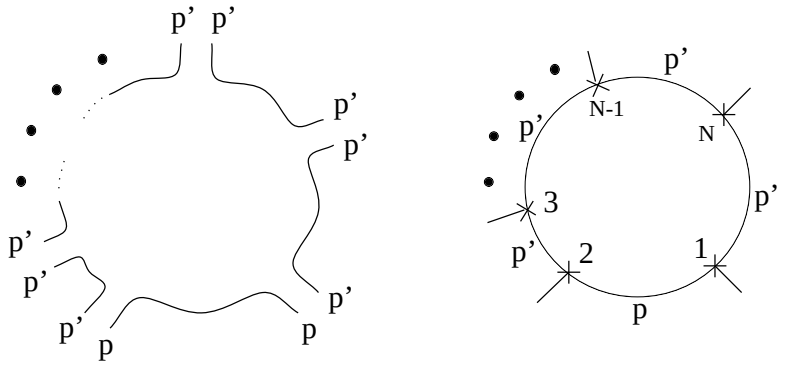

Figure 2: N point string diagram.

the vertex operators and dividing by the volume of the (super)conformal killing vectors. To obtain the amplitudes of our concern, we first locate each of the two kinds of the tachyon vertex operators $\mathbf{V}_T^+(\xi,\theta)$, $\mathbf{V}_T^-(\xi,\theta)$ discussed in the last section at $\xi=\xi_1$ and at $\xi=\xi_2$ respectively. A cut is generated on the interval between these two locations as $\mathbf{V}_T^+$ and $\mathbf{V}_T^-$ contain the twist field and the anti-twist field respectively. The worldvolume of the Dp'-brane contains this interval on which we place the $N-2$ vector emission vertex operators $\mathbf{V}_{\rm vec}(\xi,\theta)$ of the p'-p' open string. In what follows we will obtain the integral (Koba-Nielsen) representation of the amplitudes. The explicit expressions for the $N=3,4$ cases that we obtain will be exploited to determine the form of the low energy effective field theory in the subsequent section.

In manifestly supersymmetric formulation on superspace, the N point tree amplitude in question reads

$$\frac{c}{V_{\rm SCKV}}\int\prod_{a=1}^{N}d\xi_a d\theta_a\langle 0|\ \mathbf{V}_T^+(\xi_1,\theta_1;k_{1\mu})\mathbf{V}_T^-(\xi_2,\theta_2;k_{2\mu})\prod_{c=3}^{N}\mathbf{V}_{\rm vec}(\xi_c,\theta_c;k_{c\mu},\zeta_{c\mu})\ |0\rangle\ , \qquad (4.1)$$

where $V_{\rm SCKV}$ denotes the volume of the isometry group generated by the superconformal Killing vectors, namely, the graded extension of the $SL(2,\mathbf{R})$ group. We have denoted by c the overall constant which does not concern us in this paper. In eq. (4.1), the domain of ξ_a integrations is not restricted except that $\xi_3,\xi_4,\cdots,\xi_N$ are located on the cut created on the interval between ξ_1 and ξ_2. This domain falls into a sum of the $(N-2)!$ regions. In each region, an ordering among $\xi_3,\xi_4,\cdots,\xi_N$ is specified and integrals over each region give a contribution corresponding to a respective open string (dual) diagram[6]. We will evaluate the contribution from the region $\xi_2<\xi_3<\xi_4<\cdots<\xi_N<\xi_1$ and this is denoted by A_N. In most cases below, we will not write the region of integrations explicitly.

[6] Recall that we have $(N-1)!$ open string (dual) diagrams in the case of the N point amplitude of a p-p open string.

Eq. (4.1) is invariant under the graded $SL(2,\mathbf{R})$ transformations after the physical state conditions are invoked at each vertex operator. For actual evaluation of the amplitude, we first set $\theta_1 = \theta_2 = 0$ to fix the odd elements of the transformations. We then fix the even elements by giving fixed values to three of the locations of the vertex operators. These locations are chosen as ξ_1, ξ_2, and ξ_3. This amounts to factoring out the following volume element from the integration,

$$d^3F\left(\xi_1,\xi_2,\xi_3\right) d\theta_1 d\theta_2 \left(\xi_1-\xi_2\right) \ , \tag{4.2}$$

where

$$d^3F(\xi_1,\xi_2,\xi_3) \equiv \frac{d\xi_1 d\xi_2 d\xi_3}{(\xi_1-\xi_2)(\xi_2-\xi_3)(\xi_3-\xi_1)} \ . \tag{4.3}$$

Having done this, we obtain

$$A_N = c\int \frac{\prod_{a=1}^{N} d\xi_a}{d^3F(\xi_1,\xi_2,\xi_3)} \frac{\prod_{c=3}^{N} d\theta_c}{\xi_1-\xi_2} \langle 0|\, V_T^{+(-1)}(\xi_1;k_{1\mu}) V_T^{-(-1)}(\xi_2;k_{2\mu}) \prod_{c'=3}^{N} \mathbf{V}_{\rm vec}(\xi_{c'},\theta_{c'};k_{c'\mu},\zeta_{c'\mu})\,|0\rangle \ . \tag{4.4}$$

The component corresponding to the (-1)-picture has been selected at each of the tachyon vertex operators. Let us choose $\xi_1 = 0$, $\xi_2 = -\infty$ and $\xi_3 = -1$, so that the negative real axis becomes the worldsheet boundary ending on the Dp'-brane. Introducing positive real variables $x_a \equiv -\xi_a \,(= e^{\tau_a}) > 0$ and adopting eq. (3.14) for the vector emission vertex operators, we find

$$\begin{aligned} A_N = c\int & \frac{\prod_{a=1}^{N} dx_a}{d^3F(x_1,x_2,x_3)} \frac{\prod_{a'=3}^{N} d\theta_{a'} d\eta_{a'}}{x_1-x_2} \left(\frac{1}{x_2}\right)^{\sum_{I=\frac{p+2}{2}}^{p'/2} \nu_I} \\ & \times \langle 0|\, \prod_{f=1}^{N} :\exp\left[i\sum_{\mu=0}^{p} \left\{ \sqrt{\frac{\alpha'}{2}} k_{f\mu} \mathbf{X}^\mu(-x_f,\theta_f) + \frac{1}{2}\eta_f\zeta_{f\mu}\dot{\mathbf{X}}^\mu(-x_f,\theta_f) \right\} \right] : \Big|_{\substack{\zeta_{1\mu}=\zeta_{2\mu}=0 \\ \theta_1=\theta_2=0}} |0\rangle \\ & \times \langle \sigma,s|\, \prod_{c=3}^{N} :\exp\left[\sum_{I=\frac{p+2}{2}}^{p'/2} \mathbf{E}_I\left(\zeta_c,k_c,\eta_c\right) \mathbf{Z}^I(-x_c,\theta_c) + \sum_{\bar{J}=\frac{p+2}{2}}^{p'/2} \overline{\mathbf{E}}_{\overline{J}}\left(\zeta_c,k_c,\eta_c\right) \overline{\mathbf{Z}}^{\overline{J}}(-x_c,\theta_c) \right] : |\sigma,s\rangle \ , \end{aligned} \tag{4.5}$$

where $x_1 = 0$, $x_2 = \infty$ and $x_3 = 1$. Here the right hand side consists of two of the expectation values of the exponential operators: the one is obtained from the x^μ-directions ($\mu = 0,\ldots,p$) and the other is from the x^i-directions ($i = p+1,\ldots,p'$), and we have used

$$\langle \sigma_I, s_I| = \lim_{x\to\infty} x^{\nu_I} \langle 0|\, \sigma_I s_I(-x) \ . \tag{4.6}$$

Let us examine the contribution from the x^i-directions ($i = p+1,\ldots,p'$). As is explained in section II., the operators inside $\langle\sigma,s|\cdots|\sigma,s\rangle$ in eq. (4.5) are normal ordered with respect to the $SL(2,\mathbf{R})$ invariant vacuum and are not with respect to the oscillator vacuum.

Applying the reordering formula eq. (2.63), we obtain

$$\begin{aligned}
&\langle\sigma,s|\prod_{a=3}^{N}:\exp\left(\sum_{I}\mathbf{E}_{aI}\mathbf{Z}^{I}(-x_a,\theta_a)+\sum_{\overline{J}}\overline{\mathbf{E}}_{a\overline{J}}\overline{\mathbf{Z}}^{\overline{J}}(-x_a,\theta_a)\right):|\sigma,s\rangle\\
&\quad=\prod_{a=3}^{N}\exp\left[\sum_{I,\overline{J}}\mathbf{E}_{aI}\overline{\mathbf{E}}_{a\overline{J}}{\boldsymbol{\mathcal{G}}_{\rm sub}}^{I\overline{J}}((-x_a,\theta_a))\right]\\
&\qquad\times\langle\sigma,s|\prod_{c=3}^{N}:\exp\left[\sum_{I}\mathbf{E}_{cI}\mathbf{Z}^{I}(-x_c,\theta_c)+\sum_{\overline{J}}\overline{\mathbf{E}}_{c\overline{J}}\overline{\mathbf{Z}}^{\overline{J}}(-x_c,\theta_c)\right]:|\sigma,s\rangle\ ,\\
&\quad=\prod_{a=3}^{N}x_a^{-2\alpha'\sum_{I,\overline{J}}\kappa_{aI}\overline{\kappa}_{a\overline{J}}G^{I\overline{J}}}\exp\left[\mathcal{C}_a\left(\nu_I\right)+\sqrt{2\alpha'}\eta_a\sum_{I\overline{J}}e_{aI}\overline{\kappa}_{a\overline{J}}G^{I\overline{J}}\frac{\theta_a}{x_a}\right]\\
&\qquad\times\langle\sigma,s|\prod_{c=3}^{N}:\exp\left[\sum_{I}\mathbf{E}_{cI}\mathbf{Z}^{I}(-x_c,\theta_c)+\sum_{\overline{J}}\overline{\mathbf{E}}_{c\overline{J}}\overline{\mathbf{Z}}^{\overline{J}}(-x_c,\theta_c)\right]:|\sigma,s\rangle\ ,
\end{aligned}\tag{4.7}$$

where

$$\mathcal{C}_a(\nu_I)=\alpha'\sum_{I,\overline{J}}2\kappa_{aI}\overline{\kappa}_{a\overline{J}}G^{I\overline{J}}\left\{\gamma+\frac{1}{2}\Big(\boldsymbol{\psi}(\nu_I)+\boldsymbol{\psi}(1-\nu_I)\Big)\right\}\ ,\tag{4.8}$$

and $\mathbf{E}_{cI}$ and $\overline{\mathbf{E}}_{c\overline{J}}$ stand for $\mathbf{E}_I(\zeta_c,k_c,\eta_c)$ and $\overline{\mathbf{E}}_{\overline{J}}(\zeta_c,k_c,\eta_c)$ respectively. Note that the part in eq. (4.7) that corresponds to self-contractions has been given by the subtracted Green function at the coincident point:

$$\begin{aligned}
\sum_{I\overline{J}}\mathbf{E}_{aI}\overline{\mathbf{E}}_{a\overline{J}}{\boldsymbol{\mathcal{G}}_{\rm sub}}^{I\overline{J}}((-x_a,\theta_a))&\equiv\lim_{\substack{\theta_c\to\theta_a\\x_c\to x_a}}\sum_{I\overline{J}}\mathbf{E}_{cI}\overline{\mathbf{E}}_{a\overline{J}}{\boldsymbol{\mathcal{G}}_{\rm sub}}^{I\overline{J}}\left(-x_c,\theta_c|-x_a,\theta_a\right)\\
&=\mathcal{C}_a(\nu_I)+\sum_{I,\overline{J}}\left[-2\alpha'\kappa_{aI}\overline{\kappa}_{a\overline{J}}G^{I\overline{J}}\ln x_a+\sqrt{2\alpha'}\eta_a e_{aI}\overline{\kappa}_{a\overline{J}}G^{I\overline{J}}\frac{\theta_a}{x_a}\right]\ .
\end{aligned}\tag{4.9}$$

The last factor $\langle\sigma,s|\cdots|\sigma,s\rangle$ in eq. (4.7) is now calculated using the two-point function $\boldsymbol{\mathcal{G}}^{I\overline{J}}(-x_c,\theta_c|-x_{c'},\theta_{c'})$:

$$\begin{aligned}
&\langle\sigma,s|\prod_{c=3}^{N}:\exp\left[\sum_{I}\mathbf{E}_{cI}\mathbf{Z}^{I}(-x_c,\theta_c)+\sum_{\overline{J}}\overline{\mathbf{E}}_{c\overline{J}}\overline{\mathbf{Z}}^{\overline{J}}(-x_c,\theta_c)\right]:|\sigma,s\rangle\\
&\quad=\prod_{3\le c<c'\le N}\exp\left[\sum_{I,\overline{J}}\left\{\mathbf{E}_{cI}\overline{\mathbf{E}}_{c'\overline{J}}\boldsymbol{\mathcal{G}}^{I\overline{J}}(-x_c,\theta_c|-x_{c'},\theta_{c'})+\overline{\mathbf{E}}_{c\overline{J}}\mathbf{E}_{c'I}\boldsymbol{\mathcal{G}}^{I\overline{J}}(-x_{c'},\theta_{c'}|-x_c,\theta_c)\right\}\right]\\
&\quad=\prod_{3\le c<c'\le N}\exp\left[\sum_{I,\overline{J}}G^{I\overline{J}}\left[-2\alpha'\kappa_{cI}\overline{\kappa}_{c'\overline{J}}\,\mathcal{H}\left(\nu_I;\frac{x_c}{x_{c'}-\theta_c\theta_{c'}}\right)-2\alpha'\overline{\kappa}_{c\overline{J}}\kappa_{c'I}\,\mathcal{H}\left(\nu_I;\frac{x_{c'}}{x_c+\theta_c\theta_{c'}}\right)\right.\right.\\
&\qquad\left.\left.+\eta_c\sqrt{2\alpha'}\left\{e_{cI}\overline{\kappa}_{c'\overline{J}}\frac{\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}\theta_{c'}-\left(\frac{x_c}{x_{c'}}\right)^{\nu_I-1}\theta_c}{x_c-x_{c'}}+\overline{e}_{c\overline{J}}\kappa_{c'I}\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}\frac{\theta_{c'}-\theta_c}{x_c-x_{c'}}\right\}\right.\right.
\end{aligned}$$

$$
+\eta_{c'}\sqrt{2\alpha'}\left\{\kappa_{cI}\overline{e}_{c'\overline{J}}\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}\frac{\theta_{c'}-\theta_c}{x_c-x_{c'}}+\overline{\kappa}_{c\overline{J}}e_{c'I}\frac{\left(\frac{x_{c'}}{x_c}\right)^{\nu_I-1}\theta_{c'}-\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}\theta_c}{x_c-x_{c'}}\right\}
$$

$$
+\eta_c\eta_{c'}\left\{e_{cI}\overline{e}_{c'\overline{J}}\frac{\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}}{x_c-x_{c'}}\left(1-\theta_c\theta_{c'}\frac{(1-\nu_I)+\nu_I\frac{x_{c'}}{x_c}}{x_c-x_{c'}}\right)\right.
$$

$$
\left.\left.+\overline{e}_{c\overline{J}}e_{c'I}\frac{\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}}{x_c-x_{c'}}\left(1-\theta_c\theta_{c'}\frac{(1-\nu_I)+\nu_I\frac{x_c}{x_{c'}}}{x_c-x_{c'}}\right)\right\}\right]
$$

$$
-\sum_{I,\overline{J}}\epsilon(x_c-x_{c'})\frac{2\delta^{I\overline{J}}\pi b_I}{\varepsilon(1+b_I^2)}\alpha'\left(\kappa_{cI}\overline{\kappa}_{c'\overline{J}}-\overline{\kappa}_{c\overline{J}}\kappa_{c'I}\right)\Bigg] \ . \tag{4.10}
$$

The factor coming from the x^μ-directions ($\mu = 0,\ldots,p$) is handled by the two-point function $\mathbf{G}^{\mu\mu'}$:

$$
\begin{aligned}
&\langle 0|\prod_{f=1}^{N}:\exp\left[\sum_{\mu=0}^{p}\mathcal{E}_\mu\left(\zeta_f,k_f,\eta_f\right)\mathbf{X}^\mu(-x_f,\theta_f)\right]:|0\rangle \\
&\quad=\exp\left[\sum_{1\le f<f'\le N}\sum_{\mu,\mu'=0}^{p}\mathcal{E}_\mu\left(\zeta_f,k_f,\eta_f\right)\mathcal{E}_{\mu'}\left(\zeta_{f'},k_{f'},\eta_{f'}\right)\mathbf{G}^{\mu\mu'}(\mathbf{z}_f,\overline{\mathbf{z}}_f|\mathbf{z}_{f'},\overline{\mathbf{z}}_{f'})\right]\Bigg|_{\substack{z_f=\overline{z}_f=-x_f,\\ \theta_f=\overline{\theta}_f}} \\
&\qquad\times\langle 0|:\exp\left[\sum_{c=1}^{N}\sum_{\rho=0}^{p}\mathcal{E}_\rho(\zeta_c,k_c,\eta_c)\mathbf{X}^\rho(-x_c,\theta_c)\right]:|0\rangle \\
&\quad=\exp\left[\sum_{1\le f<f'\le N}\left\{\sum_{\sigma,\rho=0}^{p}\left\{\alpha' G^{\sigma\rho}k_{f\sigma}k_{f'\rho}\ln\left(x_f-x_{f'}+\theta_f\theta_{f'}\right)^2\right.\right.\right. \\
&\qquad\qquad-\sqrt{2\alpha'}\left(\eta_f G^{\sigma\rho}\zeta_{f\sigma}k_{f'\rho}+\eta_{f'}G^{\sigma\rho}k_{f\sigma}\zeta_{f'\rho}\right)\frac{\theta_f-\theta_{f'}}{x_f-x_{f'}} \\
&\qquad\qquad\left.\left.\left.+\eta_f\eta_{f'}G^{\sigma\rho}\zeta_{f\sigma}\zeta_{f'\rho}\frac{1}{(x_f-x_{f'}+\theta_f\theta_{f'})}\right\}+\sum_{i,j=1}^{p}\frac{i}{2}\,\theta^{ij}k_{fi}k_{f'j}\,\epsilon\left(x_f-x_{f'}\right)\right\}\right] \\
&\qquad\times(2\pi)^{p+1}\prod_{\mu=0}^{p}\delta\left(k_{1\mu}+\cdots+k_{N\mu}\right)\ ,
\end{aligned}
\tag{4.11}
$$

where $\mathcal{E}_\mu(\zeta_f,k_f,\eta_f)$ is a differential operator defined as

$$
\begin{aligned}
&\mathcal{E}_\mu\left(\zeta_f,k_f,\eta_f\right)=i\sqrt{\frac{\alpha'}{2}}k_{f\mu}+i\frac{1}{2}\eta_f\zeta_{f\mu}(k)\left(D_f+\overline{D}_f\right)\ , \\
&\mathcal{E}_\mu\left(\zeta_f,k_f,\eta_f\right)\mathbf{X}^\mu(-x_f,\theta_f)\equiv\mathcal{E}_\mu\left(\zeta_f,k_f,\eta_f\right)\mathbf{X}^\mu(\mathbf{z}_f,\overline{\mathbf{z}}_f)\Big|_{\substack{z_f=\overline{z}_f=-x_f\\ \theta_f=\overline{\theta}_f}}\ .
\end{aligned}
\tag{4.12}
$$

We have now evaluated the two of the expectation values in eq. (4.5) and are ready to present the integral representation of A_N. Let us first introduce several shorthand notations. We denote by $\underset{(p)}{\cdot}$ the inner product of two $(p+1)$-dimensional vectors A_i and B_j lying on the Dp-brane worldvolume with respect to the open string metric. Namely

$$A \underset{(p)}{\cdot} B = \sum_{\sigma,\rho=0}^{p} G^{\sigma\rho} A_\sigma B_\rho \ . \tag{4.13}$$

Similarly, we denote by $\underset{(p')}{\cdot}$ the inner product of two $(p'+1)$-dimensional vectors A_i and B_j lying on the Dp'-brane worldvolume with respect to the open string metric. We will also write $A \underset{(p,p')}{\cdot} B$ to denote the inner product of the last $(p'-p)$ components of the two vectors. For example, we have

$$\begin{aligned} k \underset{(p,p')}{\cdot} \zeta &= \sum_{I,\overline{J}=\frac{p+2}{2}}^{\frac{p'}{2}} \left(G^{I\overline{J}} \kappa_I \overline{e}_{\overline{J}} + G^{\overline{J}I} \overline{\kappa}_{\overline{J}} e_I \right) = \sum_{I,\overline{J}=\frac{p+2}{2}}^{\frac{p'}{2}} G^{I\overline{J}} \left(\kappa_I \overline{e}_{\overline{J}} + \overline{\kappa}_{\overline{J}} e_I \right) \ , \\ k \underset{(p,p')}{\cdot} k &= \sum_{I,\overline{J}=\frac{p+2}{2}}^{\frac{p'}{2}} 2 G^{I\overline{J}} \kappa_I \overline{\kappa}_{\overline{J}} \ , \qquad \zeta \underset{(p,p')}{\cdot} \zeta = \sum_{I,\overline{J}=\frac{p+2}{2}}^{\frac{p'}{2}} 2 G^{I\overline{J}} e_I \overline{e}_{\overline{J}} \ . \end{aligned} \tag{4.14}$$

We will use the notations $\underset{(p,p')}{\odot}$ and $\underset{(p,p')}{\times}$ which denote

$$\left(k \underset{(p,p')}{\odot} \zeta \right)_I = \sum_{\overline{J}} G^{I\overline{J}} (\kappa_I \overline{e}_{\overline{J}} + \overline{\kappa}_{\overline{J}} e_I) \ , \quad \left(k \underset{(p,p')}{\times} \zeta \right)_I = \sum_{\overline{J}} \frac{2\delta^{I\overline{J}} (\kappa_I \overline{e}_{\overline{J}} - \overline{\kappa}_{\overline{J}} e_I)}{\varepsilon(1+b_I^2)} \ , \tag{4.15}$$

etc. From these defining relations one can find that

$$\sum_I \left(k \underset{(p,p')}{\odot} \zeta \right)_I = k \underset{(p,p')}{\cdot} \zeta \ , \quad \sum_I \left(k \underset{(p,p')}{\times} \zeta \right)_I = i k \underset{(p,p')}{\cdot} J\zeta \ , \tag{4.16}$$

where J is a $(p'+1) \times (p'+1)$ antisymmetric matrix defined as

$$J = (J_\mu{}^\rho) \equiv \left(\begin{array}{ccc|cccccc} 0 & & & & & & & & \\ & \ddots & & & & & & & \\ & & 0 & & & & & & \\ \hline & & & 0 & 1 & & & & \\ & & & -1 & 0 & & & & \\ & & & & & \ddots & & & \\ & & & & & & 0 & 1 & \\ & & & & & & -1 & 0 & \end{array} \right) \begin{array}{l} 0 \\ \vdots \\ p \\ p+1 \\ p+2 \\ \vdots \\ p'-1 \\ p' \end{array} \ . \tag{4.17}$$

We will group the terms in the exponent by the number of η_a's and by the number of θ_a's, using the notation $[0,2]$, $[2,0]$, $[1,1]$, $[2,2]$. The first number in the bracket indicates the number of η_a's and the second number the number of θ_a's.

Having prepared these, we write eq. (4.5) as

$$
\begin{aligned}
A_N = c(2\pi)^{p+1}\prod_{\mu=0}^{p}\delta\left(\sum_{e=1}^{N}k_{e\mu}\right)\int\prod_{a=4}^{N}dx_a\prod_{a'=3}^{N}d\theta_{a'}d\eta_{a'}\ \exp\mathcal{C}_{a'}(\nu_I)\\
\times x_2^{-\sum_I\nu_I}(x_2-x_3)(x_3-x_1)\prod_{c''=3}^{N}x_{c''}^{-\alpha' k_{c''}\dot{(p,p')}k_{c''}}\prod_{1\le c<c'\le N}(x_{c'}-x_c)^{2\alpha' k_{c'}\dot{(p)}k_c}\\
\times\prod_{3\le c<c'\le N}\exp\left[-2\alpha'\sum_{I,\bar{J}}G^{I\bar{J}}\left\{\kappa_{cI}\overline{\kappa}_{c'\bar{J}}\mathcal{H}\left(\nu_I;\frac{x_c}{x_{c'}}\right)+\overline{\kappa}_{c\bar{J}}\kappa_{c'I}\mathcal{H}\left(\nu_I;\frac{x_{c'}}{x_c}\right)\right\}\right]\\
\times\exp\left(\mathrm{NC}\right)\exp\Big([0,2]+[2,0]+[1,1]+[2,2]\Big)\ . \qquad (4.18)
\end{aligned}
$$

Here

$$
\begin{aligned}
[0,2] = 2\alpha'\sum_{3\le c<c'\le N}\frac{\theta_c\theta_{c'}}{x_c-x_{c'}}\Bigg[&k_c\dot{(p)}k_{c'}\\
&+\frac{1}{2}\sum_I\left\{\left(k_c\underset{(p,p')}{\odot}k_{c'}\right)_I\left[\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}+\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}\right]+\left(k_c\underset{(p,p')}{\times}k_{c'}\right)_I\left[\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}-\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}\right]\right\}\Bigg]\ ,
\end{aligned}
$$

$$
\begin{aligned}
[1,1] = &\sqrt{2\alpha'}\sum_{c=3}^{N}\eta_c\theta_c\left[\frac{1}{2}\zeta_c\dot{(p,p')}(1+iJ)k_c\frac{1}{x_c}+k_1\dot{(p)}\zeta_c\frac{1}{x_1-x_c}+k_2\dot{(p)}\zeta_c\frac{1}{x_2-x_c}\right]\\
&-\sqrt{2\alpha'}\sum_{3\le c<c'\le N}\frac{1}{x_c-x_{c'}}\Bigg[\left(\eta_c\zeta_c\dot{(p)}k_{c'}+\eta_{c'}k_c\dot{(p)}\zeta_{c'}\right)(\theta_c-\theta_{c'})\\
&\quad+\frac{1}{2}\sum_I\Bigg\{\eta_c\left(\zeta_c\underset{(p,p')}{\odot}k_{c'}\right)_I\left(\left\{\left(\frac{x_c}{x_{c'}}\right)^{\nu_I-1}+\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}\right\}\theta_c-\left\{\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}+\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}\right\}\theta_{c'}\right)\\
&\qquad+\eta_{c'}\left(k_c\underset{(p,p')}{\odot}\zeta_{c'}\right)_I\left(\left\{\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}+\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}\right\}\theta_c-\left\{\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}+\left(\frac{x_{c'}}{x_c}\right)^{\nu_I-1}\right\}\theta_{c'}\right)\\
&\qquad+\eta_c\left(\zeta_c\underset{(p,p')}{\times}k_{c'}\right)_I\left(\left\{\left(\frac{x_c}{x_{c'}}\right)^{\nu_I-1}-\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}\right\}\theta_c-\left\{\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}-\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}\right\}\theta_{c'}\right)\\
&\qquad+\eta_{c'}\left(k_c\underset{(p,p')}{\times}\zeta_{c'}\right)_I\left(\left\{\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}-\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}\right\}\theta_c-\left\{\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}-\left(\frac{x_{c'}}{x_c}\right)^{\nu_I-1}\right\}\theta_{c'}\right)\Bigg\}\Bigg]\ ,
\end{aligned}
$$

$$
\begin{aligned}
[2,0] = \sum_{3\le c<c'\le N}\frac{\eta_c\eta_{c'}}{x_c-x_{c'}}\Bigg[&\zeta_c\dot{(p)}\zeta_{c'}\\
&+\frac{1}{2}\sum_I\left\{\left(\zeta_c\underset{(p,p')}{\odot}\zeta_{c'}\right)_I\left[\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}+\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}\right]+\left(\zeta_c\underset{(p,p')}{\times}\zeta_{c'}\right)_I\left[\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}-\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}\right]\right\}\Bigg]\ ,
\end{aligned}
$$

$$
[2,2] = \sum_{3\le c<c'\le N}\frac{\eta_c\theta_c\eta_{c'}\theta_{c'}}{(x_c-x_{c'})^2}\Bigg[\zeta_c\dot{(p)}\zeta_{c'}
$$

$$+\frac{1}{2}\sum_I\Bigg\{\left(\zeta_c \underset{(p,p')}{\odot}\zeta_{c'}\right)_I\left((1-\nu_I)\left\{\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}+\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}\right\}+\nu_I\left\{\left(\frac{x_c}{x_{c'}}\right)^{\nu_I-1}+\left(\frac{x_{c'}}{x_c}\right)^{\nu_I-1}\right\}\right)$$
$$+\left(\zeta_c \underset{(p,p')}{\times}\zeta_{c'}\right)_I\left((1-\nu_I)\left\{\left(\frac{x_c}{x_{c'}}\right)^{\nu_I}-\left(\frac{x_{c'}}{x_c}\right)^{\nu_I}\right\}+\nu_I\left\{\left(\frac{x_c}{x_{c'}}\right)^{\nu_I-1}-\left(\frac{x_{c'}}{x_c}\right)^{\nu_I-1}\right\}\right)\Bigg\}\Bigg] , \tag{4.19}$$

and (NC) denotes the terms containing the sign function:

$$\begin{aligned}(\mathrm{NC}) &= \sum_{1\le a<a'\le N}\frac{i}{2}\epsilon(x_a-x_{a'})\sum_{i,j=1}^{p}\theta^{ij}k_{ai}k_{a'j}\\ &\quad-\sum_{3\le c<c'\le N}\epsilon(x_c-x_{c'})\sum_{I,\overline{J}}\alpha'\frac{2\delta^{I\overline{J}}\pi b_I}{\varepsilon(1+b_I^2)}\left(\kappa_{cI}\overline{\kappa}_{c'\overline{J}}-\overline{\kappa}_{c\overline{J}}\kappa_{c'I}\right)\\ &= \sum_{1\le a<a'\le N}\frac{i}{2}\epsilon(x_a-x_{a'})\sum_{\mu,\lambda=0}^{p'}\theta^{\mu\lambda}k_{a\mu}k_{a'\lambda} ,\end{aligned} \tag{4.20}$$

with $k_{1j}=k_{2j}=0$ for $(j=p+1,\ldots,p')$. Here we have written the noncommutativity term in terms of the real variables and generalized the notation $\theta^{\mu\lambda}$ to include the time components $\theta^{0i}=0$. We also remind the readers that $\mathcal{H}\left(\nu_I;\frac{x_c}{x_{c'}}\right)$ is given in eq. (2.46).

So far, we have not exploited that $x_1=0, x_2=\infty$ and $x_3=1$ except that the oscillator vacuum $|\sigma,s\rangle$, $\langle\sigma,s|$ is obtained from the tachyon vertex operators. Firstly, by sending $x_2=\infty$, all factors in eq. (4.18) containing x_2 are removed. In fact, this is ensured by an equality

$$1-\sum_{I=\frac{p+2}{2}}^{\frac{p'}{2}}\nu_I+2\alpha'\sum_{c\neq 2}k_2\underset{(p)}{\cdot}k_c=0 , \tag{4.21}$$

which is obtained from the momentum conservation $\prod_{\mu=0}^{p}\delta\left(\sum_{c=1}^{N}k_{c\mu}\right)$ and the on-shell condition (eq. (3.8)) for the tachyon. Secondly, setting $x_1=0$, we find

$$\begin{aligned}&\prod_{\substack{1\le c<c'\le N\\ c,c'\neq 2}}\left(x_{c'}-x_c\right)^{2\alpha' k_{c'}\underset{(p)}{\cdot}k_c}\prod_{c''=3}^{N}x_{c''}^{-\alpha' k_{c''}\underset{(p,p')}{\cdot}k_{c''}}\\ &=\prod_{c=3}^{N}x_c^{-\alpha' s_c+\alpha' m_T^2}\prod_{3\le c<c'\le N}\left(x_c-x_{c'}\right)^{2\alpha' k_{c'}\underset{(p)}{\cdot}k_c} .\end{aligned} \tag{4.22}$$

Here $s_c\equiv-(k_c+k_1)\underset{(p)}{\cdot}(k_c+k_1)$ and we have used the on-shell condition for the tachyon (eq. (3.8)) and that for the vector (eq. (3.13)). Finally we would like to convert the integrations at eq. (4.18) into those over a set of $N-3$ $SL(2,\mathbf{R})$ invariant cross ratios. We choose

these cross ratios as

$$x^{(a+3)} \equiv \frac{(x_1 - x_{a+3})(x_2 - x_3)}{(x_1 - x_3)(x_2 - x_{a+3})} = \frac{x_{a+3}}{x_3}, \quad a = 1, \ldots N-3 \ . \tag{4.23}$$

We can therefore accomplish this conversion by rescaling x_{a+3} by x_3 and setting $x_3 = 1$ in eq. (4.18) without changing the form of the integrand.

Putting all these considerations together, we obtain from eq. (4.18)

$$\begin{aligned} A_N = {} & c(2\pi)^{p+1} \prod_{\mu=0}^{p} \delta\left(\sum_{e=1}^{N} k_{e\mu}\right) \int \prod_{a=4}^{N} dx_a \prod_{a'=3}^{N} d\theta_{a'} d\eta_{a'} \exp \mathcal{C}_{a'}(\nu_I) \\ & \times \prod_{c=4}^{N} \left[x_c^{-\alpha' s_c + \alpha' m_T^2} (1-x_c)^{2\alpha' k_3 \cdot k_c} \right] \prod_{4 \le c < c' \le N} (x_c - x_{c'})^{2\alpha' k_{c'} \cdot k_c} \\ & \times \prod_{3 \le c < c' \le N} \exp\left[-2\alpha' \sum_{I,\overline{J}} G^{I\overline{J}} \left\{ \kappa_{cI} \overline{\kappa}_{c'\overline{J}} \mathcal{H}\left(\nu_I; \frac{x_c}{x_{c'}}\right) + \overline{\kappa}_{c\overline{J}} \kappa_{c'I} \mathcal{H}\left(\nu_I; \frac{x_{c'}}{x_c}\right) \right\} \right] \\ & \times \exp(\mathrm{NC}) \exp\Big([0,2] + [2,0] + [1,1] + [2,2] \Big) \Big|_{x_1=0, x_2=\infty, x_3=1} . \end{aligned} \tag{4.24}$$

This expression is regarded as an $SL(2,\mathbf{R})$ invariant integral (Koba-Nielsen) representation for the amplitude of our concern. Let us list several features which are distinct from the corresponding formula in the case of a p-p open string. (See [15]).

1. The term denoted by $\exp(\mathrm{NC})$ which originated from the noncommutativity of the worldvolume extends into both the $x^1, \ldots, x^p$ directions and the remaining $x^{p+1}, \ldots, x^{p'}$ directions.

2. To each external vector leg, we have a momentum dependent multiplicative factor $\exp \mathcal{C}(\nu_I)$.

3. A new tensor J has appeared.

4. There are parts in the expression which are expressible in terms of the momenta of the tachyons, the momenta and the polarization tensors of the vectors and J alone, using the inner product with respect to the open string metric. These parts come, however, with a host of other parts which do not permit such generic description in terms of the inner product.

Let us finally compute $N = 3, 4$ cases explicitly. For $N = 3$ case, we need to pick up θ_3 and η_3 from $[1,1]$. Using the transversality of the polarization vector (eq. (3.13)) and the

$(p+1)$ dimensional momentum conservation, we find that

$$A_3 = c(2\pi)^{p+1}\prod_{\mu=0}^{p}\delta\left(\sum_{a=1}^{3}k_{a\mu}\right)\sqrt{\frac{\alpha'}{2}}\left\{(k_2-k_1)\underset{(p)}{\cdot}\zeta_3 - ik_3\underset{(p,p')}{\cdot}J\zeta_3\right\}\, e^{\mathcal{C}_3(\nu_I)}e^{\frac{i}{2}\theta^{ij}k_{1i}k_{2j}}\ . \quad (4.25)$$

For $N=4$ case, we have

$$[0,2] = 2\alpha'\frac{\theta_3\theta_4}{1-x}\left[k_3\underset{(p)}{\cdot}k_4 + \frac{1}{2}\sum_I\left\{\left(k_3\underset{(p,p')}{\odot}k_4\right)_I\left[x^{-\nu_I}+x^{\nu_I}\right] + \left(k_3\underset{(p,p')}{\times}k_4\right)_I\left[x^{-\nu_I}-x^{\nu_I}\right]\right\}\right]\ ,$$

$$\begin{aligned}[1,1] = \sqrt{2\alpha'}\Bigg[&\frac{\eta_3\theta_3}{2}\left\{\left((k_2-k_1)\underset{(p)}{\cdot}\zeta_3 - ik_3\underset{(p,p')}{\cdot}J\zeta_3\right) + k_4\underset{(p)}{\cdot}\zeta_3\right\}\\ &+\frac{\eta_4\theta_4}{2x}\left\{\left((k_2-k_1)\underset{(p)}{\cdot}\zeta_4 - ik_4\underset{(p,p')}{\cdot}J\zeta_4\right) + k_3\underset{(p)}{\cdot}\zeta_4\right\}\\ &+\frac{\theta_3-\theta_4}{1-x}\left(\eta_3 k_4\underset{(p)}{\cdot}\zeta_3 + \eta_4 k_3\underset{(p)}{\cdot}\zeta_4\right)\\ -\frac{1}{2}\frac{1}{1-x}\sum_I\Bigg\{&\eta_3\left(\zeta_3\underset{(p,p')}{\odot}k_4\right)_I\left\{\left(x^{-\nu_I+1}+x^{\nu_I}\right)\theta_3 - \left(x^{-\nu_I}+x^{\nu_I}\right)\theta_4\right\}\\ &+\eta_4\left(k_3\underset{(p,p')}{\odot}\zeta_4\right)_I\left\{\left(x^{-\nu_I}+x^{\nu_I}\right)\theta_3 - \left(x^{-\nu_I}+x^{\nu_I-1}\right)\theta_4\right\}\\ &+\eta_3\left(\zeta_3\underset{(p,p')}{\times}k_4\right)_I\left\{\left(x^{-\nu_I+1}-x^{\nu_I}\right)\theta_3 - \left(x^{-\nu_I}-x^{\nu_I}\right)\theta_4\right\}\\ &+\eta_4\left(k_3\underset{(p,p')}{\times}\zeta_4\right)_I\left\{\left(x^{-\nu_I}-x^{\nu_I}\right)\theta_3 - \left(x^{-\nu_I}-x^{\nu_I-1}\right)\theta_4\right\}\Bigg\}\Bigg]\ ,\end{aligned}$$

$$[2,0] = \frac{\eta_3\eta_4}{1-x}\left[\zeta_3\underset{(p)}{\cdot}\zeta_4 + \frac{1}{2}\sum_I\left\{\left(\zeta_3\underset{(p,p')}{\odot}\zeta_4\right)_I\left(x^{-\nu_I}+x^{\nu_I}\right) + \left(\zeta_3\underset{(p,p')}{\times}\zeta_4\right)_I\left(x^{-\nu_I}-x^{\nu_I}\right)\right\}\right]\ ,$$

$$\begin{aligned}[2,2] = \frac{\eta_3\theta_3\eta_4\theta_4}{(1-x)^2}\Bigg[&\zeta_3\underset{(p)}{\cdot}\zeta_4 + \frac{1}{2}\sum_I\Bigg\{\left(\zeta_3\underset{(p,p')}{\odot}\zeta_4\right)_I\left\{(1-\nu_I)\left(x^{-\nu_I}+x^{\nu_I}\right) + \nu_I\left(x^{-\nu_I+1}+x^{\nu_I-1}\right)\right\}\\ &+\left(\zeta_3\underset{(p,p')}{\times}\zeta_4\right)_I\left\{(1-\nu_I)\left(x^{-\nu_I}-x^{\nu_I}\right) + \nu_I\left(x^{-\nu_I+1}-x^{\nu_I-1}\right)\right\}\Bigg\}\Bigg]\ ,\end{aligned} \quad (4.26)$$

where we have set $x_3=1$ and written $x_4=x$. By picking up terms from $[2,2]+[0,2][2,0]+\frac{1}{2}[1,1]^2$, we obtain

$$\begin{aligned}A_4 = c(2\pi)^{p+1}\prod_{\mu=0}^{p}\delta\left(\sum_{a=1}^{4}k_{a\mu}\right)\int_0^1 dx\, x^{-\alpha' t+\alpha' m_T^2}(1-x)^{2\alpha' k_3\underset{(p)}{\cdot}k_4}\exp\left(\mathcal{C}_3(\nu_I)+\mathcal{C}_4(\nu_I)+(\mathrm{NC})\right)\\ \times\exp\left[-\alpha'\sum_I\left\{\left(k_3\underset{(p,p')}{\odot}k_4 + k_3\underset{(p,p')}{\times}k_4\right)_I\mathcal{H}\left(\nu_I;\frac{1}{x}\right) + \left(k_3\underset{(p,p')}{\odot}k_4 - k_3\underset{(p,p')}{\times}k_4\right)_I\mathcal{H}\left(\nu_I;x\right)\right\}\right]\\ \times\Bigg[\frac{1}{(1-x)^2}\zeta_3\underset{(p)}{\cdot}\zeta_4\left(1-2\alpha' k_3\underset{(p)}{\cdot}k_4\right)\end{aligned}$$

$$
\begin{aligned}
&+\frac{\alpha'}{2}\frac{1}{x}\left\{\left[(k_2-k_1)\underset{(p)}{\cdot}\zeta_3-ik_3\underset{(p,p')}{\cdot}J\zeta_3\right]-k_4\underset{(p)}{\cdot}\zeta_3\right\}\left\{\left[(k_2-k_1)\underset{(p)}{\cdot}\zeta_4-ik_4\underset{(p,p')}{\cdot}J\zeta_4\right]+k_3\underset{(p)}{\cdot}\zeta_4\right\}\\
&+\alpha'\frac{1}{1-x}\left\{\left[(k_2-k_1)\underset{(p)}{\cdot}\zeta_3-ik_3\underset{(p,p')}{\cdot}J\zeta_3\right]k_3\underset{(p)}{\cdot}\zeta_4-k_4\underset{(p)}{\cdot}\zeta_3\left[(k_2-k_1)\underset{(p)}{\cdot}\zeta_4-ik_4\underset{(p,p')}{\cdot}J\zeta_4\right]\right\}\\
&+\sum_I\frac{x^{-\nu_I}}{(1-x)^2}\left\{-\alpha'\left(k_3\underset{(p,p')}{\odot}k_4+k_3\underset{(p,p')}{\times}k_4\right)_I\zeta_3\underset{(p)}{\cdot}\zeta_4\right.\\
&\qquad\left.+\left(\frac{1-\nu_I}{2}-\alpha' k_3\underset{(p)}{\cdot}k_4\right)\left(\zeta_3\underset{(p,p')}{\odot}\zeta_4+\zeta_3\underset{(p,p')}{\times}\zeta_4\right)_I\right\}\\
&+\sum_I\frac{x^{\nu_I}}{(1-x)^2}\left\{-\alpha'\left(k_3\underset{(p,p')}{\odot}k_4-k_3\underset{(p,p')}{\times}k_4\right)_I\zeta_3\underset{(p)}{\cdot}\zeta_4\right.\\
&\qquad\left.+\left(\frac{1-\nu_I}{2}-\alpha' k_3\underset{(p)}{\cdot}k_4\right)\left(\zeta_3\underset{(p,p')}{\odot}\zeta_4-\zeta_3\underset{(p,p')}{\times}\zeta_4\right)_I\right\}\\
&+\sum_I\frac{\nu_I}{2}\frac{x^{-\nu_I+1}}{(1-x)^2}\left(\zeta_3\underset{(p,p')}{\odot}\zeta_4+\zeta_3\underset{(p,p')}{\times}\zeta_4\right)_I+\sum_I\frac{\nu_I}{2}\frac{x^{\nu_I-1}}{(1-x)^2}\left(\zeta_3\underset{(p,p')}{\odot}\zeta_4-\zeta_3\underset{(p,p')}{\times}\zeta_4\right)_I\\
&-\frac{\alpha'}{2}\sum_{I,L}\frac{x^{-\nu_I-\nu_L}}{(1-x)^2}\left(k_3\underset{(p,p')}{\odot}k_4+k_3\underset{(p,p')}{\times}k_4\right)_I\left(\zeta_3\underset{(p,p')}{\odot}\zeta_4+\zeta_3\underset{(p,p')}{\times}\zeta_4\right)_L\\
&-\frac{\alpha'}{2}\sum_{I,L}\frac{x^{\nu_I+\nu_L}}{(1-x)^2}\left(k_3\underset{(p,p')}{\odot}k_4-k_3\underset{(p,p')}{\times}k_4\right)_I\left(\zeta_3\underset{(p,p')}{\odot}\zeta_4-\zeta_3\underset{(p,p')}{\times}\zeta_4\right)_L\\
&-\frac{\alpha'}{2}\sum_{I,L}\frac{x^{-\nu_I+\nu_L}}{(1-x)^2}\left(k_3\underset{(p,p')}{\odot}k_4+k_3\underset{(p,p')}{\times}k_4\right)_I\left(\zeta_3\underset{(p,p')}{\odot}\zeta_4-\zeta_3\underset{(p,p')}{\times}\zeta_4\right)_L\\
&-\frac{\alpha'}{2}\sum_{I,L}\frac{x^{\nu_I-\nu_L}}{(1-x)^2}\left(k_3\underset{(p,p')}{\odot}k_4-k_3\underset{(p,p')}{\times}k_4\right)_I\left(\zeta_3\underset{(p,p')}{\odot}\zeta_4+\zeta_3\underset{(p,p')}{\times}\zeta_4\right)_L\\
&+\frac{\alpha'}{2}\sum_{I,L}\frac{x^{-\nu_I-\nu_L}}{1-x}\left(k_4\underset{(p,p')}{\odot}\zeta_3-k_4\underset{(p,p')}{\times}\zeta_3\right)_I\left(k_3\underset{(p,p')}{\odot}\zeta_4+k_3\underset{(p,p')}{\times}\zeta_4\right)_L\\
&-\frac{\alpha'}{2}\sum_{I,L}\frac{x^{\nu_I+\nu_L-1}}{1-x}\left(k_4\underset{(p,p')}{\odot}\zeta_3+k_4\underset{(p,p')}{\times}\zeta_3\right)_I\left(k_3\underset{(p,p')}{\odot}\zeta_4-k_3\underset{(p,p')}{\times}\zeta_4\right)_L\\
&+\frac{\alpha'}{2}\sum_I\frac{x^{\nu_I-1}}{1-x}\left\{\left(\left[(k_2-k_1)\underset{(p)}{\cdot}\zeta_3-ik_3\underset{(p,p')}{\cdot}J\zeta_3\right]-k_4\underset{(p)}{\cdot}\zeta_3\right)\left(k_3\underset{(p,p')}{\odot}\zeta_4-k_3\underset{(p,p')}{\times}\zeta_4\right)_I\right.\\
&\qquad\left.-\left(\left[(k_2-k_1)\underset{(p)}{\cdot}\zeta_4-ik_4\underset{(p,p')}{\cdot}J\zeta_4\right]+k_3\underset{(p)}{\cdot}\zeta_4\right)\left(k_4\underset{(p,p')}{\odot}\zeta_3+k_4\underset{(p,p')}{\times}\zeta_3\right)_I\right\}\\
&+\frac{\alpha'}{2}\sum_I\frac{x^{-\nu_I}}{1-x}\left\{\left(\left[(k_2-k_1)\underset{(p)}{\cdot}\zeta_3-ik_3\underset{(p,p')}{\cdot}J\zeta_3\right]+k_4\underset{(p)}{\cdot}\zeta_3\right)\left(k_3\underset{(p,p')}{\odot}\zeta_4+k_3\underset{(p,p')}{\times}\zeta_4\right)_I\right.\\
&\qquad\left.\left.-\left(\left[(k_2-k_1)\underset{(p)}{\cdot}\zeta_4-ik_4\underset{(p,p')}{\cdot}J\zeta_4\right]-k_3\underset{(p)}{\cdot}\zeta_4\right)\left(k_4\underset{(p,p')}{\odot}\zeta_3-k_4\underset{(p,p')}{\times}\zeta_3\right)_I\right\}\right]\ , \qquad (4.27)
\end{aligned}
$$

where t is defined as $t \equiv s_4 \equiv -(k_4+k_1)\dot{}_{(p)}(k_4+k_1)$.

V. The Zero Slope Limit and the Low Energy Effective Action

In the last section, we have evaluated the three and four point amplitudes with the initial and final tachyons and $N-2$ vectors ($N=3,4$) present. Let us try to extract physical significance from these.

The three point amplitude eq. (4.25) contains the two multiplicative factors $e^{\frac{i}{2}\theta^{ij}k_{1i}k_{2j}}$ and $e^{\mathcal{C}(\nu_I)}$ both of which are listed in the last section as prominent features. The first factor represents the noncommutativity of the Dp-brane worldvolume. The second factor will be discussed shortly. Aside from these factors and $(p+1)$-dimensional delta functions representing momentum conservation, eq. (4.25) is interpreted as coming from field theory vertex of

$$\Phi i \overset{\leftrightarrow}{\partial} \Phi^\dagger \dot{}_{(p)} A + \frac{1}{2}\Phi\Phi^\dagger J^{MN}F_{MN} \ , \tag{5.1}$$

where Φ is a complex tachyon field and A_M and F_{MN} are the gauge field and its field strength respectively. The first term is the gauge-scalar derivative interaction while the second factor is a new interaction coming from our p-p' open string system.

The four point amplitude eq. (4.27) is quite complex but one can still systematically investigate the singular behavior of the integrand around its end points $x=0,\ 1$. This behavior is sufficient to tell us the zero slope limit of the amplitude and the content of the low energy field theory. We will focus upon this in the remainder of this section. To be more accurate we consider the sum of eq. (4.27) and the one obtained from this by $k_3 \leftrightarrow k_4$, $\zeta_3 \leftrightarrow \zeta_4$ in accordance with the two open string (dual) diagrams.

The nontrivial zero slope limit is given by sending $\alpha' \to 0$ while keeping the parameter θ^{ij} of noncommutativity and the open string metric fixed. From eqs. (2.6) and (2.7) this means that [6]

$$\begin{aligned} \alpha' &\sim \varepsilon^{1/2} \to 0 \ , \\ g &\sim \varepsilon \to 0 \ , \\ |b_I| &\sim \varepsilon^{-1/2} \to \infty \ . \end{aligned} \tag{5.2}$$

In this limiting procedure, $\alpha' b_I$ becomes finite: $\alpha' b_I \to \beta_I$. Let us first look at the multiplicative factor $\mathcal{C}(\nu_I)$ defined in eq. (4.8). Using $\psi(1) = -\gamma$ and eq. (A.5), we obtain

$$\mathcal{C}(\nu_I) \to -\pi \sum_{I,\bar{J}} |\beta_I|\, \kappa_I \overline{\kappa}_{\overline{J}} G^{I\overline{J}} = -\frac{\pi}{2}\sum_I |\beta_I| \left(k \underset{(p,p')}{\odot} k \right)_I \ . \tag{5.3}$$

So this exponential multiplicative factor acts as a gaussian damping factor when vectors propagate into the $x^{p+1}, \ldots, x^{p'}$ directions.

Let us come back to the four point amplitude (4.27). If the integrand is regular, one could take the $\alpha' \to 0$ limit inside the integral and this will not give us any nontrivial contribution. If the integrand is singular at some point, it will still not give us much as long as one can avoid such singularity by a contour deformation. The nontrivial contribution in the $\alpha' \to 0$ limit, therefore, is obtained only when we have end point singularities.

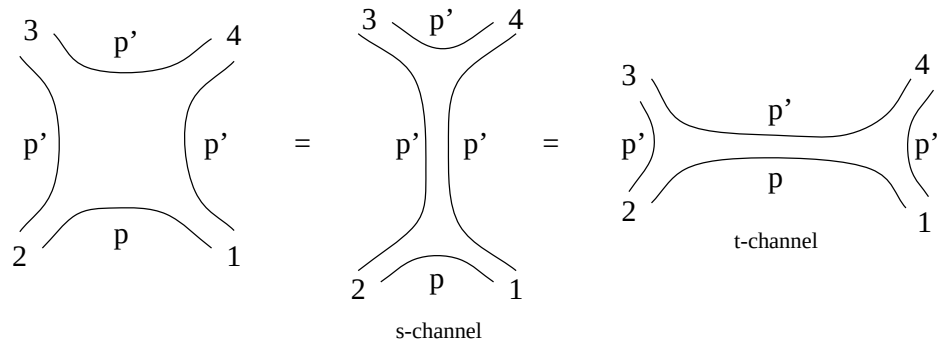

Figure 3: The string diagram corresponding to the s-channel and the t-channel.

Let us focus on the behavior of the integrand near $x = 0$ from which we can read off the mass of particles exchanged in the t-channel. Fig.3 indicates that the t-channel poles originate in the propagation of the p-p' open string. Thus the complicated behavior of the integrand near $x = 0$ should reflect the spectrum of the p-p' open string [7]. In order to identify the t-channel poles, we expand the integrand of the amplitude (4.27) around $x = 0$:

$$A_4 = \int_0^1 dx \sum_A f_A x^{-\alpha' t + K_A} \ , \tag{5.4}$$

where the coefficients f_A are functions of momenta and polarization tensors. The term $f_A x^{-\alpha' t + K_A}$ in the integrand of the above equation yields the t-channel pole at $\alpha' t = K_A + 1$, when it is integrated near $x = 0$: $\int_0^\delta dx \ldots$. From explicit computation we find that the t-channel poles exist at

$$\alpha' t = \alpha' m_T^2 + W + \sum_{I'} M_{I'}(n + 1 - \nu_{I'}) + \sum_{L'} M'_{L'}(n' + \nu_{L'}) + N \ , \tag{5.5}$$

$$\text{with} \qquad W = 0 \ , \ 1 \ , \ 1 - \nu_I \ , \ \nu_I \ , \ 1 + \nu_I \ , \ 2 - \nu_I \ ,$$
$$1 - \nu_I - \nu_L \ , \ \nu_I + \nu_L \ , \ 1 + \nu_I + \nu_L \ , \text{ or } 1 - \nu_I + \nu_L \ ,$$

where n, n', $M_{I'}$, $M'_{L'}$ and N are non-negative integers. The terms proportional to $M_{I'}$ and $M'_{L'}$ come from the exponential of the hypergeometric function $\mathcal{H}$,

$$\exp\left[-\alpha' \sum_I \left\{ \left(k_3 \underset{(p,p')}{\odot} k_4 + k_3 \underset{(p,p')}{\times} k_4 \right)_I \mathcal{H}\left(\nu_I : \frac{1}{x}\right) + \left(k_3 \underset{(p,p')}{\odot} k_4 - k_3 \underset{(p,p')}{\times} k_4 \right)_I \mathcal{H}(\nu_I : x) \right\} \right]$$

$$
\begin{aligned}
&= \exp\left(\alpha' \sum_I \pi b_I \left(k_3 \underset{(p,p')}{\times} k_4\right)_I\right) \\
&\quad \times \sum_{M=0}^{\infty} \frac{1}{M!} \left(-\alpha' \sum_{I'} \left(k_3 \underset{(p,p')}{\odot} k_4 + k_3 \underset{(p,p')}{\times} k_4\right)_{I'} \sum_{n=0}^{\infty} \frac{x^{n+1-\nu_{I'}}}{n+1-\nu_{I'}}\right)^M \\
&\quad \times \sum_{M'}^{\infty} \frac{1}{M'!} \left(-\alpha' \sum_{L'} \left(k_3 \underset{(p,p')}{\odot} k_4 - k_3 \underset{(p,p')}{\times} k_4\right)_{L'} \sum_{n'=0}^{\infty} \frac{x^{n'+\nu_{L'}}}{n'+\nu_{L'}}\right)^{M'} \quad . \qquad (5.6)
\end{aligned}
$$

The t-channel poles eq. (5.5) correspond to the spectrum of the p-p' open string [7]. In view of the analysis in [7], we expect that a large number of light states should be exchanged in the zero slope limit in which one of ν_I goes to unity and the others approach zero. In order to specify the situation, we assume, without loss of generality, that $\nu \equiv \nu_{\frac{p+2}{2}}$ goes to 1 and $\nu_{\widetilde{I}}$ ($\widetilde{I} \neq \frac{p+2}{2}$) go to 0 in the zero slope limit. In this zero slope limit many light states are realized by the poles in eq. (5.5) with $(n, n', M_{\widetilde{I}'}, M'_{\frac{p+2}{2}}, N) = 0$,

$$
W = 0 \ , \ 1-\nu \ , \ \nu_{\widetilde{I}} \ , \ 1-\nu-\nu_{\widetilde{I}} \ , \ 1-\nu+\nu_{\widetilde{I}} \ , \qquad (5.7)
$$

and $M_{\frac{p+2}{2}}$ and $M'_{\widetilde{I}}$ being arbitrary non-negative integers. Aside from the multiplicative factors and the momentum conserving delta functions, the massless pole obtained in the zero slope limit turns out to be

$$
\begin{aligned}
&\sim \left\{\frac{1}{t-m_T^2}\left[\frac{1}{2}\left\{\Big(k_2-(k_1+k_4)\Big)\underset{(p)}{\cdot}\zeta_3 - ik_3 \underset{(p,p')}{\cdot} J\zeta_3\right\}\left\{\Big((k_2+k_3)-k_1\Big)\underset{(p)}{\cdot}\zeta_4 - ik_4 \underset{(p,p')}{\cdot} J\zeta_4\right\}\right] \right. \\
&\quad + \frac{1}{t-(m_T^2+\frac{1}{\pi|\beta|})}\left[-(k_3 \underset{(p,p')}{\odot} k_4 + k_3 \underset{(p,p')}{\times} k_4)_{\frac{p+2}{2}} \zeta_3 \underset{(p)}{\cdot} \zeta_4 \right. \\
&\qquad\qquad + \Big(\frac{1}{2\pi|\beta|} - k_3 \underset{(p)}{\cdot} k_4\Big)(\zeta_3 \underset{(p,p')}{\odot} \zeta_4 + \zeta_3 \underset{(p,p')}{\times} \zeta_4)_{\frac{p+2}{2}} \\
&\qquad\qquad + \frac{1}{2}\left\{(k_2-k_1+k_4)\underset{(p)}{\cdot}\zeta_3 - ik_3 \underset{(p,p')}{\cdot} J\zeta_3\right\}(k_3 \underset{(p,p')}{\odot} \zeta_4 + k_3 \underset{(p,p')}{\times} \zeta_4)_{\frac{p+2}{2}} \\
&\qquad\qquad \left. - \frac{1}{2}\left\{(k_2-k_1-k_3)\underset{(p)}{\cdot}\zeta_4 - ik_4 \underset{(p,p')}{\cdot} J\zeta_4\right\}(k_4 \underset{(p,p')}{\odot} \zeta_3 - k_4 \underset{(p,p')}{\times} \zeta_3)_{\frac{p+2}{2}}\right] \\
&\quad + \sum_{\widetilde{I}} \frac{1}{t-(m_T^2+\frac{1}{\pi|\beta_{\widetilde{I}}|})}\left[\frac{1}{2\pi|\beta_{\widetilde{I}}|}(\zeta_3 \underset{(p,p')}{\odot} \zeta_4 - \zeta_3 \underset{(p,p')}{\times} \zeta_4)_{\widetilde{I}} \right. \\
&\qquad\qquad + \frac{1}{2}\left\{\Big(k_2-(k_1+k_4)\Big)\underset{(p)}{\cdot}\zeta_3 - ik_3 \underset{(p,p')}{\cdot} J\zeta_3\right\}\left(k_3 \underset{(p,p')}{\odot} \zeta_4 - k_3 \underset{(p,p')}{\times} \zeta_4\right)_{\widetilde{I}} \\
&\qquad\qquad \left. - \frac{1}{2}\left\{\Big((k_2+k_3)-k_1\Big)\underset{(p)}{\cdot}\zeta_4 - ik_4 \underset{(p,p')}{\cdot} J\zeta_4\right\}\left(k_4 \underset{(p,p')}{\odot} \zeta_3 + k_4 \underset{(p,p')}{\times} \zeta_3\right)_{\widetilde{I}}\right] \\
&\quad \left. - \sum_{\widetilde{I},\widetilde{L}} \frac{1}{t-(m_T^2+\frac{1}{\pi|\beta_{\widetilde{I}}|}+\frac{1}{\pi|\beta_{\widetilde{L}}|})}\frac{1}{2}\left(k_4 \underset{(p,p')}{\odot} \zeta_3 + k_4 \underset{(p,p')}{\times} \zeta_3\right)_{\widetilde{I}}\left(k_3 \underset{(p,p')}{\odot} \zeta_4 - k_3 \underset{(p,p')}{\times} \zeta_4\right)_{\widetilde{L}}\right.
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{\widetilde{I}} \frac{1}{t-(m_T^2+\frac{1}{\pi|\beta|}-\frac{1}{\pi|\beta_{\widetilde{I}}|})} \times \Bigg[-\frac{1}{2}\left(k_3 \underset{(p,p')}{\odot} k_4 + k_3 \underset{(p,p')}{\times} k_4\right)_{\frac{p+2}{2}} \left(\zeta_3 \underset{(p,p')}{\odot} \zeta_4 + \zeta_3 \underset{(p,p')}{\times} \zeta_4\right)_{\widetilde{I}} \\
&\qquad -\frac{1}{2}\left(k_3 \underset{(p,p')}{\odot} k_4 + k_3 \underset{(p,p')}{\times} k_4\right)_{\widetilde{I}} \left(\zeta_3 \underset{(p,p')}{\odot} \zeta_4 + \zeta_3 \underset{(p,p')}{\times} \zeta_4\right)_{\frac{p+2}{2}} \\
&\qquad +\frac{1}{2}\left(k_4 \underset{(p,p')}{\odot} \zeta_3 - k_4 \underset{(p,p')}{\times} \zeta_3\right)_{\frac{p+2}{2}} \left(k_3 \underset{(p,p')}{\odot} \zeta_4 + k_3 \underset{(p,p')}{\times} \zeta_4\right)_{\widetilde{I}} \\
&\qquad +\frac{1}{2}\left(k_4 \underset{(p,p')}{\odot} \zeta_3 - k_4 \underset{(p,p')}{\times} \zeta_3\right)_{\widetilde{I}} \left(k_3 \underset{(p,p')}{\odot} \zeta_4 + k_3 \underset{(p,p')}{\times} \zeta_4\right)_{\frac{p+2}{2}} \Bigg] \\
&-\sum_{\widetilde{I}} \frac{1}{t-(m_T^2+\frac{1}{\pi|\beta|}+\frac{1}{\pi|\beta_{\widetilde{I}}|})} \times \Bigg[\frac{1}{2}\left(k_3 \underset{(p,p')}{\odot} k_4 + k_3 \underset{(p,p')}{\times} k_4\right)_{\frac{p+2}{2}} \left(\zeta_3 \underset{(p,p')}{\odot} \zeta_4 - \zeta_3 \underset{(p,p')}{\times} \zeta_4\right)_{\widetilde{I}} \\
&\qquad +\frac{1}{2}\left(k_3 \underset{(p,p')}{\odot} k_4 - k_3 \underset{(p,p')}{\times} k_4\right)_{\widetilde{I}} \left(\zeta_3 \underset{(p,p')}{\odot} \zeta_4 + \zeta_3 \underset{(p,p')}{\times} \zeta_4\right)_{\frac{p+2}{2}} \Bigg] \Bigg\} \\
&\times \exp\left\{-\pi \sum_I |\beta_I| \left(k_3 \underset{(p,p')}{\odot} k_4\right)_I\right\} \ , \qquad (5.8)
\end{aligned}
$$

where we have used

$$
\begin{cases}
\nu \simeq 1 - \dfrac{1}{\pi b_{\frac{p+2}{2}}} & \Rightarrow \quad \dfrac{\alpha'}{1-\nu} \to \pi\beta_{\frac{p+2}{2}} \quad (>0) \\
\nu_{\widetilde{I}} \simeq -\dfrac{1}{\pi b_{\widetilde{I}}} & \Rightarrow \quad \dfrac{\alpha'}{\nu_{\widetilde{I}}} \to -\pi\beta_{\widetilde{I}} \quad (>0)
\end{cases} \ , \qquad (5.9)
$$

in the zero slope limit. The first term in eq. (5.8) comes from the tachyon state exchange. Here the vertices derived from three point amplitude emerge. From the t-channel diagram in Fig.4, we find that these vertices depend on momenta in a proper way. It is worth noting that combining the exponential factor in eq. (5.8) with the multiplicative factor $\exp\Big(\mathcal{C}_3(\nu_I)+\mathcal{C}_4(\nu_I)\Big)$, we obtain a gaussian damping factor in the $x^{p+1},\ldots,x^{p'}$ directions in the zero slope limit,

$$
\exp\left[-\frac{\pi}{2}\sum_I |\beta_I| \left((k_3+k_4) \underset{(p,p')}{\odot} (k_3+k_4)\right)_I\right] \ . \qquad (5.10)
$$

Next we focus on the behavior of the integrand near $x=1$ from which we can read off the s-channel poles. From Fig.3, the s-channel poles come from the propagation of the p'-p' open string. In a similar way to the t-channel, by expanding the integrand around $x=1$ and integrating it near $x=1$: $\int_{1-\delta}^{1} dx \ldots$, we find that s-channel poles correspond to the p'-p' open string spectrum. In particular, aside from the multiplicative factor and momentum conserving delta functions, the massless pole turns out to be

$$
\begin{aligned}
\sim \frac{1}{2s}\Big[& \Big\{ (k_2-k_1)_{\dot{(p)}}\zeta_3 - i(k_3+k_4)_{\dot{(p,p')}} J\zeta_3 \Big\} k_{3\,\dot{(p')}}\zeta_4 \\
& -k_{4\,\dot{(p')}}\zeta_3 \Big\{ (k_2-k_1)_{\dot{(p)}}\zeta_4 - i(k_3+k_4)_{\dot{(p,p')}} J\zeta_4 \Big\} \\
& -2\sum_I \nu_I \Big(k_3 \underset{(p,p')}{\times} k_4 \Big)_I \zeta_{3\,\dot{(p')}}\zeta_4 - 2k_{3\,\dot{(p')}} k_4 \sum_I \nu_I \Big(\zeta_3 \underset{(p,p')}{\times} \zeta_4 \Big)_I \\
& + \Big\{ -t + m_T^2 + k_{3\,\dot{(p,p')}} k_4 - \sum_I (1-2\nu_I) \Big(k_3 \underset{(p,p')}{\times} k_4 \Big)_I \Big\} \zeta_{3\,\dot{(p')}}\zeta_4 \Big] \\
& \times \exp\left[2\alpha' \sum_I \Big(k_3 \underset{(p,p')}{\odot} k_4 \Big)_I \left\{ \gamma + \frac{1}{2}\left(\boldsymbol{\psi}(\nu_I) + \boldsymbol{\psi}(1-\nu_I) \right) \right\} \right] \\
& + (k_3 \leftrightarrow k_4; \zeta_3 \leftrightarrow \zeta_4) \ , \qquad (5.11)
\end{aligned}
$$

where

$$
s \equiv -(k_3+k_4)_{\dot{(p')}}(k_3+k_4) = -2k_{3\,\dot{(p')}}k_4 \ . \qquad (5.12)
$$

Here by using eqs. (2.41), (A.1) and (A.5) we have expanded the hypergeometric function $\mathcal{H}$ around $x=1$ as

$$
\begin{aligned}
\mathcal{H}\left(\nu;\frac{1}{x}\right) &= -\frac{\pi}{2}b - \ln(1-x) \\
&\quad + \sum_{m=0}^{\infty} \frac{(-1+\nu)_m}{m!}(1-x)^m \sum_{n=0}^{\infty} \frac{(1-\nu)_n}{n!} \left\{ \psi(n+1) - \psi(n+1-\nu) \right\} (1-x)^n \ , \\
\mathcal{H}(\nu;x) &= \frac{\pi}{2}b - \ln(1-x) \\
&\quad + \sum_{m=0} \frac{(-\nu)_m}{m!}(1-x)^m \sum_{n=0}^{\infty} \frac{(\nu)_n}{n!} \left\{ \psi(n+1) - \psi(n+\nu) \right\} (1-x)^n \ . \qquad (5.13)
\end{aligned}
$$

We find that in our result eq. (5.11) the first two terms are in accordance with the vertex seen at the three point amplitude (4.25). This vertex shows up again with proper momentum dependence (see Fig.4) as well as in the t-channel pole corresponding to the tachyon exchange. Combined with the multiplicative factor $\exp\Big(\mathcal{C}_3(\nu_I) + \mathcal{C}_4(\nu)\Big)$, the exponential factor in eq. (5.11) gives us a gaussian damping factor,

$$
\exp\left[\alpha' \sum_I \left\{ \gamma + \frac{1}{2}\Big(\boldsymbol{\psi}(\nu_I) + \boldsymbol{\psi}(1-\nu_I) \Big) \right\} \left((k_3+k_4) \underset{(p,p')}{\odot} (k_3+k_4) \right)_I \right] \ . \qquad (5.14)
$$

It is noteworthy that in the zero slope limit this gaussian damping factor turns out to be the same as that of t-channel (eq. (5.10)).

While we do not try to derive here the complete action of the low energy noncommutative field theory in $p'+1$ dimensions, it is still possible to exhibit the interactions which reproduce

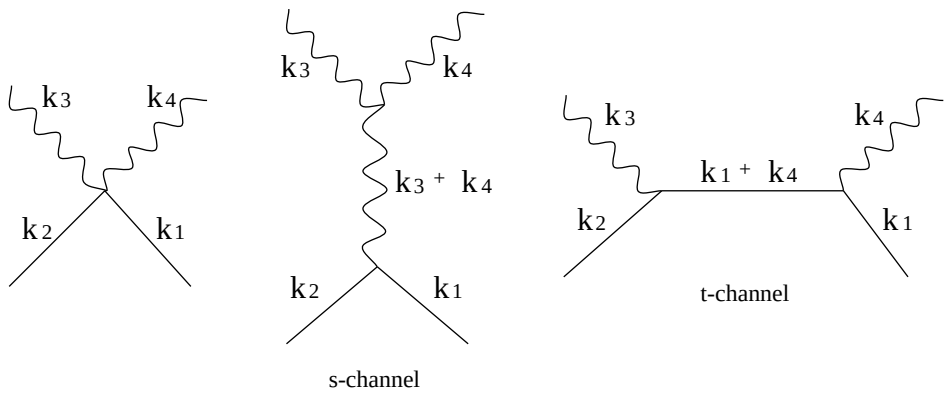

Figure 4: Field theory diagrams.

the parts of the amplitudes in the zero slope limit which are expressible in terms of the inner product with respect to the open string metric. We find that this part of the action is

$$
\begin{aligned}
& S = S_0 + S_1 \ , \\
& \text{with} \quad S_0 = \frac{1}{g_{YM}^2} \int d^{p'+1}x \sqrt{-G} \left\{ -(D_\mu \Phi)^\dagger * (D^\mu \Phi) \ - m^2 \Phi^\dagger * \Phi - \frac{1}{4} F_{MN} * F^{MN} \right\} \ , \\
& \qquad S_1 = \frac{1}{2 g_{YM}^2} \int d^{p'+1}x \sqrt{-G} \Phi^\dagger * F_{MN} J^{MN} * \Phi \ ,
\end{aligned}
\tag{5.15}
$$

where

$$
\begin{aligned}
& D_\mu \Phi = \partial_\mu \Phi - i A_\mu * \Phi \ , \qquad (D_\mu \Phi)^\dagger = \partial_\mu \Phi^\dagger + i \Phi^\dagger * A_\mu \ , \\
& F_{MN} = \partial_M A_N - \partial_N A_M - i \left[A_M, A_N \right]_* \ , \qquad \left[A_M, A_N \right]_* = A_M * A_N - A_N * A_M \ ,
\end{aligned}
\tag{5.16}
$$

the $*$ product of two functions f and g is given by

$$
f(x) * g(x) = \left. e^{\frac{i}{2} \theta^{\mu\rho} \frac{\partial}{\partial y^\mu} \frac{\partial}{\partial z^\rho}} f(y) g(z) \right|_{y,z \to x} \ , \tag{5.17}
$$

and g_{YM} is the effective Yang-Mills coupling defined by using the open sting coupling G_s and that of the closed string g_s as [6]

$$
\frac{1}{g_{YM}^2} = \frac{(\alpha')^{\frac{3-p'}{2}}}{(2\pi)^{p'-2} G_s} = \frac{(\alpha')^{\frac{3-p'}{2}}}{(2\pi)^{p'-2} g_s} \left(\frac{\det(g + 2\pi\alpha' B)}{\det G} \right)^{\frac{1}{2}} \ . \tag{5.18}
$$

In eq. (5.15) we have determined the seagull interaction corresponding to the non-pole term by invoking the noncommutative $U(1)$ invariance.

Let us finally discuss the gaussian damping factor eq. (5.10) which have originated from the exponential multiplicative factor eq. (4.8) and the lowest modes in the hypergeometric

function $\mathcal{H}$. Recall that there is no momentum conservation for the $x^{p+1}, \ldots, x^{p'}$-directions and that the tachyon momenta k_1 and k_2 are constrained to lie on the $x^0, \ldots, x^p$-directions. Without the gaussian damping factor, our picture would be that $(N-2)$ incident noncommutative $U(1)$ photons travel freely in the $x^{p+1}, \ldots, x^{p'}$-directions until they get stopped by the Dp-brane. The actual spacetime picture which we have exhibited here is that the lowest mode of the p-p' open string develops a physical scale $\sqrt{|\beta_I|}$ and that this mode creates a cloud around the Dp-brane in the zero slope limit. The noncommutative $U(1)$ photons get decelerated by the presence of this cloud, which is reflected in our damping factor eq. (5.10). The mean free paths will be measured by $\sqrt{|\beta_I|}$.

In this situation, the tachyon field in these directions should be expanded by the coherent states $\left\{\left\langle x^{p+1}, \ldots, x^{p'} | \nu_I \right\rangle\right\}$ associated with the would-be zero modes $\alpha^I_{1-\nu_I}$ (or $\overline{\alpha}^I_{\nu_I}$). On this basis, the complete analysis of low-lying states and ν_I dependent interactions obtained from the residual parts of the amplitudes will lead to the full-fledged form of the tachyon-vector interactions in these directions. The appearance of the coherent states here suggests that the fields which have originated from the p-p' open string should support noncommutative solitons on the Dp'-brane worldvolume which has recently been found in [19].

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Appendix More on the two-point function at the worldsheet boundary

The radius of convergence of the hypergeometric series $F(a,b;c;z)$ is unity. Thus we have evaluated the hypergeometric series on its convergent circle in eq. (2.43) in deriving the noncommutativity term (2.44). In this appendix we will give another derivation of eq. (2.43) to verify the noncommutativity term (2.44) and the two-point function (2.45).

Let us focus on the relation,

$$\mathcal{F}(\nu;z) = -\ln(1-z) + z^\nu \sum_{n=0}^{\infty} \frac{(\nu)_n}{n!} \left\{\boldsymbol{\psi}(n+1) - \boldsymbol{\psi}(n+\nu)\right\} (1-z)^n \ . \tag{A.1}$$

One can obtain this relation by using eq. (2.41) and a formula for the hypergeometric function [20],

$$F(a,b;a+b;z) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(n!)^2} \Big[2\boldsymbol{\psi}(n+1) - \boldsymbol{\psi}(a+n)$$

$$-\boldsymbol{\psi}(b+n)-\ln(1-z)\Big](1-z)^n\ , \qquad \text{(A.2)}$$

which is derived from the following relation by putting $c=a+b+\delta$ and by taking the limit of $\delta\to 0$:

$$\begin{aligned} F(a,b;c;z) &= \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}F(a,b;a+b-c+1;1-z) \\ &\quad +\frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}(1-z)^{c-a-b}F(c-a,c-b;c-a-b+1;1-z)\ . \end{aligned} \qquad \text{(A.3)}$$

From eq. (A.1), one can find that $\lim_{z\to 1}\left\{\mathcal{F}\left(1-\nu_I;\frac{1}{z}\right)-\mathcal{F}\left(\nu_I;z\right)\right\}$ is sensitive to the way of taking the limit. In the original expression (2.39) of the two point function, however, the way of sending $z\to 1$ is fixed in a definite way on the real axis because of the step function in front of each hypergeometric function. The constant noncommutativity term (2.44) should be more precisely described as

$$\begin{aligned} &\frac{4}{\varepsilon}\frac{\delta^{I\overline{J}}}{1+b_I^2}\left\{\lim_{\tau_1\to\tau_2+0}\mathcal{F}\left(1-\nu_I;\frac{e^{\tau_2}}{e^{\tau_1}}\right)-\lim_{\tau_1\to\tau_2-0}\mathcal{F}\left(\nu_I;\frac{e^{\tau_1}}{e^{\tau_2}}\right)\right\} \\ &\quad=\frac{4}{\varepsilon}\frac{\delta^{I\overline{J}}}{1+b_I^2}\lim_{\tau_1\to\tau_2}\left[\left\{-\ln\left|1-\frac{e^{\tau_2}}{e^{\tau_1}}\right|+\left(\frac{e^{\tau_2}}{e^{\tau_1}}\right)^{1-\nu_I}\left(\boldsymbol{\psi}(1)-\boldsymbol{\psi}(1-\nu_I)\right)\right\}\right. \\ &\qquad\qquad\left.-\left\{-\ln\left|1-\frac{e^{\tau_1}}{e^{\tau_2}}\right|+\left(\frac{e^{\tau_1}}{e^{\tau_2}}\right)^{\nu_I}\left(\boldsymbol{\psi}(1)-\boldsymbol{\psi}(\nu_I)\right)\right\}\right] \\ &\quad=\frac{4}{\varepsilon}\frac{\delta^{I\overline{J}}}{1+b_I^2}\pi b_I\ , \end{aligned} \qquad \text{(A.4)}$$

where we have used a relation for the digamma function,

$$\pi b_I=-\pi\cot(\pi\nu_I)=\boldsymbol{\psi}(\nu_I)-\boldsymbol{\psi}(1-\nu_I)\ . \qquad \text{(A.5)}$$

Thus we obtained the same noncommutativity term as eq. (2.44) through more careful treatment of the hypergeometric functions and this provides another verification to the two-point function (2.45).

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