Revised Version

# Monopoles and Family Replicated Unification

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**Abstract**

The present theory is based on the assumption that at the very small (Planck scale) distances our space-time is discrete, and this discreteness influences on the Planck scale physics. Considering our (3+1)-dimensional space-time as a regular hypercubic lattice with a parameter $a = \lambda_{\rm P}$, where $\lambda_{\rm P}$ is the Planck length, we have investigated a role of lattice artifact monopoles which is essential near the Planck scale if the Family replicated gauge group model (FRGGM) is an extension of the Standard Model at high energies. It was shown that monopoles have $N$ times smaller magnetic charge in FRGGM than in SM ($N$ is the number of families in FRGGM). These monopoles can give an additional contribution to beta-functions of the renormalisation group equations for the running fine structure constants $\alpha_{\rm i}(\mu)$ (i=1,2,3 correspond to the U(1), SU(2), and SU(3) gauge groups of the Standard Model). We have used the Dirac relation for renormalised electric and magnetic charges. Also we have estimated the enlargement of a number of fermions in FRGGM leading to the suppression of the asymptotic freedom in the non-Abelian theory. Different role of monopoles in the vicinity of the Planck scale gives rise or to AntiGUT, or to the new possibility of unification of gauge interactions (including gravity) at the scale $\mu_{\rm GUT} \approx 10^{18.4}$ GeV. We discussed the possibility of the $[{\rm SU}(5)]^3$ SUSY or $[{\rm SO}(10)]^3$ SUSY unifications.

# 1. Introduction

Trying to look insight the Nature and considering the physical processes at very small distances, physicists have made attempts to explain the well–known laws of low–energy physics as a consequence of the more fundamental laws of Nature. The contemporary physics of the electroweak and strong interactions is described by the Standard Model (SM) which unifies the Glashow-Salam-Weinberg electroweak theory with QCD – theory of strong interactions.

The gauge group of symmetry in the SM is :

$$SMG = SU(3)_{\rm c} \times SU(2)_{\rm L} \times U(1)_{\rm Y}, \tag{1}$$

which describes the present elementary particle physics up to the scale $\approx 100$ GeV.

Recently it was shown in a number of papers [1] that the family replicated gauge groups of type :

$$SU(n)^N \times SU(m)^N \tag{2}$$

play an essential role in construction of renormalizable, asymptotically free, four dimensional gauge theories that dynamically generate a fifth dimension (or fifth and sixth ones). This theory leads to the natural electroweak symmetry breaking, relying neither on supersymmetry nor on strong dynamics at the TeV scale. The new TeV physics is perturbative, and radiative corrections to the Higgs mass are finite. The Higgs scalar is an extended object – pseudo-Nambu-Goldstone boson – and a novel Higgs potential emerges naturally requiring a second light SU(2) doublet scalar.

We see that the family replicated gauge groups provide a new way to stabilize the Higgs mass in the Standard Model.

# 2. Family Replicated Gauge Group

The extension of SM with the Family Replicated Gauge Group (FRGG):

$$G = (SMG)^N = [SU(3)_{\rm c}]^N \times [SU(2)_{\rm L}]^N \times [U(1)_{\rm Y}]^N \tag{3}$$

was first suggested by C.D.Froggatt and H.B.Nielsen [2].

In Eq.(3) $N$ designates the number of quark and lepton families. If $N = 3$ (as experiment confirms), then the fundamental gauge group G is:

$$G = (SMG)^3 = SMG_{\rm 1st\ fam.} \times SMG_{\rm 2nd\ fam.} \times SMG_{\rm 3rd\ fam.}. \tag{4}$$

The generalized fundamental group:

$$G_{\rm f} = (SMG)^3 \times U(1)_{\rm f} \tag{5}$$

was suggested by the fitting of fermion masses of the SM [3].
Recently a new generalization of FRGG-model was suggested in papers [4], in which the fundamental group:

$$\begin{aligned} G_{\rm ext} &= (SMG \times U(1)_{\rm B\text{-}L})^3 \\ &\equiv [SU(3)_{\rm c}]^3 \times [SU(2)_{\rm L}]^3 \times [U(1)_{\rm Y}]^3 \times [U(1)_{\rm (B\text{-}L)}]^3 \end{aligned} \tag{6}$$

takes into account the see-saw mechanism with right-handed neutrinos, describes all modern neutrino experiments, and gives the reasonable fitting of the SM fermion masses and mixing angles. The group $G = G_{\rm ext}$ contains: $3 \times 8 = 24$ gluons, $3 \times 3 = 9$ W-bosons and $3 \times 1 + 3 \times 1 = 6$ Abelian gauge bosons.
The model is renormalisable: has no anomalies, neither gauge nor mixed.
The gauge group $G_{\rm ext}$ undergoes the spontaneous breakdown (at some orders of magnitude below the Planck scale) by 7 different Higgs fields to the gauge group which is the diagonal subgroup of $G_{\rm ext}$. Therefore, 7 Higgs fields break FRGG-model to the SM. The field $\phi_{\rm WS}$ corresponds to the Weinberg-Salam theory. Its VEV is known: $< \phi_{\rm WS} >= 246$ GeV, so that we have only 6 free parameters – six VEVs – to fit the experiment in the framework of this model.
Froggatt, Nielsen and Takanishi [4] have used them with aim to find the best fit to conventional experimental data for all fermion masses and mixing angles in the SM, also to explain the experiments in the neutrino oscillations. The typical fit was encouraging in the crude approximation. Also the neutrino masses were predicted.
Finally, we conclude that, in general, the theory with FRGG-symmetry is successful in describing of the SM experiment.

# 3. Lattice-like Structure of our Space-Time

Having an interest in the fundamental laws of physics, we can consider the two possibilities:

1. At the very small (Planck length) distances our space-time is continuous and there exists the fundamental theory with a very high symmetry.

2. At the very small distances our space-time is discrete, and this discreteness influences on the Planck scale physics.

The item 2 is an initial (basic) point of view of the present theory, but not an approximation. It is a base of the theory of physical processes proceeding at small distances of order of the Planck scale $\lambda_{\rm P} = M_{\rm Pl}^{-1}$:

$$M_{\rm Pl} = 1.22 \cdot 10^{19}\,{\rm GeV}. \tag{7}$$

In the simplest case we can imagine our (3+1) space–time as a regular hypercubic lattice with a parameter $a = \lambda_{\rm P}$. Then the lattice artifact monopoles can play an essential role near the Planck scale. But, of course, it is necessary to comment that we do not know (at least on the level of our today knowledge) what lattice-like structure (random lattice, or foam, or string lattice, etc.) plays role in the description of physical processes at very small distances [5].

The aim of the present paper is also to show that monopoles cannot be seen in the usual SM up to the Planck scale, because they have a huge magnetic charge and are completely confined. Supersymmetry does not help to see monopoles.

We suggest to consider a possibility of the existence of monopoles in our World, extending the Standard Model Group (SMG) to the Family Replicated Gauge Group $(\mathrm{SMG})^N$.

# 4. Renormalisation Group Equations for Electric and Magnetic Fine Structure Constants

J.Schwinger [6] was first who investigated the problem of renormalisation of the magnetic charge in Quantum ElectroMagnetoDynamics (QEMD), i.e. in the Abelian quantum field theory of electrically and magnetically charged particles (with charges $e$ and $g$, respectively).

Considering the "bare" charges $e_0$ and $g_0$ and renormalised (effective) charges $e$ and $g$, Schwinger obtained:

$$e/g = e_0/g_0, \tag{8}$$

what means the absence of the Dirac relation for the renormalised electric and magnetic charges.

But there exists another solution of this problem [7–9] which gives:

$$eg = e_0 g_0 = 2\pi n, \quad (n \in Z), \tag{9}$$

i.e. the existence of the Dirac relation (charge quantization condition) for both, bare and renormalised electric and magnetic charges. Here we have $n = 1$ for the minimal (elementary) charges.

These two cases lead to the two possibilities for the renormalisation group equations (RGEs) describing the evolution of electric and magnetic fine structure constants:

$$\alpha = \frac{e^2}{4\pi} \quad \text{and} \quad \widetilde{\alpha} = \frac{g^2}{4\pi}, \tag{10}$$

which obey the following RGEs containing the electric and magnetic beta-functions:

$$\frac{d(\log \alpha(\mu))}{dt} = \pm \frac{d(\log \widetilde{\alpha}(\mu))}{dt} = \beta^{(e)}(\alpha) \pm \beta^{(m)}(\widetilde{\alpha}). \tag{11}$$

In Eq.(11) we have:

$$t = \log(\frac{\mu^2}{\mu_R^2}), \tag{12}$$

where $\mu$ is the energy scale and $\mu_{\rm R}$ is the renormalisation point.

The second possibility (with minuses) in Eq.(11) corresponds to the validity of the Dirac relation for the renormalised charges. We believe only in this case considered by authors in paper [9] where we have used the Zwanziger formalism of QEMD [10].

In the present paper, excluding the Schwinger's renormalisation condition (8), we assume only the Dirac relation for running $\alpha$ and $\widetilde{\alpha}$:

$$\alpha\widetilde{\alpha} = \frac{1}{4}. \tag{13}$$

It is necessary to comment that RGEs (11) are valid only for $\mu > \mu_{\rm threshold} = m_{\rm mon}$, where $m_{\rm mon}$ is the monopole mass.

If monopole charges, together with electric ones, are sufficiently small, then $\beta$-functions can be considered perturbatively:

$$\beta(\alpha) = \beta_2(\alpha/4\pi) + \beta_4(\alpha/4\pi)^2 + ... \tag{14}$$

and

$$\beta(\widetilde{\alpha}) = \beta_2(\widetilde{\alpha}/4\pi) + \beta_4(\widetilde{\alpha}/4\pi)^2 + ... \tag{15}$$

with (see [9] and references there)

$$\beta_2 = \frac{1}{3} \quad \text{and} \quad \beta_4 = 1 \quad - \quad \text{for scalar particles,} \tag{16}$$

and

$$\beta_2 = \frac{4}{3} \quad \text{and} \quad \beta_4 \approx 4 \quad - \quad \text{for fermions.} \tag{17}$$

For scalar electric and magnetic charges we have:

$$\frac{d(\log\alpha(\mu))}{dt} = -\frac{d(\log\widetilde{\alpha}(\mu))}{dt} = \beta_2\frac{\alpha-\widetilde{\alpha}}{4\pi}(1 + 3\frac{\alpha+\widetilde{\alpha}}{4\pi} + ...) \tag{18}$$

with $\beta_2 = 1/3$, and approximately the same result is valid for fermionic particles with $\beta_2 = 4/3$. Eq.(18) shows that there exists a region when both fine structure constants are perturbative. Approximately this region is given by the following inequalities:

$$0.2 \lesssim (\alpha, \widetilde{\alpha}) \lesssim 1. \tag{19}$$

Using the Dirac relation (13), we see from Eq.(18) that in the region (19) the two-loop contribution is not larger than 30% of the one-loop contribution, and the perturbation theory can be realized in this case.

It is necessary to comment that the region (19) almost coincides with the region of phase transition couplings obtained in the lattice compact QED [11].

# 5. Evolution of Running Fine Structure Constants

The usual definition of the SM coupling constants is given in *the Modified minimal subtraction scheme*($\overline{\rm MS}$):

$$\alpha_1 = \frac{5}{3}\alpha_{\rm Y}, \quad \alpha_{\rm Y} = \frac{\alpha}{\cos^2\theta_{\overline{\rm MS}}}, \quad \alpha_2 = \frac{\alpha}{\sin^2\theta_{\overline{\rm MS}}}, \quad \alpha_3 \equiv \alpha_{\rm s} = \frac{g_{\rm s}^2}{4\pi}, \tag{20}$$

where $\alpha$ and $\alpha_{\rm s}$ are the electromagnetic and SU(3) fine structure constants respectively, Y is the hypercharge, and $\theta_{\overline{\rm MS}}$ is the Weinberg weak angle in $\overline{\rm MS}$ scheme. Using RGEs with experimentally established parameters, it is possible to extrapolate the experimental values of three inverse running constants $\alpha_{\rm i}^{-1}(\mu)$ (here i=1,2,3 correspond to U(1), SU(2) and SU(3) groups of SM) from the Electroweak scale to the Planck scale.

It is well known (see for example [12]) that (in the absence of monopoles) the one–loop approximation RGEs can be described by the following expressions:

$$\alpha_{\rm i}^{-1}(\mu) = \alpha_{\rm i}^{-1}(\mu_{\rm R}) + \frac{b_{\rm i}}{4\pi}t, \tag{21}$$

where slopes $b_{\rm i}$ are given by the following values:

$$\begin{gathered} b_{\rm i} = (b_1, b_2, b_3) = \\ (-\tfrac{4}{3}N_{\rm gen} - \tfrac{1}{10}N_{\rm S}, \quad \tfrac{22}{3}N_{\rm V} - \tfrac{4}{3}N_{\rm gen} - \tfrac{1}{6}N_{\rm S}, \quad 11N_{\rm V} - \tfrac{4}{3}N_{\rm gen}). \end{gathered} \tag{22}$$

The integers $N_{\rm gen}$, $N_{\rm S}$, $N_{\rm V}$ are respectively the numbers of generations, Higgs bosons and different vector gauge fields.

In SM we have:

$$N_{\rm gen} = 3, \quad N_{\rm S} = N_{\rm V} = 1, \tag{23}$$

and the corresponding slopes (22) describe the evolutions of $\alpha_{\rm i}^{-1}(\mu)$.

The precision of the LEP data allows to make the extrapolation of RGEs with small errors up to the Planck scale unless the new physics pops, of course. Assuming that these RGEs for $\alpha_{\rm i}^{-1}(\mu)$ contain only the contributions of the SM particles up to $\mu = \mu_{\rm Pl} \equiv M_{\rm Pl}$ and doing the extrapolation with one Higgs doublet under the assumption of a "desert" and absence of monopoles, we have the following result obtained in [13]:

$$\alpha_1^{-1}(\mu_{\rm Pl}) \approx 33.3; \quad \alpha_2^{-1}(\mu_{\rm Pl}) \approx 49.5; \quad \alpha_3^{-1}(\mu_{\rm Pl}) \approx 54.0. \tag{24}$$

The extrapolation of $\alpha_{1,2,3}^{-1}(\mu)$ up to the point $\mu = \mu_{\rm Pl}$ is shown in Fig.1 as function of the variable $x = log_{10}\mu$ (GeV). In this connection, it is very attractive to consider also the gravitational interaction.

The gravitational interaction between two particles of equal masses M is given by the usual classical Newtonian potential:

$$V_{\rm g} = -G\frac{M^2}{r} = -\left(\frac{M}{M_{\rm Pl}}\right)^2\frac{1}{r} = -\frac{\alpha_{\rm g}(M)}{r}, \tag{25}$$

which always can be imagined as a tree–level approximation of quantum gravity.
Then the quantity:

$$\alpha_{\rm g} = \left(\frac{\mu}{\mu_{\rm Pl}}\right)^2 \tag{26}$$

plays a role of the running "gravitational fine structure constant" and the evolution of its inverse quantity also is presented in Fig.1 together with the evolutions of $\alpha_{\rm i}^{-1}(\mu)$.

# 6. Dropping of the Monopole Charge in the Family Replicated Gauge Group Model (FRGGM)

In the simplest case, the scalar monopole beta-function in QEMD is (see [14] and [15]):

$$\beta(\widetilde{\alpha}) = \frac{\widetilde{\alpha}}{12\pi} + (\frac{\widetilde{\alpha}}{4\pi})^2 + ... = \frac{\widetilde{\alpha}}{12\pi}(1 + 3\frac{\widetilde{\alpha}}{4\pi} + ...). \tag{27}$$

From the last equation it follows that the theory of monopoles cannot be considered perturbatively at least for

$$\widetilde{\alpha} > \frac{4\pi}{3} \approx 4. \tag{28}$$

This limit is smaller for non-Abelian monopoles.
Using the Dirac relation, it is easy to estimate in the simple SM the Planck scale value $\widetilde{\alpha}(\mu_{\rm Pl})$ (minimal for $U(1)_{\rm Y}$ gauge group):

$$\widetilde{\alpha}(\mu_{\rm Pl}) = \frac{5}{3}\alpha_1^{-1}(\mu_{\rm Pl})/4 \approx 55.5/4 \approx 14. \tag{29}$$

This value is really very big compared with the estimate (28) and, of course, with the critical coupling $\widetilde{\alpha}_{\rm crit} \approx 1$, corresponding to the confinement-deconfinement phase transition in the lattice QED [11]. Clearly we cannot do the perturbation approximation with such a strong coupling $\widetilde{\alpha}$.
It is hard for such monopoles not to be confined.
There is an interesting way out of this problem if one wants to have the existence of monopoles, namely to extend the SM gauge group so cleverly that certain selected linear combinations of charges get bigger electric couplings than the corresponding SM couplings. That could make the monopoles which for these certain charge linear combinations couple more weakly and thus have a better chance of being allowed "to exist".
An example of such an extension of SM that can impose the possibility of the allowance of monopoles is just Family Replicated Gauge Group Model (FRGGM).
According to the FRGGM, at some point $\mu = \mu_{\rm G} < \mu_{\rm Pl}$ (or really in a couple of steps) the fundamental group $G \equiv G_{\rm ext}$ undergoes spontaneous breakdown to its diagonal subgroup:

$$G \longrightarrow G_{\rm diag.subgr.} = \{g, g, g || g \in SMG\}, \tag{30}$$

which is identified with the usual (low-energy) group SMG.

It should be said that in the FRGG-model each family has its own gluons, own W's, and own photons. The breaking just makes linear combination of a certain color combination of gluons which exists in the SM below $\mu = \mu_{\rm G}$ and down to the low energies. We can say that the phenomenological gluon is a linear combination (with amplitude $1/\sqrt{3}$ for $N = 3$) for each of the FRGG gluons of the same color combination. Then we have the following formula connecting the fine structure constants of non-Abelian FRGG-model and low energy surviving diagonal subgroup $G_{\rm diag.subg.} \subseteq (SMG)^3$:

$$\alpha^{-1}_{\rm i,diag} = \alpha^{-1}_{\rm i,1st\ fam.} + \alpha^{-1}_{\rm i,2nd\ fam.} + \alpha^{-1}_{\rm i,3rd\ fam.}. \tag{31}$$

Here i = SU(2), SU(3), and i=3 means that we talk about the gluon couplings.

Assuming that three FRGG couplings are equal to each other, we obtain:

$$\alpha^{-1}_{\rm i,diag} \approx 3\alpha^{-1}_{\rm i,one\ fam.} \equiv 3\alpha^{-1}_{\rm i,G}. \tag{32}$$

In contrast to non-Abelian theories, in which the gauge invariance forbids the mixed (in families) terms in the Lagrangian of FRGG-theory, the U(1)-sector of FRGG contains such mixed terms:

$$\frac{1}{g^2}\sum_{\rm p,q} F_{\mu\nu,\,\rm p}F^{\mu\nu}_{\rm q} = \frac{1}{g^2_{11}}F_{\mu\nu,\,1}F^{\mu\nu}_{1} + \frac{1}{g^2_{12}}F_{\mu\nu,\,1}F^{\mu\nu}_{2} + ... + \frac{1}{g^2_{23}}F_{\mu\nu,\,2}F^{\mu\nu}_{3} + \frac{1}{g^2_{33}}F_{\mu\nu,\,3}F^{\mu\nu}_{3}, \tag{33}$$

where p,q = 1,2,3 are the indices of three families of the group $(\rm SMG)^3$. Now it is easily seen that if the different families had specific equal electric charges, i.e. equal $\alpha_{\rm pq}$, then taking the diagonal subgroup we get [16]:

$$\alpha^{-1}_{\rm diag} \approx 6\alpha^{-1}_{\rm G}, \tag{34}$$

which shows that we can increase electric $\alpha$ by a factor 6 replacing it by the electric $\alpha_{\rm one\ fam.} \equiv \alpha_{\rm G}$.

Taking (34), we can get the monopole fine structure constant $\widetilde{\alpha}_{\rm G}$ which is smaller by factor 6 in comparison with $\widetilde{\alpha}$ in the SM. We can estimate at the Planck scale:

$$\widetilde{\alpha}_G(\mu_{Pl}) \approx 14/6 \approx 2.3. \tag{35}$$

But it seems (see below) that in the FRGGM we have at the Planck scale:

$$\widetilde{\alpha}_G(\mu_{Pl}) \approx 1,$$

and the perturbation theory works for $\beta$-function of scalar monopoles near the Planck scale.

The conclusion: if one wants monopoles "to exist", it is necessary to drive in the direction of a model like FRGG.

# 7. The possibility of Grand Unification Near the Planck Scale

In the AntiGUT-model by Froggatt and Nielsen [2–4] the FRGG breakdown was considered at $\mu_{\rm G} \sim 10^{18}$ GeV.

But the aim of this investigation is to show that we can see quite different consequences of the extension of SM to FRGGM if G-group undergoes the breakdown to its diagonal subgroup (i.e. SM) not at $\mu_{\rm G} \sim 10^{18}$ GeV, but at $\mu_{\rm G} \sim 10^{14}$ or $10^{15}$ GeV, i.e. before the intersection of $\alpha_2^{-1}(\mu)$ with $\alpha_3^{-1}(\mu)$ at $\mu \approx 10^{16}$ GeV.

Then in the region $\mu_{\rm G} < \mu < \mu_{\rm Pl}$ we have three $SMG \times U(1)_{\rm B\text{-}L}$ groups for three FRGG families. In this region we have a lot of fermions, mass protected or not mass protected, belonging to usual families or to mirror ones. In FRGGM the additional 6 Higgs bosons, with their large VEVs, are responsible for the mass protection of a lot new fermions appearing in the region $\mu > \mu_{\rm G}$. In this region we designate the total number of fermions $N_{\rm F}$, which is different with $N$.

Also a role of artifact monopoles can be important in the vicinity of the Planck scale. Lattice monopoles are responsible for the confinement in lattice gauge theories what is confirmed by many numerical and theoretical investigations (see review [17] and papers [18]). In the compact lattice gauge theory the monopoles are not physical objects: they are lattice artifacts driven to infinite mass in the continuum limit.

In Refs. [19–22] we have developed the Higgs Monopole Model (HMM) approximating the lattice artifact monopoles as fundamental pointlike particles described by the Higgs scalar field. Indeed, the simplest effective dynamics describing the confinement mechanism in the pure gauge lattice U(1) theory is the dual Abelian Higgs model of scalar monopoles [17], [18]. This model considers the following Lagrangian:

$$L = -\frac{1}{4g^2} F_{\mu\nu}^2(B) + \frac{1}{2}|(\partial_\mu - iB_\mu)\Phi|^2 - U(\Phi), \tag{36}$$

where

$$U(\Phi) = \frac{1}{2}\mu^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4 \tag{37}$$

is the Higgs potential of scalar monopoles with magnetic charge $g$, and $B_\mu$ is the dual gauge (photon) field interacting with the scalar monopole field $\Phi$. In this model $\lambda$ is the self-interaction constant of scalar fields, and the mass parameter $\mu^2$ is negative.

Considering the renormalization group improvement of the effective Coleman-Weinberg potential [14], written in Refs. [19–22] for the dual sector of scalar electrodynamics in the two-loop approximation for $\beta$-functions, we have calculated the U(1) critical values of the magnetic fine structure constant:

$$\tilde{\alpha}_{\rm crit} = g^2_{\rm crit}/4\pi \approx 1.20 \tag{38}$$

and (by the Dirac relation) electric fine structure constant:

$$\alpha_{crit} = \pi / g^2_{\rm crit} \approx 0.208. \tag{39}$$

These values coincide with the lattice result [11].

Writing the following RGEs for $\alpha_{\rm i}(\mu)$ containing beta-functions for the Higgs scalar monopoles:

$$\frac{d(\log \alpha_{\rm i}(\mu))}{dt} = \beta(\alpha_{\rm i}) - \beta^{(m)}(\widetilde{\alpha}_{\rm i}), \qquad {\rm i}=1,2,3, \tag{40}$$

we can use the one-loop approximation for $\beta(\alpha_{\rm i})$ because $\alpha_{\rm i}$ are small, and the two-loop approximation for dual beta-function $\beta^{(m)}(\widetilde{\alpha}_{\rm i})$ by reason that $\widetilde{\alpha}_{\rm i}$ are not very small near the Planck scale.

It was shown in a number of investigations (see for example [18] and references there), that the confinement in the SU(n) lattice gauge theory effectively comes to the same U(1) formalism. The reason is the Abelian dominance in their monopole vacuum: monopoles of the Yang-Mills theory are the solutions of the U(1)-subgroups, arbitrary embedded into the SU(n) group. After a partial gauge fixing – Abelian projection by 't Hooft [23] – SU(n) gauge theory is reduced to the Abelian $U(1)^{n-1}$ theory with $n-1$ different types of Abelian monopoles. Choosing the Abelian gauge for dual gluons, it is possible to describe the confinement in the lattice SU(n) gauge theories by the analogous dual Abelian Higgs model of scalar monopoles.

Using the Abelian gauge by 't Hooft and taking into account that the direction in the Lie algebra of monopole fields are gauge dependent, we have found in [21] an average over these directions and obtained the group dependence relation between the phase transition fine structure constants for the groups U(1) and $\rm SU(n)/Z_n$ :

$$\alpha^{-1}_{\rm n,\ crit} = \frac{n}{2}\sqrt{\frac{n+1}{n-1}}\alpha^{-1}_{\rm U(1),\ crit}. \tag{41}$$

We have calculated this relation using only the one–loop approximation diagrams of non-Abelian theories.

According to Eq.(41), we have the following relations:

$$\alpha^{-1}_{\rm U(1),\ crit} : \alpha^{-1}_{\rm 2,\ crit} : \alpha^{-1}_{\rm 3,\ crit} = 1 : \sqrt{3} : 3/\sqrt{2}. \tag{42}$$

Near the Planck scale we are in the vicinity of the critical points [19–22].

Finally, taking into account that in the non-Abelian sectors of FRGG we have the Abelian artifact monopoles, we obtain the following RGEs:

$$\frac{d(\alpha^{-1}_{\rm i}(\mu))}{dt} = \frac{b_{\rm i}}{4\pi} + \frac{N_{\rm M}}{\alpha_{\rm i}}\beta^{(m)}(\widetilde{\alpha}_{\rm U(1)}), \tag{43}$$

where $b_{\rm i}$ are given by the following values:

$$\begin{array}{c} b_{\rm i} = (b_1, b_2, b_3) = \\ (-\frac{4}{3}N_{\rm F} - \frac{1}{10}N_{\rm S}, \quad \frac{22}{3}N_{\rm V} - \frac{4}{3}N_{\rm F} - \frac{1}{6}N_{\rm S}, \quad 11N_{\rm V} - \frac{4}{3}N_{\rm F}). \end{array} \tag{44}$$

The integers $N_{\rm F}$, $N_{\rm S}$, $N_{\rm V}$, $N_{\rm M}$ are respectively the total numbers of fermions, Higgs bosons, vector gauge fields and scalar monopoles in FRGGM considered in our theory. Approximating artifact monopoles by the Higgs scalar fields with a magnetic charge $g$, we have the following Abelian monopole beta-function in the two-loop approximation [21]:

$$\beta^{(m)}(\widetilde{\alpha}_{\rm U(1)}) = \frac{\widetilde{\alpha}_{\rm U(1)}}{12\pi}(1 + 3\frac{\widetilde{\alpha}_{\rm U(1)}}{4\pi}). \tag{45}$$

Using the Dirac relation $\alpha\widetilde{\alpha} = 1/4$, we have:

$$\beta^{(m)} = \frac{\alpha^{-1}_{\rm U(1)}}{48\pi}(1 + 3\frac{\alpha^{-1}_{\rm U(1)}}{16\pi}), \tag{46}$$

and the group dependence relation (41) gives:

$$\beta^{(m)} = \frac{C_{\rm i}{\alpha_{\rm i}}^{-1}}{48\pi}(1 + 3\frac{C_{\rm i}{\alpha_{\rm i}}^{-1}}{16\pi}), \tag{47}$$

where

$$C_{\rm i} = (C_1, C_2, C_3) = (\frac{5}{3}, \frac{1}{\sqrt{3}}, \frac{\sqrt{2}}{3}). \tag{48}$$

Finally we have the following RGEs:

$$\frac{d(\alpha_{\rm i}^{-1}(\mu))}{dt} = \frac{b_{\rm i}}{4\pi} + N_{\rm M}\frac{C_{\rm i}{\alpha_{\rm i}}^{-2}}{48\pi}(1 + 3\frac{C_{\rm i}{\alpha_{\rm i}}^{-1}}{16\pi}), \tag{49}$$

where $b_{\rm i}$ and $C_{\rm i}$ are given by Eqs.(44) and (48), respectively.
In our FRGG model:

$$\begin{array}{l} N_{\rm V} = 3, \ N_{\rm M} = 6 \ - \text{for i=1}, \\ N_V = N_M = 3 \quad \ - \text{for i=2,3}, \end{array} \tag{50}$$

because we have 3 times more gauge fields ($N = 3$), in comparison with usual SM and one Higgs scalar monopole in each family.
Assuming 6 scalar Higgs bosons ($N_S = 6$) breaking FRGG to SMG, and the total number of fermions $N_{\rm F} = 2N_{\rm tot}$ (usual and mirror families), $N_{\rm tot} = N \cdot N_{\rm gen} = 3 \times 3 = 9$ (three SMG groups with three generations in each group), we have obtained the evolutions of $\alpha_{\rm i}^{-1}(\mu)$ near the Planck scale by numerical calculations for $N_{\rm F} = 18$ and $\mu_{\rm G} = 10^{14}$ GeV. Fig.2 shows the existence of the unification point. We see that in the region $\mu > \mu_{\rm G}$ a lot of new fermions and a number of monopoles near the Planck scale change the one-loop approximation behaviour of $\alpha_{\rm i}^{-1}(\mu)$ which we had in SM. In the vicinity of the Planck scale these evolutions begin to decrease, approaching the Planck scale $\mu = \mu_{\rm Pl}$, what means the suppression of the asymptotic freedom in the non-Abelian theories.

Fig.3 demonstrates the unification of all gauge interactions, including gravity (the intersection of $\alpha_{\rm g}^{-1}$ with $\alpha_{\rm i}^{-1}$), at

$$\alpha_{\rm GUT}^{-1} \approx 27 \quad \text{and} \quad x_{\rm GUT} \approx 18.4. \tag{51}$$

It is easy to calculate that for one family we have:

$$\widetilde{\alpha}_{\rm GUT,\ one\ fam.} = \frac{\alpha_{\rm GUT,\ one\ fam.}^{-1}}{4} = \frac{\alpha_{\rm GUT}^{-1}}{4 \cdot 6} \approx \frac{27}{24} \approx 1.125, \tag{52}$$

and

$$\alpha_{\rm GUT,\ one\ fam.} \approx 0.22, \tag{53}$$

what means that at the GUT scale electric and monopole charges are not large and can be considered perturbatively.
Here we can expect the existence of $[{\rm SU}(5)]^3$ SUSY, or $[{\rm SO}(10)]^3$ SUSY unification with superparticles of masses:

$$M \approx 10^{18.4} \quad \text{GeV}. \tag{54}$$

The scale $\mu_{\rm GUT} = M$, given by Eq.(54), can be considered as a SUSY breaking scale.
The unification theory with $[{\rm SU}(5)]^3$-symmetry was suggested first by S.Rajpoot [24].
Considering the predictions of such a theory for the low-energy physics and cosmology, maybe in future we shall be able to answer the question: "Does the unification of $[{\rm SU}(5)]^3$ SUSY or $[{\rm SO}(10)]^3$ SUSY really exist near the Planck scale?"

# 8. Conclusions

In the present paper we have shown:

1. That the existence of monopoles in Nature leads to the consideration of the Family Replicated Gauge Groups of symmetry as an extension of the Standard Model in sense that using of monopoles corresponding to the family replicated gauge fields we can bring the monopole charge down from the unbelievably large value which it gets in the simple SM, according to the Dirac relation.

2. If our (3+1)–dimensional space–time is discrete and has a lattice–like structure, then the lattice artifact monopoles play an essential role near the Planck scale if the FRGGM works there. We have approximated these artifact monopoles by the Higgs scalar fields.

3. The breakdown of FRGG at $\mu_{\rm G} \sim 10^{14}$ GeV produces a lot of fermions in the region $\mu_{\rm G} < \mu < \mu_{\rm Pl}$ which gives the depression of asymptotic freedom near the Planck scale.

4. In contrast to the AntiGUT by Froggatt–Nielsen, predicting the absence of supersymmetry and unification up to the Planck scale, these fermions, together with monopoles, lead to the possible existence of unification of all interactions (including gravity) at

$$\mu_{\rm GUT} = 10^{18.4}\ GeV$$

and

$$\alpha_{\rm GUT}^{-1} = 27.$$

5. The possibility of $[{\rm SU}(5)]^3$ SUSY or $[{\rm SO}(10)]^3$ SUSY unifications was discussed in the present investigation.

# 9. Acknowledgements

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# References


[1] N. Arkani-Hamed, A. G. Cohen, H. Georgi et al., hep-th/0104005; hep-ph/0105239; hep-ph/0206020; hep-ph/0206021

[2] H. B. Nielsen, "Dual Strings. Fundamental of Quark Models", in: *Proceedings of the XYII Scottish University Summer School in Physics*, St.Andrews, 1976, p.528.
C. D. Froggatt, H. B. Nielsen, *Origin of Symmetries*, Singapore: World Scientific, 1991.

[3] C. D. Froggatt, G. Lowe, H. B. Nielsen, Phys. Lett. B**311**, 163 (1993);
Nucl. Phys. B **414**, 579 (1994); ibid B **420**, 3 (1994);
C. D. Froggatt, H. B. Nielsen, D. J. Smith, Phys. Lett. B **358**, 150 (1996);
C. D. Froggatt, M. Gibson, H. B. Nielsen, D. J. Smith, Int. J. Mod. Phys. A **13**, 5037 (1998).

[4] H. B. Nielsen, Y. Takanishi, Nucl. Phys. B **588**, 281 (2000); ibid, B **604**, 405 (2001); Phys. Lett. B **507**, 241 (2001); hep-ph/0011168, hep-ph/0101181, hep-ph/0101307.
C. D. Froggatt, H. B. Nielsen, Y. Takanishi, "Family Replicated Gauge Groups and Large Mixing Angle Solar Neutrino Solution", preprints DESY 02-006, GUPTA/01/12/01, NBI-HE-01-12, hep-ph/0201152, January 2002.

[5] L. V. Laperashvili, D. A. Ryzhikh, *"Phase Transition in Gauge Theories and the Planck Scale Physics"*, Festschrift for H. B. Nielsen, Ljubljana, 2001; preprint ITEP 24-01, Moscow, 2001, 60pp.; hep-th/0110127.

[6] J.Schwinger, Phys.Rev. **144**, 1087 (1966); **151**, 1048; 1055 (1966); **173**, 1536 (1968); Science **165**, 757 (1969); **166**, 690 (1969).

[7] S. Coleman, in: *The Unity of the Fundamental Interactions*, Eriche Lectures (1982), ed. A.Zichichi (Plenum, 1983), p.21.

[8] M. Blagojevic and P. Senjanovic, Phys. Rept. **157**, 234 (1988).

[9] L. V. Laperashvili and H. B. Nielsen, Mod. Phys. Lett. A **14**, 2797 (1999).

[10] D. Zwanziger, Phys. Rev. D **3**, 343 (1971); R. A. Brandt, F. Neri, D. Zwanziger, Phys. Rev. D **19**, 1153 (1979).

[11] J. Jersak, T. Neuhaus and P. M. Zerwas, Phys. Lett. B **133**, 103 (1983); Nucl. Phys. B **251**, 299 (1985); H. G. Everetz, T. Jersak, T. Neuhaus, P. M. Zerwas, Nucl. Phys. B **251**, 279 (1985); J. Jersak, T. Neuhaus, H. Pfeiffer, Phys. Rev. D **60**, 054502 (1999); Nucl. Phys. Proc. Suppl. **83-84**, 491 (2000).

[12] M. B. Voloshin and K. A. Ter-Martirosyan, *Theory of Gauge Interactions of Elementary Particles*, Energoatomizdat, 1984.

[13] P. Langacker, N. Polonsky, Phys. Rev. D **47**, 4028 (1993); ibid, D **49**, 1454 (1994); ibid, D **52**, 3081 (1995).

[14] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).

[15] D. R. T. Jones, Nucl. Phys. B **75**, 531 (1974); Phys. Rev. D **25**, 581 (1982).

[16] D. L. Bennett and H. B. Nielsen, Int. J. Mod. Phys. A **9**, 5155 (1994); ibid, A **14**, 3313 (1999).

[17] T. Suzuki, Progr. Theor. Phys. Suppl. **131**, 633 (1998).

[18] M. N. Chernodub, M. I. Polikarpov, in *"Cambridge 1997, Confinement, Duality and Non-perturbative Aspects of QCD"*, p.387, ed. by Pierre van Baal, Plenum Press, 1998; hep-th/9710205;
M. N. Chernodub, F. V. Gubarev, M. I. Polikarpov, A. I. Veselov, Prog. Ther. Phys. Suppl. **131**, 309 (1998); hep-lat/9802036;
M. N. Chernodub, F. V. Gubarev, M. I. Polikarpov, V. I. Zakharov, Nucl. Phys. B **592**, 107, 2000; Nucl. Phys. B **600**, 163 (2001).

[19] L. V. Laperashvili and H. B. Nielsen, "Multiple Point Principle and Phase Transition in Gauge Theories", in: *Proceedings of the International Workshop on "What Comes Beyond the Standard Model"*, Bled, Slovenia, 29 June - 9 July 1998; Ljubljana 1999, p.15.

[20] L. V. Laperashvili and H. B. Nielsen, Int. J. Mod. Phys. A **16**, 2365 (2001).

[21] L. V. Laperashvili, H. B. Nielsen and D. A. Ryzhikh, Int. J. Mod. Phys. A **16**, 3989 (2001).

[22] L. V. Laperashvili, H. B. Nielsen and D. A. Ryzhikh, Yad. Fiz. **65**, 377 (2002); Phys. Atom. Nucl. **65**, 353 (2002).

[23] G. 't Hooft, Nucl. Phys. B **190**, 455 (1981).

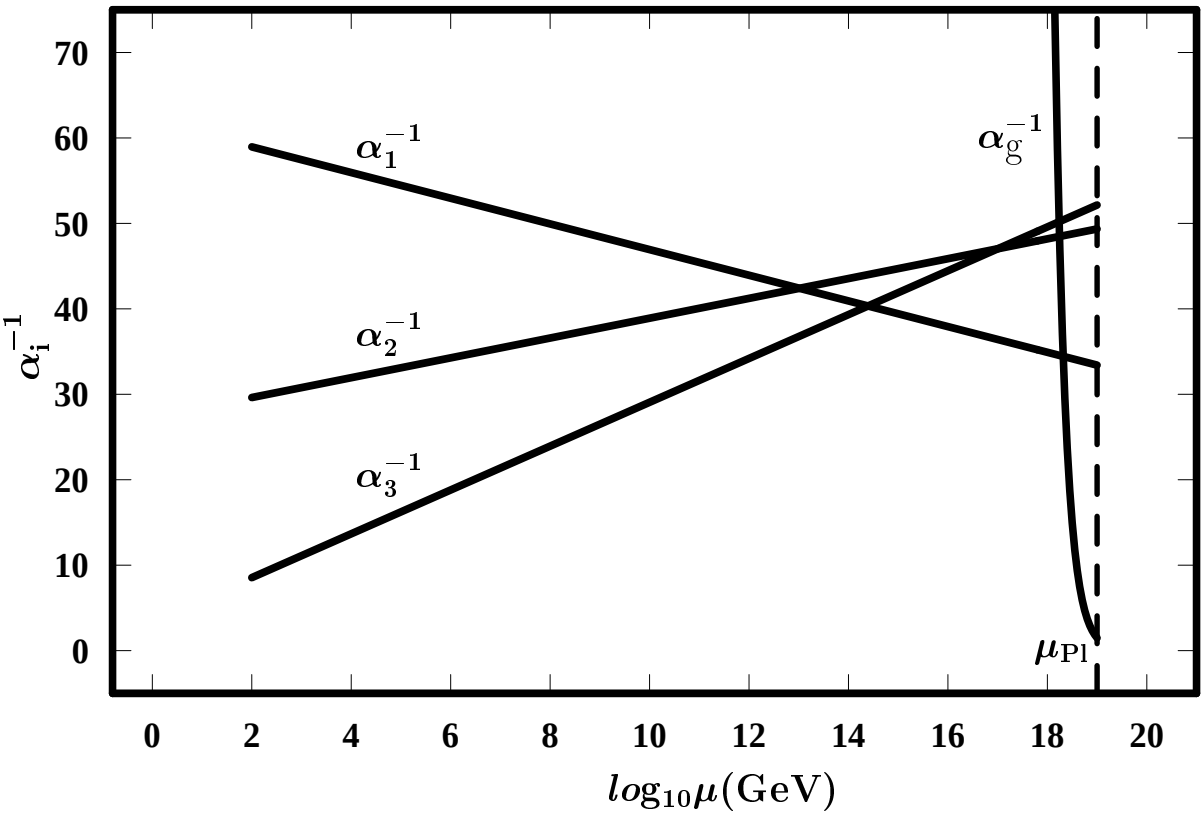
[24] S. Rajpoot, Nucl. Phys. Proc. Suppl. **51A**, 50 (1996).

# Figure captions

**Figure 1.** The evolution of three inverse running constants $\alpha_i^{-1}(\mu)$, where i=1,2,3 correspond to U(1), SU(2) and SU(3) groups of the SM. The extrapolation of their experimental values from the Electroweak scale to the Planck scale was obtained by using the renormlization group equations with one Higgs doublet under the assumption of a "desert". The precision of the LEP data allows to make this extrapolation with small errors. The intersection of the inverse "gravitational finestructure constant" $\alpha_g^{-1}(\mu)$ with $\alpha_1^{-1}(\mu)$ occurs at the point $(x_0,\ \alpha_0^{-1})$: $\alpha_0^{-1} \approx 34.4$, and $x_0 \approx 18.3$, where $x = \log_{10}\mu(\mathrm{GeV})$.

**Figure 2.** The evolution of finestructure constants $\alpha_{1,2,3}^{-1}(\mu)$ beyond the Standard model in the Family replicated gauge group model (FRGGM) with influence of monopoles near the Planck scale.

**Figure 3.** The evolution of $\alpha_{1,2,3}^{-1}(\mu)$ in the Standard Model (SM) and beyond it. The breakdown of FRGG occurs at $\mu_G \sim 10^{14}$ GeV. It is shown the possibility of the $[SU(5)]^3$ SUSY unification of all gauge interactions, including gravity, at $\alpha_{GUT}^{-1} \approx 27$ and $x_{GUT} \approx 18.4$, where $x = \log_{10}\mu$ (GeV).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60
50
40
30
20
10
0
$\alpha_i^{-1}$
$\alpha_1^{-1}$
$\alpha_2^{-1}$
$\alpha_3^{-1}$
$\alpha_g^{-1}$
$\mu_{Pl}$
0
2
4
6
8
10
12
14
16
18
20
$log_{10}\mu(\mathrm{GeV})$

Fig.1.

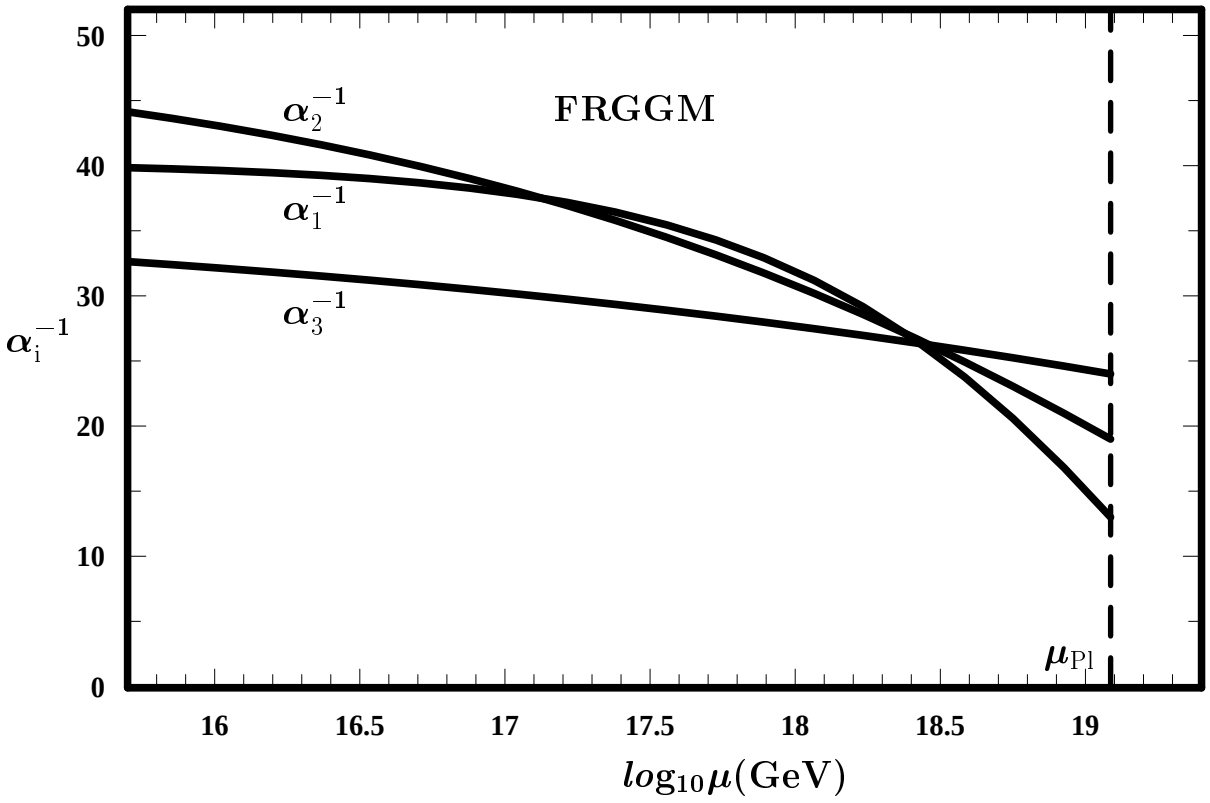
FRGGM
$\alpha_2^{-1}$
$\alpha_1^{-1}$
$\alpha_3^{-1}$
$\alpha_i^{-1}$
$\mu_{Pl}$
50
40
30
20
10
0
16
16.5
17
17.5
18
18.5
19
$log_{10}\mu(GeV)$

Fig.2.

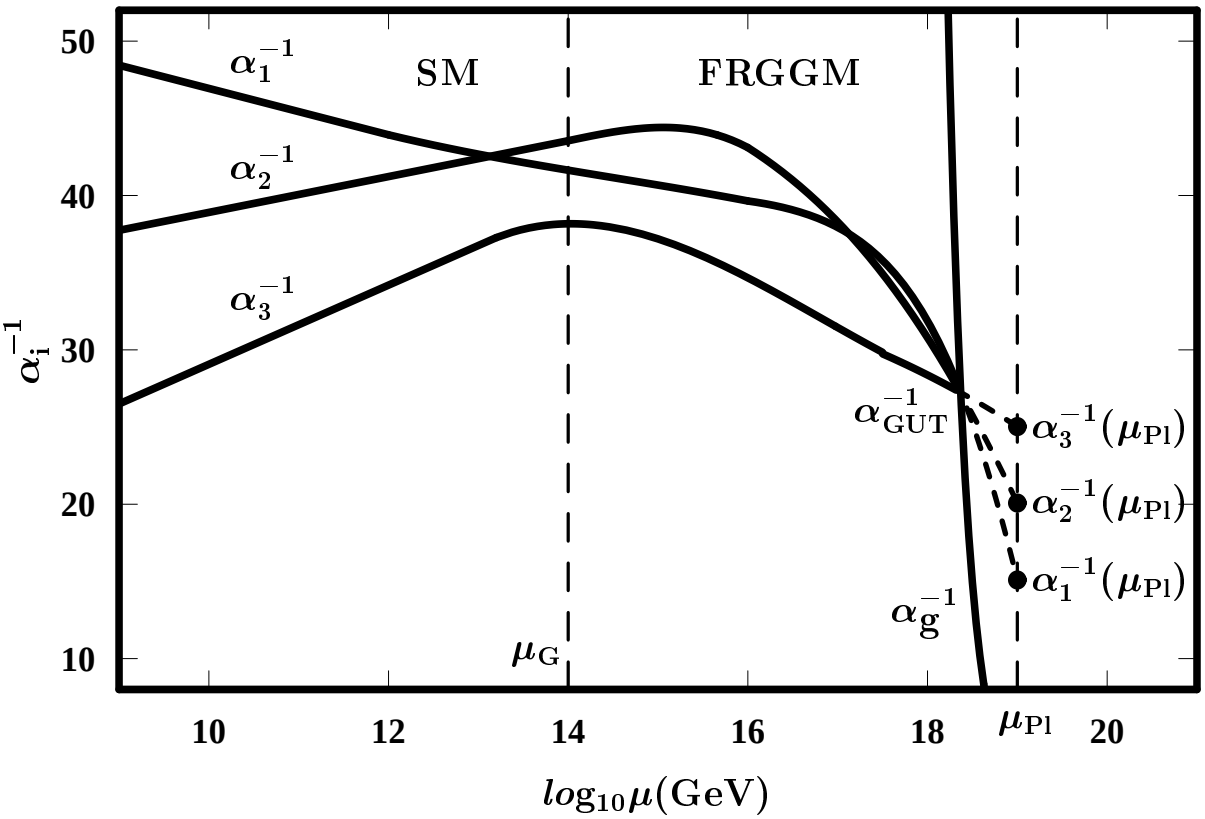
50
40
30
20
10
$\alpha_i^{-1}$
$\alpha_1^{-1}$
$\alpha_2^{-1}$
$\alpha_3^{-1}$
SM
FRGGM
$\alpha_{\rm GUT}^{-1}$
$\alpha_3^{-1}(\mu_{\rm Pl})$
$\alpha_2^{-1}(\mu_{\rm Pl})$
$\alpha_1^{-1}(\mu_{\rm Pl})$
$\alpha_{\rm g}^{-1}$
$\mu_{\rm G}$
$\mu_{\rm Pl}$
10
12
14
16
18
20
$log_{10}\mu({\rm GeV})$

**Fig.3.**