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THE CONFINEMENT MECHANISM IN YANG-MILLS THEORY?

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Abstract:

Using the recently proposed non-linear gauge condition $\frac{1}{2}[\partial \cdot D + D \cdot \partial](\partial \cdot A) = (\partial \cdot D)(\partial \cdot A) = 0$, we show the area law behavior of the Wilson loop and the linear dependence of the instantaneous gluon propagator. The field configurations responsible for confinement are those in the non-linear sector of the gauge-fixing condition (the linear sector being the Coulomb gauge). The non-linear sector is actually composed of "Gribov horizons" on the parallel surfaces $\partial \cdot A^a = f^a \neq 0$. In this sector, the gauge field A^a_μ can be expressed in terms of f^a and a new vector field t^a_μ. The effective dynamics of f^a suggests non-perturbative effects. This was confirmed by showing that all spherically symmetric (in 4-D Euclidean) f^a are classical solutions and averaging these solutions using a qaussian distribution (thereby treating these fields as random) led to confinement. In essence the confinement mechanism is not quantum mechanical in nature but simply a statistical treatment of classical spherically symmetric fields on the "horizons" of $\partial \cdot A^a = f^a(x)$ surfaces.

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I. **Introduction**

The explicit proof of confinement in non-Abelian theories has long eluded particle theorists. Although there have been intimations of such behaviour in lattice computations[1], the exact mechanism of confinement has not yet been satisfactorily exhibited. There are those who point to the existence of Gribov copies[2] as the confinement mechanism[3], which is contradicted by others[4]. Still for others, confinement is due to quantum mechanics around non-trivial classical

configurations such as instantons[5] and constant chromoelectric/magnetic[6] fields. Unfortunately, these field configurations failed to confine quarks inside hadrons.

Shortly after Parisi and Sourlas[7] showed dimensional reduction in a spin system in random magnetic field, Olesen[8] conjectured that the same mechanism may be responsible for quark confinement. And this would be an elegant proof of confinement because QCD, at least it s non-perturbative aspect, would be effectively reduced to a 2-dimensional theory if indeed such a mechanism takes place. Unfortunately, Olesen s conjecture still remains a conjecture.

In three previous papers[9,10,11] this author argued the necessity of generalizing the Coulomb gauge because field configurations that satisfy $\partial \cdot A^a = f^a \neq 0$ and $(\partial \cdot D)^{ab} f^b = 0$ are not gauge transformable to the Coulomb surface. These field configurations, essentially, are the "Gribov" horizons on the surfaces $\partial \cdot A^a = f^a$, which are surfaces parallel to the Coulomb surface $\partial \cdot A^a = 0$. In references (10) and (11), it was also shown that the gauge transform of a field configuration on the horizon of $\partial \cdot A^a = f^a$ surface never leave the surface and the horizon. Thus, if only the Coulomb gauge is used, then all the field configurations on the horizons of the parallel surfaces $\partial \cdot A^a = f^a \neq 0$ will be left out of the path-integral and physical effects associated with these brushed aside.

The above arguments show that the Coulomb gauge needs to be generalized to the non-linear condition

$$(\partial \cdot D)(\partial \cdot A) = 0 \quad . \quad (1)$$

This gauge condition contains the Coulomb gauge (the linear sector) and the horizons of each of the parallel surfaces $\partial \cdot A^a = f^a$ (the non-linear sector).

In reference (11), it was shown that the gauge condition defined by equation (1) can no longer be generalized to a gauge-fixing with higher derivatives and higher order in $A^a_{,\mu}$ that contains the Coulomb gauge. That paper also derived the new degrees of freedom in the non-linear sector of the gauge condition defined by equation (1). The effective dynamics was also derived and it hints of non-perturbative effects.

This paper will make use of the new degrees of freedom and their effective dynamics to show confinement. We will prove the area law behaviour of the Wilson loop and the linear dependence of the instantaneous "gluon" propagator. These follow from the fact that all spherically symmetric functions $f^a(x)$ (in 4-D Euclidean space-time) are classical solutions. Since there is no particular spherically symmetric function that is a solution, the contributions of all these solutions must be taken into account. This is done through a suitable averaging prescription and the most commonly used is the gaussian averaging. In essence, this means the spherically symmetric classical solution are treated as random fields. From these considerations, we derive confinement.

Before we show in detail the confinement mechanism we will first review the relevant results of reference (11) in Section II. Section III will present the computation of the "gluon" propagator and the Wilson loop. Section IV will summarize the results and present the conclusion.

II. **The Effective Dynamics of the New Degrees of Freedom**

In 4-D Euclidean space-time the non-linear sector of the non-linear gauge condition defined by equation (1) is given by

$$(\partial_\mu D_\mu)^{ab} f^b = 0 \quad , (2a)$$

$$\partial_\mu A^a_\mu = f^a \neq 0 \quad . (2b)$$

Equations (2; a,b) are solved by

$$\ell_o^2 D^{ab}_\mu f^b = (A^a_\mu - \partial_\mu \frac{1}{2} f^a) - t^a_\mu \quad , (3)$$

where a length scale ℓ_o is introduced for dimensional reasons (f^a has dimension (mass)^2) and t^a_μ is transverse. Equation (3) can be solved for A^a_μ in terms of f^a and t^a_μ giving as

$$A^a_\mu = \frac{1}{(1+g^2\ell_o^4 \vec{f}\cdot\vec{f})}\left\{\delta^{ab} + g\ell_o^2 \in^{abc} f^c + g^2\ell_o^4 f^a f^b\right\}\times\left(\ell_o^2\partial_\mu f^b + \partial_\mu \frac{1}{2} f^b + t^b_\mu\right) . (4)$$

Rescaling the fields by $g\ell_o^2 f^a \to f^a$ so that the new field f^a is dimensionless and defining a new vector field

$$t'^a_\mu = t^a_\mu + \frac{1}{g\ell_o^2}\partial_\mu \frac{1}{2} f^a \quad , (5)$$

and dropping the prime on the new field vector field, we get

$$A^a_\mu = \frac{1}{(1+\vec{f}\cdot\vec{f})}\left\{\delta^{ab} + \in^{abc} f^c + f^a f^b\right\}\left(\frac{1}{g}\partial_\mu f^b + t^b_\mu\right) . (6)$$

The new vector field t^a_μ is no longer transverse but satisfies

$$\partial_\mu t^a_\mu = \frac{1}{g\ell_o^2} f^a \quad . (7)$$

For A^a_μ to satisfy $\partial_\mu A^a_\mu = \frac{1}{g\ell_o^2} f^a$ (because f^a was rescaled), the following constraint must be

met

$$\mathcal{A}^a = \left[\epsilon^{abc} + \epsilon^{abd}\ f^d f^c + \epsilon^{adc}\ f^d f^b + f^a\ \epsilon^{bcd}\ f^d\right] \partial_\mu f^b t^c_\mu = 0 \quad . (8)$$

With equations (7) and (8), the number of degrees of freedom from twelve $A^a_{\mu i s}$ satisfying three constraints $\partial_\mu A^a_{\mu i} = \frac{1}{g \ell_o^2} f^a$ to twelve $t^a_{\mu i s}$ plus three $f^a s$, which satisfy six constraints given by (7) and (8) remains the same.

The shift in $t^a_{\mu i}$ as given in equation (5), which was not carried out in reference (11), is important for it removes the non-locality in the effective Lagrangian of the new degrees of freedom. Also when we put $f^a = 0$, we get back the Coulomb gauge.

In terms of the new degrees of freedom, the path-integral is

$$PI = \int (df^a)(dt^a_{\mu i})\ \delta(\partial \cdot t^a - \frac{1}{g \ell_o^2} f^a)\ \delta(\mathcal{A}^a)\ \det(1 + \vec{f} \cdot \vec{f})\ \det \mathcal{A}\ e^{-S_{YM}(f^a; t^a_{\mu i})} \quad , (9)$$

Where q is the Fadeev-Popov determinant of the non-linear gauge condition given by

$$\mathcal{A}^{ad} = (D \cdot \partial)^{ab}\ (\partial \cdot D)^{bd} - \epsilon^{abc}\left[\partial(\partial \cdot A^b)\right] \cdot D^{cd} \quad . (10)$$

It is understood that equation (6) will be substituted in the above expressions.

The field strength tensor expressed in terms f^a and $t^a_{\mu i}$ has the following form

$$F^a_{\mu\nu} = \frac{1}{g} Z^a_{\mu\nu}(f) + L^a_{\mu\nu}(f,t) + g\ Q^a_{\mu\nu}(f,t) \quad , (11)$$

where $Z^a_{\mu\nu}, L^a_{\mu\nu}$ and $Q^a_{\mu\nu}$ are zeroth order, linear and quadratic in $t^a_{\mu i}$ respectively (at this point the explicit forms of Z, L and Q are not important). The Yang-Mills action becomes

$$S_{YM} = \int d^4x \left\{ \frac{1}{g^2} Z^a_{\mu\nu}(f) Z^a_{\mu\nu}(f) + \frac{2}{g} Z^a_{\mu\nu}(f) L^a_{\mu\nu}(f,t) \right.$$
$$\left. + \left[2 Z^a_{\mu\nu}(f) Q^a_{\mu\nu}(f,t) + L^a_{\mu\nu}(f,t) L^a_{\mu\nu}(f,t)\right] + 2 g L^a_{\mu\nu}(f,t) Q^a_{\mu\nu}(f,t) + g^2 Q^a_{\mu\nu}(f,t) Q^a_{\mu\nu}(f,t) \right\} . \quad (12)$$

The effective dynamics of the new degrees of freedom are solved from equation (12). We notice

that the pure f term has a typical non-perturbative behaviour $\left(\frac{1}{g^2}\right)$. It is extremely non-linear because of the $(1+\vec{f}\cdot\vec{f})^{-1}$ factor in A^a_μ and it goes like $(\partial f)^4$.

III. The Confinement Mechanism

Since the effective dynamics of the f^a's suggest non-perturbative effects, we will consider the pure *f* sector first and see if there is/are physical effect(s) associated with it. It is important to emphasize that what will be done here is not the same as doing a background decomposition about a classical configuration where we write

$$A^a_\mu = \tilde{A}^a_\mu + a^a_\mu, \quad (13a)$$

and the action given by

$$S(A) = S_O(\tilde{A}) + \frac{1}{2}\frac{\delta^2 S}{\delta A^2}\bigg|_{A=\tilde{A}} a^2 + \ldots \quad (13b)$$

In section II, we re-expressed A^a_μ in terms of new degrees of freedom and equations (6) and (12) are not in the same "spirit" as equations (13a) and (13b) respectively.

The action for the pure *f* term is given by

$$S_O(f) = \frac{1}{4}\int d^4x\, Z^a_{\mu\nu}(f)\, Z^a_{\mu\nu}(f), \quad (14a)$$

where

$$Z^a_{\mu\nu}(f) = X^{abc}\,\partial_\mu f^b\,\partial_\nu f^c, \quad (14b)$$

$$X^{abc} = \frac{1}{(1+\vec{f}\cdot\vec{f})^2}\Big\{-(1+2\vec{f}\cdot\vec{f})\in^{abc} + 2\,\delta^{ab}f^c - 2\,\delta^{ac}f^b$$

$$+ 3\in^{abd} f^d f^c - 3\in^{acd} f^d f^b + \in^{bcd} f^a f^d\Big\}. \quad (14c)$$

Note that $Z^a_{\mu\nu}$ is proportional to $(\partial f)^2$ and X^{abc} is anti-symmetric w.r.t. the last two indices. The classical equation of motion is

$$\frac{\delta S_O(f)}{\delta f^d(x)} = X^{abc}X^{adc'}\Big\{\quad 2f^b\,\partial_\nu f^c\,\partial_\nu f^{c'} + \partial_\mu f^b\,\partial_\mu\partial_\nu f^c\,\partial_\nu f^{c'}$$

$$+\partial_{\mu}f^{b}\partial_{\nu}f^{c}\partial_{\mu}\partial_{\nu}f^{c'}\Big)+\partial_{\mu}\Big[X^{abc}X^{adc'}\Big]\partial_{\mu}f^{b}\partial_{\nu}f^{c}\partial_{\nu}f^{c'}$$

. (15)

Although equation (15) is a complicated, second-order, non-linear partial differential equation, it is easy to see that all spherically symmetric , are solutions of equation (15). The result follows simply from $\partial_{\mu}f^{a}(x)=\frac{x_{\mu}}{x}\frac{df^{a}}{dx}$ and the fact that X^{abc} is anti-symmetric w.r.t. interchange of the last two indices.

Given that there is not just one classical configuration but a whole class of classical configurations (all spherically symmetric functions), how do we take into account their contributions in physical processes? The simplest idea is to average the contributions of the spherically symmetric solutions using a gaussian distribution functional.

$$P[f]=N(\alpha)\exp\left\{-\alpha\int_{0}^{\infty}ds\,f^{a}(s)f^{a}(s)\right\}, \quad (16)$$

Where $N(\alpha)$ is a normalization factor and a has dimension of (length)$^{-1}$. It is tempting to identify α^{-1} with f^{a}, the length scale introduced in equation (3). However, at this point, we will leave these parameters arbitrary and not even speculate on their relationship with the confinement length scale (about 1 fermi). Anyway, we are only after the general and not the exact confinement behaviour.

We will now compute the gluon propagator . In this computation, it really does not matter if we refer to $\alpha\frac{\pi}{4}$ or A^{a}_{μ} as the gluon because both these fields have divergence proportional to $f^{a}(x)$. Since the spherically symmetric $f^{a}(x)$ is effectively a gaussian white noise, we find

$$\left\langle A^{a}_{\mu}(x)\,A^{b}_{\nu}(0)\right\rangle_{f}=\delta^{ab}\;\delta_{\mu\nu}|x|, \quad (17)$$

where $\langle\ \rangle_{f}$ means averaging using the gaussian distribution functional given by equation (16). Equation (17) follows from the fact that

$$\lim_{x'\to 0}\partial^{x}_{\mu}\partial^{x'}_{\nu}\left\langle A^{a}_{\mu}(x)\,A^{b}_{\nu}(x')\right\rangle_{f}=\lim_{x'\to 0}\left\langle f^{a}(x)f^{b}(x')\right\rangle$$

. (18)

Note the delta function in equation (18) is one-dimensional with the invariant lengths *x* and *x* as arguments. When we take (the instantaneous limit, we find the linear potential in spatial distance.

Another way of getting this result is by first computing the 4-D Euclidean propagator in momentum space. This is given by

$$g_E(k) = \int d^4x \;\; x\, e^{ik\cdot x} \boxed{} \qquad , (19)$$

Computation yields

$$g_E(k) \sim \frac{1}{(\vec{k}\cdot\vec{k} + k_4^2)^{5/2}} \quad , (20a)$$

and analytically continuing to Minkowski space gives

$$g_M(\vec{k}, k_o) \sim \frac{1}{(\vec{k}\cdot\vec{k} - k_o^2)^{5/2}} \quad . (20b)$$

The Minkowski space-time propagator is then given by

$$g_M(\vec{x}, x_o) \sim \int_{-\infty}^{\infty} dk_o \int d^3k \frac{e^{i\vec{k}\cdot\vec{x} - k_o x_o}}{(\vec{k}\cdot\vec{k} - k_o^2)^{5/2}} \quad . (21)$$

Evaluating equation (21) is rather involved (and since not needed for this paper s purpose, will not be evaluated) but the instantaneous gluon propagator $^{(x_o = 0)}$ is rather trivial to compute. The result is

$$g_M(\vec{x}; x_o = 0) \sim \int d^3k \frac{e^{i\vec{k}\cdot\vec{r}}}{(\vec{k}\cdot\vec{k})^2} \quad . (22)$$

Note the characteristic $\frac{1}{|\vec{k}|^4}$ dependence, which signals the linear potential.

Next we consider the Wilson loop given by

$$W = T_r\, P \exp. \left\{ i \oint A\cdot dz \right\}, (23)$$

Where *P* stands for path-ordering. We will use the rectangular path given in the Figure. The vector field $^{A_\mu}$ is given by equation (6) with $^{t^a_\mu = 0}$. We will then take the average of the Wilson loop using the distribution functional given by equation (16), i.e.,

$$\langle W \rangle_f = \int (df^a)\, TrP \exp.\left\{ i \oint A\cdot dz \right\} N(\alpha) \exp.\left\{ -\alpha \int_0^{\infty} ds\, f^a(s) f^a(s) \right\}$$

[illegible] (24)

To evaluate (24), we discretize the paths by dividing ℓ into N components with $\Delta z = \frac{\ell}{N},\ z_o = 0,\ z_j = j\Delta z,\ z_{N+1} = \ell$; and T into $M > N$ components with $\Delta t = \Delta z$ (in natural units, why this assumption is made will become clear later), $t_o = 0,\ t_j = j\Delta z,\ \ t_{M+1} = T$. The opposite sides of the legs on the z and t axes have coordinates given by $s_j = (T^2 + z_j^2)^{1/2}$ and $s'_j = (\ell^2 + t_j^2)$ respectively (see Figure). The path-ordered integral is given by

$$\begin{aligned} P\exp\left\{i\oint A\cdot dz\right\} &= \left[(1+i\,A_3(0)\Delta z)(1+i\,A_3(z_1)\Delta z)\ldots(1+i\,A_3(z_{N-1})\Delta z)(1+i\,A_3(z_N)\Delta z)\right]_{a\to b} \\ &\times\left[(1+i\,A_4(s'_o)\Delta t)(1+i\,A_4(s_2)\Delta t)\ldots(1+i\,A_4(s'_{M-1})\Delta t)(1+i\,A_4(s'_M)\Delta t)\right]_{b\to c} \\ &\times\left[(1+i\,A_3(s_{N+1})\Delta z)(1+i\,A_3(s_N)\Delta z)\ldots.(1+i\,A_3(s_2)\Delta z)(1+i\,A_3(s_1)\Delta z\right]_{c\to d} \\ &\times\left[(1+i\,A_4(T)\Delta t)(1+i\,A_4(t_M)\Delta t)\ldots(1+i\,A_4(t_2)\Delta t)(1+i\,A_4(t_1)\Delta t)\right]_{d\to a} \end{aligned} \quad (25)$$

Figure. The rectangular path used in evaluating the Wilson loop.

Note that the space lattice points have spacing equal to $\Delta z = \frac{\ell}{N}$ while the time lattice points have spacing equal to $\Delta t = \frac{T}{M}$. For convenience in identifying points with the same invariant space-time length, we put $\Delta z(s) = \frac{1}{[\ell^2 - \ell + t_j^2]^{1/2}}$.

In equation (25), we will substitute the expression for $t^a_{\mu's}$ given in equation (6) with $t^a_{\mu} = 0$. The derivatives are given by the difference equations

$$\partial_z f^a(z_j) = \frac{f^a(z_{j+1}) - f(z_j)}{\Delta z}, \quad (26a)$$

$$\partial_z f^a(s_j) = \frac{\partial s_j}{\partial z_j}\frac{\partial f^a(s_j)}{\partial s_j} = \left(\frac{z_j}{s_j}\right)\left(\frac{[f^a(s_{j+1}) - f^a(s_j)]}{\Delta s}\right), \quad (26b)$$

$$\frac{z_j}{s_j}\frac{\Delta z}{\Delta s} = 1. \quad (26c)$$

Now we will see why we put $\Delta z = \Delta t$. Note that all spherically symmetric $f^a(x)$ are classical configurations. On the rectangular lattice, there is an infinite number of points on adjacent sides a ®

b and *d* ® *a*; and *b*® *c* and *c* ® *d* that have the same invariant distance. For easy identification of these points, the *t* and *z* spacings are made equal. These are the lattice points $z_j = t_j; \quad s_j = s'_k$ (with $k = \left(M^2 - N^2 + j^2\right)^{\frac{1}{2}}$).

When we discretize the gaussian functional distribution, we must take into account the fact that $f^a(t_j) = f^a(z_j)$ and $f^a(s_j) = f^a(s'_k)$. This can be done by writing

$$\int_0^\infty ds\, f^a(s)\, f^a(s) = \sum f^a(w_i)\, f^a(w_i)\Delta w \quad , (27)$$

where it is understood that w_i are points on the lattice sites that have the same invariant distances, $\Delta w = [(\Delta z)(\Delta t)]^{1/2}$ and $\alpha = \left(\frac{1}{\ell T}\right)^{1/2}$. In essence, we are using the geometric mean of the parameters to take into account the fact that corresponding points on adjacent legs have the same field f^a.

The lowest, non-trivial contributions, which are of order $(1/g^2)$ come from the following terms:

i.
   i. Two consecutive fields on each of the four segments *a* ® *b*, *b* ® *c*, *c* ® *d* and *d* ® *a*; because of the derivatives $\partial_z f$ and $\partial_t f$.
   ii. Two fields on adjacent sides involving because of spherical *f s*with the same invariant space-time length symmetry.

In the case of (i), we need to evaluate the following four averages: $\left\langle A_3^a(z_j)\, A_3^a(z_{j+1})\right\rangle_f$, $\left\langle A_4^a(s'_j)\, A_4^a(s'_{j+1})\right\rangle_f$, $\left\langle A_3^a(s_{j+1})\, A_3^a(s_j)\right\rangle$ and $\left\langle A_4^a(t_{j+1})\, A_4^a(t_j)\right\rangle_f$. In the case of (ii), the following have non-zero contributions: $\left\langle A_3^a(z_j)\, A_4^a(t_j)\right\rangle_f$, $\left\langle A_3^a(z_j)\, A_4^a(t_{j+1})\right\rangle_f$, $\left\langle A_3^a(z_j)\, A_4^a(t_{j-1})\right\rangle_f$ and similar terms on the adjacent sides *b* ® *c* and *c* ® *d*. Altogether, there are ten (10) averages to compute.

Let us write down two of the typical averages:

$$\left\langle A_3^a(z_j)\, A_3^a(z_{j+1})(\Delta z)^2\right\rangle_f = -\left\langle \frac{1}{(1+\vec{f}\cdot\vec{f})z_j}\, \frac{1}{(1+\vec{f}\cdot\vec{f})z_{j+1}} \Big[f^a(z_{j+1})\, f^a(z_{j+1}) \right.$$
$$\left. + f^a(z_j) f^b(z_j) f^a(z_{j+1}) f^b(z_{j+1}) + \left(f^a(z_{j+1}) f^b(z_{j+1})\right)^2 \right.$$

$$+ f^a(z_j) f^b(z_j) f^a(z_{j+1}) f^b(z_{j+1}) f^c(z_{j+1}) f^c(z_{j+1}) \Big] \Big\rangle_f \text{, (28a)}$$

$$\left\langle A_3^a(z_j) A_4^a(t_j) \Delta z \Delta t \right\rangle_f = -\Big\langle \frac{1}{(1+\vec{f}\cdot\vec{f})z_j} \frac{1}{(1+\vec{f}\cdot\vec{f})t_j} \Big[f^a(z_{j+1}) f^a(t_{j+1})$$
$$+ f^a(z_j) f^b(t_j) + f^a(z_j) f^a(t_j) f^b(z_{j+1}) f^b(t_{j+1})$$
$$+ f^a(z_j) f^b(t_j) f^a(z_j) f^b(t_j) + f^a(z_j) f^b(z_j) f^a(t_{j+1}) f^b(z_{j+1})$$
$$+ f^a(z_j) f^b(z_j) f^b(z_j) f^a(t_j) + f^a(z_j) f^a(t_j) f^b(z_j) f^b(z_{j+1})$$
$$\times f^c(t_j) f^c(t_{j+1}) + f^a(z_j) f^a(t_j) f^b(z_j) f^b(z_j) f^c(t_j) f^c(t_j) \Big] \Big\rangle_f \text{. (28b)}$$

In evaluating equations (28 a,b), we note that all averages except those on specified lattice points will give unity because of the normalization factor. The terms containing the specified lattice points are integrals of the form (after integrating out the angular part) $\frac{1}{N}\int_0^\infty dr \frac{r^{2n} e^{-\sigma r^2}}{(1+r^2)^p}$ where *p* is either 1 or 2 and n = 1, 2, 3, or 4. The normalization *N* is given by $N = \int d^3r\, e^{-\sigma r^2} = \pi^{3/2} \sigma^{-3/2}$.

The integrals can be evaluated using the following integral from Gradsteyn and Ryzhik (page 338)[12]

$$\int_0^\infty \frac{e^{-\sigma r^2}}{r^2 + \beta^2} dr = \left[1 - \Phi(\sigma^{1/2}\beta)\right] \frac{\pi}{2\beta} e^{\beta^2 \sigma} \quad (29)$$

where $\mathrm{Re}\,\beta > 0$ and $\arg \sigma < \frac{\pi}{4}$. The function F is the error function given by

$$\Phi(\sigma^{1/2}\beta) = \frac{2}{\sqrt{\pi}} \int_0^{\beta\pi^{1/2}} e^{-t^2} dt \quad (30)$$

Differentiating equation (29) with respect to s and/or b and then putting b = *1* yield the integrals needed to compute the averages. In the following, we will just show two of the integrals.

$$\int_0^\infty dr \frac{r^2 e^{-\sigma r^2}}{(1+r^2)} = \frac{\pi^{1/2}}{2\sigma^{1/2}} - \frac{\pi}{2} e^{\sigma} (1 - \Phi(\sigma^{1/2})) \quad (31a)$$

$$\int_0^\infty dr\,\frac{r^8\,e^{-\sigma r^2}}{(1+r^2)^2} = \frac{3}{8}\pi^{1/2}\sigma^{-5/2} - \frac{\pi^{1/2}}{2}\sigma^{-3/2} + \frac{3}{2}\pi^{1/2}\sigma^{-1/2}$$
$$+ \frac{\pi^{1/2}}{2}\sigma^{1/2} - \frac{\pi}{4}e^{\sigma}(7+2\sigma)\left[1-\Phi(\sigma^{1/2})\right]. \quad (31b)$$

From the distribution functional, we read $\sigma = \omega\Delta w = \left(\frac{\Delta z\Delta t}{\ell T}\right)^{1/2}$. Note that this term is proportional to $\left(\frac{1}{NM}\right)^{1/2}$, thus, it is an infinitesimal number and a good expansion parameter. The last ingredient needed is the expansion of the error function

$$\Phi(\sigma^{1/2}) = \frac{2}{\sqrt{\pi}}\left(\sigma^{1/2} - \frac{1}{3}\sigma^{3/2} + \frac{1}{10}\sigma^{5/2} - \ldots\right). \quad (32)$$

Using all these in the computations, we find a series expansion in terms of s , with the dominant term being those with negative powers of s . The results are:

$$\left\langle A_3^a(z_j)A_3^a(z_{j+1})(\Delta z)^2\right\rangle_f = -\frac{1}{2}\sigma^{-1} - 2 + \frac{4}{3}\sigma + \ldots \quad , (33a)$$

$$\left\langle A_3^a(z_j)A_4^a(z_j)(\Delta z)(\Delta t)\right\rangle_f = 2\sigma^{-1} - 2 + \frac{10}{3}\pi^{1/2}\sigma^{1/2} - \frac{29}{3}\sigma + \ldots \quad (33b)$$

$$\left\langle A_3^a(z_j)A_4^a(t_{j-1})\Delta z\Delta t\right\rangle_f = \left\langle A_3^a(z_j)A_4^a(t_{j+1})\Delta z\Delta t\right\rangle_f = -\frac{1}{2}\sigma^{-1} - 2 + \frac{2}{3}\sigma + \ldots \quad (33c)$$

Equation (33a) gives also the results for the consecutive points in the other three segments *b*® *c*; *c*® *d*; and *d* ® *a*. Equations (33b) and (33c) also give the same results for points on segments b ® c and c ® d that have the same invariant distance.

When we sum the contributions, we must note that the consecutive points on sides *a*® *b* and *c*® *d* total *(N-1) » N* each. Those on sides b ® c and d ® a (time path) give *(M-1) » M* for each. The contributions from adjacent legs with the same invariant distance total *N* for each type of term. Taking all these into account, we get

$$\left\langle T_r P\, e^{i\oint A\cdot dz}\right\rangle_f^{Euc} = 1 - \frac{1}{g^2}\left\{\left[-\frac{1}{2}\sigma^{-1} - 2 + \frac{4}{3}\sigma + \ldots\right](2N+2M)\right.$$
$$\left. + \left[2\sigma^{-1} - 2 + \frac{10}{3}\pi^{1/2}\sigma^{1/2} - \frac{29}{3}\sigma + \ldots\right]2N + \left[-\frac{1}{2}\sigma^{-1} - 2 + \frac{2}{3}\sigma + \ldots\right]4N\right\}$$
$$= 1 - \frac{1}{g^2}\left\{-\sigma^{-1}M + \sigma^{-1}N + \ldots\right\} \quad , (34)$$

where the superscript Euc means Euclidean space. Rotating this to Minkowski space, the time component *(~M)* will acquire an extra (-) sign and thus, we get the leading term of the Wilson loop

$$\left\langle T_r P\, e^{i\oint A\cdot dz}\right\rangle_f^{Min} = \exp.\left\{-\frac{1}{g^2}\left(\frac{\Delta x\Delta t}{\ell T}\right)^{-1/2}(M+N)\right\}. \quad (35)$$

Strictly speaking, since our computation is relativistic, albeit in Euclidean space, ℓ must be equal to T (in natural units) or equivalently M = N. This will give $M+N=(MN)^{1/2}=2\left(\frac{\ell T}{\Delta z\,\Delta t}\right)^{1/2}$ and the Wilson loop averaged over all classical configurations has an area law behaviour. However, to compare with lattice computations where M is typically equal to N or at most 2N, we find that $M+N\approx(MN)^{1/2}=\left(\frac{\ell T}{\Delta z\,\Delta t}\right)^{1/2}$, still resulting in an area law behaviour for the Wilson loop.

IV . **Conclusion**

We have exhibited the possible confinement mechanism in SU(2) Yang-Mills theory. We have shown that the instantaneous gluon propagator behaves linearly with spatial distance and the Wilson loop has an area law dependence.

It is clear from the paper that the confinement mechanism is not due to quantum mechanics about a classical configuration but by treating statistically a class of classical configurations (spherically symmetric) of the field f^a. The logical question to ask is why is confinement transparent in the non-linear gauge condition defined in equation (1) and not in the usual linear gauges. The reason is simple, linuar gauges are i~sufficient gaugu-fixing conditions because they miss completely the fiel configurations on the horizons on each of the parallel surface $\partial\cdot A^a = f^a$. As conjectured in reference (11) and shown in this paper, the physics of these field configurations are non-perturbative. In particular, the statistical treatment of these classical configurations may be responsible for confinement.

The possible confinement mechanism exhibited in this paper is similar but not exactly the same as the "Copenhagen vacuum" (see Olesen, reference 8). Both make use of random fields. However, whereas the "Copenhagen vacuum s" random fluctuations (chromoelectric/magnetic) are quantum in nature, the random fields in this paper are purely classical. Also, since the random fields in this paper are scalars, it may possibly lead to the realization of the Parisi-Sourlas mechanism in QCD. The derivation of the linear potential (the "Coulomb" potential in 1+1 space-time) clearly hints of dimensional reduction.

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