 Communications in Theoretical Physics

Commun. Theor. Phys. **72** (2020) 015001 (8pp) https://doi.org/10.1088/1572-9494/ab5448

Integrable boundary conditions for the B_2 model*

Panpan Xue[1], **Guang-Liang Li**[1], **Junpeng Cao**[2,3,4,8], **Kun Hao**[5,6], **Tao Yang**[5,6,7], **Wen-Li Yang**[5,6,7,8] **and Kangjie Shi**[5]

[1] Shaanxi Province Key Laboratory of Quantum Information and Quantum Optoelectronic Devices, Xian Jiaotong University, Xian 710049, China
[2] Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China
[3] School of Physical Sciences, University of Chinese Academy of Sciences, Beijing, China
[4] Songshan Lake Materials Laboratory, Dongguan, Guangdong 523808, China
[5] Institute of Modern Physics, Northwest University, Xian 710069, China
[6] Shaanxi Key Laboratory for Theoretical Physics Frontiers, Xian 710069, China
[7] School of Physics, Northwest University, Xian 710069, China

E-mail: junpengcao@iphy.ac.cn and wlyang@nwu.edu.cn

Received 16 September 2019, revised 19 October 2019
Accepted for publication 31 October 2019
Published 31 December 2019

Abstract

New integrable B_2 model with off-diagonal boundary reflections is proposed. The general solutions of the reflection matrix for the B_2 model are obtained by using the fusion technique. We find that the reflection matrix has 7 free boundary parameters, which are used to describe the degree of freedom of boundary couplings, without breaking the integrability of the system. The new quantization conditions will induce the novel structure of the energy spectrum and the boundary states. The corresponding boundary effects can be studied based on the results in this paper. Meanwhile, the reflection matrix of high rank models associated with B_n algebra can also be obtained by using the method suggested in this paper.

Keywords: Bethe Ansatz, lattice integrable models, Yang-Baxter equation, reflection equation

1. Introduction

The exactly solvable model has many applications in condensed matter physics, theoretical and mathematical physics [1, 2]. The typical quantization conditions of the energy spectrum is the periodic boundary conditions [3, 4]. In 1988, Sklyanin proposed the reflection matrix and reflection equations to describe the integrable open boundary conditions [5]. From then on, the exactly solvable models with boundary reflection have attracted a lot of interest.

The integrable quantization conditions are very important. Many interesting phenomena such as the boundary bound states and novel elementary excitations are found in the integrable models with open boundary conditions. For this purpose, the first step during the study is to obtain the solution of the reflection equation. From the reflection matrix, one can construct the corresponding exactly solvable models. Meanwhile, if the reflection matrix has the off-diagonal elements, the $U(1)$-symmetry of the system is broken and the number of particles with an intrinsic degree of freedom are not conserved, which induces many interesting phenomena such as the spin-flipped effect and helical elementary excitations. Motivated by these aspects, many works have been done on the solving the reflection equation [6–14].

The B_n vertex model can be related to the $O(2n+1)$ quantum spin chain and have many applications in the $O(2n+1)$-sigma models. For an example, the $O(3)$ vertex model is equivalent to the isotropic spin-1 XXX model or the $O(3)$-sigma model, which has been well studied. For the higher

* Supported by the National Program for Basic Research of MOST (Grant Nos. 2016YFA0300600 and 2016YFA0302104), the National Natural Science Foundation of China (Grant Nos. 11934015, 11975183,11547045, 11774397, 11775178, 11775177 and 91536115), Australian Research Council (Grant No. DP 190101529), the Major Basic Research Program of Natural Science of Shaanxi Province (Grant Nos. 2017KCT-12, 2017ZDJC-32), and the Double First-Class University Construction Project of Northwest University.
[8] Authors to whom any correspondence should be addressed.

rank case, the model with period and diagonal boundary have been solved by using the analytical or algebraic Bethe ansatz method. However, the integrable B_n open chain associated with off-diagonal reflection matrices has more integrable boundary interactions terms and the corresponding eigen-states involve more structures. Therefore, it is interesting to investigate the most generic (or off-diagonal) reflection matrices of the B_n models.

In this paper, we study the integrable quantization conditions of the B_2 model by using the method of fusion. Starting from the fundamental spinorial representation, we obtain the general solutions of reflection equations. We find that the reflection matrix has 7 free boundary parameters, which are used to describe the degree of freedom of boundary couplings, without breaking the integrability of the system. Based on them, we give the model Hamiltonian of integrable B_2 model with off-diagonal boundary reflections. The new quantization conditions will induce the novel structure of the energy spectrum.

The advantage of method suggested in this paper is as follows. The symmetry of the system can not be broken by the fusion. Thus, the number of free boundary parameters remain unchanged during the fusion. The number can be calculated easily from the fundamental representation because the dimension of the fundamental one is lower. All the algebras of the fused representations are the same. Second, all the parameterizations are uniform by using the fusion. Third, the fusion can also be used to solve the energy spectrum. For example, the fusion can supply the closed operator production identities of the fused transfer matrices, which are necessary to construct the $T-Q$ relations. Forth, the results can be generalized to the B_n case easily.

This paper is organized as follows. In section 2, we introduce the spinorial R-matrix of the B_2 model. The general solution of the spinorial reflection equation is also obtained. In section 3, we generate the vectorial R-matrix of the B_2 model by using the fusion. In section 4, we calculate the general solution of the vectorial reflection matrix. In section 5, we discuss the degenerate cases. In section 6, we construct the integrable model Hamiltonian based on all the above ingredients. Concluding remarks are given in section 7.

2. Spinorial representation

The fundamental R-matrix of the B_2 model is the spinorial one defined in the space $V_{1'}\otimes V_{2'}$ with the form [15]

$$\begin{aligned}
R^{ss}_{1'2'}(u)^{ii}_{ii} &= a_2(u) = \left(u+\frac{1}{2}\right)\left(u+\frac{3}{2}\right),\\
R^{ss}_{1'2'}(u)^{ij}_{ij} &= b_2(u) = u\left(u+\frac{3}{2}\right), \quad i\neq j,\bar{j},\\
R^{ss}_{1'2'}(u)^{i\bar{i}}_{\bar{i}i} &= c_2(u) = u+\frac{3}{4},\\
\xi_i\xi_{\bar{j}}R^{ss}_{1'2'}(u)^{i\bar{i}}_{j\bar{j}} &= d_2(u) = -\frac{u}{2}, \quad i\neq j,\bar{j},\\
R^{ss}_{1'2'}(u)^{i\bar{i}}_{i\bar{i}} &= e_2(u) = u(u+1),\\
R^{ss}_{1'2'}(u)^{ij}_{ji} &= g_2(u) = \frac{u}{2}+\frac{3}{4}, \quad i\neq j,\bar{j},
\end{aligned} \tag{1}$$

where $\{i,j\}=\{1,2,3,4\}$, $i+\bar{i}=5$, $\xi_i=1$ if $i\in\{1,2\}$ and $\xi_i=-1$ if $i\in\{3,4\}$. The $R^{ss}_{1'2'}$ matrix has the following properties

$$\begin{aligned}
regularity: R^{ss}_{1'2'}(0) &= \rho_1(0)^{\frac{1}{2}}\mathcal{P}_{1'2'},\\
unitarity: R^{ss}_{1'2'}(u)R^{ss}_{2'1'}(-u) &= \rho_1(u),\\
crossing\ unitarity: R^{ss}_{1'2'}(u)^{t_1}R^{ss}_{2'1'}(-u-3)^{t_1} &= \rho_1\left(u+\frac{3}{2}\right),
\end{aligned}$$

where $\rho_1(u)=\left(u^2-\frac{1}{4}\right)\left(u^2-\frac{9}{4}\right)$, $\mathcal{P}_{1'2'}$ is the exchange operator with the definition $[\mathcal{P}_{1'2'}]^{ij}_{kl}=\delta_{il}\delta_{jk}$, $\{i,j,k,l\}=\{1,2,3,4\}$ and t_i denotes the transposition in the space V_i. The spinorial R-matrix satisfies the Yang-Baxter equation

$$\begin{aligned}
&R^{ss}_{1'2'}(u-v)R^{ss}_{1'3'}(u)R^{ss}_{2'3'}(v)\\
&\quad = R^{ss}_{2'3'}(v)R^{ss}_{1'3'}(u)R^{ss}_{1'2'}(u-v).
\end{aligned} \tag{2}$$

In order to describe the boundary reflection, we should introduce the reflection matrix $K^{s-}(u)$ defined in the auxiliary space. The spinorial R-matrix (1) and the reflection equation $K^{s-}(u)$ satisfy the reflection equation

$$\begin{aligned}
&R^{ss}_{1'2'}(u-v)K^{s-}_{1'}(u)R^{ss}_{2'1'}(u+v)K^{s-}_{2'}(v)\\
&\quad = K^{s-}_{2'}(v)R^{ss}_{1'2'}(u+v)K^{s-}_{1'}(u)R^{ss}_{2'1'}(u-v),
\end{aligned} \tag{3}$$

where $K^{s-}_{1'}(u)=K^{s-}(u)\otimes id$, $K^{s-}_{2'}(u)=id\otimes K^{s-}(u)$ and id means the unitary matrix. we get a K^s-matrix:

$$K^{s-}(u)=\begin{pmatrix} 1-\zeta u & c_1u & c_2u & c_3u\\ c_4u & 1-\eta u & c_7u & -c_2u\\ c_5u & c_8u & 1+\eta u & c_1u\\ c_6u & -c_5u & c_4u & 1+\zeta u\end{pmatrix}, \tag{4}$$

where η, ζ and c_i, $(i=1,2,\cdots,8)$ are the boundary parameters with the constrains

$$\begin{aligned}
c_2c_8 &= c_3c_5+c_1(\zeta+\eta),\\
c_7c_1 &= -c_3c_4+c_2(\zeta-\eta),\\
c_2c_6 &= c_5c_7-c_4(\zeta+\eta).
\end{aligned} \tag{5}$$

The dual reflection matrix K^{s+} is constructed by the reflection matrix K^{s-} as

$$K^{s+}(u)=K^{s-}\left(-u-\frac{3}{2}\right)|_{\zeta,\eta,c_i\to\tilde{\zeta},\tilde{\eta},\tilde{c}_i}, \tag{6}$$

where $\tilde{\eta}$, $\tilde{\zeta}$ and $\tilde{c}_i$, $(i=1,2,\cdots,8)$ are the boundary parameters characterizing the boundary couplings at the other side. They are not independent and satisfy the constrains

$$\begin{aligned}
\tilde{c}_2\tilde{c}_8 &= \tilde{c}_3\tilde{c}_5+\tilde{c}_1(\tilde{\zeta}+\tilde{\eta}),\\
\tilde{c}_7\tilde{c}_1 &= -\tilde{c}_3\tilde{c}_4+\tilde{c}_2(\tilde{\zeta}-\tilde{\eta}),\\
\tilde{c}_2\tilde{c}_6 &= \tilde{c}_5\tilde{c}_7-\tilde{c}_4(\tilde{\zeta}+\tilde{\eta}).
\end{aligned} \tag{7}$$

The dual reflection matrix satisfies the dual reflection equation

$$R^{ss}_{1'2'}(-u+v)K^{s+}_{1'}(u)R^{ss}_{2'1'}(-u-v-3)K^{s+}_{2'}(v)$$
$$=K^{s+}_{2'}(v)R^{ss}_{1'2'}(-u-v-3)K^{s+}_{1'}(u)R^{ss}_{2'1'}(-u+v), \quad (8)$$

where $K^{s+}_{1'}(u)=K^{s+}(u)\otimes id$ and $K^{s+}_{2'}(u)=id\otimes K^{s+}(u)$.

Next, we consider the degenerate cases of the general solutions (4) and (6). If the boundary parameters

$$c_1=c_2=\cdots=c_8=0,\quad \zeta=\eta,$$
$$\tilde{c}_1=\tilde{c}_2=\cdots=\tilde{c}_8=0,\quad \tilde{\zeta}=\tilde{\eta}, \quad (9)$$

the reflection matrices (A.1) and (6) degenerate into the diagonal ones

$$\bar{K}^{s-}(u)=diag[\zeta'-u,\,\zeta'-u,\,\zeta'+u,\,\zeta'+u],$$
$$\bar{K}^{s+}(u)=diag\left[\tilde{\zeta}'+u+\frac{3}{2},\,\tilde{\zeta}'+u+\frac{3}{2},\,\tilde{\zeta}'-u-\frac{3}{2},\,\tilde{\zeta}'-u-\frac{3}{2}\right], \quad (10)$$

where $\zeta'=\zeta^{-1}$ and $\tilde{\zeta}'=\tilde{\zeta}^{-1}$.

The second degenerate case is that the reflection matrices (4) and (6) have the off-diagonal constant solution. Rewrite reflection matrices (4) as

$$K^{s-}(u)=\frac{1}{u}\tilde{K}^{s-}(u). \quad (11)$$

Taking the limit of $u\to\infty$ and neglecting the common factor of the reflection matrix, we obtain a constant reflection matrix

$$\tilde{K}^{s-}=\begin{pmatrix}-\hat{\zeta} & \hat{c}_1 & \hat{c}_2 & \hat{c}_3\\ \hat{c}_4 & -\hat{\eta} & \hat{c}_7 & -\hat{c}_2\\ \hat{c}_5 & \hat{c}_8 & \hat{\eta} & \hat{c}_1\\ \hat{c}_6 & -\hat{c}_5 & \hat{c}_4 & \hat{\zeta}\end{pmatrix}, \quad (12)$$

where $\hat{\eta}$, $\hat{\zeta}$ and $\hat{c}_i$, $(i=1,2,\cdots,8)$ are the new parameterization of boundary parameters with constrains

$$\hat{c}_2\hat{c}_8=\hat{c}_3\hat{c}_5+\hat{c}_1(\hat{\zeta}+\hat{\eta}),$$
$$\hat{c}_7\hat{c}_1=-\hat{c}_3\hat{c}_4+\hat{c}_2(\hat{\zeta}-\hat{\eta}),$$
$$\hat{c}_2\hat{c}_6=\hat{c}_5\hat{c}_7-\hat{c}_4(\hat{\zeta}+\hat{\eta}). \quad (13)$$

The dual transformation between $\tilde{K}^{s+}$ and $\tilde{K}^{s-}$ remains unchanged,

$$\tilde{K}^{s+}=\tilde{K}^{s-}|_{\hat{\zeta},\hat{\eta},\hat{c}_i\to\tilde{\hat{\zeta}},\tilde{\hat{\eta}},\tilde{\hat{c}}_i}. \quad (14)$$

We note that the number of free parameter in (12) and (14) is 7.

3. Fusion

In this section, we consider the fusion [16–23], which can be carried out because the spinorial R-matrix (1) degenerates into the projector operator at the point of $u=-\frac{1}{2}$,

$$R^{ss}_{1'2'}\left(-\frac{1}{2}\right)=P^{ss(5)}_{1'2'}\times S, \quad (15)$$

where S is a constant matrix (we omit its expression because we do not need it), and $P^{ss(5)}_{1'2'}$ is the 5-dimensional projector

$$P^{ss(5)}_{1'2'}=\sum_{i=1}^{5}|\tilde{\psi}_i\rangle\langle\tilde{\psi}_i|, \quad (16)$$

with

$$|\tilde{\psi}_1\rangle=\frac{1}{\sqrt{2}}(|12\rangle-|21\rangle),\quad |\tilde{\psi}_2\rangle=\frac{1}{\sqrt{2}}(|31\rangle-|13\rangle),$$
$$|\tilde{\psi}_3\rangle=\frac{1}{2}(|14\rangle-|41\rangle+|23\rangle-|32\rangle),$$
$$|\tilde{\psi}_4\rangle=\frac{1}{\sqrt{2}}(|24\rangle-|42\rangle),\quad |\tilde{\psi}_5\rangle=\frac{1}{\sqrt{2}}(|34\rangle-|43\rangle).$$

Taking fusion in the quantum space, we have

$$P^{ss(5)}_{2'3'}R^{ss}_{1'2'}\left(u+\frac{1}{2}\right)R^{ss}_{1'3'}(u)P^{ss(5)}_{2'3'}$$
$$=u(u+1)(u+2)R^{sv}_{1'\langle 2'3'\rangle}\left(u+\frac{1}{4}\right). \quad (17)$$

Denote $\langle 1'2'\rangle\equiv 1$ and the dimension of fused space V_1 is 5. The fused matrix $R^{sv}_{1'2}$ has the following properties

$$R^{sv}_{1'2}(u)R^{vs}_{21'}(-u)=\rho_2(u),$$
$$R^{sv}_{1'2}(u)^{t_1}R^{vs}_{21'}(-u-3)^{t_1}=\rho_2\left(u+\frac{3}{2}\right),$$
$$R^{ss}_{1'2'}(u-v)R^{sv}_{1'3}(u)R^{sv}_{2'3}(v)=R^{sv}_{2'3}(v)R^{sv}_{1'3}(u)R^{ss}_{1'2'}(u-v), \quad (18)$$

where $\rho_2(u)=\frac{25}{16}-u^2$.

Next, taking the fusion in the auxiliary space, we obtain the vectorial R-matrix of B_2 model

$$P^{ss(5)}_{1'2'}R^{sv}_{2'3}(u)R^{sv}_{1'3}\left(u-\frac{1}{2}\right)P^{ss(5)}_{1'2'}=R^{vv}_{\langle 1'2'\rangle 3}\left(u-\frac{1}{4}\right), \quad (19)$$

which is defined in the 5-dimensional auxiliary space and 5-dimensional quantum space. Detailed calculation shows that the elements of the vectorial R-matrix read [24]

$$R^{vv}_{12}(u)^{ij}_{kl}=u\left(u+\frac{3}{2}\right)\delta_{ik}\delta_{jl}+\left(u+\frac{3}{2}\right)\delta_{il}\delta_{jk}-u\delta_{j\bar{i}}\delta_{k\bar{l}}, \quad (20)$$

where $\{i,j,k,l\}=\{1,2,3,4,5\}$ and $i+\bar{i}=6$. The vectorial R-matrix has the following properties

$$regularity: R^{vv}_{12}(0)=\rho_3(0)^{\frac{1}{2}}\mathcal{P}_{12},$$
$$unitarity: R^{vv}_{12}(u)R^{vv}_{21}(-u)=\rho_3(u),$$
$$crossing\ unitarity: R^{vv}_{12}(u)^{t_1}R^{vv}_{21}(-u-3)^{t_1}=\rho_3\left(u+\frac{3}{2}\right),$$

where $\rho_3(u)=(u^2-1)\left(u^2-\frac{9}{4}\right)$. The R_{12}^{vv} matrix satisfies the Yang-Baxter equation

$$\begin{aligned}&R_{1'2}^{sv}(u-v)R_{1'3}^{sv}(u)R_{23}^{vv}(v)=R_{23}^{vv}(v)R_{1'3}^{sv}(u)R_{1'2}^{sv}(u-v),\\&R_{12}^{vv}(u-v)R_{13}^{vv}(u)R_{23}^{vv}(v)=R_{23}^{vv}(v)R_{13}^{vv}(u)R_{12}^{vv}(u-v).\end{aligned}\tag{21}$$

4. General solution of vectorial reflection matrix

Now, it is clear that the vectorial R-matrix (20) of B_2 model can be obtained by the spinorial one (1) by using the fusion. The integrable model Hamiltonian is constructed by the vectorial representation. For the open boundary case, we should introduce the vectorial reflection matrix K^{v-}, which satisfies the reflection equations

$$\begin{aligned}&R_{1'2}^{sv}(u-v)K_{1'}^{s-}(u)R_{21'}^{vs}(u+v)K_2^{v-}(v)\\&=K_2^{v-}(v)R_{1'2}^{sv}(u+v)K_{1'}^{s-}(u)R_{21'}^{vs}(u-v),\\&R_{12}^{vv}(u-v)K_1^{v-}(u)R_{21}^{vv}(u+v)K_2^{v-}(v)\\&=K_2^{v-}(v)R_{12}^{vv}(u+v)K_1^{v-}(u)R_{21}^{vv}(u-v).\end{aligned}\tag{22}$$

The general solution of the reflection matrix can be obtained by solving the above reflection equations directly. In this paper, we use the fusion technique to obtain the solution. We first note that the dimension of the auxiliary space of the vectorial R-matrix is 5, thus the corresponding reflection matrix defined in the auxiliary space should be 5×5. That is to say, we need to fuse the 4×4 spinorial reflection matrices $K^{s\pm}$ into the 5×5 vectorial reflection matrices $K^{v\pm}$.

The vectorial reflection matrix K^{v-} can be obtained by taking the fusion in the auxiliary space,

$$\begin{aligned}&P_{1'2'}^{5}K_{2'}^{s-}\left(u+\frac{1}{4}\right)R_{1'2'}^{ss}(2u)K_{1'}^{s-}\left(u-\frac{1}{4}\right)P_{1'2'}^{5}\\&=\left(u-\frac{1}{4}\right)\left(u+\frac{3}{4}\right)K_{\langle 1'2'\rangle}^{v-}(u).\end{aligned}\tag{23}$$

Substituting the general solution of spinorial reflection matrices K^{s-} with the parameterization (A.20) into (23) and after some calculations, we obtain the explicit matrix form of K^{v-} as

$$K^{v-}(u)=\begin{pmatrix}K_{11}^{v-}(u)&K_{12}^{v-}(u)&K_{13}^{v-}(u)&K_{14}^{v-}(u)&K_{15}^{v-}(u)\\K_{21}^{v-}(u)&K_{22}^{v-}(u)&K_{23}^{v-}(u)&K_{24}^{v-}(u)&K_{25}^{v-}(u)\\K_{31}^{v-}(u)&K_{32}^{v-}(u)&K_{33}^{v-}(u)&K_{34}^{v-}(u)&K_{35}^{v-}(u)\\K_{41}^{v-}(u)&K_{42}^{v-}(u)&K_{43}^{v-}(u)&K_{44}^{v-}(u)&K_{45}^{v-}(u)\\K_{51}^{v-}(u)&K_{52}^{v-}(u)&K_{53}^{v-}(u)&K_{54}^{v-}(u)&K_{55}^{v-}(u)\end{pmatrix},\tag{24}$$

where

$$\begin{aligned}K_{11}^{v-}(u)&=-\frac{1}{8}\{32(\eta u+\zeta u-1)+(4u+1)\\&\quad\times[c_3c_6+c_7c_8+\eta^2+\zeta^2\\&\quad+2c_2c_5-8\eta\zeta u+2c_1c_4(4u+1)]\},\\K_{12}^{v-}(u)&=u[(c_2c_4+c_7\zeta)(4u+1)-4c_7],\\K_{13}^{v-}(u)&=\frac{u}{\sqrt{2}}[(c_1c_7-c_3c_4+c_2\zeta+c_2\eta)(4u+1)-8c_2],\\K_{14}^{v-}(u)&=u[(c_3\eta-c_1c_2)(4u+1)-4c_3],\\K_{15}^{v-}(u)&=-(c_2^2+c_3c_7)u(4u+1),\\K_{21}^{v-}(u)&=u[(c_1c_5+c_8\zeta)(4u+1)-4c_8],\\K_{22}^{v-}(u)&=-\frac{1}{8}\{32(\zeta u-\eta u-1)+(4u+1)\\&\quad\times[c_3c_6+c_7c_8+\eta^2+\zeta^2\\&\quad+2c_1c_4+8\eta\zeta u+2c_2c_5(4u+1)]\},\\K_{23}^{v-}(u)&=\frac{u}{\sqrt{2}}[(c_3c_5+c_2c_8+c_1\zeta-c_1\eta)(4u+1)-8c_1],\\K_{24}^{v-}(u)&=(c_3c_8-c_1^2)u(4u+1),\\K_{25}^{v-}(u)&=u[(c_3\eta-c_1c_2)(4u+1)+4c_3],\\K_{31}^{v-}(u)&=\frac{u}{\sqrt{2}}[(c_4c_8-c_1c_6+c_5\zeta+c_5\eta)(4u+1)-8c_5],\\K_{32}^{v-}(u)&=\frac{u}{\sqrt{2}}[(c_2c_6+c_5c_7+c_4\zeta-c_4\eta)(4u+1)-8c_4],\\K_{33}^{v-}(u)&=-\frac{1}{8}[(c_7c_8+c_3c_6+\zeta^2+\eta^2)(4u+1)^2\\&\quad-2(c_1c_4+c_2c_5)(16u^2-1)-32],\\K_{34}^{v-}(u)&=\frac{u}{\sqrt{2}}[(c_3c_5+c_2c_8+c_1\zeta-c_1\eta)(4u+1)+8c_1],\\K_{35}^{v-}(u)&=\frac{u}{\sqrt{2}}[(c_1c_7-c_3c_4+c_2\zeta+c_2\eta)(4u+1)+8c_1],\\K_{41}^{v-}(u)&=u[(c_6\eta-c_4c_5)(4u+1)-4c_6],\\K_{42}^{v-}(u)&=(c_6c_7-c_4^2)u(4u+1),\\K_{43}^{v-}(u)&=\frac{u}{\sqrt{2}}[(c_2c_6+c_5c_7+c_4\zeta-c_4\eta)(4u+1)+8c_4],\\K_{44}^{v-}(u)&=-\frac{1}{8}\{32(\eta u-\zeta u-1)+(4u+1)\\&\quad\times[c_3c_6+c_7c_8+\eta^2+\zeta^2\\&\quad+2c_1c_4+8\eta\zeta u+2c_2c_5(4u+1)]\},\\K_{45}^{v-}(u)&=u[(c_2c_4+c_7\zeta)(4u+1)+4c_7],\\K_{51}^{v-}(u)&=-(c_6c_8+c_5^2)u(4u+1),\\K_{52}^{v-}(u)&=u[(c_6\eta-c_4c_5)(4u+1)+4c_6],\\K_{53}^{v-}(u)&=\frac{u}{\sqrt{2}}[(c_4c_8-c_1c_6+c_5\zeta+c_5\eta)(4u+1)+8c_5],\\K_{54}^{v-}u)&=u[(c_1c_5+c_8\zeta)(4u+1)+4c_8],\\K_{55}^{v-}(u)&=-\frac{1}{8}\{-32(\eta u+\zeta u+1)\\&\quad+(4u+1)[c_3c_6+c_7c_8+\eta^2+\zeta^2\\&\quad+2c_2c_5-8\eta\zeta u+2c_1c_4(4u+1)]\}.\end{aligned}\tag{25}$$

The corresponding dual reflection matrix is

$$K^{v+}(u) = K^{v-}\left(-u - \frac{3}{2}\right)|_{\zeta,\eta,c_i\to\tilde{\zeta},\tilde{\eta},\tilde{c}_i}, \tag{26}$$

which satisfies the dual reflection equations

$$\begin{aligned}
&R^{sv}_{1'2}(-u+v)K^{s+}_{1'}(u)R^{vs}_{21'}(-u-v-3)K^{v+}_2(v)\\
&\quad= K^{v+}_2(v)R^{sv}_{1'2}(-u-v-3)K^{s+}_{1'}(u)R^{vs}_{21'}(-u+v),\\
&R^{vv}_{12}(-u+v)K^{v+}_1(u)R^{vv}_{21}(-u-v-3)K^{v+}_2(v)\\
&\quad= K^{v+}_2(v)R^{vv}_{12}(-u-v-3)K^{v+}_1(u)R^{vv}_{21}(-u+v).
\end{aligned} \tag{27}$$

5. Degenerate cases

Now, we consider some special cases of the general solutions (24) and (26). If the boundary parameters satisfy

$$\begin{aligned}
&c_1 = c_2 = \cdots = c_8 = 0, \quad \zeta = \eta,\\
&\tilde{c}_1 = \tilde{c}_2 = \cdots = \tilde{c}_8 = 0, \quad \tilde{\zeta} = \tilde{\eta},
\end{aligned} \tag{28}$$

the reflection matrices (24) and (26) degenerate into the diagonal ones [25]

$$\begin{aligned}
\bar{K}^{v-}(u) = diag[&(\zeta'+1-4u)(\zeta'-1-4u),\\
&(\zeta'+1+4u)(\zeta'-1-4u),\\
&(\zeta'+1+4u)(\zeta'-1-4u),\\
&(\zeta'+1+4u)(\zeta'-1-4u),\\
&(\zeta'+1+4u)(\zeta'-1+4u)],
\end{aligned} \tag{29}$$

$$\begin{aligned}
\bar{K}^{v+}(u) = diag[&(\tilde{\zeta}'+7+4u)(\tilde{\zeta}'+5+4u),\\
&(\tilde{\zeta}'-5-4u)(\tilde{\zeta}'+5+4u),\\
&(\tilde{\zeta}'-5-4u)(\tilde{\zeta}'+5+4u),\\
&(\tilde{\zeta}'-5-4u)(\tilde{\zeta}'+5+4u),\\
&(\tilde{\zeta}'-7-4u)(\tilde{\zeta}'-5-4u)],
\end{aligned} \tag{30}$$

where $\zeta' = \zeta^{-1}$ and $\tilde{\zeta}' = \tilde{\zeta}^{-1}$.

For the constant solution (12) of spinorial reflection matrix, the fusion relation (23) degenerates

$$\begin{aligned}
&P^5_{1'2'}\tilde{K}^{s-}_{2'}\left(u+\frac{1}{4}\right)R^{ss}_{1'2'}(2u)\tilde{K}^{s-}_{1'}\left(u-\frac{1}{4}\right)P^5_{1'2'}\\
&\quad=\left(u+\frac{3}{4}\right)\tilde{K}^{v-}_{\langle 1'2'\rangle}(u),
\end{aligned} \tag{31}$$

which gives another degenerated vectorial reflection matrix $\tilde{K}^{v-}$ with the elements

$$\begin{aligned}
\tilde{K}^{v-}_{11}(u) &= \frac{1}{2}[-2\hat{c}_2\hat{c}_5 - \hat{c}_7\hat{c}_8 - \hat{c}_3\hat{c}_6\\
&\quad - 2\hat{c}_1\hat{c}_4(4u+1) - \hat{\zeta}^2 + 8u\hat{\zeta}\hat{\eta} - \hat{\eta}^2],\\
\tilde{K}^{v-}_{12}(u) &= 4u(\hat{c}_2\hat{c}_4 + \hat{c}_7\hat{\zeta}),\\
\tilde{K}^{v-}_{13}(u) &= -2\sqrt{2}u[\hat{c}_3\hat{c}_4 - \hat{c}_1\hat{c}_7 - \hat{c}_2(\hat{\zeta}+\hat{\eta})],\\
\tilde{K}^{v-}_{14}(u) &= -4u(-\hat{c}_3\hat{\eta} + \hat{c}_1\hat{c}_2),\\
\tilde{K}^{v-}_{15}(u) &= -4u(\hat{c}_2^2 + \hat{c}_3\hat{c}_7),\\
\tilde{K}^{v-}_{21}(u) &= 4u(\hat{c}_1\hat{c}_5 + \hat{c}_8\hat{\zeta}),\\
\tilde{K}^{v-}_{22}(u) &= \frac{1}{2}[-2\hat{c}_1\hat{c}_4 - \hat{c}_7\hat{c}_8 - \hat{c}_3\hat{c}_6\\
&\quad - 2\hat{c}_2\hat{c}_5(4u+1) - \hat{\zeta}^2 - 8u\hat{\zeta}\hat{\eta} - \hat{\eta}^2],\\
\tilde{K}^{v-}_{23}(u) &= 2\sqrt{2}u[\hat{c}_3\hat{c}_5 + \hat{c}_2\hat{c}_8 + \hat{c}_1(\hat{\zeta}-\hat{\eta})],\\
\tilde{K}^{v-}_{24}(u) &= (\hat{c}_3\hat{c}_8 - \hat{c}_1^2)4u,\\
\tilde{K}^{v-}_{25}(u) &= -4u(-\hat{c}_3\hat{\eta} + \hat{c}_1\hat{c}_2),\\
\tilde{K}^{v-}_{31}(u) &= 2\sqrt{2}u[\hat{c}_4\hat{c}_8 - \hat{c}_1\hat{c}_6 + \hat{c}_5(\hat{\zeta}+\hat{\eta})],\\
\tilde{K}^{v-}_{32}(u) &= 2\sqrt{2}u[\hat{c}_5\hat{c}_7 + \hat{c}_2\hat{c}_6 - \hat{c}_4(-\hat{\zeta}+\hat{\eta})],\\
\tilde{K}^{v-}_{33}(u) &= \frac{1}{2}[2(\hat{c}_1\hat{c}_4 + \hat{c}_2\hat{c}_5)(4u-1)\\
&\quad - (4u+1)(\hat{\zeta}^2 + \hat{\eta}^2 + \hat{c}_7\hat{c}_8 + \hat{c}_3\hat{c}_6)],\\
\tilde{K}^{v-}_{34}(u) &= 2\sqrt{2}u[\hat{c}_3\hat{c}_5 + \hat{c}_2\hat{c}_8 - \hat{c}_1(-\hat{\zeta}+\hat{\eta})],\\
\tilde{K}^{v-}_{35}(u) &= -2\sqrt{2}u[\hat{c}_3\hat{c}_4 - \hat{c}_1\hat{c}_7 - \hat{c}_2(\hat{\zeta}+\hat{\eta})],\\
\tilde{K}^{v-}_{41}(u) &= -4u(\hat{c}_4\hat{c}_5 - \hat{c}_6\hat{\eta}),\\
\tilde{K}^{v-}_{42}(u) &= (\hat{c}_6\hat{c}_7 - \hat{c}_4^2)4u,\\
\tilde{K}^{v-}_{43}(u) &= 2\sqrt{2}u[\hat{c}_5\hat{c}_7 + \hat{c}_2\hat{c}_6 - \hat{c}_4(-\hat{\zeta}+\hat{\eta})],\\
\tilde{K}^{v-}_{44}(u) &= \frac{1}{2}[-2\hat{c}_1\hat{c}_4 - \hat{c}_7\hat{c}_8 - \hat{c}_3\hat{c}_6\\
&\quad - 2\hat{c}_2\hat{c}_5(4u+1) - \hat{\zeta}^2 - 8u\hat{\zeta}\hat{\eta} - \hat{\eta}^2],\\
\tilde{K}^{v-}_{45}(u) &= 4u(\hat{c}_2\hat{c}_4 + \hat{c}_7\hat{\zeta}),\\
\tilde{K}^{v-}_{51}(u) &= -(\hat{c}_6\hat{c}_8 + \hat{c}_5^2)4u,\\
\tilde{K}^{v-}_{52}(u) &= -4u(\hat{c}_4\hat{c}_5 - \hat{c}_6\hat{\eta}),\\
\tilde{K}^{v-}_{53}(u) &= 2\sqrt{2}u[\hat{c}_4\hat{c}_8 - \hat{c}_1\hat{c}_6 + \hat{c}_5(\hat{\zeta}+\hat{\eta})],\\
\tilde{K}^{v-}_{54}(u) &= 4u(\hat{c}_1\hat{c}_5 + \hat{c}_8\hat{\zeta}),\\
\tilde{K}^{v-}_{55}(u) &= \frac{1}{2}[-2\hat{c}_2\hat{c}_5 - \hat{c}_7\hat{c}_8 - \hat{c}_3\hat{c}_6\\
&\quad - 2\hat{c}_1\hat{c}_4(4u+1) - \hat{\zeta}^2 + 8u\hat{\zeta}\hat{\eta} - \hat{\eta}^2].
\end{aligned} \tag{32}$$

All the matrix elements are the polynomials of u. We note that the degree of polynomials in equation (32) is lower than that in equation (25). Again, the dual transformation (26) between $\tilde{K}^{v+}$ and $\tilde{K}^{v-}$ remains unchanged

$$\tilde{K}^{v+}(u) = \tilde{K}^{v-}\left(-u - \frac{3}{2}\right)|_{\hat{\zeta},\hat{\eta},\hat{c}_i\to\tilde{\hat{\zeta}},\tilde{\hat{\eta}},\tilde{\hat{c}}_i}. \tag{33}$$

6. The B_2 model with integrable open boundaries

Now, we have all the ingredients to construct the integrable B_2 model with open boundary conditions. The interaction in the bulk is described by the monodromy matrix constructed by the vectorial R-matrices as

$$T_0(u) = R_{01}^{vv}(u)R_{02}^{vv} \cdots R_{0N}^{vv}(u), \tag{34}$$

where V_0 is the 5-dimensional auxiliary space and $V_1 \otimes V_2 \cdots \otimes V_N$ is the 5^N-dimensional physical space. Considering the boundary reflection, the transfer matrix of the system is

$$t(u) = tr_0\{K_0^{v+}(u)T_0(u)K_0^{v-}(u)T_0^{-1}(-u)\}. \tag{35}$$

From the Yang-Baxter equation (21), reflection equation (22) and dual reflection equation (27), one can prove that the transfer matrices with different spectral parameters commute with each other, $[t(u), t(v)] = 0$. Therefore, $t(u)$ serves as the generating function of all the conserved quantities of the system. The model Hamiltonian can be obtained by taking the derivative of the logarithm of the transfer matrix

$$\begin{aligned} H &= \frac{\partial \ln t(u)}{\partial u}|_{u=0} \\ &= \sum_{k=1}^{N-1} H_{kk+1} + \frac{1}{2\xi}K_1^{v-\prime}(0) \\ &\quad + \frac{tr_0\{K_0^{v+}(0)H_{N0}\}}{tr_0 K_0^{v+}(0)} + constant. \end{aligned} \tag{36}$$

Here the first term $H_{kk+1} = \mathcal{P}_{kk+1}[R_{kk+1}^{vv}(u)]'|_{u=0}$ describes the coupling in the bulk. The second and third terms describe the external magnetic fields at two boundaries with $K_1^{v-}(0) = \xi \times id$. Substituting the values of vectorial R-matrix (20), the reflection matrix (24) and the dual one (26) into the Hamiltonian (36) and choosing the suitable parameterization, the explicit form of the Hamiltonian can be obtained. The corresponding eigen-energy could be calculated by the off-diagonal Bethe ansatz method [26–28].

7. Discussion

In this paper, we study the integrable quantization conditions of the B_2 model. Starting from the fundamental spinorial R-matrix, we obtain the vectorial R-matrix by using the fusion technique. The general solutions of the vectorial reflection equations are obtained and the corresponding structures are analyzed. We find that the reflection matrix has 7 free boundary parameters, which are used to describe the degree of freedom of boundary couplings, without breaking the integrability of the system. Based on them, we give the model Hamiltonian of integrable B_2 model with off-diagonal boundary reflections.

Appendix

The general solution of the matrix $K^{s-}(u)$ may have 16 non-zero elements

$$K^{s-}(u) = \begin{pmatrix} 1 + c_{11}u & c_{12}u & c_{13}u & c_{14}u \\ c_{21}u & 1 + c_{22}u & c_{23}u & c_{24}u \\ c_{31}u & c_{32}u & 1 + c_{33}u & c_{34}u \\ c_{41}u & c_{42}u & c_{43}u & 1 + c_{44}u \end{pmatrix}, \tag{A.1}$$

where c_{ij} are the non-zero boundary parameters. Substituting equation (4) into the reflection equation (3), the left hand side of equation (3) should be equal to the right hand side of equation (3), which give the constraints among the boundary parameters. For simplicity, we use the index method

$$\begin{aligned} &[\text{left hand side of equation(3) }]_{jl}^{ik} \\ &=[\text{right hand side of equation(3)}]_{jl}^{ik}, \end{aligned} \tag{A.2}$$

where i, k are the row indices and j, l are column indices. Considering the matrix element $\binom{11}{24}$, we have

$$\frac{1}{4}c_{14}(c_{12} - c_{34})uv(u - v)(u + v)(3 + 2u - 2v) = 0. \tag{A.3}$$

Because equation (A.3) is valid for all the values of u and v, thus the coefficient must be zero, which gives $c_{34} = c_{12}$. We note that $c_{14} \neq 0$. From the condition that the matrix element $\binom{11}{34}$ in both sides of equation (3) should be equal, we have

$$\frac{1}{4}c_{14}(c_{13} + c_{24})uv(u - v)(u + v)(3 + 2u - 2v) = 0, \tag{A.4}$$

which gives $c_{24} = -c_{13}$. From the matrix element $\binom{22}{13}$, we have

$$\frac{1}{4}c_{23}(c_{21} - c_{43})uv(u - v)(u + v)(3 + 2u - 2v) = 0, \tag{A.5}$$

which gives $c_{43} = c_{21}$. The matrix element $\binom{12}{33}$ gives

$$\frac{1}{4}c_{32}(c_{31} + c_{42})uv(u - v)(u + v)(3 + 2u - 2v) = 0, \tag{A.6}$$

which reads $c_{42} = -c_{31}$. From the element $\binom{24}{12}$, we have

$$\begin{aligned} &\frac{1}{8}uv(u - v)(3 + 2u - 2v)(3c_{11}c_{14} + 3c_{12}c_{24} + 3c_{13}c_{34} \\ &\quad + 3c_{14}c_{44} + 2c_{11}c_{14}u + 4c_{12}c_{24}u \\ &\quad + 4c_{13}c_{34}u + 2c_{14}c_{44}u + 2c_{11}c_{14}v \\ &\quad + 2c_{44}c_{14}v + 4c_{12}c_{24}v + 4c_{13}c_{34}v) = 0. \end{aligned} \tag{A.7}$$

Substituting relations (A.3) and (A.4) into (A.7), we obtain

$$\begin{aligned} &\frac{1}{8}c_{14}(c_{11} + c_{44})uv(u - v) \\ &\times (3 + 2u + 2v)(3 + 2u - 2v) = 0, \end{aligned} \tag{A.8}$$

which gives $c_{11} = -c_{44}$. Considering the element $\binom{21}{13}$, we have

$$-\frac{1}{8}uv(u-v)(3+2u-2v)(3c_{12}c_{31}+3c_{22}c_{32} + 3c_{32}c_{33}+3c_{34}c_{42}+2c_{22}c_{32}u+4c_{12}c_{31}u + 4c_{42}c_{34}u+2c_{32}c_{33}u+2c_{22}c_{32}v + 2c_{33}c_{32}v+4c_{12}c_{31}v+4c_{34}c_{42}v) = 0. \quad (A.9)$$

Substituting relations (A.3) and (A.6) into (A.9), we have

$$-\frac{1}{8}c_{32}(c_{22}+c_{33})uv(u-v) \times(3+2u+2v)(3+2u-2v)=0, \quad (A.10)$$

which gives $c_{22} = -c_{33}$. From the element $\binom{12}{12}$, we obtain

$$\frac{1}{8}uv(u-v)(3+2u-2v)(3c_{11}c_{12}+3c_{12}c_{22}+3c_{13}c_{32} + 3c_{14}c_{42}+2c_{11}c_{12}u+2c_{12}c_{22}u + 2c_{14}c_{31}u+2c_{13}c_{32}u+4c_{14}c_{42}u + 2c_{11}c_{12}v+2c_{12}c_{22}v+2c_{14}c_{31}v + 2c_{13}c_{32}v+4c_{14}c_{42}v) = 0. \quad (A.11)$$

Substituting relations (A.6), (A.8) and (A.10) into (A.11), we have

$$-\frac{1}{8}(c_{14}c_{31}-c_{13}c_{32}+c_{12}c_{33}+c_{12}c_{44})uv \times(u-v)(3+2u+2v)(3+2u-2v)=0, \quad (A.12)$$

which means

$$c_{44} = \frac{1}{c_{12}}(c_{13}c_{32}-c_{12}c_{33}-c_{14}c_{31}). \quad (A.13)$$

The matrix element $\binom{11}{12}$ gives the constraint

$$-\frac{1}{8}uv(u-v)(3+2u-2v)(3c_{11}c_{21}+3c_{21}c_{22}+3c_{23}c_{31} + 3c_{24}c_{41}+2c_{11}c_{21}u+2c_{21}c_{22}u + 2c_{23}c_{31}u+2c_{13}c_{41}u+4c_{24}c_{41}u + 2c_{11}c_{21}v+2c_{21}c_{22}v+2c_{23}c_{31}v + 2c_{13}c_{41}v+4c_{24}c_{41}v) = 0. \quad (A.14)$$

With the help of relations (A.4), (A.8), (A.10) and (A.12), we obtain

$$\frac{1}{8c_{12}}(-c_{14}c_{21}c_{31}-c_{12}c_{23}c_{31}+c_{13}c_{21}c_{32}+c_{12}c_{13}c_{41}) \times uv(u-v)(3+2u+2v)(3+2u-2v)=0. \quad (A.15)$$

Then we have

$$c_{32} = \frac{1}{c_{13}c_{21}}(c_{14}c_{21}c_{31}+c_{12}c_{23}c_{31}-c_{12}c_{13}c_{41}). \quad (A.16)$$

Considering the element $\binom{13}{11}$, we have

$$\frac{1}{8}uv(u-v)(3+2u-2v)(3c_{11}c_{13}+3c_{12}c_{23}+3c_{13}c_{33} + 3c_{14}c_{43}+2c_{11}c_{13}u-2c_{14}c_{21}u + 2c_{12}c_{23}u+2c_{13}c_{33}u+4c_{14}c_{43}u + 2c_{11}c_{13}v-2c_{14}c_{21}v+2c_{12}c_{23}v + 2c_{13}c_{33}v+4c_{14}c_{43}v) = 0. \quad (A.17)$$

With the help of relations (A.5), (A.8), (A.12) and (A.15), we get

$$\frac{1}{8c_{21}}(c_{14}c_{21}^2+c_{12}c_{21}c_{23}-c_{13}c_{23}c_{31}+2c_{13}c_{21}c_{33}+c_{13}^2c_{41}) \times uv(u-v)(3+2u+2v)(3+2u-2v)=0, \quad (A.18)$$

which gives

$$c_{33} = \frac{1}{2c_{13}c_{21}}(c_{13}c_{23}c_{31}-c_{13}^2c_{41}-c_{14}c_{21}^2-c_{12}c_{21}c_{23}). \quad (A.19)$$

After long calculations of all the other matrix elements, we can not obtain new constraint any more. Therefore, the 16 non-zero elements in the reflection matrix $K^{s-}(u)$ must satisfy above 9 constraints and only 7 parameters of them are free. For simplicity and comparing with previous results, we denote

$$c_{11}=-c_{44}=-\zeta,\quad c_{22}=-c_{33}=-\eta,\quad c_{12}=c_{34}=c_1,\\ c_{13}=-c_{24}=c_2,\quad c_{21}=c_{43}=c_4,\quad c_{31}=-c_{42}=c_5,\\ c_{14}=c_3,\quad c_{41}=c_6,\quad c_{23}=c_7,\quad c_{32}=c_8. \quad (A.20)$$

Then we arrive at equation (4).

References

[1] Korepin V E, Bogoliubov N M and Izergin A G 1993 *Quantum Inverse Scattering Method and Correlation Function* (Cambridge: Cambridge University Press)
[2] Alcaraz F C, Barber M N, Batchelor M T, Baxter R J and Quispel G R W 1987 *J. Phys.* A **20** 6397
[3] Sklyanin E K and Faddeev L D 1978 *Sov. Phys. Dokl* **23** 902
[4] Takhtadzhan L A and Faddeev L D 1979 *Rush. Math. Surveys* **34** 11
[5] Sklyanin E K 1988 *J. Phys.* A **21** 2375
[6] Malara R and Lima-Santos A 2006 *J. Stat. Mech.* **2006** P09013
[7] de Vega H J and González-Ruiz A 1993 *J. Phys.* A **26** L519
[8] de Vega H J and González-Ruiz A 1994 *J. Phys.* A **27** 6129
[9] Lima-Santos A 1999 *Nucl. Phys.* B **558** 637
[10] Shi K, Li G-L, Fan H and Hou B Y 2000 *High Energy Phys. and Nucl. Phys.* **24** 11 (in Chinese)
[11] de Vega H J and González-Ruiz A 1994 *Nucl. Phys.* B **417** 553
[12] de Vega H J and González-Ruiz A 1994 *Mod. Phys. Lett.* A **09** 2207
[13] Li G-L, Yue R H and Hou B Y 2000 *Nucl. Phys.* B **586** 711
[14] Lima-santos A and Malara R 2003 *Nucl. Phys.* B **675** 661
[15] Chicherin D, Derkachov S and Isaev A P 2013 *J. Phys.* A **46** 485201

[16] Frahm H, Seel A and Wirth T 2008 *Nucl. Phys.* B **802** 351
[17] Karowski M 1979 *Nucl. Phys.* B **153** 244
[18] Kulish P P, Reshetikhin N Y and Sklyanin E K 1981 *Lett. Math. Phys.* **5** 393
[19] Kulish P P and Sklyanin E K 1982 *Lect. Notes Phys.* **151** 61
[20] Kirillov A N and Reshetikhin N Y 1986 *J. Sov. Math.* **35** 2627
[21] Kirillov A N and Reshetikhin N Y 1987 *J. Phys.* A **20** 1565
[22] Mezincescu L and Nepomechie R I 1992 *Nucl. Phys.* B **372** 597
[23] Zhou Y-K 1996 *Nucl. Phys.* B **458** 504
[24] Martins M J and Ramos P B 1997 *Nucl. Phys.* B **500** 579
[25] Lima-santos A 2003 *Nucl. Phys.* B **654** 466
[26] Wang Y, Yang W-L, Cao J and Shi K 2015 *Off-Diagonal Bethe Ansatz for Exactly Solvable Models* (Berlin: Springer)
[27] Cao J, Yang W-L, Shi K and Wang Y 2014 *JHEP* **04** 143
[28] Cao J, Cui S, Yang W-L, Shi K and Wang Y 2015 *JHEP* **04** 036