

# Comparative analysis for rock slope stability with different constitutive models of rock materials

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**Abstract.** Based on the principle of reducing shear strength of rock materials, a rock slope stability with different constitutive models of rock materials is performed by using finite element method. The safety factor of rock slope stability is computed based on convergence property of elastic-plastic numerical simulations by reducing shear strength of rock materials until the numerical computing is divergent or plastic zone is connected. Three kinds of Drucker-Prager yield criterions with different yield surfaces are utilized to investigate the influence of yield criterions with different yield surfaces to safety factor of rock slope stability. The investigation shows that the computed safety factors of rock slope stability by using Drucker-Prager yield criterions with different yield surfaces are approximately equal. The maximum relative error for safety factors is less 10% by using different Drucker-Prager yield criterions.

## 1. Introduction

Rock slope stability is commonly affected by shear strength parameters of rock materials. Mohr-Coulomb yield criterion and Drucker-Prager yield criterion are widely used to evaluate rock slope stability. However, yield criterions with different yield surfaces will affect safety factor of rock slope stability. Raghuvanshi comprehensively reviewed on governing parameters and various stability analysis techniques for plane mode of failure in rock slopes [1]. Elmo introduced an improved terminology commonly used by the discrete fracture network community to define rock bridge intensity relative to the sampling region. In rock engineering, the measurement of rock bridges is exacerbated by the fact that rock bridges are not visible unless the rock mass is exposed by human activities or by natural events such as rockfalls [2]. Schlotfeldt proposed an integrated approach of designing overhanging rock slopes where the relative dimensions of the slope exceed the scale of fracturing and the rock mass failure needs to be considered rather than kinematic release of individual blocks[3]. Tang developed a model that combines the FEM and DDA approaches. The main concept of this approach is to first apply FEM to model crack growth behaviour and then automatically switch to the DDA module to model the post-failure process when the slip surface forms [4]. Lv proposed a procedure to determine the shear strength parameters through back analysis based on stability and failure status before and after earthquake and monitoring data after earthquake [5]. Belghali adopted a pseudo-static method to account for the inertial forces induced in the rock mass by seismic events. The strength properties of the rock material are described by a modified Hoek-Brown strength criterion, whereas the passive bolts are modelled as bar-like inclusions that exhibit only resistance to tensile-



compressive forces [6]. Chen presented a case history of the 205m high intake slope at the Huangjinping (HJP) hydropower station on the Dadu River. Deformation and cracks developed on the cut slope during excavation, and the deformation extended onto the natural slope above the cut, giving rise to serious safety concerns [7]. Stead emphasized the importance of structural geology to slope stability assessments, reviewing how structures control slope failure mechanisms, how engineering geologists measure structures and include them in slope stability analyses, and how numerical simulations of slopes incorporate geological structures and processes [8]. Wong developed the numerical manifold method as a tool to investigate the progressive failure in rock slopes. The entire processes of the progressive slide surface development related to crack initiation, propagation, coalescence and degradation to eventual catastrophic failure are successfully captured [9]. The objective of the paper is to analyse the influence of yield criterions with different yield surfaces to safety factor of rock slope stability and evaluate rock slope stability for Bijia mountain.

## 2. Determination of parameters of constitutive models for rock materials

Mohr-Coulomb yield criterion is widely used to characterize soil and rock yielding property and is found to hold well in actual tests, and is expressed as

$$\frac{\sigma_1 - \sigma_3}{2} = \left[ \frac{\sigma_1 + \sigma_3}{2} + \frac{C}{\tan \varphi} \right] \sin \varphi \quad (1)$$

Where  $C$  and  $\varphi$  denote the cohesion and internal friction angle of rock mass, respectively.  $\sigma_1$  and  $\sigma_3$  denote maximum principal stress and minimum principal stress.  $C$  and  $\varphi$  can be determined from direct shear tests of rock specimens with different normal pressure. Main mechanical parameters of rock materials of some rock slope are listed in Table 1.

**Table 1.** Main mechanical parameters of rock materials of some rock slope.

| Parameter | $\rho/\text{kg/m}^3$ | $C/\text{MPa}$ | $\varphi/^\circ$ | $E/\text{GPa}$ | $\mu$ |
|-----------|----------------------|----------------|------------------|----------------|-------|
| Values    | 2300                 | 0.20           | 35               | 15.0           | 0.30  |

The yield surface defined in Mohr-Coulomb yield criterion has a serious drawback due to its angular nature in the principal stress space. In order to deal with the problem, Drucker-Prager proposed smooth surface by writing the yield criterion as a continuous relationship between the stress invariants. Drucker-Prager yield criterion with linear yield surface is expressed as

$$\begin{cases} F = t - p \tan \beta - d = 0 \\ t = \frac{q}{2} \left[ 1 + \frac{1}{k} - \left( 1 - \frac{1}{k} \right) \left( \frac{r}{q} \right)^3 \right] \end{cases} \quad (2)$$

Where  $\beta$ ,  $q$ ,  $d$ ,  $p$ ,  $k$  and  $r$  are parameters of Drucker-Prager yield criterion, and can be determined from following equations and Table 2 when parameters ( $C$  and  $\varphi$ ) in Mohr-Coulomb yield criterion are known.  $k$  is strength ratio between tensile and compressive tests, and  $0.778 < k < 1.0$ .  $p$  is average stress.

$$\begin{aligned} p &= -\frac{1}{3}(\sigma_1 + 2\sigma_3) \\ q &= \sigma_1 - \sigma_3 \\ r &= -q \end{aligned} \quad (3)$$

$$d = \begin{cases} \left( 1 - \frac{1}{3} \tan \beta \right) \sigma_c & \text{uniaxial compressive test} \\ \left( \frac{1}{k} - \frac{1}{3} \tan \beta \right) \sigma_t & \text{uniaxial tensile test} \\ \frac{\sqrt{3}}{2} \tau \left( 1 + \frac{1}{k} \right) & \text{direct shear test} \end{cases} \quad (4)$$

Drucker-Prager yield criterion with parabolic yield surface is expressed as

$$\begin{cases} F = \sqrt{l_0^2 + q^2} - p \tan \beta - d = 0 \\ l_0 = d' |_{\sigma_t} - p_t |_{\sigma_t} \tan \beta \end{cases} \quad (5)$$

$$d = \begin{cases} \sqrt{l_0^2 + \sigma_c^2} - \frac{\sigma_c}{3} \tan \beta & \text{uniaxial compressive test} \\ \sqrt{l_0^2 + \sigma_t^2} + \frac{\sigma_t}{3} \tan \beta & \text{uniaxial tensile test} \\ \sqrt{l_0^2 + d^2} & \text{direct shear test} \end{cases} \quad (6)$$

Drucker-Prager yield criterion with exponent yield surface is expressed as

$$F = aq^b - p - p_t = 0 \quad (7)$$

Where a and b are model parameters and can be determined from Table 2.

$$p_t = \begin{cases} a\sigma_c^b - \frac{\sigma_c}{3} & \text{uniaxial compressive test} \\ a\sigma_t^b + \frac{\sigma_t}{3} & \text{uniaxial tensile test} \\ ad^b & \text{direct shear test} \end{cases} \quad (8)$$

**Table 2.** Relationship between parameters in Mohr-Coulomb yield criterion and parameters in Drucker-Prager yield criterions (Associated flow rule).

|                                 |   |  |  |
|---------------------------------|---|--|--|
| Linear<br>Drucker-<br>Prager    | $\tan \beta = \frac{\sqrt{3} \sin \varphi}{\sqrt{1 + \frac{1}{3} \sin^2 \varphi}}$              | -----  | $\frac{d}{c} = \frac{\sqrt{3} \cos \varphi}{\sqrt{1 + \frac{1}{3} \sin^2 \varphi}}$  |
| Parabolic<br>Drucker-<br>Prager | $\tan \beta = \left( \frac{\sqrt{3} \sin \varphi}{\sqrt{1 + \frac{\sin^2 \varphi}{3}}} \right)$ | $\frac{d}{c} = \frac{\sin \varphi}{\sqrt{3 + \sin^2 \varphi}}$ | $\sigma_c = \frac{c \sqrt{3} \cos \varphi}{\left(1 - \frac{\sin \varphi}{\sqrt{3 + \sin^2 \varphi}}\right) \sqrt{1 + \frac{1}{3} \sin^2 \varphi}}$ |
| Exponent<br>Drucker-<br>Prager  | $a = \frac{\sqrt{1 + \frac{1}{3} \sin^2 \varphi}}{\sqrt{3} \sin \varphi}$                       | $b = 1$  | $\sigma_c = \frac{c \sqrt{3} \cos \varphi}{\left(1 - \frac{\sin \varphi}{\sqrt{3 + \sin^2 \varphi}}\right) \sqrt{1 + \frac{1}{3} \sin^2 \varphi}}$ |

When parameters in Mohr-Coulomb yield criterion are known, parameters in Drucker-Prager yield criterions can be determined.

### 3. Practical application of Drucker-Prager yield criterions to slope stability analysis

Some cross section of Bijia mountain, located in Jinzhou city, Liaoning province, is shown in Figure 1. The slope angles are 50° and 60° in left and right sides, respectively. The height of FEM model is 61.5 m. The widths of FEM model are 20m and 106.8m in upper and bottom. Main mechanical parameters of rock materials are listed in Table 1.

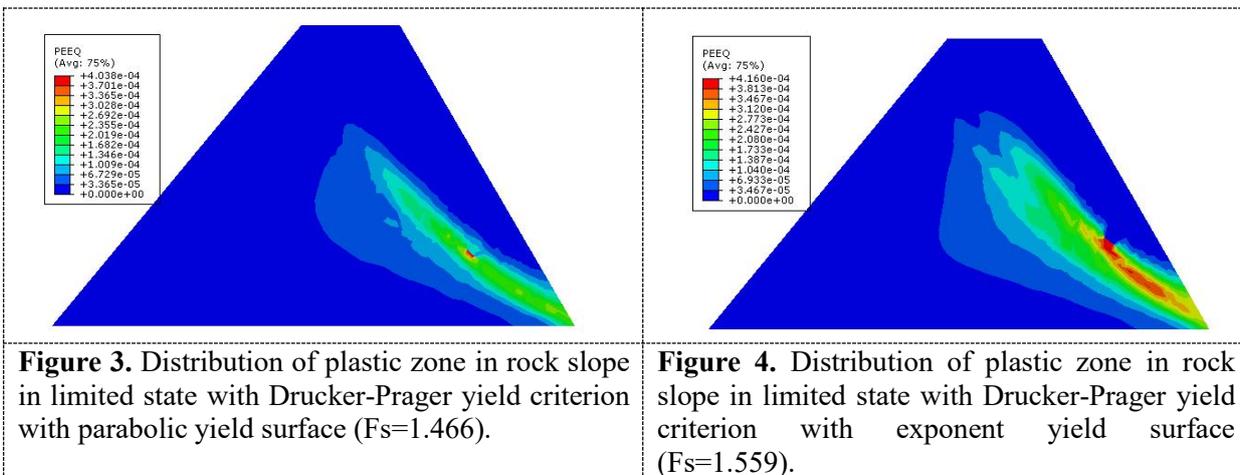
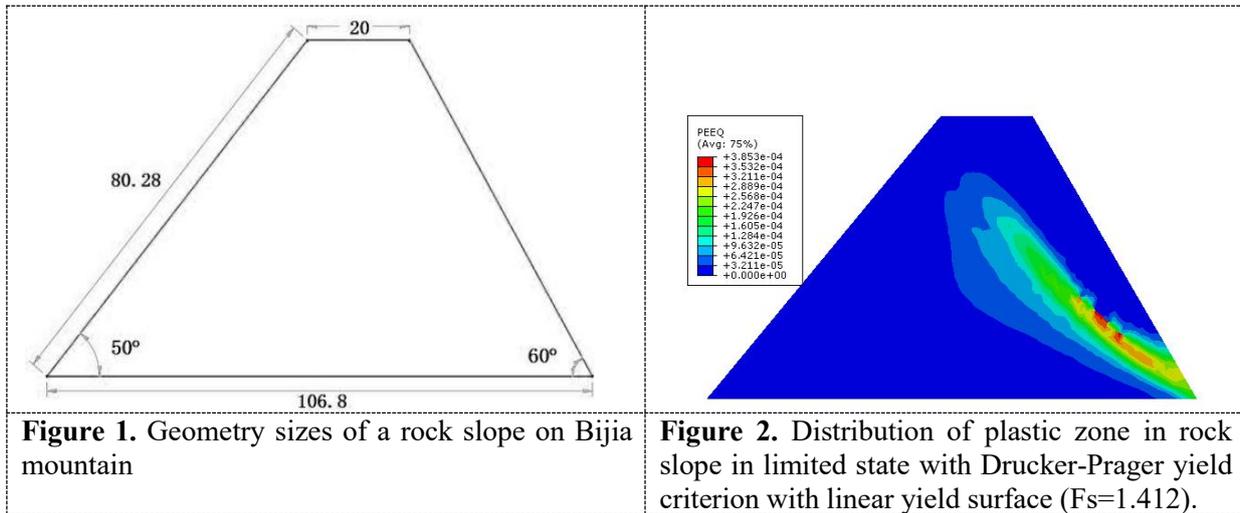
The shear strength reducing method is used to simulate rock slope stability and determine the safety factor of rock slope. Three kinds of constitutive models for Drucker-Prager yield criterion are used to investigate rock slope stability and evaluate the safety factor of rock slope. The relationship between reduced shear strength of rock materials and the safety factor of rock slope is expressed as

$$C_r = \frac{C}{F_s} \quad (9)$$

Where  $F_s$  denotes the safety factor of rock slope,  $C_r$  is cohesion of rock materials after reducing when considering safety factor  $F_s$ .

$$\tan \varphi_r = \frac{\tan \varphi}{F_s} \tag{10}$$

Where  $\varphi_r$  is internal friction angle of rock materials after reducing. Distributions of plastic zone in rock slope in limited state with different Drucker-Prager yield criterions are shown in Figure 2, 3 and 4.



It can be observed from Figure 2, 3 and 4 that the shapes of distribution of plastic zones in rock slope in limited state for three kinds of Drucker-Prager yield criterions are identical, and the safety factors of rock slope with different Drucker-Prager yield criterions are 1.412, 1.466 and 1.559, respectively. The maximum relative error is less 10% by using different Drucker-Prager yield criterions. The safety factors of rock slope with Drucker-Prager yield criterion with linear yield surface is minimum by comparing with parabolic yield surface and exponent yield surface.

**4. Conclusions**

Finite element method in conjunction with Drucker-Prager yield criterions is validated to be a reliable numerical procedure for evaluating the factor of safety of rock slopes. This procedure can conveniently compute the factor of safety of rock slopes based on the evaluating criterions of slope stability with unconverged FE solution or appearing inflexion for deformation of FEM model by reducing shear strength of rock materials. The investigation shows that the safety factors of rock slope

with different Drucker-Prager yield criterions are slightly different and the maximum relative error is less 10% by using different Drucker-Prager yield criterions.

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