Faulty node modeling and diagnosis in interconnection networks (extended abstract)

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Abstract. Faulty processing node analysis, in particular, self-diagnostic paradigm, is an important topic in the area of interconnection network studies. Several diagnostic models have been introduced, with the PMC and MM* models being two of the most popular ones. Researchers have also proposed various extensions and enhancements of fault-tolerance models to better capture the distribution, and identification, of faulty nodes in realistic scenarios. One of them is the g-extra fault-tolerance model, where each cluster in a network with faulty nodes contains at least g+1 fault-free nodes. This paper, following an analytical and constructive approach based on applied graph theory, suggests a general process to identify the maximum number of faulty nodes in a network in terms of the g-extra fault-tolerance model, and as a demonstrative example, provides a specific result for the (n, k)-star graphs.

1. Introduction

Nowadays, thanks to the rapid progress in computer engineering technology, multi-processor systems have become a common reality in both our work and lives, where an interconnection network plays a crucial role. It is unavoidable that some of the processing nodes in such a network will become faulty, sometimes disabling the normal function of the computer system. Thus, there is often a need to identify, and then correct/replace, these faulty nodes in such a system to restore its connectivity, and the normal operation of the computer system. For obvious reasons, one would want to have a self-diagnosable system where the computing nodes are able to detect faulty ones themselves. It turns out that we could simulate such a diagnostic task of an engineering nature via a mathematical process, with three components: the associated topology of such a system, the intended fault-tolerance model, and the diagnostic model.

The topology of a network system is usually modeled with a connected graph $G(V, E)$, where V represents a collection of processing nodes, and E the connections in between pairs of nodes. Many network topologies have been suggested and studied in the literature, including the hypercube [7], the star graph [1], and the (n, k)-star graph [5], denoted by $S_{n,k}$ in the rest of this paper.

Let $G(V, E)$ represent an interconnection network, we use a *faulty set* to identify a collection of faulty processing nodes, which is just a collection of vertices $F \subseteq V$, effectively removed from G. Since such an unrestrictive fault-tolerance model could imply the highly unlikely situation that all the neighbors of a vertex would be faulty, several more sophisticated fault-tolerance models have been suggested in the literature. With the *conditional faulty set* model [8], every vertex, faulty or not, has at least one *fault-free* neighbor in the *survival graph G-F*. With a *g-good neighbor faulty set* [12] every fault-free vertex has at least g fault-free neighbors in the survival graph; and, with a *g-extra faulty set* [18], every component in the survival graph contains at least g+1 vertices. In this paper, we will focus on this latter fault-tolerance model associated with a g-extra faulty set.

By a *neighbor* of a vertex, v, in G, we mean a vertex u such that (u, v) is an edge in G. The MM* model [14], which extends the *comparison diagnostic model,* i.e., the MM model [10,11] is a rather popular diagnostic model, where each processing node, called a *testing node*, sends a test message to each and every pair of its distinct neighbors, called a *tested node*, and then compares their responses. Based on all such comparison results, the fault status of the system, as well as the faulty nodes, can be determined. With another diagnostic model, the PMC model [13], a node sends a test message to each of its neighbors and judges its fault status based on the received responses. Various efficient algorithms to identify such faulty sets have been proposed in, e.g. [6,14]. We notice that, with both the MM* and PMC models, when a testing node is faulty, responses from those tested nodes will be unreliable.

A collection of all the aforementioned test results obtained with a diagnostic model is called a *syndrome* of the diagnosis. A subset F ($\subseteq V$) is *compatible with a syndrome* [8] if the latter can be generated when all the vertices in F are faulty and all those in $V \setminus F$, i.e., the difference between V and F, are fault-free. Since faulty testing nodes lead to unreliable results, as observed in [10, 11], two faulty sets may be compatible with the same syndrome, thus making such a faulty set unidentifiable. This observation leads to the notion of a graph being *t-diagnosable* [14] when up to t faulty vertices in G can be identified. And the *diagnosability* of a graph G, denoted by $t(G)$, is defined to be the maximum number of faulty vertices that G can guarantee to identify.

Many diagnosability results have appeared in literature, including [8,9,12,15,18], where we notice that much of the derivation details as reported in those papers devoted to different structures are essentially shared among themselves, and even with those used to derive results on the g-good-neighbor diagnosability. We strongly believe this practice is unnecessary. In this paper, we will continue our work in [4], and outline a general process to derive diagnosability results under various fault-tolerance, and diagnostic, models.

The rest of this paper proceeds as follows: We will start with some basic definitions regarding the self-diagnostic paradigm at a system level in the following section, and describe a general process of seeking both upper bounds, and lower bounds, of general diagnosability results in Section 3, and demonstrate the value of such a general process in Section 4. We conclude this paper with some final remarks in Section 5.

2. Basic notions of general diagnosability

Let $G(V, E)$ represent an interconnection network, and let M stand for a certain fault-tolerance model. *An M-faulty set* is a faulty set, $F \subseteq V$, consistent with M. For example, F ($\subseteq V$) is a g-extra faulty set if every component in G-F contains at least g+1 vertices. G is said to be *M t-diagnosable in terms of a diagnostic model D*, if G is diagnosable for any M-faulty set of size at most t in D, where D refers to either the PMC model or the MM* model in the rest of this paper.

Let F_1 and F_2 be two distinct M-faulty sets, $F_1 \subseteq V$, $F_2 \subseteq V$, (F_1, F_2) is *distinguishable* in G if and only if they do not lead to the same syndrome. They are *indistinguishable* otherwise. Then, $t_M(G, D)$, the *M-diagnosability of G, in terms of a diagnostic model D*, equals the maximum number t such that G is M t-diagnosable, i.e., for all distinct M-faulty set pairs (F_1, F_2), such that $F_1 \subseteq V$, $F_2 \subseteq V$, and $|F_1| \leq t$, $|F_2| \leq t$, (F_1, F_2) is distinguishable in terms of D.

Thus, the diagnosability problem really comes down to a decision problem of whether two faulty sets are distinguishable within a diagnostic model. In this regard, the following result specifies a necessary and sufficient condition of two faulty sets being distinguishable in the MM* model, where $F_1 \Delta F_2$ stands for $(F_1 \setminus F_2) \cup (F_2 \setminus F_1)$, i.e., the symmetric difference of F_1 and F_2.

Theorem 2.1. (following as [14]) Let $G(V, E)$ be a connected graph. For any two sets F_1, F_2 such that $F_1 \subseteq V$, $F_2 \subseteq V$ and $F_1 \neq F_2$, F_1 and F_2 are distinguishable under the MM* model if and only if at least one of the following three conditions holds: 1) there are two distinct vertices v and w, $\{v, w\} \subseteq V \setminus (F_1 \cup F_2)$ and there is a vertex x, $x \in F_1 \Delta F_2$ such that (v, w, x) is a path in G; 2) there are two distinct vertices v and x, $\{v, x\} \subseteq F_1 \setminus F_2$ and there is a vertex w, $w \in V \setminus (F_1 \cup F_2)$ such that (v, w, x) is a path in G; and 3) there are two distinct vertices v and x, $\{v, x\} \subseteq F_2 \setminus F_1$ and there is a vertex w, $w \in V \setminus (F_1 \cup F_2)$ such that (v, w, x) is a path in G.

The conditions associated with the PMC model are somewhat simpler, as expected, since the PMC model is a special case of the MM* model when the two vertices being tested are the same.

Theorem 2.2. (following as [6]) Let $G(V, E)$ be a graph. For any two sets F_1, F_2 such that $F_1 \subseteq V$, $F_2 \subseteq V$ and $F_1 \neq F_2$, F_1 and F_2 are distinguishable under the PMC model if and only if there exist a vertex u, $u \in V \setminus (F_1 \cup F_2)$, and another vertex v in $F_1 \Delta F_2$, such that (u, v) is an edge of G.

3. A general process of deriving diagnosability results

Recall that an M-faulty set is just a faulty vertex set F in a graph $G(V, E)$, related to a certain fault-tolerance model M. It is also an *M-cut* if G-F is disconnected. Although an M-faulty set does not need to be an M-faulty cut, the construction of such an M-faulty cut in G turns out to be a crucial step to derive the upper bound of the M t-diagnosability of G.

The size of a minimum M-faulty cut of a graph G is referred to as its *M-connectivity*, denoted by $\kappa_M(G)$. The derivation of the M-connectivity of a graph depends on its topology, and is often tedious and challenging. On the other hand, this quantity plays a critical role in deriving the lower bound of the related M t-diagnosability of a graph. Many results to this regard have appeared in literature. Readers are referred to [4] for the connection between the g-good neighbor connectivity of the arrangement graph and its g-good-neighbor diagnosability, $g \in [1, 2]$. The 2-extra connectivity of the bubble-sort graph is derived in [16], where its connection to its g-extra diagnosability is also given. The g-good-neighbor connectivity, $g \in [0, n\text{-}k]$, of the (n, k)-star graph is given in [17], and its connection to its g-good-neighbor diagnosability is explored in [4]. Recently, the g-extra connectivity of the (n, k)-star graph, and its connection to its g-extra diagnosability, are discussed in [9]. The major contribution of this paper is to provide an alternative and largely structure independent, approach to deriving a diagnosability result for a graph, once its related connectivity result is available.

Given a graph $G(V, E)$, we use $N(v)$ to denote the neighbors of v in V, i.e., $N_G(v) = \{w: (v, w) \in E\}$. Let $S \subseteq V$, we use $N_G(S)$ to denote all the neighbors of vertices in S, excluding those in S; and use $N^c{}_G(S)$ to denote $N_G(S) \cup S$. Notice that $N_G(S)$ and $N^c{}_G(S)$ are also referred to as *open* and *closed neighborhood* of S, respectively, in literature. We will drop the subscript G when the context is clear.

To derive an upper bound of $t_M(G, D)$, we carefully choose a set $Y(\subsetneq V)$ such that $V \neq N^c(Y)$ and both $N(Y)$ and $N^c(Y)$ are M-faulty sets.

The following result follows from Theorems 2.1 and 2.2. (Due to page restrictions, we have to omit technical proofs of all the results as contained in this paper, which will be included in its full version.)

Proposition 3.1. Let $G(V, E)$ be a connected graph, M stand for a certain fault-tolerance model, and let $Y \subsetneq V$. If both $N(Y)$ and $N^c(Y)$ are M-faulty sets, and $V \setminus N^c(Y) \neq \emptyset$, then $t_M(G, D) \leq |N^c(Y)|$-1.

It turns out that we can derive the following result for two specific fault-tolerance models.

Corollary 3.1. Let $G(V, E)$ be a connected graph, and let Y be a subset of V. If $N(Y)$ is an M-faulty set, where M stands for either the g-good-neighbor or the g-extra fault-tolerance model, and $V \setminus N^c(Y) \neq \emptyset$. Then $t_M(G, D) \leq |N^c(Y)|$-1.

Recall that $S_{n,\,k}$ stands for the (n, k)-star graph, the theorem below gives one specific result, obtained via such a construction, where we use $t_g(G, D)$ to denote the g-good-neighbor diagnosability of G in terms of a *diagnostic model D*.

Theorem 3.1. (following as **[**4]) For $n \geq 4$, $k \in [2, n)$, $g \in [0, n\text{-}k]$, $t_g(S_{n,\,k}, D) \leq n+g(k\text{-}1)\text{-}1$.

To show that t is a lower bound of $t_M(G, D)$, i.e., $t_M(G, D) \geq t$, we need to show that, for any two distinct M-faulty sets, F_1, F_2, such that $|F_1| \leq t$, $|F_2| \leq t$, and $V \setminus (F_1 \cup F_2) \neq \emptyset$, (F_1, F_2) is distinguishable in G in terms of the diagnostic model D. We start with a useful, and indeed a critical, result.

Proposition 3.2. Let $G(V, E)$ be a connected graph, M stand for a fault-tolerance model, and let F_1, $F_2 \subsetneq V$. If both F_1 and F_2 are M-faulty sets in G, so is $F_1 \cap F_2$.

The structure independent part of the general process of deriving a lower bound of M-diagnosability of a graph G in terms of the PMC model can then be summarized as follows.

Proposition 3.3. Let $G(V, E)$ be a connected graph, $\kappa_M(G)$ be the M-connectivity of G, and F_1 and F_2 be a pair of distinct M-faulty sets, such that $|F_1| \leq t$, $|F_2| \leq t$, and $V \setminus (F_1 \cup F_2) \neq \emptyset$. Assuming that $F_1 \setminus F_2 \neq \emptyset$ (Since $F_1 \neq F_2$, either $F_1 \setminus F_2 \neq \emptyset$ or $F_2 \setminus F_1 \neq \emptyset$). If $t \geq |F_1 \setminus F_2| + \kappa_M(G)$ leads to a contradiction with the PMC model assumption, then $t_M(G, \text{PMC}) \geq t$.

In particular, let $\tau_g(G, D)$ stand for the g-extra diagnosability of G in terms of D, we have the following result.

Corollary 3.2. Let $G(V, E)$ be a connected graph, and let $\kappa_g(G)$ be its g-extra connectivity, $g \geq 1$. If $|V| > 2\,(\kappa_g(G) + g)$, then $\tau_g(G, \text{PMC}) \geq \kappa_g(G) + g$.

We give a sketch of a proof of the above result as follows: Let (F_1, F_2) be a pair of distinct g-extra faulty sets in G such that $|F_1| \leq t = \kappa_g(G) + g$ and $|F_2| \leq t = \kappa_g(G) + g$. We want to show that such a pair is always distinguishable in terms of the PMC model, thus G is t-diagnosable. Just assume such a pair (F_1, F_2) is indistinguishable in terms of the PMC model.

Assume $F_1 \setminus F_2 \neq \emptyset$, and let C be a component that shares vertices with $F_1 \setminus F_2$. Since, (F_1, F_2) is assumed indistinguishable in terms of the PMC model, by Theorem 2.2, no vertex outside $F_1 \cup F_2$ is adjacent to any vertex in $F_1 \setminus F_2$. Hence, $C \subseteq F_1 \setminus F_2$, and, as a result, $|F_1 \setminus F_2| \geq |C| \geq g+1$. Since G is connected, we would have to conclude that any vertex u of $V \setminus (F_1 \cup F_2)$ has to go through a vertex in $F_1 \cap F_2$ to be connected to another vertex in $F_1 \setminus F_2$. Thus, $F_1 \cap F_2$ is a cut. By Proposition 3.2, since both F_1 and F_2 are g-extra faulty sets, $F_1 \cap F_2$ is also a g-extra faulty set, thus a g-extra cut. In other words, $|F_1 \cap F_2| \geq \kappa_g(G)$. Finally, the assumption $\kappa_g(G) + g = t \geq |F_1 \setminus F_2| + \kappa_g(G) \geq (g+1) + \kappa_g(G)$ leads to a contradiction, hence (F_1, F_2) has to be distinguishable in terms of the PMC model, and the result follows.

The process of deriving a lower bound result of the M-diagnosability in terms of the MM* diagnostic model is essentially the same as that for the PMC model, except that we also need to show that no isolated vertex exists in $V \setminus (F_1 \cup F_2)$, where (F_1, F_2) is the pair of indistinguishable M-faulty sets that we would use to construct the desired contradiction. The reason for this additional requirement is that, for this MM* case, if a vertex u is isolated in $V \setminus (F_1 \cup F_2)$, it can be adjacent to some vertex in $F_1 \Delta F_2$. Then, $F_1 \cap F_2$ would not be a cut, and we could not use the argument that we just used in proving Corollary 3.2 to reach the desired conclusion. We instead have the following parallel result.

Proposition 3.4. Let $G(V, E)$ be a connected graph, $\kappa_M(G)$ be the M-connectivity of G, and let F_1, F_2 be two M-faulty sets in terms of the MM* diagnostic model, such that $|F_1| \leq$ t, $|F_2| \leq$ t, and the non-empty set of $V \setminus (F_1 \cup F_2)$ contains no isolated vertex. Assume that $F_1 \setminus F_2 \neq \emptyset$, if $t \geq |F_1 \setminus F_2| + \kappa_M(G)$ leads to a contradiction, then $t_M(G, \text{MM*}) \geq t$.

It turns out that no vertex in $V \setminus (F_1 \cup F_2)$ could be isolated when $g \geq 2$ for both the g-good-neighbor and the g-extra fault-tolerance models, as the following result shows.

Corollary 3.3. Let $G(V, E)$ be a connected graph, and let $\kappa_g(G)$ be the g-extra connectivity of G, $g \geq 2$. If $|V| > 2(\kappa_g(G)+g)$, then $\tau_g(G, \text{MM*}) \geq \kappa_g(G) + g$.

4. The g-extra diagnosability of the (n, k)-star graph

The star graph, denoted by S_n, was proposed in [1] as an attractive alternative to the well-known hypercube structure when used as an interconnection network. However, the requirement that the number of vertices in the star graph be n! results in a large size gap between S_n and S_{n+1}.To address this scalability issue, the (n, k)-star graph was suggested in [5], which brings in a flexibility in choosing its size, while preserving many attractive properties of the star graph, including vertex symmetry. This class of graphs has been well studied in the literature, together with its fault-tolerance properties, e.g., [3,4,9,17].

Vertices of $S_{n,k}$, $n \geq 3$, $k \in [2, n)$, is simply the collection of all the k-permutations taken out of {1, 2, …, n}. Thus, $S_{n,k}$ contains $n!/(n-k)!$ vertices. Let $u = [p_1, \ldots, p_k]$, $v = [q_1, \ldots,, q_k]$, (p, q) is an edge of $S_{n,k}$ either, for some $i \in [2, k]$, v can be obtained from u by swapping p_1 and p_i (called an *i-edge*}); or, for some symbol $e \in \{1, 2, \ldots, n\} \setminus \{p_1, \ldots, p_k\}$, v can be obtained from u by replacing p_1 with e (called a*1-edge*). Thus, a vertex is incident to k-1 i-edges, $i \in [2, k]$; and n-k 1 edges, a total of n-1 edges. As a result, $S_{n,k}$ contains a total of $((n-1)n!/[2(n-k)!]$ edges.

As shown in Figure 1, $S_{4,2}$ contains 4!/2! (= 12) vertices, and 18 edges. For example, [1, 2] (=12) is a vertex in $S_{4,2}$, and ([1, 2], [2, 1]), ([1, 2], [3, 2]), and ([1, 2], [4, 2]) are all edges of this graph. In particular, [1, 2] is incident to three edges, including two 1-edges: ([1, 2],[3, 2]) and ([1, 2], [4, 2]) , and [1, 2] is also incident to exactly one 2-edge ([1, 2], [2, 1]).

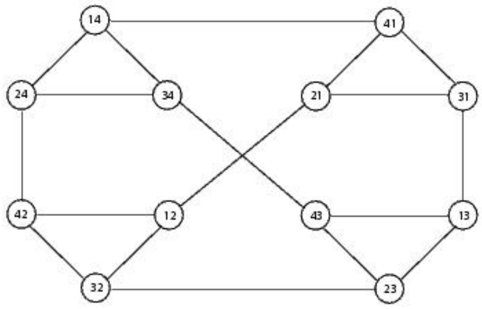

Figure 1. $S_{4,2}$

Issues related to the g-good-neighbor diagnosability of the (n, k)-star graphs have been addressed in [4,15]. Its g-extra diagnosability has also been derived recently in [9] by following a structure dependent approach. It turns out that these results can all be derived following the general approach that we have described in the previous section.

Recall that $t_g(G, D)$ stands for the g-good-neighbor diagnosability of a graph G in terms of a diagnostic model D, and $\tau_g(G, D)$ its g-extra diagnosability. We start with the following result, since a vertex u has at least one neighbor if and only if the component containing u has at least two vertices.

Lemma 4.1. (following as [16]) Let $G(V, E)$ be a connected graph, then $\tau_1(G, D) = t_1(G, D)$.

Since it has already been proved that $t_1(S_{n,k}, D) = n+k-2$ (Theorem 5.3 in [4] for the PMC case and [15] for the MM* case), by Lemma 4.1, we immediately have the following result.

Corollary 4.1. Let $n \geq 4$, $k \in [2, n)$, $\tau_1(S_{n,k}, D) = n+k-2$.

For the cases of $g \geq 2$, the following result provides an upper bound of $\tau_g(S_{n,k}, D)$ since if a fault-free node has at least g neighbors, thus the component contains at least g+1 vertices.

Lemma 4.2. (following as [16]) Let G be a connected graph, let $g \geq 0$, then $\tau_g(G, D) \leq t_g(G, D)$.

We then obtain an upper bound result for the g-extra diagnosability of the (n, k)-star graph, by Theorem 3.1 as follows.

Lemma 4.3. For $n \geq 4$, $k \in [2, n)$, $g \in [0, n-k]$, $\tau_g(S_{n,k}, D) \leq n+g(k-1)-1$.

As pointed out, to obtain a hopefully tight lower bound of $\tau_g(G, D)$, we need a result of its M-connectivity, which could be derived by making use of its super-connectedness property. More specifically, a graph is *super m-vertex connected of order q* if, with at most m vertices being deleted, the survival graph is either connected or it consists of a large component and the small components containing at most q vertices altogether [2,17].

For a detailed discussion about this inspiring and important structural property of a graph, and its close relationship to various fault-tolerance properties, including g-good-neighbor connectivity, g-extra connectivity, component connectivity, cyclic connectivity, as well as the Menger connectedness, readers are referred to [2]. To address our current issue, we make the following observation.

Theorem 4.1. (following as Theorem 8 of [17]) Let n, k, and r be positive integers such that $k \in [2, n)$ and $r \in [1, n\text{-}k\text{+}1]$. If F is a set of vertices of $S_{n,k}$ such that $|F| \leq n+(r-1)k-2r$, then $S_{n,k} - F$ is either connected or has a large component and small components with at most r-1 vertices in total.

Taking $g = r-1$, if we want to have a component, besides the larger component, in a survival graph $S_{n,k} - F$, which contains at least g+1 (=r) vertices, $g \in [0, n\text{-}k]$, we have to take at least $n+g(k-2)-1$ vertices. As a result, any g-extra cut must contain at least $n+g(k-2)-1$ vertices. In particular, $\kappa_g(S_{n,k}) \geq n+g(k-2)-1$.

It is easy to show that $|V(S_{n,k})| > 2(\kappa_g(S_{n,k})+g) = 2(n+g(k-1)-1)$. Thus, by Corollaries 3.2, 3.3, 4.1, and Lemma 4.3, we have the following result.

Theorem 4.2. Let $n \geq 4$, $k \in [3, n)$, $g \in [0, n\text{-}k]$, $\tau_g(S_{n,k}, D) = n+g(k-1)-1$.

Incidentally, since $S_{n,n-1}$ is isomorphic to the star graph (Lemma 4 in [5]), and $S_{n,n-2}$ is isomorphic to the alternating group network [3], denoted by AN_n, the g-extra diagnosability results of these latter two graphs immediately follow.

Corollary 4.1. Let $n \geq 4$, $\tau_g(S_n, D) = \tau_g(S_{n,n-1}, D) = 2n-3$.

Corollary 4.2. Let $n \geq 4$, $g \in [1, 2]$, $\tau_g(AN_n, D) = \tau_g(S_{n,n-2}, D) = n+g(n-2)-1$.

5. Concluding remarks

In this paper, we summarized and unified a process that we can effectively apply to derive diagnosability results for multiple fault-tolerance models in terms of such mainstream diagnostic models as the PMC and the MM* models. Simply put, to derive such a diagnosability result for a graph G in terms of a fault-tolerance model M, and a diagnostic model D, we need to obtain $\kappa_M(G)$, the M-connectivity of G, and the details of the rest of the process in terms of model D are mostly structure independent, thus can be spared.

We also demonstrated the value of such a general process by obtaining the g-extra diagnosability of the (n, k)-star graphs. Although this result has already been reported in the literature, this alternative derivation is straightforward, and even mechanical, thus has its clear advantage.

As future research topics, besides attempting to achieve the diagnosability results of other interconnection structures under the existing fault-tolerance models in light of this general process, we will also look into other appropriate fault-tolerance models, and the feasibility of applying this general process to derive various fault-tolerance property under such alternative models.

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