A Low Complexity Robust Adaptive Beamforming Method with Orthogonality Constraint

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Abstract. Performance of many adaptive beamforming methods degrades when antenna array steering vector mismatch exists and popular optimization-based methods suffer from heavy computation complexity. In this paper, we investigate robust adaptive beamforming technique with low complexity. An optimization problem is constructed based on orthogonality constraint instead of conventional norm constraint; therefore, semidefinite programming (SDP) relaxation is avoided. We present the approach to obtain closed-form solution by imposing Lagrange multiplier methodology. Numerical experiments demonstrate that proposed low complexity method outperforms other previously developed beamforming methods and reduces computation time significantly.

1. Introduction

Adaptive beamforming has been widely utilized in many fields, i.e., wireless communications, radar, astronomy and medical imaging [1-2]. If the knowledge of desired signal steering vector is known exactly, conventional adaptive beamforming methods can suppress interference effectively; otherwise, desired signal may be suppressed as an interference, resulting in drastically reduced antenna array output signal-to-interference-plus-noise ratio (SINR) [3-5]. Steering vector mismatch including pointing error and array geometry perturbations is the common case in practical situations, hence many approaches have been proposed for improving robustness against steering vector mismatch.

Eigenspace-based beamforming [6] is a prevalent robust adaptive technique, however, it doesn't work well at low signal-to-noise ratio (SNR) because of subspace swap phenomenon. With the presumed steering vector and the prior information that the mismatch vector is norm bounded, the worst-case-based beamforming [7] is proposed as an optimization-based technique. An approach utilizing orthogonal interference-plus-noise (IN) subspace projection matrix is introduced in[8] and the dimension of estimated IN subspace is determined by an artificial energy percentage parameter. Sparse reconstruction[9] has been recently introduced as a new beamforming design principle. The algorithm performs perfectly only for ideal array geometry and presence of array perturbations would lead to severe performance degradation. Besides, methods in [7-9] are optimization-based and the computation complexity is high.

In this paper, a low complexity robust adaptive beamforming method with orthogonality constraint is proposed. By analyzing the advantage of orthogonality constraint, we construct a new optimization problem. Low complexity Lagrange multiplier methodology is employed to obtain closed-form solution. Proposed low complexity method is shown to outperform existing beamforming methods by simulation results. Furthermore, proposed method reduces computation complexity significantly.

2. The Signal model

We consider a uniform linear array (ULA) with M omni-directional sensors. The received signal $\mathbf{x}(k)$ is given by:

$$\mathbf{x}(k)=\mathbf{s}(k)+\mathbf{i}(k)+\mathbf{n}(k)=s_0(k)\mathbf{a}_0+\mathbf{i}(k)+\mathbf{n}(k)$$

where $\mathbf{a}_0$ is desired steering vector, k is time index , $s_0(k)$ is waveform of desired signal, $\mathbf{s}(k)$, $\mathbf{i}(k)$ and $\mathbf{n}(k)$ represent desired signal, interference, and noise, respectively. $\mathbf{s}(k)$ and $\mathbf{i}(k)$ are assumed to be statistically independent to each other. The output of beamforming method can be written as

$$y(k)=\mathbf{w}^H\mathbf{x}(k)$$

where $\mathbf{w}$ is the weight vector of the array and $(\cdot)^H$ stands for the Hermitian transpose. The optimal weight vector can be formulated as:

$$\mathbf{w}_{\mathrm{opt}}=\frac{\mathbf{R}_{i+n}^{-1}\mathbf{a}_0}{\mathbf{a}_0^H\mathbf{R}_{i+n}^{-1}\mathbf{a}_0} \quad \text{* MERGEFORMAT (1)}$$

where $\mathbf{R}_{i+n}$ $E\left\{\left(\mathbf{i}(k)+\mathbf{n}(k)\right)\left(\mathbf{i}(k)+\mathbf{n}(k)\right)^H\right\}$ is IN covariance matrix. In practical situations, $\mathbf{R}_{i+n}$ and $\mathbf{a}_0$ may be unknown, therefore, sample covariance matrix $\hat{\mathbf{R}}=1/N\sum_{k=1}^{N}\mathbf{x}(k)\mathbf{x}^H(k)$ with N snapshots and presumed steering vector $\overline{\mathbf{a}}$ are employed to replace them. is transformed to sample matrix inversion (SMI) adaptive beamforming:

$$\mathbf{w}_{\mathrm{SMI}}=\frac{\hat{\mathbf{R}}^{-1}\overline{\mathbf{a}}}{\overline{\mathbf{a}}^H\hat{\mathbf{R}}^{-1}\overline{\mathbf{a}}}. \quad \text{* MERGEFORMAT (2)}$$

If gap exists between aprior $\overline{\mathbf{a}}$ and actual $\mathbf{a}_0$, the beampattern of $\mathbf{w}_{\mathrm{SMI}}$ won't point to the direction where desired signal is arriving. Furthermore, with limited snapshots, sample covariance matrix cannot precisely represent $\mathbf{R}_{i+n}$, then desired signal may be regarded as interference and be suppressed. Influenced by above two drawbacks, SINR of degrades severely compared to optimal weight vector. The objective of this paper is to obtain better estimated desired signal vector $\mathbf{a}$ to increase output SINR.

3. Proposed Method

3.1. Beamforming Based on Orthogonality

In [10], A. Khabbazibasmenj et al. proposed an optimization problem to estimate desired signal vector based on norm constraint:

$$\begin{aligned} \min_{\mathbf{a}} \quad & \mathbf{a}^H\hat{\mathbf{R}}^{-1}\mathbf{a} \\ \text{s.t.} \quad & \|\mathbf{a}\|^2=M \\ & \mathbf{a}^H\tilde{\mathbf{C}}\mathbf{a}\le\Delta_0=\max_{\theta\in\Theta}\mathbf{d}^H(\theta)\tilde{\mathbf{C}}\mathbf{d}(\theta) \end{aligned} \quad \text{* MERGEFORMAT (3)}$$

where $\tilde{\mathbf{C}}=\int_{\tilde{\Theta}}\mathbf{d}(\theta)\mathbf{d}^H(\theta)d\theta$, $\mathbf{d}(\theta)$ is the steering vector corresponding to direction θ under the assumption of ideal array geometry, Θ is angular sector where desired signal is located, $\tilde{\Theta}$ is the complement of Θ. Unfortunately, optimization problem is non-convex due to non-convex equality constraint $\|\mathbf{a}\|^2=M$. The solution can be obtained by imposing SDP relaxation and (4) is transformed into:

$$\begin{aligned} \min_{\mathbf{A}} \quad & \mathrm{Tr}\left(\hat{\mathbf{R}}^{-1}\mathbf{A}\right) \\ \text{s.t.} \quad & \mathrm{Tr}\left(\mathbf{A}\right) = M \\ & \mathrm{Tr}\left(\tilde{\mathbf{C}}\mathbf{A}\right) \le \Delta_0 \\ & \mathbf{A} \ge \mathbf{0} \end{aligned}$$ * MERGEFORMAT (4)

where $\mathbf{A}$ $\mathbf{a}\mathbf{a}^H$. The optimization problem is not equivalent to because $\mathbf{A}$ in may be not rank-one. Furthermore, is usually solved with matlab toolbox such as Sedumi and CVX, resulting in high computation complexity.

The non-convex property of results from non-convex equality constraint, hence the problem without non-convex equality constraint is convex and can be expressed as:

$$\min_{\mathbf{a}} \quad \mathbf{a}^H\hat{\mathbf{R}}^{-1}\mathbf{a} \quad \text{s.t.} \quad \mathbf{a}^H\tilde{\mathbf{C}}\mathbf{a} \le \Delta_0 .$$ * MERGEFORMAT (5)

Obviously, the solution to is $\mathbf{a} = \mathbf{0}$. To avoid this trivial solution, another constraint is needed. Let mismatch vector $\mathbf{e} = \mathbf{a}_0 - \bar{\mathbf{a}}$, $\mathbf{e}$ can be decomposed as $\mathbf{e} = \mathbf{e}_\perp + \mathbf{e}$. $\mathbf{e}_\perp$ represents the part orthogonal to $\bar{\mathbf{a}}$ and $\mathbf{e}$ denotes the part parallel to $\bar{\mathbf{a}}$. Since norm of $\bar{\mathbf{a}}$ does not influence accuracy of weight vector $\bar{\mathbf{a}}$, only the estimation of $\mathbf{e}_\perp$ is essential. Based on the physical meaning of $\mathbf{e}_\perp$, we have:

$$\bar{\mathbf{a}}^H\mathbf{e}_\perp = 0 .$$ * MERGEFORMAT (6)

can be rewritten as:

$$\bar{\mathbf{a}}^H\left(\mathbf{a} - \bar{\mathbf{a}}\right) = 0 .$$ * MERGEFORMAT (7)

Combined with and , the new optimization problem is given by:

$$\begin{aligned} \min_{\mathbf{a}} \quad & \mathbf{a}^H\hat{\mathbf{R}}^{-1}\mathbf{a} \\ \text{s.t.} \quad & \bar{\mathbf{a}}^H\left(\mathbf{a} - \bar{\mathbf{a}}\right) = 0, \quad \mathbf{a}^H\tilde{\mathbf{C}}\mathbf{a} \le \Delta_0 \end{aligned}$$ * MERGEFORMAT (8)

Compared to , is a convex optimization problem and SDP relaxation is not needed. Consider

$$\bar{\mathbf{a}}^H\left(\mathbf{a} - \bar{\mathbf{a}}\right) = \bar{\mathbf{a}}^H\mathbf{a} - \bar{\mathbf{a}}^H\bar{\mathbf{a}} = \bar{\mathbf{a}}^H\mathbf{a} - M .$$

is transformed as:

$$\begin{aligned} \min_{\mathbf{a}} \quad & \mathbf{a}^H\hat{\mathbf{R}}^{-1}\mathbf{a} \\ \text{s.t.} \quad & \bar{\mathbf{a}}^H\mathbf{a} = M, \quad \mathbf{a}^H\tilde{\mathbf{C}}\mathbf{a} \le \Delta_0 . \end{aligned}$$ * MERGEFORMAT (9)

Mention that both and are quadratically constrained quadratic program (QCQP) problem.

3.2 Closed-form Solution

Consider the function:

$$f_1\left(\mathbf{a}, \lambda, \mu\right) = \mathbf{a}^H\hat{\mathbf{R}}^{-1}\mathbf{a} + \lambda\left(\mathbf{a}^H\tilde{\mathbf{C}}\mathbf{a} - \Delta_0\right) + \mu\left(2M - \bar{\mathbf{a}}^H\mathbf{a} - \mathbf{a}^H\bar{\mathbf{a}}\right)$$ * MERGEFORMAT (10)

where $\lambda \ge 0$, $\mu \ge 0$. Assuming

$$\mathbf{a}^H\tilde{\mathbf{C}}\mathbf{a} \le \Delta_0 .$$ * MERGEFORMAT (11)

The second constraint in is satisfied and the solution to is given by:

$$\mathbf{a} = \frac{M\hat{\mathbf{R}}\bar{\mathbf{a}}}{\bar{\mathbf{a}}^H\hat{\mathbf{R}}\bar{\mathbf{a}}} .$$ * MERGEFORMAT (12)

By inserting into , the condition in is equivalent to:

$$\frac{M^2\bar{\mathbf{a}}^H\hat{\mathbf{R}}\tilde{\mathbf{C}}\mathbf{R}\bar{\mathbf{a}}}{\bar{\mathbf{a}}^H\hat{\mathbf{R}}\bar{\mathbf{a}}\bar{\mathbf{a}}^H\hat{\mathbf{R}}\bar{\mathbf{a}}} \le \Delta_0 \quad . \quad \text{* MERGEFORMAT (13)}$$

Therefore, if holds, is the solution to optimization problem . However, if doesn't hold, second constraint in is active. For fixed λ and μ, the minimization of results in:

$$\hat{\mathbf{a}}_{\lambda,\mu} = \mu\left(\hat{\mathbf{R}}^{-1} + \lambda\tilde{\mathbf{C}}\right)^{-1}\bar{\mathbf{a}} \quad . \quad \text{* MERGEFORMAT (14)}$$

Through the differential of subject to λ and μ, solution to the maximization of $f_1(\mathbf{a},\lambda,\mu)$ with respect to λ and μ is given by:

$$\hat{\mu} = \frac{M}{\bar{\mathbf{a}}^H\left(\hat{\mathbf{R}}^{-1} + \lambda\tilde{\mathbf{C}}\right)^{-1}\bar{\mathbf{a}}} \quad . \quad \text{* MERGEFORMAT (15)}$$

$$g_1(\lambda) \quad \frac{\bar{\mathbf{a}}^H\left(\hat{\mathbf{R}}^{-1} + \lambda\tilde{\mathbf{C}}\right)^{-1}\tilde{\mathbf{C}}\left(\hat{\mathbf{R}}^{-1} + \lambda\tilde{\mathbf{C}}\right)^{-1}\bar{\mathbf{a}}}{\bar{\mathbf{a}}^H\left(\hat{\mathbf{R}}^{-1} + \lambda\tilde{\mathbf{C}}\right)^{-1}\bar{\mathbf{a}}\bar{\mathbf{a}}^H\left(\hat{\mathbf{R}}^{-1} + \lambda\tilde{\mathbf{C}}\right)^{-1}\bar{\mathbf{a}}} = \frac{\Delta_0}{M^2} \quad . \quad \text{* MERGEFORMAT (16)}$$

Optimal Lagrange multiplier $\hat{\lambda}$ can be solved with the Newton's method. In the following part, we try to obtain $\hat{\lambda}$. Obviously, $g_1(\lambda)$ is a monotonically decreasing function with respect to λ when $\lambda \ge 0$. The Newton's method can be adopted upon until the upper bound and lower bound of $\hat{\lambda}$ is obtained. When $\lambda = 0$, based on ,

$$g_1(0) = \frac{\bar{\mathbf{a}}^H\hat{\mathbf{R}}\tilde{\mathbf{C}}\hat{\mathbf{R}}\bar{\mathbf{a}}}{\left(\bar{\mathbf{a}}^H\hat{\mathbf{R}}\bar{\mathbf{a}}\right)^2} > \frac{\Delta_0}{M^2} \quad \text{* MERGEFORMAT (17)}$$

hence $\lambda = 0$ can be regarded as the lower bound. As $\lambda \to \infty$,

$$g_1(\lambda) \to g_1(\infty) = \frac{\bar{\mathbf{a}}^H\tilde{\mathbf{C}}^{-1}\bar{\mathbf{a}}}{\bar{\mathbf{a}}^H\tilde{\mathbf{C}}^{-1}\bar{\mathbf{a}}\bar{\mathbf{a}}^H\tilde{\mathbf{C}}^{-1}\bar{\mathbf{a}}} < \rho \quad \frac{\bar{\mathbf{a}}^H\tilde{\mathbf{C}}\bar{\mathbf{a}}}{\bar{\mathbf{a}}^H\bar{\mathbf{a}}\bar{\mathbf{a}}^H\bar{\mathbf{a}}} < \frac{\Delta_0}{M^2} \quad . \quad \text{* MERGEFORMAT (18)}$$

Based on and , we learn that optimal Lagrange multiplier $\hat{\lambda}$ always exists and the solution to $g_1(\lambda) = \rho$ is large than $\hat{\lambda}$, hence the upper bound of solution to $g_1(\lambda) = \rho$ is also the upper bound of $\hat{\lambda}$. We perform following eigenvalue composition:

$$\hat{\mathbf{R}}^{1/2}\tilde{\mathbf{C}}\left(\hat{\mathbf{R}}^{1/2}\right)^H = \mathbf{U}\boldsymbol{\Gamma}\mathbf{U}^H \quad , \quad \text{* MERGEFORMAT (19)}$$

where $\mathbf{U}$ denotes the set of eigenvectors, $\boldsymbol{\Gamma} = \mathrm{diag}(\gamma_1, \cdots, \gamma_M)$ and γ_i are eigenvalues sorted in descending order. can be rewritten as:

$$\tilde{\mathbf{C}} = \hat{\mathbf{R}}^{-1/2}\mathbf{U}\boldsymbol{\Gamma}\mathbf{U}^H\left(\hat{\mathbf{R}}^{-1/2}\right)^H \quad . \quad \text{* MERGEFORMAT (20)}$$

Insert into and let $\mathbf{h} = \mathbf{U}\hat{\mathbf{R}}^{1/2}\bar{\mathbf{a}}$, we have

$$g_1(\lambda) = \frac{\sum_{i=1}^{M}\frac{|h_i|^2\phi_i}{(\phi_i + \lambda)^2}}{\left(\sum_{i=1}^{M}\frac{|h_i|^2\phi_i}{\phi_i + \lambda}\right)^2} \quad \text{* MERGEFORMAT (21)}$$

where h_i is the ith variable of $\mathbf{h}$, $\phi_i = \frac{1}{\gamma_i}$ for $i = 1, \cdots, M$ and are sorted in ascending order. Then

$$\rho \le \frac{\frac{1}{(\phi_1+\lambda)^2}\sum_{i=1}^{M}|h_i|^2\phi_i}{\frac{1}{(\phi_M+\lambda)^2}\left(\sum_{i=1}^{M}|h_i|^2\phi_i\right)^2} = \frac{(\phi_M+\lambda)^2}{(\phi_1+\lambda)^2\sum_{i=1}^{M}|h_i|^2\phi_i}.$$

Upper bound of $\hat{\lambda}$ is given by:

$$\lambda_{up} = \frac{c_2\phi_1 - \phi_M}{1 - c_2}$$

where $c_2 = \sqrt{\rho\sum_{i=1}^{M}|h_i|^2\phi_i}$. With lower bound and upper bound of optimal Lagrange multiplier $\hat{\lambda}$, the Newton's method is employed in $\left[0, \lambda_{up}\right]$ to obtain $\hat{\lambda}$ satisfying $g_1\left(\hat{\lambda}\right) = \Delta_0 / M^2$.
Inserting into , we have:

$$\mathbf{a} = \frac{M\left(\hat{\mathbf{R}}^{-1} + \hat{\lambda}\tilde{\mathbf{C}}\right)^{-1}\bar{\mathbf{a}}}{\bar{\mathbf{a}}^H\left(\hat{\mathbf{R}}^{-1} + \hat{\lambda}\tilde{\mathbf{C}}\right)^{-1}\bar{\mathbf{a}}}. \quad \text{* MERGEFORMAT (22)}$$

The weight vector is given by:

$$\hat{\mathbf{w}} = \frac{\hat{\mathbf{R}}^{-1}\mathbf{a}}{\mathbf{a}^H\hat{\mathbf{R}}^{-1}\mathbf{a}}. \quad \text{* MERGEFORMAT (23)}$$

For proposed method, if holds, the computation complexity of estimating actual steering vector in can be neglected, hence the total computation complexity originates from the matrix inversion operation when computing weight vector in , which is $O(M^3)$. However, if doesn't hold, another matrix inversion operation is performed in . Since the binary search and Newton's method only involves several multiplications and derivative calculations, the total computation complexity is still $O(M^3)$. By solving QCQP problems and with optimization software, the computation complexity is $O(M^{3.5})$. Overall, the proposed method has a remarkable advantage to QCQP methods in the view of the computation complexity and is competent in real-time application scenarios.

4. Experimental results and analysis

Basic simulation conditions are the same as Example 1. All simulations are performed using matlab 2016b running on an Intel Core i7 Duo, 2.5 GHz processor with 4GB of memory, under Windows 7 Service Pack 1. 100 Monte-Carlo runs are performed.

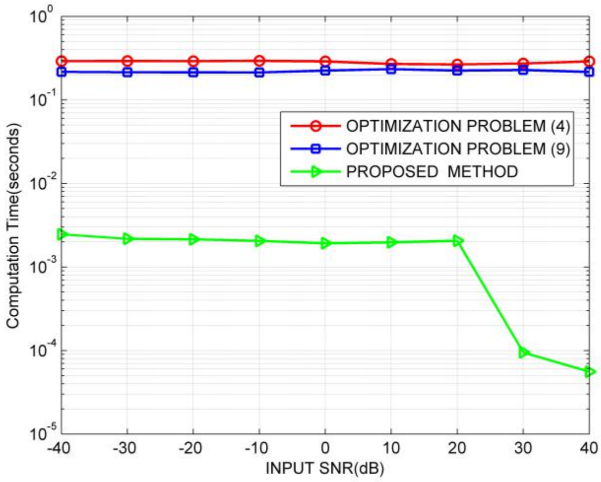

Figure 1. The computation time comparison.

Simulation Example 1: Computation Time. In this example, the computation time of optimization problem , optimization problem solved by CVX matlab Toolbox and proposed low complexity method is displayed in Figure 1. For optimization problem , the number of unknown variables is M times of optimization problem solved by CVX matlab Toolbox, hence more iterations are needed for the convergence of . Proposed low complexity method significantly reduces the computation complexity compared with optimization-based methods. When input SNR is higher than 30dB, the computation complexity is reduced further since is satisfied. However, the computation complexity has no matter with input SNR for optimization-based methods.

The proposed method is compared to the eigenspace-based method[6], the worst-case-based method[7], IN subspace projection matrix method[8], the sparse reconstruction method[9] and optimization problem in terms of the output SINR.

Simulation Example 2: Exactly known desired steering vector. In this example, the desired steering vector $\mathbf{a}_0$ is assumed to be equivalent with presumed steering vector $\bar{\mathbf{a}}$. It can be seen from Figure 2 that proposed low complexity method outperforms other beamforming techniques except for sparse reconstruction method. Under the assumption that desired steering vector is known exactly, the ideal IN can be reconstructed and the output SINR is almost equal to optimal value. At low SNR, eigenspace-based method suffers from severe performance degradation because of a high probability of subspace swap. Proposed method performs better than optimization problem , verifying that SDP relaxation results in performance loss.

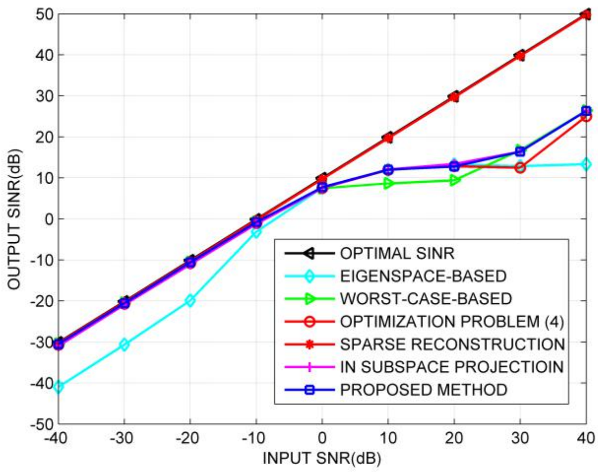

Figure 2. Output SINR versus SNR with Exactly known desired steering vector.

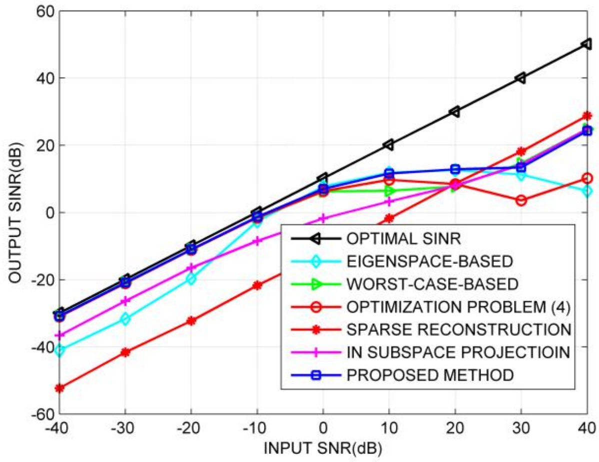

Figure 3. Output SINR versus SNR with pointing error, gain error and phase error.

Simulation Example 3: Pointing error, gain error and phase error. In this example, the presumed DOA of desired signal is not known exactly. Also, array geometry is distorted by gain and phase error. The DOA of desired signal is uniformly distributed in $[\bar{\theta}_s - 3°, \bar{\theta}_s + 3°]$. Array gain and phase error are uniformly distributed in [-3dB, 3dB] and [-20°, 20°], respectively. As can be observed from Figure 3, the performance of [8] is terrible. Since the construction of IN matrix is based on ideal array geometry, the occurrence of array geometry, i.e., gain and phase error, results in severe performance degradation. Besides, compared with proposed method, the disadvantage of SDP relaxation is evident.

5. Conclusion

In this paper, a new beamforming method is proposed. We construct a convex optimization problem by imposing orthogonality constraint. Lagrange multiplier methodology is employed to obtain closed-form solution. As a low complexity method without SDP relaxation, the proposed method is demonstrated to outperform existing beamforming methods and takes less computation time by simulation results.

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