

# FFT based unambiguous acquisition method for BOC modulation signal

Bin Li<sup>1,2</sup>, Dan Qi<sup>1</sup>, Shaozhen Zhang<sup>1</sup>, Dan Cheng<sup>1</sup>, Jun Mu<sup>1</sup> and Zheng Yan<sup>1</sup>

<sup>1</sup>Space Star Technology Co, Ltd, Beijing, China

<sup>2</sup>E-mail: libin@spacestar.com.cn

**Abstract.** This paper presents an unambiguous acquisition algorithm for Binary Offset Carrier (BOC) modulation signals, which is adopted by modernized Global Navigation Satellite System (GNSS). By utilizing FFT, the spectrum of BOC signals is derived. This paper proposes an unambiguous acquisition method based on FFT to get rid of the BOC modulation drawback. By utilizing FFT, the spectrum of BOC signal is derived. The up and down sideband are separated in frequency domain by rectangular function, which means sideband filtering. Due to sparsity of filtered BOC spectrum, frequency compression method is proposed to reduce the number of IFFT points. Theoretical and simulation results show that this technique completely removes the ambiguity threat in acquisition process.

## 1. Introduction

With rapid development of Global Navigation Satellite System (GNSS), several systems have to coexist in a limited number of frequency bands. In order to reduce the interference with Binary Phase Shift Keying (BPSK) modulated signal, John Betz developed Binary Offset Carrier modulation (BOC modulation) [1]. BOC modulation method enhances interoperability and achieves high accuracy code estimation in multi-path condition. Currently, this modulation method has been adopted by almost all GNSSs. However, the ambiguous problem challenge signal acquisition, which caused by notches and peaks within  $\pm$  one chip in the correlation envelope.

Some unambiguous acquisition techniques are developed getting rid of these drawbacks. Sideband filtering in time domain can reduce the effect of side peaks [3,4]. However, filtering and the dual sideband processing increase the implementation complexity and cause a loss of power. Autocorrelation Side-Peak Cancellation Technique (ASPeCT) algorithm is proposed to realize unambiguous acquisition [5]. However, it is only dedicated to sin-BOC (n, n) signals. Similarly, Gamma-Ray Ablation Sensing System (GRASS) method is only suitable for sin-BOC [6]. In addition, both ASPeCT and GRASS method use non-coherent integral value to reconstruct correlation function, which will cause extra noise and reduce acquisition sensitivity.

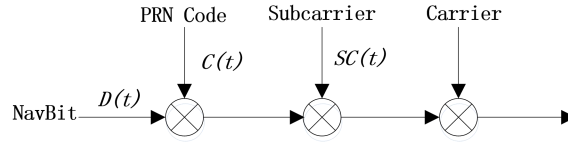
In this paper, an unambiguous acquisition based on fast Fourier transform (FFT) is proposed. This method eliminates side peaks by employing a frequency domain filter and is suitable for all BOC modulated signals. The remainder of this paper is organized as follows. Firstly, BOC signal model and spectral property is discussed. Next the unambiguous acquisition is explained in detail. At last, simulation results is given and analyzed. It is shown that proposed method can eliminate side peak effectively and achieve signal acquisition.

## 2. BOC Signal Model

As shown in Fig.1, the subcarrier modulation step is introduced into BOC modulation, which is



different from BPSK modulation. Signal energy will be separate into 2 parts, and shift away from the carrier frequency. Therefore, BOC modulation is also known as a split-spectrum modulation.



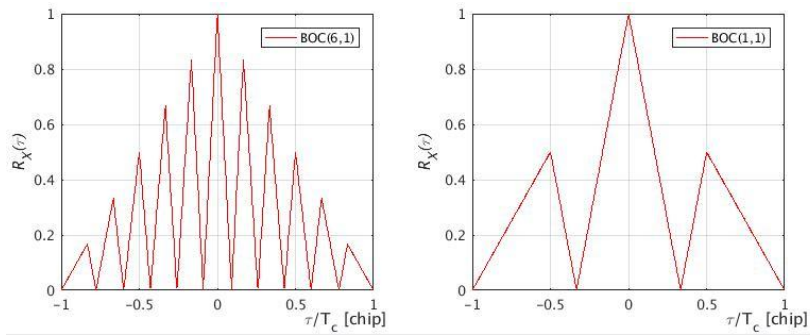
**Figure 1.** BOC Signal Modulation Block Diagram.

There are two type of subcarriers, sine subcarrier  $SC_{\sin}(t)$  and cosine subcarrier  $SC_{\cos}(t)$ , which are expressed as:

$$SC_{\sin}(t) = \text{sign}(\sin(2\pi f_s t)) \quad (1)$$

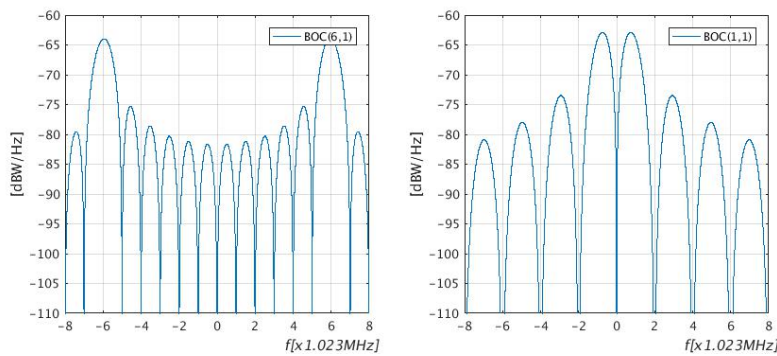
$$SC_{\cos}(t) = \text{sign}(\cos(2\pi f_s t)) \quad (2)$$

where  $f_s$  is subcarrier frequency. BOC modulated signal is denoted as  $\text{BOC}(m, n)$ , where  $m$  means the ratio of the subcarrier frequency  $f_{sc}$  to 1.023MHz, and  $n$  represents the ratio of the spreading chip rate  $f_c$  to 1.023MHz. Moreover, the ratio  $N = 2m/n$  is referred to as BOC-modulation order.



**Figure 2.** Correlation function of BOC modulated signals.

Fig.2 illustrates the correlation function of BOC modulated signals. For both BOC (1,1) and BOC (6,1) modulation, subcarriers of BOC signals create side peaks on both sides of the center peak of the sharp autocorrelation function (ACF), which leads to ambiguities or notches. These are very challenging in the acquisition process. Moreover, according to the simulation results and theoretical analysis, the number of correlation peaks is  $2N-1$ , which depends on the BOC modulation order.



**Figure 3.** Spectrum of BOC modulated signals.

The spectrum of BOC modulation signals is shown in Fig.3. The main spectral energy of BOC signal concentrates two main lobes which locate at the side of the carrier frequency. The band width

between main lobes depends on modulation order. Especially, two main lobes of BOC (1,1) signals locate next to each other. The signal characteristic raises challenges to filter design. It is impossible to implement a filter in time domain, which can remove one of main lobes without energy loss. It leads to acquisition performance reduction.

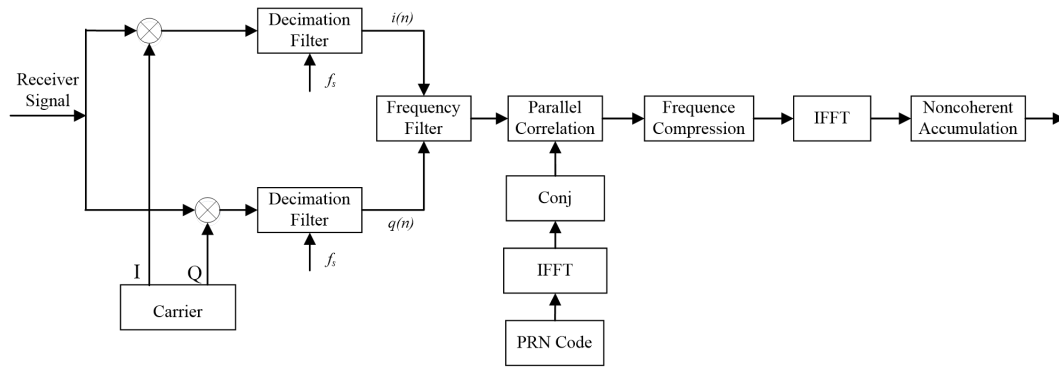
### 3. FFT Based Unambiguous Acquisition Method

The flow chart of FFT based unambiguous acquisition method is shown in Fig.4. Proposed acquisition method has 6 parts, carrier cancellation, down sampling, frequency-domain filtering, parallel correlation, frequency compression and non-coherent accumulation.

The received signal is sampling data of intermediate frequency, which is expressed as:

$$s_i = c(t - \tau)sc(t - \tau) \cos[2\pi(f_{IF} + f_D)t + \theta] \quad (3)$$

where  $c(t - \tau)$  is modulated Pseudo-Random Noise (PRN) code,  $sc(t - \tau)$  denotes subcarrier,  $f_{IF}$  is the intermediate frequency,  $f_D$  is the doppler frequency,  $\theta$  is carrier phase. The carrier modulated in received signal is cancelled by multiplying the in-phase and quad-phase local carrier.



**Figure 4.** FFT based unambiguous acquisition structure.

In order to reduce the burden of calculating correlation function, a decimation filter is introduced to down sampling. After carrier cancellation, intermediate signals transfer to complex signals, and down sampling in complex field is adopted. According to the sampling theorem, sample frequency  $f_s$  should be more than  $2(f_c + f_{sc})$ , which is BOC signal band width. For example, the sample frequency of BOC (1,1) signals should be over 4.096MHz.

#### 3.1. Frequency-domain filtering

The rectangular function is used as frequency-domain filter, which can separate up and down sideband signals. As shown below, the amplitude-frequency response of filter is expressed as:

$$G_u(f) = \begin{cases} 1, & (f_{sc} - f_c) \leq f \leq (f_{sc} + f_c) \\ 0, & \text{others} \end{cases} \quad (4)$$

$$G_d(f) = \begin{cases} 1, & -(f_{sc} + f_c) \leq f \leq -(f_{sc} - f_c) \\ 0, & \text{others} \end{cases} \quad (5)$$

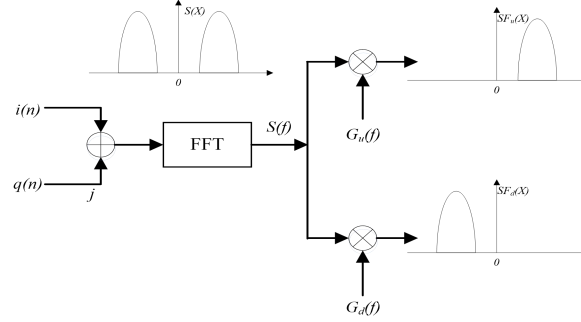
where,  $f_{sc}$  is subcarrier frequency, and  $f_c$  is chip frequency.

As shown in Fig.5, the down sampling data of in-phase and quad-phase signals is handled as complex number at first. And then, FFT is used to transform signals from time to frequency domain. The spectrum of signal multiply up-sideband filtering function  $G_u(f)$  and down-sideband filtering function  $G_d(f)$ , separately. The filtered signals are expressed as:

$$SF_u(f) = S(f)G_u(f) = \begin{cases} S(f), & (f_{sc} - f_c) \leq f \leq (f_{sc} + f_c) \\ 0, & \text{others} \end{cases} \quad (6)$$

$$SF_d(f) = S(f)G_d(f) = \begin{cases} S(f), & -(f_{sc} + f_c) \leq f \leq -(f_{sc} - f_c) \\ 0, & \text{others} \end{cases} \quad (7)$$

Obviously, there is only one main lobe in the band. And the filtered signals can be handled as BPSK signals.

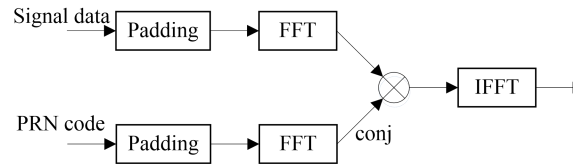


**Figure 5.** Block diagram of frequency-domain filter.

### 3.2. Parallel Code Correlation

In order to reuse filtered spectral data, proposed acquisition technique adopts a parallel code correlation. According to the properties of convolution, the correlation function is calculated by frequency multiplying. However, cyclic convolution method is not suitable for calculation correlation function, since GNSS PN code period is not a power of 2.

To avoid code phase mismatch of cyclic convolution, zero padding method is introduced in correlation step. Zero padding parallel code correlation method is shown in Fig.6. For example, 2ms received signal have 4092 points after down sampling. It extends to 4096 points by padding 4 zero elements. Similarly, 2046 local chips will pad 2050 zeros to form 4096 points sequence. Calculating 4096-point FFT for both 4096-point sequences and then computing the complex conjugate of the follow one. By taking multiplying and inverse fast Fourier transform (IFFT), the correlation values are obtained.



**Figure 6.** Block diagram of zero padding parallel code correlation.

### 3.3. Frequency Compression

It is noted that the multiplying results have lots of zeros elements due to the rectangular function. By utilizing the sparse feature of multiplying results, frequency compression method is developed to reduce the number of IFFT points. As shown in Fig.5, only up-band or down-band energy is retained in filtered spectrum. And there is no energy in other frequency band. It seems that the signal energy is compressed. Thus, only nonzero frequency is transformed to time domain, and the number of IFFT is reduced. It seems that the frequency band is compressed. Take BOC (1,1) signal as example. The down sampling frequency  $f_s$  is 8.184MHz, which is 8 times of chip frequency. After filtering, up or down band is only 2 times of chip frequency, 2.046MHz. Assuming 2ms signal data is used. The FFT points  $N_{FFT}$  is 8192 and the IFFT points  $N_{IFFT}$  is 2048. The ratio of compression is given as  $R_{fc} = N_{FFT}/N_{IFFT}$ .

The sample frequency in time domain is reduced after frequency compression since only pass band is transformed by taking IFFT. According to FFT/IFFT definition,  $f_s$  decreases to  $B/2$ , where  $B$  is the pass bandwidth.

### 3.4. Non-coherent Accumulation

By transforming the results of frequency compression to time domain, we can correlation results of received signal and replica code which are complex numbers. The correlation values are denoted as  $i(n)+jq(n)$ ,  $i(n)$  and  $q(n)$  are the in-phase and quad-phase correlation value. The expressions are:

$$i(n) = aR(\tau)\text{sinc}(f_e T_{coh})\cos(\phi_e) \quad (8)$$

$$q(n) = aR(\tau)\text{sinc}(f_e T_{coh})\sin(\phi_e) \quad (9)$$

where  $a$  is the signal amplitude,  $\tau$  is the delay between received code phase and local replica code phase,  $f_e$  is the frequency error between received signal and local carrier and  $\phi_e$  is the phase error. It is noticed that subcarriers are eliminated after filtering.

Because the phase-frequency response of proposed filter is nonlinear, the value of  $\phi_e$  is different for up-band and down-band signal. Therefore, the up-band and down-band is expressed as:

$$s_{up} = i_{up} + jq_{up} = aR(\tau)\text{sinc}(f_e T_{coh})e^{-j\phi_{up}} \quad (10)$$

$$s_{dn} = i_{dn} + jq_{dn} = aR(\tau)\text{sinc}(f_e T_{coh})e^{-j\phi_{dn}} \quad (11)$$

Here  $s_{up}$  and  $s_{down}$  are complex expression. Generally,  $\phi_{up}$  is not equal to  $\phi_{dn}$ , so the signals can not be merged directly. Non-coherent integral is introduced to eliminate the effect of phase error. And the test criterion used in this technique is given by:

$$V = \sqrt{i_{up}^2 + q_{up}^2} + \sqrt{i_{dn}^2 + q_{dn}^2} \quad (12)$$

The criterion  $V$  has no relation with phase error, so the nonlinear of phase-frequency response will not affect signal detection.

## 4. Simulation Results and Discussion

Proposed method is simulated on MATLAB platform. The signal power is -130dBm, intermediate frequency is 46.42MHz, doppler frequency is 500Hz, and the sampling frequency is 62MHz. In addition, the PRN code design refers to BDS B1C signal, the code period is 10230 and chip frequency is 1.023MHz, and the initial code phase is 6270.

As shown in Fig.6, the peak of correlation function is obvious, and there are no side peaks. Therefore, proposed algorithm can eliminate the side peaks effectively for both high and low order BOC signals. Moreover, sin-BOC signal and cos-BOC modulation signal are also simulated. The simulation shows similar results.

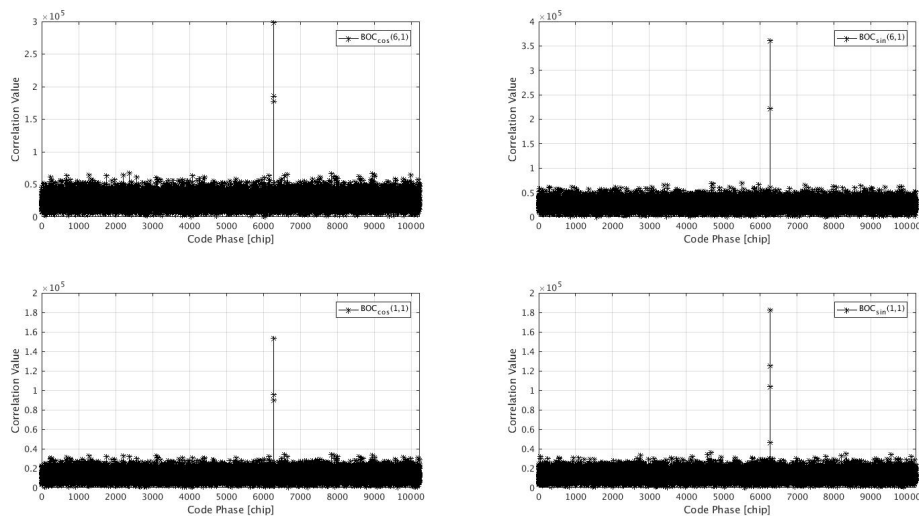
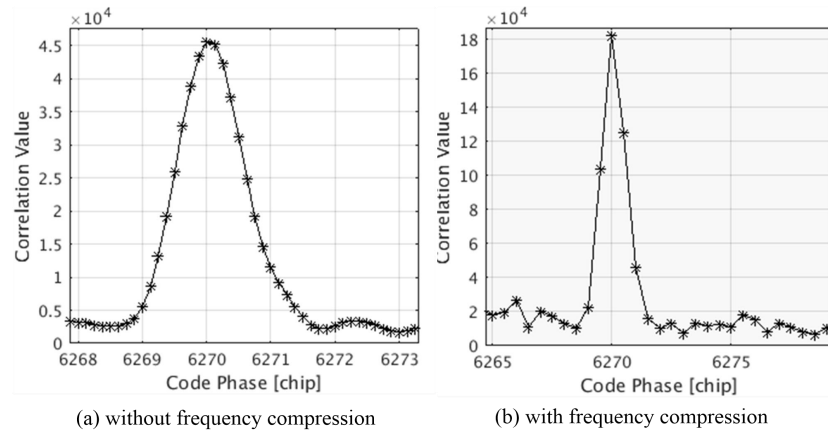


Figure 7. Block diagram of zero padding parallel code correlation.

The simulation results in details with and without frequency compression are shown in Fig.7. BOC (1,1) signal is used to simulate frequency compression. The sample frequency depends on the frequency bandwidth. There are 8 sampling points within one chip. After adopting frequency compression, sampling points number decreases to 2. Both with and without frequency compression, correct code phase is obtained by proposed filtering. Therefore, it is possible to adjust the ratio of frequency compression according to code phase searching accuracy.



**Figure 8.** Acquisition results at correlation peak.

### Conclusion

In order to eliminate side peak of BOC signals correlation function, an unambiguous acquisition method is proposed, which introduce a frequency domain filter based on FFT. To reduce calculation burden, parallel code correlation based on data reuse and frequency compression are developed. Simulation results show proposed acquisition method can eliminate side peak effectively and exact code phase can be identified. The number of frequency compression points is determined according to search accuracy. The experiment results show proposed algorithm can be applied to all kinds of BOC modulated signal.

### Acknowledgments

The authors express their appreciation to many members of ASIC Design Center whose innovative suggestions and hard work contributed to the implementation of proposed unambiguous algorithm. The ASIC Design Center also provides experimental facilities and simulation platform. This work was supported by BeiDou Navigation Receiver Special Program of Space Star Technology Co. Ltd.

### References

- [1] Betz J W 1999 The offset carrier modulation for GPS modernization *Proc. ION GPS P* 639-648
- [2] Lohan E S, Burian A and Renfors M 2008 Low-complexity unambiguous acquisition methods for BOC-modulated CDMA signals *Int. J. Commun. Syst. Network* **26** p 503-522
- [3] Fishman P, Betz J W 2000 Predicting performance of direct acquisition for the M code signal *Proc. ION NTM* p 574-582
- [4] Martin N, Leblond V, Guillotel G and Heiries V 2003 BOC (x, y) signal acquisition techniques and performances *Proc. ION GNSS* p 188-198
- [5] Julien O, Macabiau C, Cannon E and Lachapelle G 2007 ASPeCT: unambiguous sine-BOC(n, n) acquisition/tracking technique for navigation applications *IEEE Trans. Aero. Electron. Syst.* **43** p 150-162
- [6] Zheng Yao, Mingquan Lu and Zhenming Feng 2010 Unambiguous Sine-Phased Binary Offset Carrier Modulated Signal Acquisition Technique *IEEE Trans on Wireless Communicatins* **9** (2) 577-580