Robust beamforming optimization for magnetic resonance coupling wireless power transfer systems

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Abstract. Magnetic Resonance Coupling (MRC) wireless power transfer (WPT) is a promising technique for portable devices and Internet of Things applications, which can transfer power to energy-constrained terminals in mid-range with relatively high efficiency. To enhance the transfer performance, by applying more coils in transceivers, multiple-input single-output (MISO) MRC-WPT system has attracted much attention in recent years. In this paper, a MISO MRC-WPT system with multiple transmitters (TXs) and a single receiver (RX) is studied and its robust magnetic beamforming problem is investigated. The optimal robust beamforming design of the currents of TXs is investigated to maximize the power received by RX with the mutual inductance error between TXs and RX, whereas the total power required is less than a certain value and each TXs has a certain limited transmitting power. By using certain transformation techniques, the optimization problem is converted into a relaxed semidefinite programming (SDP) problem which can be solved efficiently. We further prove that the solution of the relaxed SDP problem is always rank-one, which indicates that the relaxation is tight, and the optimal solution for the problem was obtained. Numerical results demonstrate that the proposed beamforming design can achieve up to 97% transfer efficiency by reducing the measurement errors.

1. Introduction

Recently, Wireless power transfer (WPT) [1-2] has attracted widely attention because it make possible a reliability, convenience and safety power supply method for the portable devices and Internet of Things applications. For the technology of wireless power transfer, magnetic resonance coupling (MRC) has been considered as a promising technique since it can provide longer transmission distance, higher transmission efficiency and wide working frequency band [3-6].

In 2014, J. Jadidian and D. Katabi of the Massachusetts Institute of Technology proposed the Magnetic multiple-input multiple-output (MIMO) theory [7], applying multiple power transmitters (TXs) and one power receiver (RX) to improve the charging efficiency and charging flexibility. Base on the MIMO structure, magnetic beamforming is anticipated to enhance the transfer performance of WPT system. In [8], magnetic beamforming for a MISO-WPT system with multiple TXs and a single RX is investigated, and the receiving efficiency is significantly improved by optimizing the TXs currents. However, the beamforming design in [8] based on the perfect knowledge of the mutual inductance between TXs and RX. In practical scenarios, perfect knowledge of the mutual information between TXs and RX may not be available due to the influence of environment and positional deviation. Therefore, the robust beamforming design over the mutual inductance error for MRC-WPT system needs to be investigated.

In this paper, we assume that the measured mutual inductance has a norm bounded error, then consider an MRC-WPT system with one RX and multiple TXs whose source current (or corresponding source voltage) can be adjusted so that the induced magnetic field can be optimally superimposed at RX to achieve maximum power transfer. The robust beamforming is designed by jointly optimizing the currents of all TXs to maximize the received power while under the limit of total transmit power constraints and the power constraints of each transmitter circuit. According to convex optimization theory, the proposed optimization problem is a non-convex quadratically constrained quadratic programming (QCQP) [9]. By means of inequality method and S-procedure method, the original optimization problem is transformed into a semi-positive definite programming problem with rank 1. Then, by applying semidefinite relaxation (SDR), we show that the optimal solution of the semidefinite programming (SDP) problem is always rank-one, which means that the relaxation is tight, and the optimal solution of the original problem can be obtain.

2. MISO-WPT system model

The system model discussed in this paper is shown in Figure 1, A MISO MRC-WPT system with multiple power transmitters (TXs) and one power receiver (RX) is considered. For each TXs and RX, an air-core induction coils is applied. Specially, the transceiver circuit comprise a series RLC resonant circuits and the coil is considered as the inductor. $L_n > 0$, $C_n > 0$, $n = 1,\dots,N$ and $L_r > 0$, $C_r > 0$ denote the self-inductance and the capacitance in the resonant circuit of the *n*-th TX and the RX respectively. The capacitances in each resonant circuit are set to make the all circuit resonant at the resonance angular frequency $\omega = \omega_0 = 1/\sqrt{LC}$. Moreover, the *n*-th TX is connected to a stable power source supplying sinusoidal voltage over time given by $\tilde{v}_{tx,n}(t) = \mathrm{Re}\left\{v_{tx,n}e^{j\omega t}\right\}$, where $v_{tx,n}$ denoting the complex voltage and $\omega > 0$ denoting the operating angular frequency. Thus, the steady-state current flowing through the *n*-th TX is defined as $\tilde{i}_{tx,n}(t) = \mathrm{Re}\left\{i_{tx,n}e^{j\omega t}\right\}$, with the complex current $i_{tx,n}$. This current produces a time-varying magnetic flux in the transmitter's electromagnetic coil, which passes through the RX coil and induces time-varying currents in it. We denote $\tilde{i}_{rx}(t) = \mathrm{Re}\left\{i_{rx}e^{j\omega t}\right\}$, with complex current i_{rx}, as the steady state current at receiver.

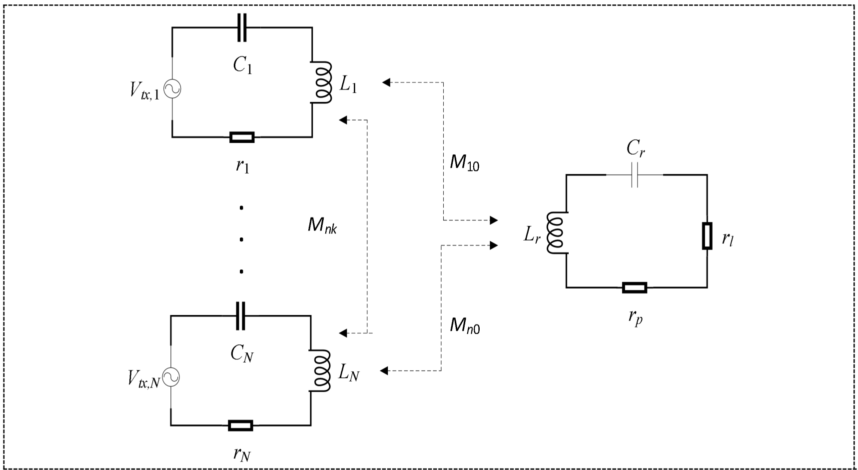

Figure 1. MISO-WPT system model.

Define M_{n0} as the mutual inductance between the *n*-th TX coil and the RX coil, and M_{nk} ($n \neq k$) as the mutual inductance between the *n*-th TX coil and the *k*-th TX coil respectively. The mutual inductances is a positive or negative real number, which depend on the physical characteristics of each pair of TX and RX coils such as their relative distance, orientations, etc.

The source resistance of the n-th TX is denoted by $r_{tx,n} > 0$. Then the diagonal resistance matrix is defined as $\mathbf{R} \quad \mathrm{diag}\left\{r_{tx,1}, r_{tx,1}, \ldots, r_{tx,N}\right\}$. Similarly, the total resistance of RX is denoted by r_{rx} , which is given by $r_{rx} = r_{rx,p} + r_{rx,l}$, where $r_{rx,p} > 0$ is the parasitic resistance and $r_{rx,l} > 0$ is the load resistance. It should be noted that $r_{rx,l}$ is a real value resistor and $r_{rx,l} \quad r_{rx,p}$.

Assume that there is a controller applied in the MRC-WPT system, thus the information of system pre-measured parameters can be obtained. Define the complex TX current $i_{tx,n}$ as the design variable, by optimization, the controller can realize magnetic beamforming.

By applying Kirchhoff's circuit law, the coupled current i_{rx} of to the RX is given by

$$i_{rx} = \frac{j\omega}{r_{rx}} \sum\nolimits_{n=1}^{N} M_{n0} i_{tx,n} . \tag{1}$$

Define the vector of all TXs currents as $\mathbf{i} = [i_{tx,1}, \cdots, i_{tx,N}]^T$, and the vector of mutual inductances between the RX coil and TXs coils as $\mathbf{m} = [M_{10}, \cdots, M_{N0}]^T$. From (1), the receiving power of RX can be given by

$$p_r = \frac{1}{2}\left|i_{rx}\right|^2 r_{rx,l} = \frac{\omega^2 r_{rx,l}}{2 r_{rx}^{\ 2}} \mathbf{i}^H \mathbf{m}\mathbf{m}^T \mathbf{i} . \tag{2}$$

Similarly, by applying Kirchhoff's circuit law to each TX, the source voltage of the n-th TX can be expressed as

$$v_{tx,n} = \left(r_{tx,n} + \frac{M_{n0}^{\ 2}\omega^2}{r_{rx}} \right) i_{tx,n} + \sum\nolimits_{k \neq n} \left(j\omega M_{nk} + \frac{M_{k0} M_{n0} \omega^2}{r_{rx}} \right) i_{tx,k} . \tag{3}$$

Based on (3), the total transmitted power of TXs can be expressed as

$$p_t = \frac{1}{2}\mathrm{Re}\left\{ \sum\nolimits_{k=1}^{N} v_k \overset{*}{i_k} \right\} = \frac{1}{2}\left(\mathbf{i}^H \mathbf{R}\mathbf{i} + \frac{\omega^2}{r_{rx}^{\ 2}} \mathbf{i}^H \mathbf{m}\mathbf{m}^T \mathbf{i} \right) . \tag{4}$$

3. Problem formulation

In this section, we design the MRC beamforming to maximize the power delivered to the RX load over transmit currents vector $\mathbf{i}$, subject to the following constraints: the total source power should meet a maximum level $\beta > 0$,i.e., $p_t \le \beta$ and each TX n should meet the largest sustainable power value p_n . This is to ensure the normal operation of the transmitter and avoid damage due to excessive power. In practical application scenarios, the receiver load is expected to achieve the maximum energy receiving efficiency under the limited power of the transmitter.

3.1. Norm-bounded model

Define $\mathbf{Q}_n(i,j)$ as $\mathbf{Q}_n(i,j) = \begin{cases} r_{tx,n}, \ i = j = n \\ 0, \ \text{otherwise} \end{cases}$, based on the discussion above, the problem P1 is formulated as follows

$$\max_{\mathbf{i}} \ \frac{\omega^2 r_{rx,l}}{2 r_{rx}^{\ 2}} \mathbf{i}^H \mathbf{m}\mathbf{m}^T \mathbf{i} , \tag{5}$$

$$s.t. \quad \frac{1}{2}\left(\mathbf{i}^H \mathbf{R}\mathbf{i} + \frac{\omega^2}{r_{rx}} \mathbf{i}^H \mathbf{m}\mathbf{m}^T \mathbf{i} \right) \le \beta , \tag{6}$$

$$\frac{1}{2}\mathbf{i}^H \mathbf{Q}_n \mathbf{i} \le p_n , n = 1, 2, \ldots N . \tag{7}$$

For the optimization problem P1, perfect channel information is assumed. However, in the practical scenario, the mutual inductances cannot be measured precisely, thus the measurement errors must be considered in the magnetic beamforming design. Generally, there are two error models for mutual inductance measurement, namely a norm boundary model and a probability model. The norm-bounded error model generates a worst case of robust design, which can provide a performance boundary of the system. In this paper, the actual mutual inductance between the TXs and RX coils are assumed to be two parts: pre-measured mutual inductance and errors. By applying norm-bounded model to mutual inductance measurement, the actual mutual inductance vector m can be expressed as

$$\mathbf{m} = \mathbf{m}_q + \Delta\mathbf{m}, \tag{8}$$

$$\left\|\Delta\mathbf{m}\right\| \le \varepsilon, \tag{9}$$

where $\mathbf{m}_q$ is the pre-measured mutual inductances vector between TXs and RX, $\Delta\mathbf{m}$ is the error vector of the mutual inductances, and ε is the radius of the uncertainty region.

To take the error vector of mutual inductance measurement into account, the optimization problem P1 based on worst-case criterion can be formulated as problem P2, which is given by

$$\text{P2:} \quad \max_{\mathbf{i}} \quad \frac{\omega^2 r_{rx,l}}{2{r_{rx}}^2}\mathbf{i}^H\left(\mathbf{m}_q+\Delta\mathbf{m}\right)\left(\mathbf{m}_q+\Delta\mathbf{m}\right)^T\mathbf{i}, \tag{10}$$

$$\text{s.t.} \quad \frac{1}{2}\left(\mathbf{i}^H\mathbf{R}\mathbf{i}+\frac{\omega^2}{r_{rx}}\mathbf{i}^H\left(\mathbf{m}_q+\Delta\mathbf{m}\right)\left(\mathbf{m}_q+\Delta\mathbf{m}\right)^T\mathbf{i}\right) \le \beta, \tag{11}$$

$$\frac{1}{2}\mathbf{i}^H Q_n \mathbf{i} \le p_n, \quad \text{n=1,2,...,N}, \tag{12}$$

$$\left\|\Delta\mathbf{m}\right\| \le \varepsilon. \tag{13}$$

According to the convex optimization theory, (P2) is in general a non-convex QCQP problem [9] due to the objective function in (10). It is well known that the general nonconvex QCQP problem is NP-hard and thus, intractable. However, as we will show in the following, due to the special structure of the objective function and the constraints, problem P2 can be reformulated as a convex SDP problem and solved optimally.

3.2. Reformulation and optimization

Before optimizing, the problem P2 is transformed into a more tractable form. For the objective function of P2 in (10), we obtain

$$\left\|\left(\mathbf{m}_q+\Delta\mathbf{m}\right)^T\mathbf{i}\right\| = \left\|{\mathbf{m}_q}^T\mathbf{i}+\Delta\mathbf{m}^T\mathbf{i}\right\| \ge \left\|{\mathbf{m}_q}^T\mathbf{i}\right\| - \left\|\Delta\mathbf{m}^T\mathbf{i}\right\|. \tag{14}$$

Then applying the Cauchy-Schwarz inequality to the second term in the right-hand-side of (14), we have

$$\left\|\Delta\mathbf{m}^T\mathbf{i}\right\| \le \left\|\Delta\mathbf{m}^T\right\|\left\|\mathbf{i}\right\| \le \varepsilon\left\|\mathbf{i}\right\|. \tag{15}$$

Applying (15) to (14) and defining $\mathbf{I}$ as the identity matrix, we then have that

$$\begin{aligned}\left\|\left(\mathbf{m}_q+\Delta\mathbf{m}\right)^T\mathbf{i}\right\|^2 &\ge \left(\left\|{\mathbf{m}_q}^T\mathbf{i}\right\| - \varepsilon\left\|\mathbf{i}\right\|\right)^2 = \left\|{\mathbf{m}_q}^T\mathbf{i}\right\|^2 - 2\varepsilon\left\|{\mathbf{m}_q}^T\mathbf{i}\right\|\left\|\mathbf{i}\right\| + \varepsilon^2\left\|\mathbf{i}\right\|^2 \\ &\ge \left\|{\mathbf{m}_q}^T\mathbf{i}\right\|^2 - 2\varepsilon\left\|{\mathbf{m}_q}^T\mathbf{i}\right\|\left\|\mathbf{i}\right\|^2 + \varepsilon^2\left\|\mathbf{i}\right\|^2 = \mathbf{i}^H\left(\mathbf{m}_q{\mathbf{m}_q}^T + \varepsilon\left(\varepsilon - 2\left\|{\mathbf{m}_q}^T\mathbf{i}\right\|\right)\mathbf{I}\right)\mathbf{i}.\end{aligned} \tag{16}$$

For the constraint function of (11), it can be solved with the help of the S-procedure [9]. The S-procedure can be presented as the following lemma.

Lemma1: Let A_1, $\mathrm{A}_1 \in \mathrm{h}\ \mathrm{H}^n$, b_1, $\mathrm{b}_2 \in \mathrm{C}^{n\times 1}$, c_1, $\mathrm{c}_2 \in \mathrm{C}$, and suppose there exists an $\mathbf{x}$ with $\mathbf{x}^H\mathbf{A}_1\mathbf{x}+2\mathrm{Re}\left\{{\mathbf{b}_1}^T\mathbf{x}\right\}+\mathrm{c}_1<0$. Then, the following relationship:

$$\mathbf{x}^H\mathbf{A}_1\mathbf{x}+2\mathrm{Re}\left\{{\mathbf{b}_1}^T\mathbf{x}\right\}+\mathrm{c}_1 \ge 0, \tag{17}$$

$$\mathbf{x}^H \mathbf{A}_2 \mathbf{x} + 2\mathrm{Re}\left\{\mathbf{b}_2{}^T \mathbf{x}\right\} + \mathrm{c}_2 \geq 0, \tag{18}$$

holds if and only if there exists a $\lambda > 0$ such that

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{b}_1 \\ \mathbf{b}_1{}^T & \mathrm{c}_1 \end{bmatrix} - \begin{bmatrix} \mathbf{A}_2 & \mathbf{b}_2 \\ \mathbf{b}_2{}^T & \mathrm{c}_2 \end{bmatrix} \pm 0 \tag{19}$$

Define Hermitian matrix $\mathbf{X} = \mathbf{i}\mathbf{i}^H$. By expanding equation (11) and equation (13), we can obtain that

$$\begin{cases} \frac{r_{rx}}{\omega^2} Tr(\mathbf{RX}) + \mathbf{m}_q{}^T \mathbf{X} \mathbf{m}_q + 2\mathrm{Re}\left\{\mathbf{m}_q{}^T \mathbf{X} \Delta \mathbf{m}\right\} + \Delta \mathbf{m}^T \mathbf{X} \Delta \mathbf{m} - \frac{2\beta\, r_{rx}}{\omega^2} \leq 0 \\ \|\Delta \mathbf{m}\| \leq \varepsilon \end{cases}. \tag{20}$$

Applying Lemma 1, we then equivalently reformulate

$$\Phi(\mathbf{X}, \lambda, \varepsilon) = \begin{bmatrix} \lambda \mathbf{I} - \mathbf{X} - \mathbf{X}\mathbf{m}_q \\ -\mathbf{m}_q{}^T \mathbf{X} - \mathbf{m}_q{}^T \mathbf{X} \mathbf{m}_q - \frac{r_{rx}}{\omega^2} Tr(\mathbf{RX}) - \lambda \varepsilon^2 + \frac{2\beta r_{rx}}{\omega^2} \end{bmatrix} \pm 0 \ . \tag{21}$$

In summary, problem P2 can be equivalently reformulated to problem P3-SDP as follow

$$\max_{\mathbf{X}} \quad \frac{\omega^2 r_{rx,l}}{2 r_{rx}{}^2} \mathrm{Tr}(\mathbf{MX}), \tag{22}$$

$$\text{s.t.} \quad \Phi(\mathbf{X}, \lambda, \varepsilon) \pm 0, \tag{23}$$

$$\mathrm{Tr}(\mathbf{Q}_n \mathbf{X}) \leq 2p_n, \quad n = 1, 2, \ldots N, \tag{24}$$

$$\mathrm{Rank}(\mathbf{X}) = 1, \mathbf{X} \pm 0, \tag{25}$$

where $\mathbf{M} = \mathbf{m}_q \mathbf{m}_q{}^T + \varepsilon\left(\varepsilon - 2\left\|\mathbf{m}_q{}^T \mathbf{i}\right\|\right)\mathbf{I}$.

By ignoring the rank-one constraint in (25), we obtain the SDR of problem P3−SDP, denoted by (P3−SDR), which becomes a linear optimization problem that can be solved by existing convex optimization tools. At this point, an important question is that whether the optimal solution of (P3−SDR), denoted as $\mathbf{X}^*$, is rank-one. If the solution $\mathbf{X}^*$ to (P3−SDR) satisfies $\mathrm{Rank}(\mathbf{X}) = 1$, $\mathbf{X}^*$ is also the optimal solution to (P3−SDP). In the following part, we will prove that the rank of the solution of (P3−SDR) is exactly 1.

Theorem 1: The optimal solution $\mathbf{X}^*$ for problem (P3−SDR) is rank-one.

Proof: Let $\mathbf{A}_q \pm 0, \boldsymbol{\rho} = [\rho_1, \ldots, \rho_N]^T \geq 0$ and $\mathbf{S} \pm 0$ be the dual variables corresponding to the constraints given in (23), (24), and (25), respectively. Thus, the Lagrangian of (P3−SDR) is written as

$$L(\mathbf{X}, \mathbf{A}_q, \boldsymbol{\rho}, \mathbf{S}) = -\frac{\omega^2 r_{rx,l}}{2 r_{rx}{}^2} \mathrm{Tr}(\mathbf{MX}) + \mathrm{Tr}\left(\mathbf{A}_q \Phi(\mathbf{X}, \lambda, \varepsilon)\right) + \sum_{n=1}^{N} \rho_n \left(\mathrm{Tr}(\mathbf{Q}_n \mathbf{X}) - 2p_n\right) - \mathrm{Tr}(\mathbf{SX}). \tag{26}$$

Moreover, let $\boldsymbol{\Omega}_q = [\mathbf{I}\mathbf{m}_q]$, the Karush−Kuhn−Tucker (KKT) conditions [9] of (P3−SDR) are given by

$$\nabla_{\mathbf{X}} L(\mathbf{X}^*, \mathbf{A}_q{}^*, \boldsymbol{\rho}^*, \mathbf{S}^*) = -\frac{\omega^2 r_{rx,l}}{2 r_{rx}{}^2} \mathbf{M} + \boldsymbol{\Omega}_q \mathbf{A}_q \boldsymbol{\Omega}_q{}^H + \sum_{n=1}^{N} \rho_n \mathbf{Q}_n - \mathbf{S} = 0 \ , \tag{27}$$

$$\mathbf{S}^* \mathbf{X}^* = 0. \tag{28}$$

Next, by multiplying (27) by X* on both sides and substituting (28) into the obtained equation, we have

$$-\frac{\omega^2 r_{rx,l}}{2 r_{rx}{}^2} \mathbf{MX}^* + \left(\boldsymbol{\Omega}_q \mathbf{A}_q \boldsymbol{\Omega}_q{}^H + \sum_{n=1}^{N} \rho_n \mathbf{Q}_n\right) \mathbf{X}^* = 0. \tag{29}$$

Then, we obtain

$$\mathrm{rank}\left(\left(\boldsymbol{\Omega}_q \mathbf{A}_q \boldsymbol{\Omega}_q{}^H + \sum_{n=1}^{N} \rho_n \mathbf{Q}_n\right) \mathbf{X}^*\right) \leq \mathrm{rank}\,(\mathbf{M}) = 1 \tag{30}$$

Since the matrix $\mathbf{\Omega}_q \mathbf{A}_q \mathbf{\Omega}_q{}^H \pm 0$ and $\mathbf{Q}_n$ must have full rank. Hence, $\left(\mathbf{\Omega}_q \mathbf{A}_q \mathbf{\Omega}_q{}^H + \sum_{n=1}^{N} \rho_n \mathbf{Q}_n\right)$ has full rank. Together with (30), $\mathrm{Rank}(\mathbf{X}^*) = 1$ can be proved.

4. Numerical results

In this section, we consider the MRC-WPT system with N=5 TX coils and one RX coil, each of which has 100 turns and a radius of 0.1 meter and cooper wire with radius of 0.1 millimeter was used. The resistance of all TXs is set identically as 0.336Ω. For the RX, its parasitic resistance and load resistance are $r_{rx,p} = 0.336$ Ω and $r_{rx,l} = 50$ Ω, respectively. The self and mutual inductances are given in Table 1.

Table 1. Mutual/Self inductances (μH).

	TX1	TX2	TX3	TX4	TX5
TX1	47700	2.2970	0.8074	2.2970	6.5741
TX2	2.2970	47700	2.2970	0.8074	6.5741
TX3	0.8074	2.2970	47700	2.2970	6.5741
TX4	2.2970	0.8074	2.2970	47700	6.5741
TX5	6.5741	6.5741	6.5741	6.5741	47700

The load power of RX versus different levels of uncertainty bounds is given in Figure 2. It can be seen from the figure that the receiving power decreases gradually as the uncertainty bounds increases, therefore, the uncertain boundary should not be too large to guarantee the load power.

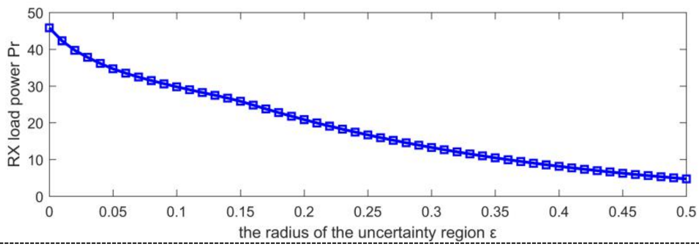

Figure 2. RX load power versus error region.

Figure 3 and Figure 4 show the total source power efficiency and load power of RX under the uncertainty bounds. It can be seen from Figure 3 that, the beamforming design can achieve 97% efficiency when uncertainty region ε=0 , however, the efficiency decreases significantly with the increase of ε . The results show that the beamforming design can work well only when the measurement error is small. Furthermore, as shown in Figure 3, when the total transmitted power increase to a certain extent, the efficiency will decrease. The reason of this phenomenon is that when the when the total transmits power increases, the maximum limit of each transmitter power becomes the main constrains of the beamforming design. As shown in Figure 4, when the total TXs power is small, the received power in RX increase with the increase of the total TXs power, this mean that the maximum total TXs power is the main constrain of the beamforming design under this condition. Like the results in Figure 3, when the total transmits power increases to a certain extent, the received power tends to be flat, which means the maximum limit of each transmitter power becomes the main constrains.

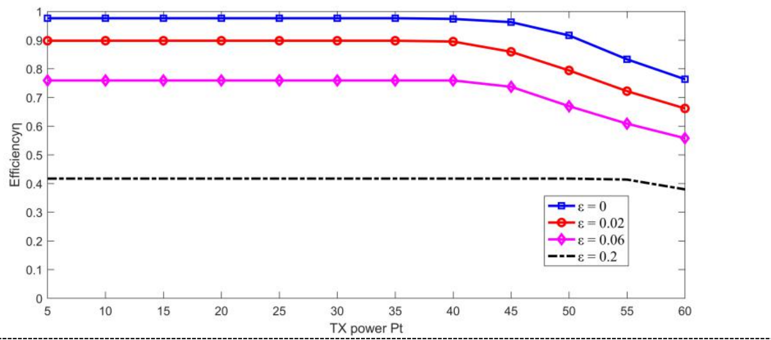

Figure 3. Efficiency versus TX power.

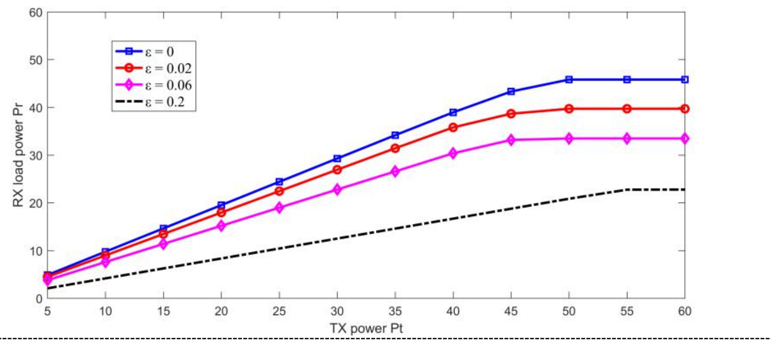

Figure 4. RX load power versus TX power.

5. Conclusion

In this paper, we investigated the robust beamforming design for the MISO MCR-WPT system with the mutual inductance error between TXs and RX. The problem of maximize the power received by RX while limit the total power required and the transmitting power of each TXs is solved by means of semidefinite relaxation and thus the optimal solution of the problem is obtained. Simulation results show that the beamforming design can achieve up to 97% transfer efficiency and the measurement error mutual inductances significantly reduce the beamforming performance. Therefore, in order to guarantee the beamforming performance, mutual inductances estimation errors should be minimized as possible.

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Acknowledgments

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