

# Quantum localization and electronic transport in covalently functionalized carbon nanotubes

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## Abstract

Carbon nanotubes are of central importance for applications in nano-electronics thanks to their exceptional transport properties. They can be used as sensors, for example in biological applications, provided that they are functionalized to detect specific molecules. Due to their one-dimensional geometry the carbon nanotubes are very sensitive to the phenomenon of Anderson localization and it is therefore essential to know how the functionalization modifies their conduction properties and if they remain good conductors. Here we present a study of the quantum localization induced by functionalization in metallic single walled carbon nanotubes (SWCNT) with circumferences up to 15 nm. We consider resonant and non-resonant adsorbates that represent two types of covalently functionalized groups with strong and moderate scattering properties. The present study provides a detailed analysis of the localization behaviour and shows that the localization length can decrease down to 20–50 nm at concentrations of about 1 percent of adsorbates. On this basis we discuss the possible electronic transport mechanisms which can be either metallic like or insulating like with variable range hopping.

Keywords: carbon nanotube, quantum transport, electronic localization

 Supplementary material for this article is available [online](#)

(Some figures may appear in colour only in the online journal)

## 1. Introduction

Single walled carbon nanotubes (SWCNTs) are quasi-one-dimensional cylindrical shape materials with  $sp^2$  hybridized carbon atoms, that exhibit unique physical and chemical properties. Since their discovery in 1991, they have attracted much attention for their fundamental properties and their wide range of potential applications, such as for energy storage [1, 2], solar cells [3–5], nano-electronics and sensors devices [6, 7].

SWCNT get easily bundled forming agglomeration of nanotubes as a result of the strong Van der Waals energy interaction [8], and because of that it's always challenging to manipulate them efficiently for experiments. Yet surface functionalization allows to overcome this problem since ad-atoms and molecules covalently attached to it's surface can greatly reduce the effect of the strong Van der Waals interaction. The functionalization also allows the compatibility toward other molecules to create new composite materials based on carbon nanotubes [9, 10].

In particular, it has been reported that SWCNT is ideal to create reliable and accurate electrochemical biosensor, faster than conventional ones [2, 6, 11, 12], thanks to its low dimensionality, nanometric size, large surface area, high sensitivity and fast response. The two main components in such device, are the biological recognition element that detects a biological molecule and the transducer that collects a charge transferred from the biological molecule to convert it into an electrical signal unique to a specific detected element. Since a covalent functionalization can drastically alter the electronic properties of carbon nanotubes [13, 14], it can affect the detection process in a biosensor.

Many theoretical analyses of electronic transport in SWCNT have been carried out using different numerical techniques, such as real space Kubo–Greenwood method for bulk systems [15–17], and Laudauer type formalisms for finite systems coupled to leads [14, 18–26]. Most of them discussed the importance of the Anderson localization due to charged defects or local defects such as vacancies, ad-atoms or ad-molecules. However, a global understanding of localization process based on numerical calculations that takes into account the effects of quantum interferences without approximations is still missing. In this paper we investigate these effects for SWCNT of diameter up to 4–5 nm and large concentrations of functionalized sites up to 1 or a few percent. In particular we address the question of the coherent electronic transport in the nanotube and the occurrence of localization effect which can diminish the conduction properties of the functionalized nanotube [27–29]. Since strong localization effects are inherent to low dimensional conductors [13, 26] it is important to have a clear description of its effect on these electronic devices. Our approach allows to propose a detailed analysis of the conductivity and its correction due to quantum localization for a wide range of concentrations of resonant and non-resonant defects. It is confirmed that quantum localization is the key to understand the wide variety of transport regimes depending on the nanotube diameter, the adsorbates concentration and the type of functionalization. In particular we show that at low adsorbates concentration the system obeys the Thouless relation, typical of a one-dimensional system, which links the number of conducting channels, the elastic mean-free path and the localization length. The values of these fundamental lengths are estimated for resonant and non-resonant adsorbates with concentration ranging from 0.1% to 4%, which corresponds to different types of applications such as sensors. Yet we show also that these nanotubes can present a variety of other behaviors that emerge at sufficiently high concentration of adsorbates. For example we find that a transition from 1D localization behavior to 2D localization behavior can occur for resonant scatterers at a concentration of the order of 1%. Also, depending on the adsorbate concentration and nanotube diameter we discuss the occurrence of a regime of variable range hopping which could possibly be observed at room temperature.

The paper is organized as follows. First section presents the tight-binding model and the basic concepts used to analyze the quantum localization effect on transport. The numerical

method to study SWCNT is presented section 2 and in the supplementary material ([stacks.iop.org/JPhysCM/32/115301/mmedia](https://stacks.iop.org/JPhysCM/32/115301/mmedia)). Then we analyze the localization in the cases of carbon nanotube functionalized with non-resonant (section 3) or resonant (section 4) adsorbates with various concentrations in the range of 0.1% to 4%. A discussion is given section 5 about the transport mechanism metallic like or insulating like and in particular the possibility of variable range hopping.

## 2. Model and methodology

### 2.1. Modeling resonant and non-resonants adsorbates in SWCNT

A single walled carbon nanotube (SWCNT) is equivalent to a rolled up stripe of a 2D graphene sheet. The orientation of rolling up graphene, is defined by the chiral vector  $\vec{C}_h = n\vec{a}_1 + m\vec{a}_2$ , where  $\vec{a}_1$  and  $\vec{a}_2$  are the unit cell vectors of graphene, the  $n$  and  $m$  are coefficients used to define the chirality of the nanotube. The SWCNT diameter is  $d = \frac{C_h}{\pi}$  where  $C_h = \|\vec{C}_h\| = a\sqrt{3(n^2 + m^2 + nm)}$  the circumference of the nanotube, with  $a = 0.142$  nm the distance between two neighboring carbon atoms. The ratio between the length and diameter  $\frac{L}{d} \sim \left(\frac{\text{length}}{\text{diameter}}\right)$  can be as large as  $10^4$  to  $10^5$  [30] and carbon nanotube can be considered as a quasi-one-dimensional nano-structure. In this study the SWCNT has an infinite length which avoids edges effects.

In this paper we consider the metallic zigzag single walled carbon nanotube  $(n, 0)$  where the shape of the SWCNT cross section is zigzag and with ‘ $n$ ’ is a multiple integer of 3. We took  $n = 30$ ,  $n = 60$  and  $n = 90$ . We show detailed results for  $n = 60$  in the article and compare them with the results for  $n = 30$  and  $n = 90$  that are summarised in the supplementary material.

We study the electronic properties of SWCNT using a first neighbour tight-binding hamiltonian

$$\tilde{H} = -t \sum_{ij} \left( c_i^\dagger c_j + c_i c_j^\dagger \right), \quad (1)$$

where  $c_i^\dagger$  ( $c_i$ ) is the creation (annihilation) operator. The hopping integral  $t = 2.7$  eV allows the electron to hop from an atom site to one of its three first neighbours. We consider that all the onsite energies are the same and they are all set to zero ( $\varepsilon_0 = 0$  eV).

When an ad-atom or molecule is attached to the nanotube surface it creates an adsorbate and a covalent bond. This covalent bond is equivalent to taking out a  $p_z$  orbital. It has been established that adsorption on top of a carbon atom allows to keep the  $sp^2$  hybridization [31, 32]. Therefore we model adsorption on top of a carbon site by simply removing the corresponding  $p_z$  orbital in equation (1). We therefore assume that an adsorbate can be seen as a carbon vacancy present in the SWCNT without disturbing the other carbon atoms positions. Note that an adsorbate is a local defect and produces inter-valley scattering which leads to Anderson localization [33–35].

We consider two types of local defects obtained by removing one isolated  $p_z$  orbital or two neighboring  $p_z$  orbitals. These two types of defects correspond respectively to resonant and non-resonant adsorbates. Indeed the effect of an isolated local defect is entirely described by an energy dependent T-matrix  $T(E)$ . Resonant adsorbates are modeled by removing one isolated  $p_z$  orbitals and the corresponding T-matrix presents a strong resonance at the charge neutrality point (CNP) creating a peak in the density of states around the CNP. A similar peaks exist in graphene for which  $T(E)$  even presents a divergence at the CNP. Non-resonant adsorbates, are modelled by removing two neighbouring orbitals at the same time. In that case the T-matrix varies smoothly with energy and does not show resonance close to the CNP. Therefore no peak appears in the DOS around the CNP. Resonant and non-resonant adsorbates affect differently the electronic transport, and in particular the resonant adsorbates produce a stronger scattering than the non-resonant adsorbates close to the CNP.

## 2.2. Analysis of quantum localization

Following the real space Kubo–Greenwood method [36–41] for transport properties of electron in the nanotube, we compute  $X^2(E, t)$  which is the average square of the quantum spreading along the direction of the nanotube, after a time  $t$  and for states at energy  $E$ . That method has been also used to study graphene monolayer [42], bilayer [43, 44]. For an electron in a 1D system propagating in a static disordered potential we know that all states are localized and quantum diffusion is limited by the localization length  $\xi(E)$ . Yet the time evolution of  $X^2(E, t)$  contains much information about the localization phenomena. In order to do this analysis we simulate how inelastic scattering affects diffusion and conductivity. At the simplest level the effect of inelastic scattering is described by introducing an inelastic scattering time  $\tau_{ie}$  and a time of phase coherence of the wave-function  $\tau_\Phi$ . These two times can be different and their relative value can depend on the studied system. In carbon nanotubes it has been argued that  $\tau_\Phi$  can be smaller than  $\tau_{ie}$  because loss of phase coherence can be due predominantly to long wavelength acoustic modes which have a small effect on  $\tau_{ie}$  [17]. In the present study we use the relaxation time approximation (RTA) which consist in assuming that the velocity correlation function of the system is destroyed by inelastic scattering and the associated loss of wave-function coherence. In this picture there is no distinction between  $\tau_\Phi$  and  $\tau_{ie}$ . We therefore introduce a parameter  $\tau_i$  such that  $\tau_i \simeq \tau_{ie} \simeq \tau_\Phi$ . Associated to  $\tau_i$  there is a length scale called the inelastic mean-free path  $L_i(E, \tau_i)$  which is the typical propagation length at energy  $E$  after a time  $\tau_i$  (see supplementary material for precise definition). Beyond  $\tau_i$  the transport regime is diffusive because of the loss of coherence, and the diffusivity is given by  $D(E, \tau_i) = \frac{L_i^2(E, \tau_i)}{\tau_i}$ . The expression of electrical conductivity becomes

$$\sigma(E_F, \tau_i) = 2e^2 n(E_F) D(E_F, \tau_i), \quad (2)$$

where  $n(E_F)$  is the density of states per unit length (per spin) at the Fermi energy  $E_F$ . As shown below the variation of the

conductivity  $\sigma(E_F, \tau_i)$  with  $L_i(E, \tau_i)$  or with  $\tau_i$  contains essential information about the quantum transport. It allows to extract also the elastic mean-free path  $l_e(E)$  and the localization length  $\xi(E)$ , and will be discussed in detail. For example an important quantity is  $\sigma_M(E_F)$  the maximum of the conductivity as a function of  $L_i(E, \tau_i)$ . In the following  $\sigma_M(E_F)$  is named the microscopic conductivity. It corresponds to a situation where the inelastic mean-free path is of the order of the elastic mean-free path so that the effect of quantum localization cannot develop and decrease the conductivity.  $\sigma_M(E_F)$  is given in term of the mean-free path  $l_e(E)$  by

$$\sigma_M(E_F) = G_0 N_{ch}(E_F) l_e(E_F), \quad (3)$$

where  $G_0 = 2e^2/h$  is the quantum of conductance and  $N_{ch}(E) = n(E)/n_0$  is the ratio between the density of states per unit length and per spin  $n(E)$  at the energy  $E$  and the density of states for one channel  $n_0 = 2/hV_G$  where  $V_G$  is the velocity in graphene at the CNP.  $n_0$  is half the density of states at the zero energy in the nanotube (there are two conduction channels near  $E = 0$  one close to each Dirac point). Therefore  $N_{ch}(E)$  is close to the number of channel at the energy  $E$  but tends to be higher because the contribution to the density of states of a channel close to its minimum energy (Van Hove singularity) is higher than  $n_0 = 2/hV_G$ .

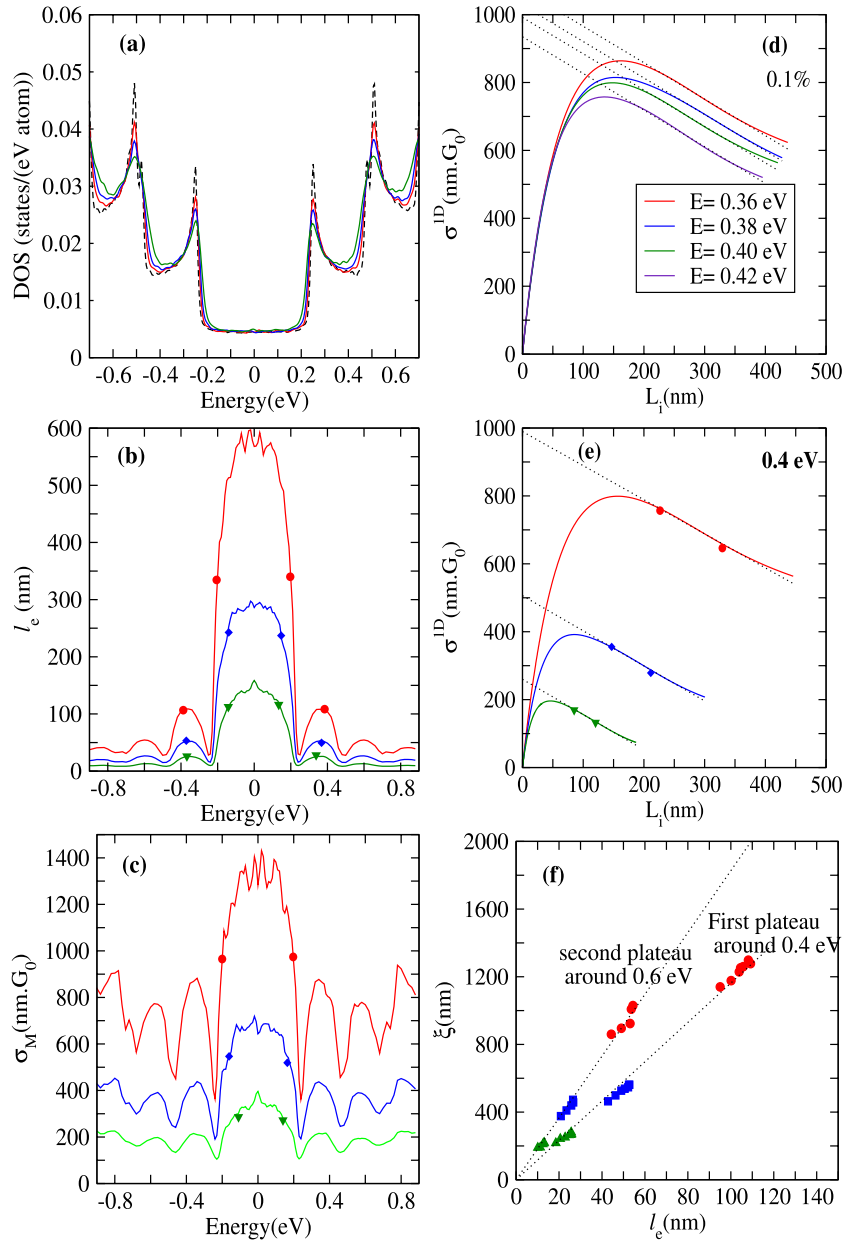
## 3. Electronic transport for non-resonant adsorbates

We investigate the electronic behavior of metallic SWCNT (60, 0) with a large diameter  $d = 4.75$  nm and a circumference  $C_h = 14.94$  nm, in presence of non-resonant adsorbates. We consider first the low concentration cases (0.1%, 0.2% and 0.4%) (figure 1) and then higher concentrations cases (1%, 2% and 4%) (figure 2).

Figure 1(a) shows the total density of states versus energy. For low energy near the CNP, the DOS is weakly affected by disorder. At larger energy and in particular close to Van Hove singularity (VHS), the modification of the DOS is more important.

Figure 1(b), shows the elastic mean-free path  $l_e$  as a function of energy  $E$ .  $l_e$  is maximum around the CNP energy and decreases by crossing a VHS. It reaches a new maximum in between two successive VHS. Indeed the probability of scattering increases as a function of energy according to the DOS and therefore the mean scattering time and the mean-free path decrease. In addition the new channel that appears at a VHS has a low band velocity. The probability of scattering also increases by increasing the concentration of adsorbates which results in a shorter elastic mean-free path. The results show that  $l_e$  is inversely proportional to the concentration of adsorbates, which is in accordance with the Fermi Golden rule.

The microscopic conductivity  $\sigma_M$  versus energy is shown in figure 1(c). This quantity is given by equation (3) and is proportional to the product of results in figures 1(a) and (b). The evolution of  $\sigma_M$  as a function of energy, follows globally the behaviour of  $l_e$  as a function of energy. As for the elastic mean-free path the increase of concentration of adsorbates decreases the conductivity  $\sigma_M$  which is inversely proportional



**Figure 1.** Low concentration of non-resonant adsorbates: 0.1% (red/filled circle), 0.2% (blue/filled square) and 0.4% (green/filled triangle). (a) Density of states versus energy, (b) Elastic mean-free path  $l_e$  versus energy, (c) Microscopic conductivity  $\sigma_M$  versus energy, (d) Electronic conductivity  $\sigma$  versus inelastic mean-free path  $L_i$  for 0.1% of non-resonant adsorbates, (e) Electronic conductivity  $\sigma$  versus inelastic mean-free path  $L_i$  at  $E = 0.4$  eV, and (f) Localization length  $\xi$  versus elastic mean-free path  $l_e$  (Thouless relation).

to the concentration of adsorbates.  $\sigma_M$  is a semi-classical quantity corresponding to the diffusive regime when  $L_i$  and  $l_e$  are comparable. Thus, it does not depend on the quantum localization effects.

Figures 1(d) and (e) present the electronic conductivity as a function of the inelastic mean-free path. At low  $L_i$ ,  $\sigma(E_F, \tau_i)$  increases in the ballistic regime. Then it reaches its maximum and for large  $L_i$  decreases because of Anderson localization. In the localization regime, we find that the conductivity behaves according to the 1D scale dependent conductivity [45] given by

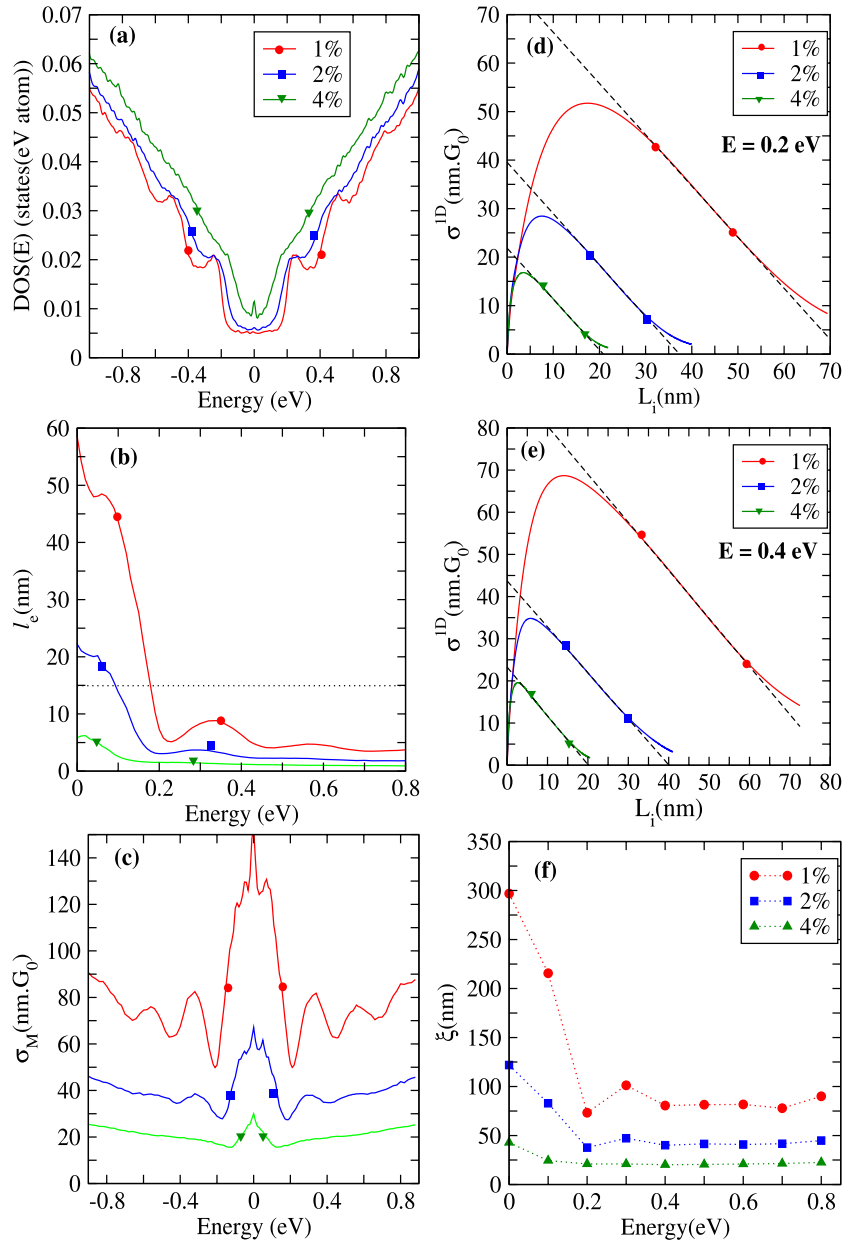
$$\sigma(E_F, L_i) = \sigma_M(E_F) - G_0 (L_i - L_e). \quad (4)$$

Here  $L_e$  is defined as the value of  $L_i$  for which the extension of the linear regime crosses the maximum value of  $\sigma_M(E_F)$ .  $L_e$  is expected to be of the order of the elastic mean-free path  $l_e$  and indeed we find numerically that  $L_e \simeq 2l_e$ . The behavior given by equation (4) is consistent with the standard theory of localization in one dimensional conductors [45].

The localization length  $\xi(E)$  is defined as the value of  $L_i$  for which the extrapolation of equation (4) cancels. Therefore using equation (3) and since  $L_e \simeq 2l_e$  one get

$$\xi(E) \simeq (N_{ch}(E) + 2)l_e(E), \quad (5)$$

where, in this Thouless relation [46],  $N_{ch}(E) = n(E)/n_0$ . As explained previously  $N_{ch}(E)$  is close to the number of channels



**Figure 2.** High concentration of non-resonant adsorbates: 1% (red/filled circle), 2% (blue/filled square) and 4% (green/filled triangle). (a) Density of states versus energy, (b) Elastic mean-free path  $l_e$  versus energy, (c) Microscopic conductivity per unit surface  $\sigma_M/C_h$  versus energy, (d) Electronic conductivity  $\sigma$  versus inelastic mean-free path  $L_i$  at  $E = 0.2$  eV, (e) Electronic conductivity  $\sigma$  versus inelastic mean-free path  $L_i$  at  $E = 0.4$  eV, and (f) Localization length  $\xi$  versus energy.

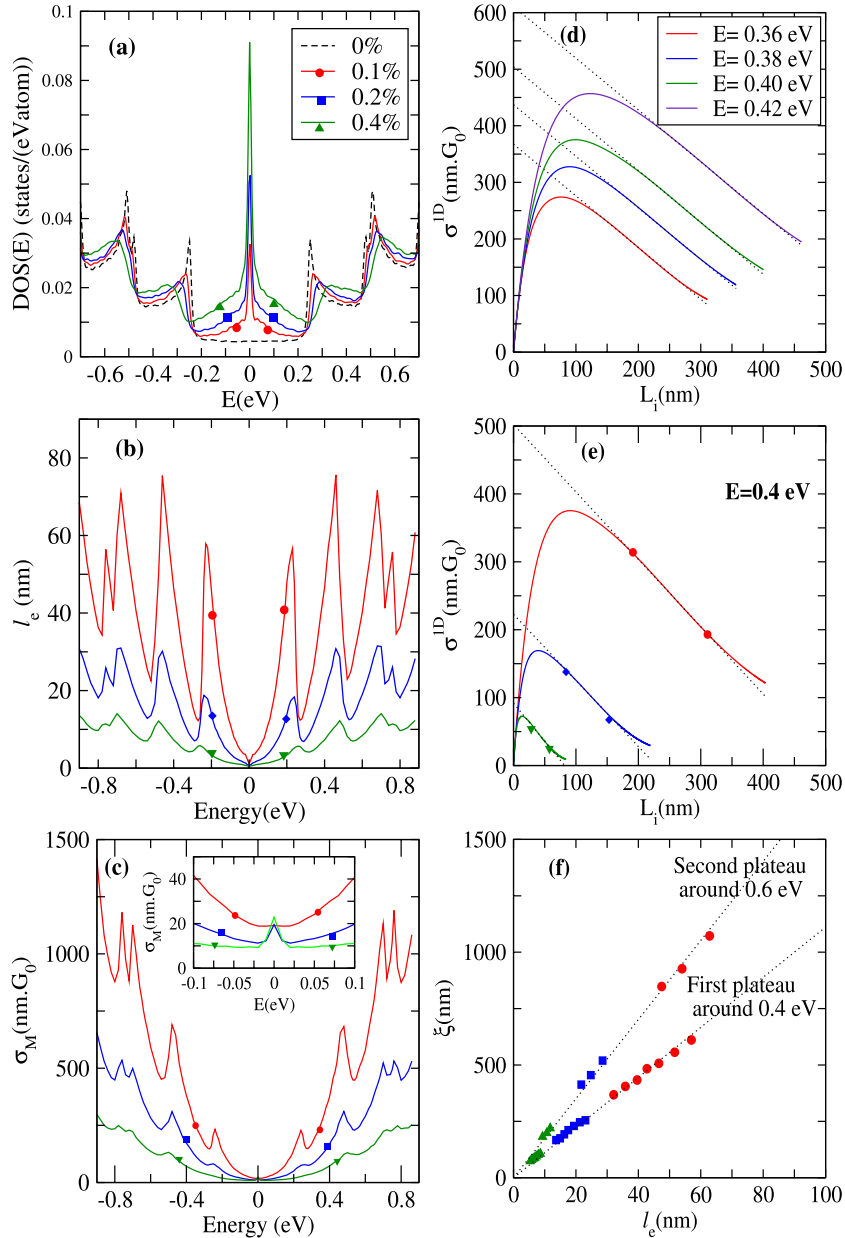
at the energy  $E$  but tends to be higher. As shown in the figure 1(f), the localization length  $\xi$  evolves as a linear function of elastic mean-free path  $l_e$  as predicted by the Thouless relation with a slope that increases with the number of conduction channels  $N_{ch}$ . For the first plateau around 0.4 eV,  $N_{ch} \approx 6$  and  $N_{ch} \approx 10$  for the second plateau around 0.6 eV. These results confirm the accuracy of our numerical calculations.

Figure 2 presents a similar study as figure 1 but for higher concentrations (1%, 2% and 4%), the modification of the DOS by disorder becomes more important and the VHS are smeared out. In addition for the concentration (4%) the density of states is strongly modified even close to the CNP. Despite these differences the variations of  $l_e(E)$ ,  $\xi(E)$  and  $\sigma_M(E)$  remain similar

and are still inversely proportional to the concentration. The linear variation of conductivity with  $L_i$  is also given by equation (4). Note that even at high concentration of non-resonant adsorbates the localization length is always larger than the circumference of the nanotube  $\xi > C_h$  as shown in figure 2(f).

We conclude therefore that for the non-resonant adsorbates the nanotube behaves as a 1D conductor with a relatively small amount of disorder, for which the microscopic conductivity  $\sigma_M(E)$  and the elastic mean-free path  $l_e(E)$  are obtained from the Fermi golden rule. In addition our results confirm the Thouless relation between the localization length  $\xi(E)$ , the elastic mean-free path and the number of conduction channels.





**Figure 3.** Low concentration of resonant adsorbates: 0.1% (red/filled circle), 0.2% (blue/filled square) and 0.4% (green/filled triangle). (a) Density of states versus energy, (b) Elastic mean-free path  $l_e$  versus energy, (c) Microscopic conductivity  $\sigma_M$  versus energy, (d) Electronic conductivity  $\sigma$  versus inelastic mean-free path  $L_i$  for 0.1% of resonant adsorbates, (e) Electronic conductivity  $\sigma$  versus inelastic mean-free path  $L_i$  at  $E = 0.4$  eV, and (f) Localization length  $\xi$  versus elastic mean-free path  $l_e$  (Thouless relation).

#### 4. Electronic transport for resonant adsorbates

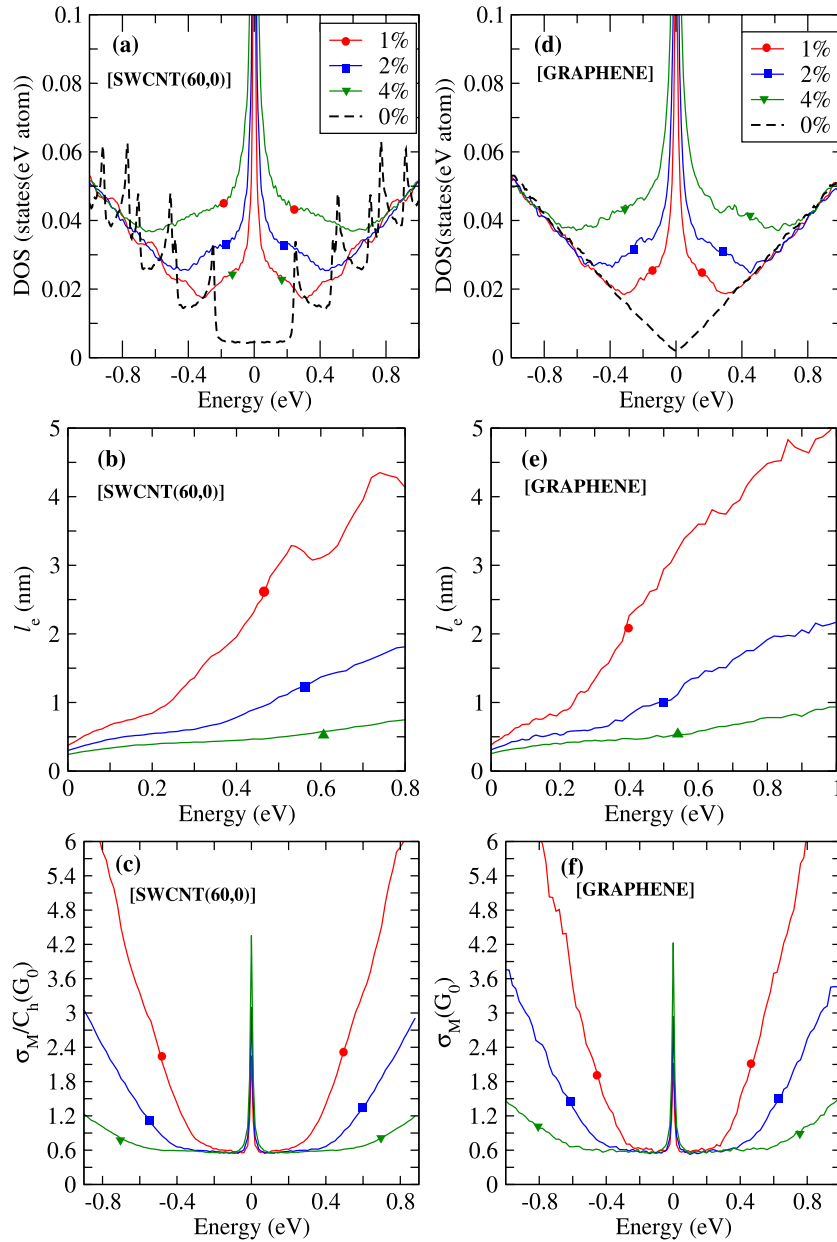
We now investigate the electronic behavior of the metallic SWCNT (60,0) in presence of resonant adsorbates. We consider first the low concentration cases (0.1%, 0.2% and 0.4%) (figure 3) and then higher concentrations cases (1%, 2% and 4%) (figure 4).

In figure 3(a), the density of states versus energy for resonant adsorbates shows more dramatical effects with a pic of resonant states around the Fermi's energy, a shift in the plateau of the DOS for larger energy and the positions and height of the VHS are greatly affected.

For resonants adsorbates as shown in figure 3(b), the mean-free path increases as a function of energy within a plateau

of energy and by crossing a VHS, since the probability of scattering is much higher for low energies close to the resonant states where  $l_e$  is at its minimal value.  $l_e$  is shown to be always larger than the circumference of the nanotube which confirms that the electronic behavior of SWCNT (60,0) is one dimensional.

The microscopic conductivity  $\sigma_M$  expressed in unit of quantum conductance  $G_0 = 2e^2/h$  versus energy is shown in figure 3(c) for low concentration of resonant adsorbates. The increase of concentration of adsorbates decreases the conductivity  $\sigma_M$ . The evolution of  $\sigma_M$  as a function of energy, follows the general behavior of  $l_e$  as a function of energy.  $\sigma_M$  is a semi-classical quantity computed in the diffusive regime when  $L_i$  and  $l_e$  are comparable. Thus, it does not take into account the



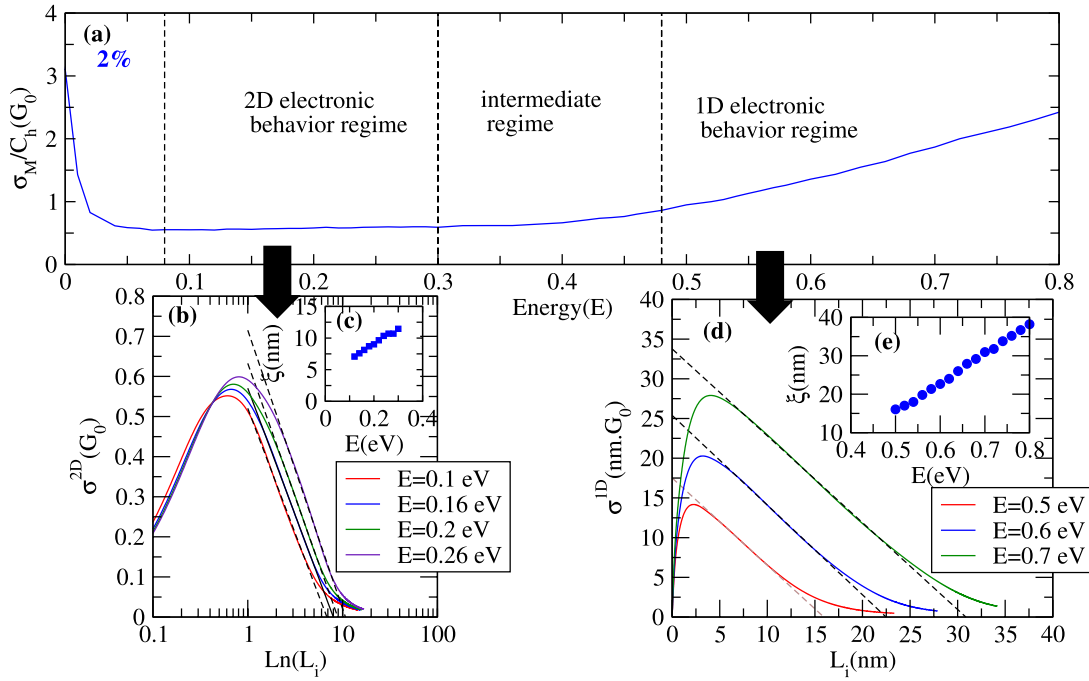
**Figure 4.** A comparative representation of electronic transport properties of SWCNT (60,0) and graphene at high concentration of resonant adsorbates: 1% (red/filled circle), 2% (blue/filled square) and 4% (green/filled triangle). (SWCNT(60,0)) (a) Density of states, (b) Elastic mean-free path  $l_e$ , and (c) Microscopic conductivity per unit of surface,  $\sigma_M/C_h$ , versus energy. (GRAPHENE) (d) Density of states, (e) Elastic mean-free path  $l_e$ , and (f) Microscopic conductivity  $\sigma_M$ , versus energy.

quantum effects (localization effects). The right column of figure 3 shows the occurrence of localization effects similar to those shown for the non-resonant states. This behavior corresponds to the laws of one-dimensional conductors as given by equations (4) and (5).

Figure 4(a), shows that at high concentration of resonant adsorbates (1%, 2% and 4%), the VHS disappear from the density of states around the Fermi's energy, because the effects of electronic scattering is so dramatic to the point that the quantification of the  $\vec{k}$  vector in the direction of  $\vec{C}_h$  is broken. The new shape of the DOS is actually close to the DOS of graphene shown in figure 4(d).

We can see in figure 4(b) that the elastic mean-free path is much shorter than the circumference  $C_h$ , for all energies. As a result, we can expect a transition from one-dimensional towards two-dimensional electronic behavior in a SWCNT induced by increasing the concentration of adsorbates. Indeed if the characteristic lengths are shorter than its circumference the nanotube should behave like a sheet of graphene.

This dimensionality crossover should be identified by investigating the effects on the microscopic conductivity and the scale dependent conductivity. Since the beginning, the electrical conductivity  $\sigma$  is expressed in units of length for one-dimensional system  $\sigma^{1D}$ , we redefine this quantity for a



**Figure 5.** For SWCNT (60,0) with 2% of resonant defects: (a) Microscopic conductivity per unit of surface  $\sigma_M/C_h$  versus energy, (b) Electronic conductivity versus inelastic mean-free path in log scale, (c) Localization length versus energy shows  $\xi < C_h$ , (d) Electronic conductivity versus inelastic mean-free path, and (e) Localization length versus energy shows  $\xi > C_h$ .

two-dimensional system by dividing it by  $C_h$ , so  $\sigma^{2D} = \frac{\sigma^{1D}}{C_h}$  is expressed in units of surface.

As shown in figure 4(c) the high concentration of resonant adsorbates leads to a plateau of minimum conductivity around  $0.6 G_0$ , similar to the reported results on the 2D graphene (see figure 4(f)) in other studies [42]. This value of minimum conductivity is universal as long the diameter of zigzag SWCNT is large enough and the concentration of resonants adsorbates is high enough to actually create a dimensionality crossover in the electronic behavior.

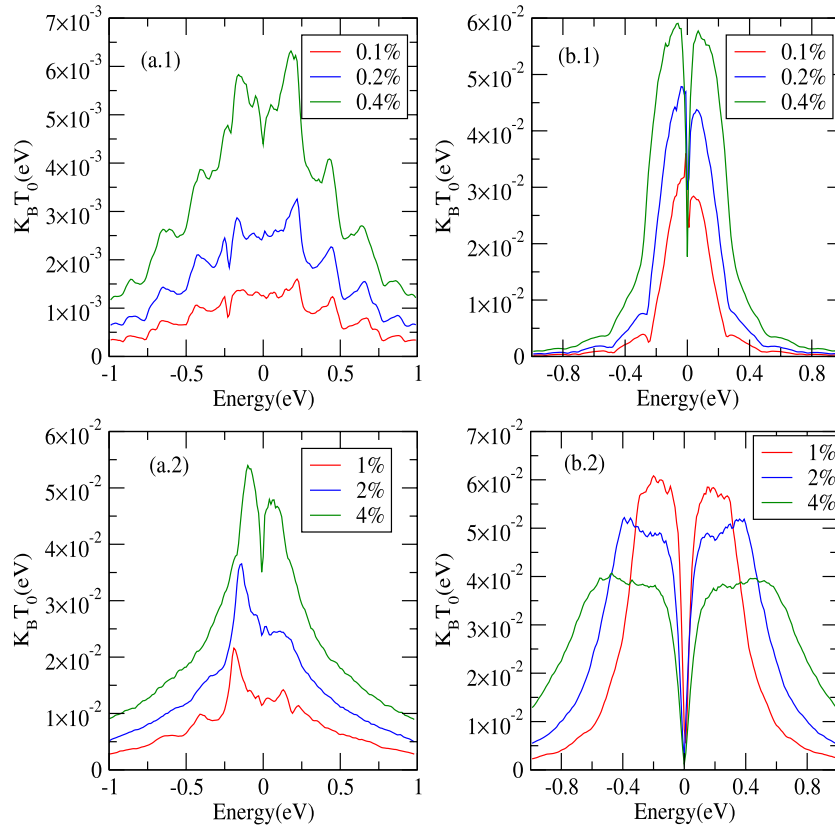
At 2% of resonant adsorbates, figure 5 shows clearly the transition of the electronic behavior in the microscopic conductivity. For low energy in figure 5(b) the 2D scale dependent conductivity:  $\sigma_{2D}(L_i) = \sigma_0 - \beta G_0 \ln\left(\frac{L_i}{L_c}\right)$ , fits the decreasing behavior of the electronic conductivity towards the localization regime in the logarithmic scale for large  $L_i$ . The slope in log scale according to  $\sigma_{2D}(L_i)$  shows a new universal slope,  $\beta = 1/\pi$ , similar to what we observe in 2D graphene [42], which confirms the 2D electronic behavior in presence of high concentration of resonant adsorbates. Based on our estimation of localization length shown in figure 5(c), this 2D electronic behavior appears when the localization length is shorter than the circumference  $\xi < C_h$ . For high energy as shown in figure 5(d), the 1D scale dependent conductivity confirms the 1D electronic behavior when  $\xi > C_h$ . Therefore the 1D/2D crossover is governed by the ratio between the localisation length  $\xi$  and the circumference  $C_h$ . Thus we can define as a function of energy within the microscopic conductivity three regimes (1) a 2D regime when  $\xi < C_h$ , (2) An intermediate regime when  $\xi \approx C_h$  and (3) A 1D regime when  $\xi > C_h$ .

## 5. Discussion

We now discuss the case of non-resonant and resonant adsorbates by combining the results on the (60,0) nanotube and those briefly presented in the supplementary material on (30,0) and (90,0) nanotubes. We focus particularly on the localization length which is a measure of the importance of localization effects and their impact on transport. Strictly at low temperature the conductance of a nanotube becomes exponentially small if the length of the nanotube is larger than the localization length. Equivalently if this length is smaller than the localization length then the nanotube can conduct. Yet at finite temperature the inelastic scattering can destroy the localization effect on a length  $L_i$ . Therefore if  $\tau_i$  is smaller than the time needed for localization  $\tau_L$  the nanotube can conduct. We have discussed this in the supplementary material and find that the time needed for localization  $\tau_L$  varies between  $10^{-12}$  s and a few  $10^{-11}$  s.

For non-resonant scatterers we find that in all cases the Anderson localization is well described by the standard 1D transport theory in a low concentration limit. This implies that the elastic mean-free path obeys the Fermi golden rule and varies inversely proportionally to the adsorbate concentrations. In addition the relation between the elastic mean-free path and the localization length obeys the Thouless relation. Depending on the Fermi energy and the adsorbate concentrations the localization length varies between 1200 nm and 30 nm. These values are similar for all three nanotubes. Yet a noticeable difference is that for smaller diameters the energy range where there are only two channels is more important. In this two channels zone the localization length tends to be





**Figure 6.** Temperature  $k_B T_0$  as function of energy  $E$  for low and high concentrations. Results in the left column are for non-resonant defects and in the right column for resonant defects. Room temperature  $T_R$  corresponds to  $k_B T_R \simeq 2.5 \times 10^{-2}$  eV.

larger than at higher energies. Therefore nanotubes with non-resonant adsorbates will present better transport when the Fermi energy is closer to the CNP.

For the resonant adsorbates the situation is opposite. Indeed the scattering matrix  $T(E)$  presents a strong resonance close to the charge neutrality point and this leads to a short mean-free path and a short localization length at energies close to the charge neutrality point. Therefore in the case of resonant adsorbates the transport properties are more deteriorated close to the CNP and better transport properties will be obtained if the Fermi energy is sufficiently far from the CNP. At low concentrations the behaviour is also well described by the 1D quantum transport theory except very close to the neutrality point where even the density of states is modified by the adsorbates. However at higher concentrations and for sufficiently large diameter carbon nanotubes can behave like a 2D system and not like a 1D conductor. This happens when the elastic mean-free path and the localization length become smaller than the circumference of the nanotube. For large nanotubes with diameter of 4–5 nanometers this regime can be reached with resonant scatterers at concentrations of defects of the order of one percent and for energies of a fraction of an electron Volt away from the charge neutrality level ( $E < 0.3$  eV at 2%). This 2D behaviour is comparable with the electronic behaviour in graphene, in which  $\sigma(L_i)$  decreases for large  $L_i$  accordingly to the 2D scale dependent conductivity and leading to the creation of minimum microscopic conductivity around the Dirac energy. Yet at higher energies

( $E > 0.5$  eV at 2%) the scattering by resonant states is less efficient and one recovers a typical 1D behaviour. Depending on energy and concentration the localization length varies between 1000 nm and 10 nm. These values are about the same in all three nanotubes.

Finally we discuss the possibility of a transport mechanism that is based on hopping. This type of transport is well documented in nanotubes and bundles or two dimensional arrays of nanotubes. At the very low temperatures of a few Kelvin, the electron-electron interaction can play a strong role but at higher temperatures the thermally activated hopping between localized states is usually due to electron-phonon coupling [47, 48]. We shortly discuss this regime now.

In the nearest neighbor hopping regime the transport is thermally activated and in the variable range hopping regime (which occurs at lower temperature) the conductivity  $\sigma$  is typically proportional to  $\propto \exp(-(T_0/T)^{1/2})$  [49–51]. This regime of variable range hopping appears typically at temperature below  $T_0 = 4/(n(E)\xi(E))$  where  $\xi(E)$  is the localization length and  $n(E)$  the density of states per unit length and per spin. Figure 6 shows the variation of this temperature  $T_0$  as a function of concentration and energy  $E$  for resonant and non-resonant defects. These results show that transport through the variable range Hopping transport could occur even at room temperature in systems with sufficiently large defects concentration provided that the Fermi energy is sufficiently close to the charge neutrality point. Note that for resonant states the  $T_0(E)$  decreases in the immediate vicinity of the CNP due to the

higher density of states caused by the resonant states. Around the CNP,  $T_0(E)$  increases as function of concentration of resonant and non-resonant adsorbates, however beyond 1% of resonant adsorbates (at 2% and 4%) as shown in figure 6(b.2),  $T_0(E)$  decreases which coincide with the appearance of the strong 2D electronic behavior previously discussed.

## 6. Conclusion

We have presented a study of the quantum localization in single walled carbon nanotubes (SWCNT), with resonants and non-resonants adsorbates that represent two extreme types of covalently functionalized groups with moderate and strong scattering properties. In the present study the concentration of these adsorbates varies between 0.1%, and 4% and we consider nanotubes of average and large circumferences (up to 15 nm). Our study shows that the type of adsorbates, the circumference of the nanotube and the Fermi energy are determinant for the localization phenomenon. Depending on these parameters the localization length can become small of the order of a few 10 nm. The localization can even be smaller than the circumference of the nanotube in the case where there are resonant scatterers, a Fermi energy close to the CNP and a large nanotube. In this special case the localization effect follows the 2D laws. Quite generally we find that non-resonant adsorbates have better transport properties for Fermi energy close to the CNP where it is the opposite with resonant adsorbates. These systems could also present a transport through nearest neighbor hopping or even through Variable Range Hopping even at room temperature provided that the Fermi energy is sufficiently close from the Charge Neutrality Point and also for sufficiently high concentration of adsorbates.

We believe that the present study can help to understand the localization effect in a given device and to determine in which regime it is. Therefore this work should be useful for obtaining efficient nano-sensors based on covalent functionalisation of carbon nanotubes.

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