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Overlap method for the performance evaluation of coordinate measurement systems and the calibration of one-dimensional artifacts

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Abstract
This article describes a method called 'overlap' to perform two main tasks. The first is the performance evaluation of a coordinate measurement system (CMS) with a measuring range larger than the length of artifacts. The second is the calibration of one-dimensional artifacts using a CMS with a measuring range smaller than the length of the artifacts. Examples of one-dimensional artifacts are ball bars, step gauges, or scale bars with lines or circle marks. For the first task, the method uses the translation between errors in a line inside the volume of the CMS. For the second task, it uses the rotation and translation between 3D coordinates of elements belonging to the artifact. Validation of the method for the first task is done by evaluating the performance of a coordinate measuring machine (CMM) in parallel lines to two of its axes. For this, we use a step gauge and a ball bar covering the range of the CMM's lines under evaluation not using overlap. Later, we evaluate the same lines using a segment of the same step gauge and ball bar but this time using overlap. For the second task, the validation is similar; we use a ball bar whose length falls inside the measurement range of a high-accuracy CMM, then the ball bar can be measured in its entire length with the CMM not using overlap and later in segments using overlap.

Keywords: overlap, one-dimensional artifacts, calibration, performance evaluation, coordinate measurement systems

(Some figures may appear in colour only in the online journal)

1. Introduction

One of the main tasks of a National Metrology Institute (NMI) is to provide calibration services for other laboratories and industries. Many of these services represent a metrological challenge that entails the development of new measurement methods. Mexico's NMI (CENAM) received one of these challenges related to the dimensional calibration of ball bars with a length of 3 m and 5 m owned by Volkswagen, see figure 1. However, CENAM did not have the equipment with enough range and accuracy to measure such ball bars. These long bars are being used for the performance evaluation of large coordinate measuring machines (CMMs). Transporting the bars represents a cost for the company and its manipulation represents risks for the user. The challenge of calibrating these bars as well as the reason for which they are intended

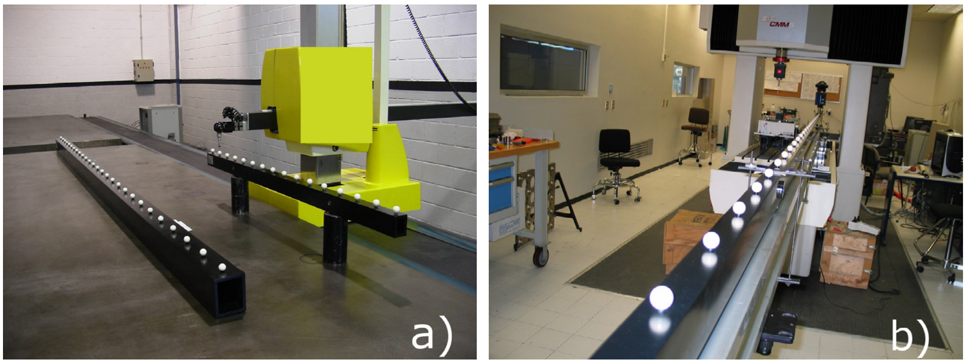

Figure 1. (a) Performance evaluation of a large CMM with a small ball bar. (b) Setup for 5 m ball bar measurement using a moving table CMM. The CMM has a measurement range of 1100 mm only.

allowed us to develop a method called 'overlap'. The overlap method solves two problems: the first and most novel was the performance evaluation of large coordinate measuring systems (CMSs) using small size artifacts, see figure 1(a); the second was the dimensional calibration of large artifacts having a small CMS, see figure 1(b).

Due to technical reasons, documentary standards for the performance evaluation of CMSs typically require calibrated lengths to be similar in length to the range of the CMS under test. Because of this, we have large one-dimensional artifacts like step gauges, ball bars, or scale bars with lines or circle marks. These documentary standards also require that the length of the artifacts must be known with an expanded $(k = 2)$ uncertainty that is at least four times smaller than the manufacturer's maximum permissible error (MPE) specification of the instrument under test [1]. Examples of documentary standards for the performance evaluation of CMS are ISO 10360-2:2009 [2] and ASME B89.4.10360.2-2008 [3]. These require that the minimum size of the artifacts to evaluate a coordinate measuring machine (CMM) must be at least 66% of the largest axis of the CMM. Other standards such as ASME B89.4.19-2006 [1] and ISO 10360-10:2016 [4], require that the artifact for the performance evaluation of a laser tracker (LT) must be greater or equal to 2.1 m, etc.

The calibration of large artifacts is complicated, as there are typically few high accuracy CMS with the appropriate measurement ranges to calibrate them directly. In addition to the overlapping method reported here, methods such as multilateration can be used with LT to calibrate large artifacts, Sandwith *et al* [5]. Also, large CMM with a line inside its volume geometrically compensated by an interferometer, a calibrated standard or any other technique, ISO 15530:3:2011 [6], Trapet *et al* [7] or Schwenke *et al* [8] can be used. In their approach the repeatability of the CMM is the greatest influence on the uncertainty of measurement. The overlap method will allow the calibration of these large artifacts and the performance evaluation of CMS using artifacts whose length does not cover the measurement range of the CMS.

The performance evaluation of CMS applying the overlap method uses the translation between errors in a line inside the volume of the CMS. The linear movement of the position of the artifact allow us to cover the full range of the CMS and perform its evaluation. The overlap method assumes that the user is able to move the artifact used as a reference on a line inside a volume where CMS's errors do not change dramatically. To validate the performance evaluation using the overlap method, we will measure this line again using a ball bar that covers the entire line of the CMM.

For the calibration of a one-dimensional artifact, the overlap method uses a CMM and the rotation and translation between the 3D coordinate of common points within adjacent segments of the same artifact. The movement of the artifact within a line of the CMM to cover its entire length produces the roto-translation. The geometric errors in the line where the artifact is placed affect not only the roto-translation measurement performed in the overlapping method but also affect the measurement without overlapping of any piece in that line. It is recommended that the CMM be compensated for its geometry errors before overlapping. Future work can look for the consequences of significant geometry errors in the measurement line. To validate the calibration with the overlap method, we will measure a small ball bar that fits within the measurement volume of a CMM using and not using overlap.

In the rest of the article, we describe our approach to performing the overlap method in a CMM. In section 2, we review the scientific literature related to our problem. Then, in sections 3 and 5, we describe the method we employ to do the overlap. In section 4 we show how to evaluate the uncertainty. We present our results in section 6 and discussion in section 7. Finally, we conclude by summarizing our findings and delineating future directions of inquiry.

2. Prior work

Methods such as multilateration using an LT, Takatsuji *et al* [9, 10], could be used to calibrate large one-dimensional artifacts instead of the method proposed here. The bundled adjustment method allows the use of LT to calibrate ball bars of considerable length with small enough uncertainty, Sandwith *et al* [5]. The problem with these methods is that the errors of an LT increases when using the encoders to measure the geometrical elements of one-dimensional artifacts. Also, it is not possible to measure step gauges or scales bars with line or circle marks with small enough uncertainty using an LT. For step gauges, LTs can incorporate a probe tip as in a CMM, however, the orientation of the probe with respect to

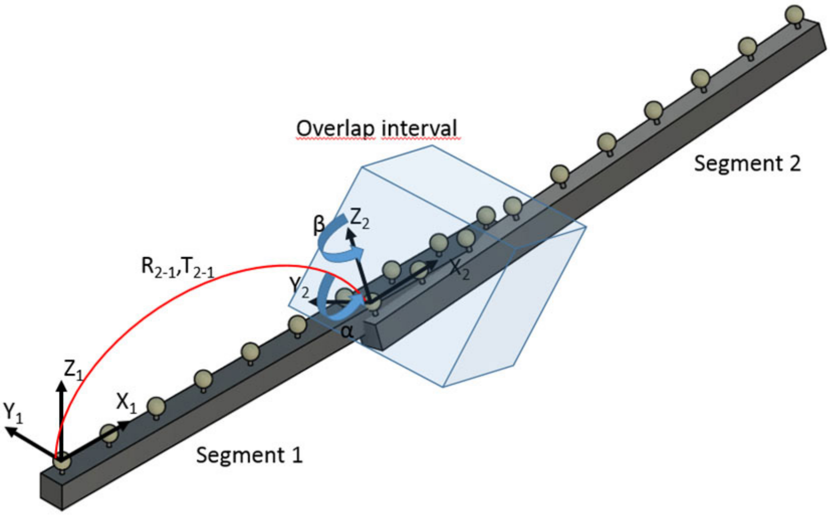

Figure 2. Degrees of freedom caused by overlap during the calibration of large ball bars or during the performance evaluation of large volume CMS.

the LT introduce more errors in LT's measurement. In the case of lines or circle marks, an LT cannot see the marks with a scanner or with its SMR probe, therefore it is not possible to measure the marks.

Another method to calibrate large one-dimensional artifacts could be to use photogrammetric-based techniques. Luhmann [11], Gale *et al* [12], and Cuypers *et al* [13] present examples of systems based on such technology. If we want to measure ball bars with this technique, we should paste several markers over the surface of the spheres. However, the accuracy in the measurement of the ball bar using this technique is not sufficient to evaluate a CMM. For the case of steps gauges or scale bars, photogrammetric-based methods are not appropriate.

An articulated arm coordinate measuring machine (AACMM) may also be used to calibrate such large artifacts using the best fit between common points. The essence of the overlap method presented here is a particular best fit between shared 3D coordinates points measured in different coordinate systems. Like the other methods we mentioned, the problem with AACMM is the accuracy, Romdhani *et al* [14]. So the uncertainty to calibrate such large artifacts is not enough for the performance evaluation of a CMM, for example.

Cox *et al* [15] is the only author that we found to have used the overlap method using a CMM. Cox named the method 'repositioning' and uses tapped bores that must be part of the workpiece to attach at least three reference spheres, to perform the repositioning. The spheres are used as registration points to pass from one reference coordinate system to another; he mentioned that this registration points should not be collinear. Butler *et al* [16] implement the repositioning using FORTRAN software. Nevertheless, the software is too complicated for practical everyday usage with proprietary CMM software and requires strong mathematical and programming skills from end-users. The work proposed by Cox and Buttler is a more general method for repositioning a workpiece. It is a best-fit method to measure large workpieces from different coordinate systems linked with geometrical elements.

The advantages of our overlap method over Cox and Butler's method are that we do not modify the measurement work piece by placing reference spheres, instead, we use the elements belonging to the work piece. The manual movement of the work piece under measurement and the imperfections of the geometric element's position avoid the collinearity of the registration points.The mathematical model using trigonometric ratios (TR) to perform overlap, shown in section 5.1, is a novel method compared with Cox's method or singular value decomposition (SVD) method also described in section 5.2. The last two methods are commonly used for registration and can be implemented with any computer software. Finally, and as the most relevant point, we present a novel method for the performance evaluation of large volume CMS and its uncertainty evaluation.

As mentioned, there are several one-dimensional artifacts of large dimensions. For the remainder of this paper, we will concentrate only on just one type of these artifacts, the large-sized ball bar. However, the overlap method applies to other one-dimensional artifacts that allow the measurements of its elements in 3D coordinates, such as gauges with line or circle marks using optical CMMs. Also, within the available CMS, the CMM is the equipment so far with greater accuracy, therefore, we carried out all the experiments using this machine instead of an LT or an AACMM.

3. Performance evaluation of CMS with the overlap method

The overlap method was first used for the calibration of elements whose dimensions can ot be covered by a CMS of a particular size. How ever, this method also allows us to do the opposite. That is, to evaluate the performance of CMS whose linear dimensions are not covered by at least 66% as indicated by the ISO 10360-2 standard [2] for the case of CMM. Figure 2 shows one ball bar overlapped with itself to extend the covered range in a line inside the volume of a CMS.

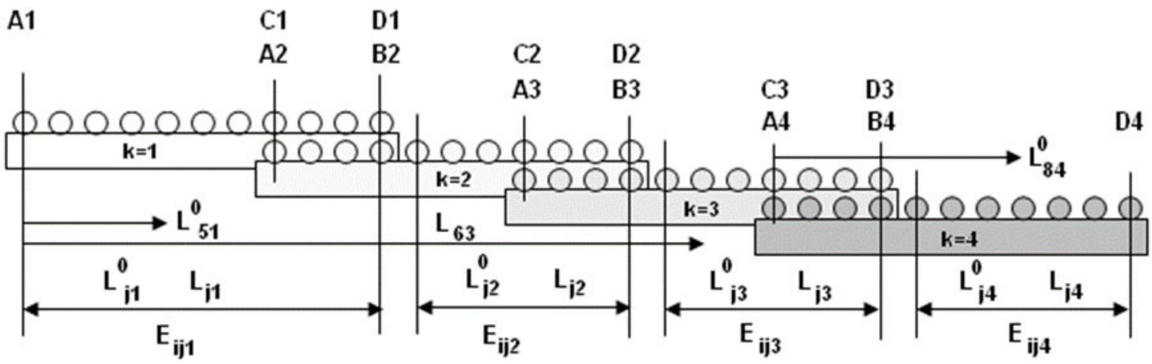

Figure 3. Model that exemplifies the overlap of ball bars with $k = 4$ for performance evaluation of a CMS.

The method is used to convert a measurement gauge of a certain length into one of greater extent, by overlapping geometric elements with itself. The length extension of the artifact requires at least two common elements in the overlap interval. However, the more features in the overlap interval will ensure a better connection because there is an average of the deviations in such a range. Figure 3 is a simple representation of the overlap. However, we know that such an overlap can be difficult to perform, such as, on a diagonal line inside CMM's volume. One way to solve this problem is to use a jig, supported with tripods, as large as 66% of the diagonal of the CMM and with the reference gauge placed on this jig. Another way would be to use tripods that support the reference gauge and guide the measurement of the gauge using a previous written program in the CMM. The program would allow us to place the tripod that supports the gauge so that we can guide it to perform the overlap.

Figure 3 shows how to overlap using a ball bar and the terminology used by the equations developed for the overlap of one-dimensional elements. Let k be the position of the gauge inside the measuring volume of the machine and $C_k - 1$ to $D_k - 1$ with A_k to B_k the interval of overlap for each of the k positions of the measurement line that we wish to extend. $(k = 2, 3, ..., n)$.

Equations (1)–(4) shows the evaluation error in $k = 3$ positions of a ball bar, where M is the measured value of the gauge reported by the CMS and N is the calibrated value:

$$E^0_{ijk} = M_{ijk} - N_j \tag{1}$$

$$E_{ij1} = M_{ij1} - N_j = E^0_{ij1} \tag{2}$$

$$E_{ij2} = M_{ij2} - N_j + \frac{1}{I}\sum_{j=C_1}^{D_1}\sum_{i=1}^{I} E_{ij1} - \frac{1}{I}\sum_{j=A_2}^{B_2}\sum_{i=1}^{I} E^0_{ij2} \tag{3}$$

$$E_{ij3} = M_{ij3} - N_j + \frac{1}{I}\sum_{j=C_2}^{D_2}\sum_{i=1}^{I} E_{ij2} - \frac{1}{I}\sum_{j=A_3}^{B_3}\sum_{i=1}^{I} E^0_{ij3} \tag{4}$$

$$E_{ijk} = M_{ijk} - N_j + \frac{1}{I}\sum_{j=C_{k-1}}^{D_{k-1}}\sum_{i=1}^{I} E_{ijk-1} - \frac{1}{I}\sum_{j=A_k}^{B_k}\sum_{i=1}^{I} E^0_{ijk} \tag{5}$$

where A_k is the first element measured in position k (reference element of this position). B_k is the last element that is part of the overlap in position k. C_k is the first element with which the overlap begins in position k. D_k is the last element measured in position k. I is the total number of measurements by position. k is the number of positions. i is the current number of repetitions. j is the current number of elements in a position. M_{ijk} is the length measured in position k, measuring element j in repetition i, covering the first element in this position. N_j is the calibrated length ad distance from element j to element 1. E^0_{ijk} is the length deviation measured in position k, measuring element j in repetition i, covering the first element in this position. E_{ijk} is the deviation of length measured in position k, measuring element j in repetition i, without covering the position of the first element.

In figure 3, L^0_{jk} is the nominal distance between an element j in position k, and the first element in the same position, L_{jk} is the nominal distance that exists from an element j in position k to the first element of the first position.

Equation (5) expresses the general model for performance evaluation of CMS using overlap. Equation (5) only consider the errors coming from the CMS and uses the errors in the interval area to perform the overlap.

Another way for the performance evaluation of CMS using overlap consists of using equations (19)–(24) (TR method) or equations (25)–(30) (SVD method). For this, it is recommended to use nominal coordinates for the positions of the balls of the bar in the segments one, two, three, etc, and add to these positions, the errors founded during the performance evaluation of the CMS. Once we add the errors to nominal coordinates, equations from the TR method or SVD method can be applied directly.

For example, if the distance between balls of the bar is approximately 100 mm, then the nominal coordinates of the bar would be 0, 0, 0; 100, 0, 0; etc in X, Y, Z coordinates, respectively. To these values add the measured errors, that is, the calibrated minus the measured distance. An error of 0.010 mm for the ball in position 100 would represent a position in X, Y, Z coordinates of 100.01, 0, 0 in that position. Note how the values of Y, Z are always zero since we are evaluating a CMS and these coordinates are not of interest, that is, rotational errors are not considered. In the case of performance evaluation of CMS, the measurand of interest is the error estimated with the length measured by the CMS under evaluation minus the calibrated value of that length.

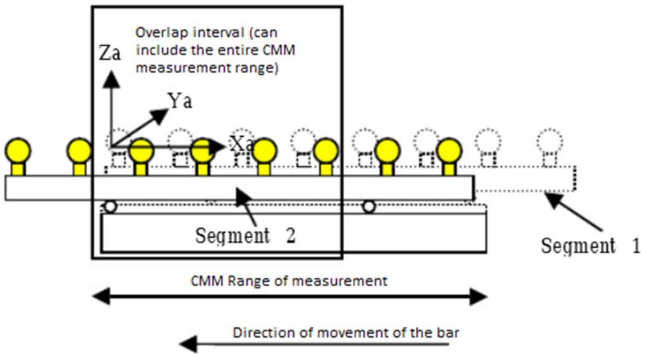

Figure 4. Overlap method applied to large ball bar calibration.

4. Uncertainty for the performance evaluation of CMS and calibration of large ball bars

The uncertainty for the performance evaluation of CMS is evaluated, taking into account the following factors according to GUM [19]: the gauge uncertainty, the repeatability in the overlap area, as well as the gauge's temperature. These are also the main factors used in the standard ISO/TS 23165:2006 [20], the only difference is the repeatability in the overlap area that increases the uncertainty from where the overlap was applied.

The gauge uncertainty and the repeatability in the overlap area are described by the following equations:

$$U^0(L) = c + a \cdot L \tag{6}$$

$$U^0_{jk} = U^0(L_{j1}) = c + a \cdot L_{j1} \tag{7}$$

$$U_{j2} = 2 \cdot \sqrt{\left[\frac{1}{2}U_1^{\max} + \frac{1}{2}U^0(L^0_{j2})\right]^2 + \frac{\sigma_1^2 + \sigma_2^2}{n_{A2B2} \cdot I}} \tag{8}$$

$$U_{j3} = 2 \cdot \sqrt{\left[\frac{1}{2}U_2^{\max} + \frac{1}{2}U^0(L^0_{j3})\right]^2 + \frac{\sigma_2^2 + \sigma_3^2}{n_{A3B3} \cdot I}} \tag{9}$$

$$U_{jk} = 2 \cdot \sqrt{\left[\frac{1}{2}U_{k-1}^{\max} + \frac{1}{2}U^0(L^0_{jk})\right]^2 + \frac{\sigma_3^2 + \sigma_k^2}{n_{A4B4} \cdot I}} \tag{10}$$

$$\sigma_k = \sqrt{\frac{1}{\Sigma_{k=1}^{K} J_k}\Sigma_{k=1}^{K}\Sigma_{j=1}^{J_k}\sigma_{jk}^2} \tag{11}$$

$$\sigma_{jk} = \sqrt{\frac{1}{I-1}\Sigma_{i=1}^{I}[M_{ijk} - M_{jk}]^2} \tag{12}$$

$$M_{jk} = \frac{1}{I}\Sigma_{i=1}^{I}[M_{ijk}]. \tag{13}$$

$U^0(L)$ is the gauge uncertainty coming from the calibration certificate of the gauge. U_{jk} is the contribution to the uncertainty coming from E_{ijk}. U^0_{jk} is the contribution to the uncertainty coming from E^0_{ijk}. a is a factor depending on the length. c is a constant. J_k is the number of elements in position k. σ_k is the standard deviation of all elements in position k. σ_{jk} is the standard deviation of element j in position k. n_{AkBk} is the number of elements in the overlap area between position k and $k-1$. $U_k^{\max}$ is the maximum uncertainty after an overlap, which is taken into account for the next uncertainty evaluation. The nomenclature of equations (6) to (13) is based on figure 3.

To calculate the influence of temperature, we have three terms, which we must add quadratically so that they are considered as variances and therefore as the best estimate of the variation of the mean:

$$\frac{U_\alpha}{2} \cdot (T - 20) \cdot L_j. \tag{14}$$

U_α is the uncertainty coming from the thermal expansion coefficient of the material, T is the average temperature during the measurement of the bar in position k, $T = (T_{\text{start}} - T_{\text{end}})/2$, L_j is the length of any element j:

$$\alpha \cdot L_j \cdot \frac{U_{ct}}{2}. \tag{15}$$

α is the thermal expansion coefficient, U_{ct} is the uncertainty of the temperature sensor found in the calibration certificate:

$$\alpha \cdot L_j \cdot \frac{VT_{\text{obj}}}{\sqrt{12}}. \tag{16}$$

VT_{obj} is the significant temperature variation of the bar during all the measurements in all the positions.

Finally, the uncertainty evaluation for temperature is

$$U(\text{temp}) = 2 \cdot \sqrt{\left[\frac{U_\alpha}{2} \cdot (T-20) \cdot L_j\right]^2 + \left[\alpha \cdot L_j \cdot \frac{U_{ct}}{2}\right]^2 + \left[\alpha \cdot L_j \cdot \frac{VT_{\text{obj}}}{\sqrt{12}}\right]^2}. \tag{17}$$

The final uncertainty of the test for the performance evaluation of a CMS is

$$U(\text{test}) = \sqrt{U_{jk}^2 + U(\text{temp})^2}. \tag{18}$$

The uncertainty for the calibration of large one-dimensional artifacts take into account exactly the same factors as the performance evaluation of CMS. But, in the calibration case, the gauge uncertainty comes from the gauge used to get the scale factor correction of the line where the artifact under calibration is set in the CMM. It means that if an interferometric laser is used for scale compensation then the uncertainty from the laser is the gauge uncertainty. Another gauge for scale factor could be a step gauge or gauge block calibrated with low uncertainty in a reference laboratory.

5. Calibration of large one-dimensional artifacts using the overlap method

To calibrate large one-dimensional artifcats (ball bars) in a particular CMS like a CMM, we proceed as follows. We divide the one-dimensional artifcats (ball bars) into two or more segments (segment 1, segment 2, segment 3, ..., segment n) in such a way that we measure each segment in the interval of the CMM, see figure 4. To join the results of the measurement of the segments, it is necessary that a certain number of geometric elements (at least two and preferably four, see figures 6(d)–(f)) are common in both segments. These elements will be part of the overlap interval. Cox *et al* [15] mentioned three methods to reposition ball bars. The first is the rotation about a vertical axis using a rotary table, where the ball bar is fixed directly onto the rotary table. The second is the translation along a horizontal axis using a jig. We must mount the piece under measurement on a jig that can be fixed to the CMM table in various positions overhanging at either end of the CMM table. We must manipulate the artifact on the jig from one position to another. This method was the approach adopted in this work to calibrate large ball bars, see figure 1. The last method is the rotation about a horizontal axis, used when cylindrical-type artifacts must be measured.

5.1. A mathematical model for overlap using trigonometric ratios

The trigonometric ratios (TR) method is the simplest and novel way to perform overlap. It uses the X, Y and Z coordinates of the elements involved in the overlap area to evaluate rotation and translation between segments. In the TR method, we establish the coordinate system of the object in the first segment, and the geometric features that are part of it are measured. Then, the artifact is moved in the direction of the same axis, leaving some features in the overlap interval (at least two). Then, we establish a new coordinate system of the object, and the geometric elements of segment two are measured. The measurements from part two to part one are connected once the results from part two are corrected (it means putting everything in the coordinate system of part one). The correction comes from six possible errors in the overlap area: three translations and three rotations. The translation errors are corrected, averaging the results obtained from segment two to one, employing

$$G_{k,c} = \frac{\sum_{i=1}^{n} C_i}{n} \tag{19}$$

$$\mathrm{corr}T_c = G_{1,c} - G_{2,c} \tag{20}$$

where $G_{k,c}$ is the center of gravity X, Y, Z of each segment evaluated with the balls in the overlap interval. C is the X, Y, Z coordinate of each ball in the overlap interval. k is the number of segments. n is the number of balls or geometrical elements in the overlap interval. $\mathrm{corr}T_c$ is the translation correction X, Y, Z value to be applied to balls outside the overlap interval of the next segment, in this case, segment two. The positions of the balls in segment one remain uncorrected, but we must correct the balls of the next segment that are outside the overlap area with equation (20).

To correct the rotational errors, we firstly determine the angles α, and β shown in figure 2 using the following equations:

$$\alpha = \frac{\sum_{i=1}^{n}(Z_{2,1} - Z_{1,i})(X_{2,i} - G_{2,x})}{\sum_{i=1}^{n}(X_{2,i} - G_{2,X})^2} \tag{21}$$

$$\beta = \frac{\sum_{i=1}^{n}(Y_{2,1} - Y_{1,i})(X_{2,i} - G_{2,x})}{\sum_{i=1}^{n}(X_{2,i} - G_{2,X})^2} \tag{22}$$

where α and β are the rotational angles around Y_2 and Z_2 axis respectively. $Z_{2,i}$ and $Y_{2,i}$ are Z and Y coordinates of segment two for each of the elements in the overlap interval. $Z_{1,i}$ and $Y_{1,i}$ are Z and Y coordinates of segment one for each of the elements in the overlap interval. $X_{2,i}$ is the X coordinate of segment two for each of the elements in the overlap interval.

Then, to correct Z and Y coordinates of all the geometric elements outside the interval area of segment two, the following expressions are applied:

$$CorrZ_\alpha = \alpha(X_{2,i} - G_{2,X}) \tag{23}$$

$$CorrY_\beta = -\beta(X_{2,i} - G_{2,X}). \tag{24}$$

The rotation errors are typically small because we use a jig to move the ball bar in a line within the CMM.

The results of all the geometric elements, from segment two to segment one, are connected with the corrections found by expressions (20), (23) and (24). We add these corrections to the coordinates of elements outside the overlap interval of the segment two measurements. Then, we use coordinates of segment one with the corrected coordinates of segment two (elements outside the overlap interval), and as a result, we obtain a final measurement of the large artifact. If we divide the artifact into more than two segments, now the combination of segment one with the corrected segment two takes the place of a new segment one. Segment three takes the place of segment two and we proceed to carry out the measurements and calculations again.

The TR method does not correct the rotational angle around the axis of translation of the ball bar (rotation around X). The next subsection evaluates all the three translations and rotations to perform the overlap method. Cox *et al* [15] explains that registration points (overlap balls) must not be collinear, however, using the equations (19)–(24), the registration points must be as collinear as possible to reduce overlap error. With the collinearity, the rotational errors from equations (21) and (22) are negligible, so only the translation of the piece is taken into account. Collinearity does not occur due to imperfections in the ball's positions and the manual movement of the bar over the jig. The jig is used to support the bar always at the same points to avoid deformations of the bar. We must attach the jig in some way to the CMM table, and the material should be as rigid as possible.

5.2. A mathematical model for overlap using SVD

The overlap method using singular value decomposition (SVD) is based on the least-squares fitting implemented by Arun *et al* [17] and Icasio *et al* [18]. Recall that segment one has the coordinate system, X_1, Y_1, Z_1, and segment two the coordinate system, X_2, Y_2 and Z_2. The objective of the overlap method is to find the rotation and translation between both coordinate systems as shown in figure 2. Once the roto-translation is calculated, the X_1, Y_1, Z_1 coordinates of segment one remain the same, and to these coordinates, we add the X_2, Y_2, Z_2 coordinates corrected by rotation and translation of segment two. With this, we managed to calibrate the large bar. If due to the size of the bar we divide the artifact into more than two segments, now the combination of segment one with the corrected segment two takes the place of a new segment one. Segment three takes the place of segment two and we proceed to carry out the measurements and calculations again.

The algorithm to solve the roto-translation consists of the following steps:

Step 1: evaluate the centroids of each segment, taking into account only the balls in the overlap area:

$$\text{centroid} = \sum_{i=1}^{n} \frac{(X_i, Y_i, Z_i)}{n}. \tag{25}$$

Step 2: calculate the $3X3$ matrix:

$$Q = \left(\begin{bmatrix} X_1 & Y_1 & Z_1 \\ \vdots & \vdots & \vdots \\ X_i & Y_i & Z_i \end{bmatrix}_1 - \text{centroid}_1\right)^T \cdot \left(\begin{bmatrix} X_1 & Y_1 & Z_1 \\ \vdots & \vdots & \vdots \\ X_i & Y_i & Z_i \end{bmatrix}_2 - \text{centroid}_2\right). \tag{26}$$

Q is the rank 3 matrix, centroid_1 and centroid_2 are the centroids of balls in segments one and two respectively and i is the number of balls in the overlap area.

Step 3: find the SVD of Q:

$$[U, S, V] = SVD(Q). \tag{27}$$

Step 4: calculate

$$X = V \cdot U^T. \tag{28}$$

V is the right singular values of matrix Q and U^T is the transpose of the left singular values of matrix Q.

Step 5: calculate, $det(X)$, the determinant of X:

if $det(X) = +1$, then $R_{(2-1)} = X$

if $det(X) = -1$, then we have co-planar or collinear registration points, so in this case the third column of V must be multiplied by -1 and then $R_{(2-1)} = V \cdot U^T$

Step 6: evaluate translation ($T_{(2-1)}$) between segment two and one:

$$T_{(2-1)} = \text{centroid}_1 - (R_{(2-1)} \cdot \text{centroid}_2). \tag{29}$$

Step 7: finally, apply rotation and translation to the balls outside the overlap area from the next segment, in this case, segment two:

$$\text{Corr}_{\text{RotTras}} = R_{(2-1)}(X_i, Y_i, Z_i)_2 + T_{(2-1)}. \tag{30}$$

6. Experimental results

6.1. Experimental setup

In our experiments, we employed a CMM Mitutoyo model BJ 1015. It has a measurement range of 1500 mm, 1000 mm, and 800 mm in orthogonal axes. Its spatial resolution along any one of the axes is 0.0005 mm, with a positioning accuracy of 0.001 mm. The other employed CMM is a Mitutoyo Legex, model 9106. It has a measurement range of 1010 mm, 910 mm and 610 mm in the orthogonal axes. Its spatial resolution along any one of the axes is 0.000 01 mm, with a positioning accuracy of 0.0001 mm.

Two ball bars were employed; one was 1500 mm long and the other was 700 mm. Both were made of carbon fiber. The 1500 mm long bar has spheres every 100 mm and the 700 mm long bar has spheres every 50 mm. The step gauge used for performance evaluation with the overlap method is a Koba step gauge 1020 mm long with steps every 20 mm, made of steel.

In these experiments, we do not use a jig to hold the ball bar because we use a segment of the same ball bar or step gauge to perform the overlap. In a normal measurement using overlap, we use a rectangular aluminum profile that is approximately 5100 mm long, 100 mm wide, and 120 mm high, see figure 1. Besides, we use a carbon fiber profile with a length of 3200 mm, a width of 80 mm and a height of 10 mm.

6.2. Results for the performance evaluation of CMS with the overlap method

Figure 5 show the results for the performance evaluation along the axes of a CMM. A step gauge was used to evaluate Y-axis error of a CMM and a ball bar was used to evaluate X-axis error of the same CMM. We evaluate Y-axis and X-axis errors without the overlap method and the errors when using the overlap method. The step gauge has gauge blocks every 20 mm. To avoid the influences of the probe tip diameter error, we assume that step gauge has gauge blocks every 40 mm so only unidirectional gauge blocks were measured. The step gauge was 1020 mm long and the ball bar was 1500 mm long. The ball bar has solid ceramic spheres every 100 mm.

For the step gauge case, the overlap method was realized using 600 mm long segment of the 1020 mm gauge. In this case, we overlapped the last six gauge blocks of step gauge segment from 400 mm to 600 mm, with the first six gauge blocks of the step gauge segment from 0 mm to 200 mm. For the ball bar case the overlap method was realized using 900 mm long segment of the 1500 mm gauge. In this case, we overlapped the last five balls of the ball bar segment from

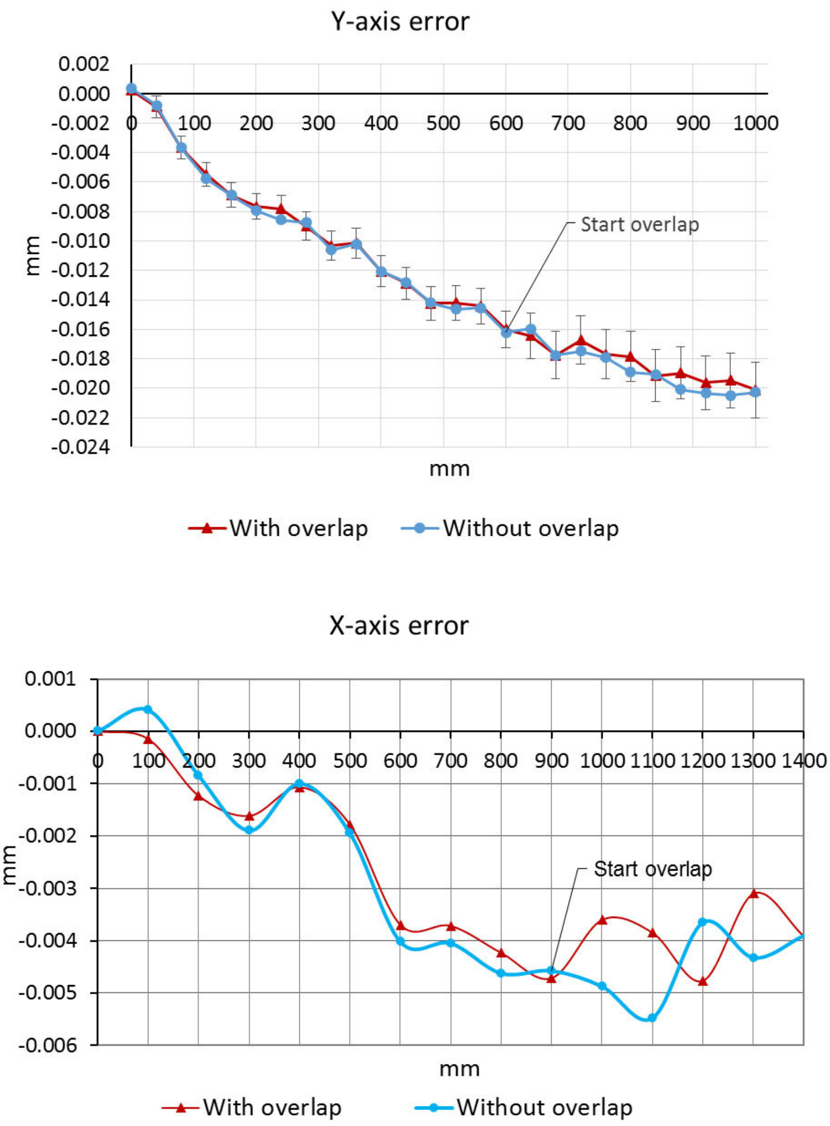

Figure 5. Deviations of two CMM's axis using a step gauge (top) and a ball bar (bottom) without overlap and with overlap.

500 mm to 900 mm, with the first five balls of the ball bar segment from 0 mm to 400 mm.

Figure 5 shows that overlap starts at 600 mm for the step gauge case and at 900 mm for the ball bar case, from where there are differences on the order of CMM repeatability between the errors using a segmented overlap and the errors with the step gauge and ball bar measured without using overlap. Figure 5 (top) also shows uncertainty vertical bars for the performance of one CMM axis. After 600 mm uncertainty increases not because of linear dependency in the step but because of overlap zone, see figure 8. The magnitude of the deviations for the *Y*-axis and *X*-axis from CMM is good enough considering the repeatability of the machine is smaller than 0.001 mm.

Figures 6(a)–(c) shows the error when we studied overlap measuring a 'large bar' as a function of the angular orientation of the ball bar during overlap. We move the bar by some angle in the second position relative to the first position. Both the SVD and TR methods show no dependency in error according to the ball bar angular position. However, figures 6(a)–(c) shows that an angle of 0 degrees is better for TR and SVD methods in *X*, *Y* and *Z* coordinates.

Figures 6(d)–(f) shows the error measuring a 'large bar' as a function of the number of balls during overlap. In each case, we considered a different quantity of balls in the overlap zone. Figures 6(d)–(f) shows that four balls are sufficient to overlap with the SVD or TR method. TR method works with only two balls, but the SVD method needs at least three balls. In all the cases, the principal coordinate is *X*, and for this coordinate, the SVD method has less error than the TR method.

6.3. Results for the calibration of large one-dimensional artifacts using the overlap method

Figure 7 shows the difference in distance between spheres of a 700 mm ball bar measured without using overlap (the bar fit the measuring range of a high accuracy CMM), and the distance between spheres of the same 700 mm ball bar but now with two methods for overlap. The bar has balls at approximately 50 mm intervals.

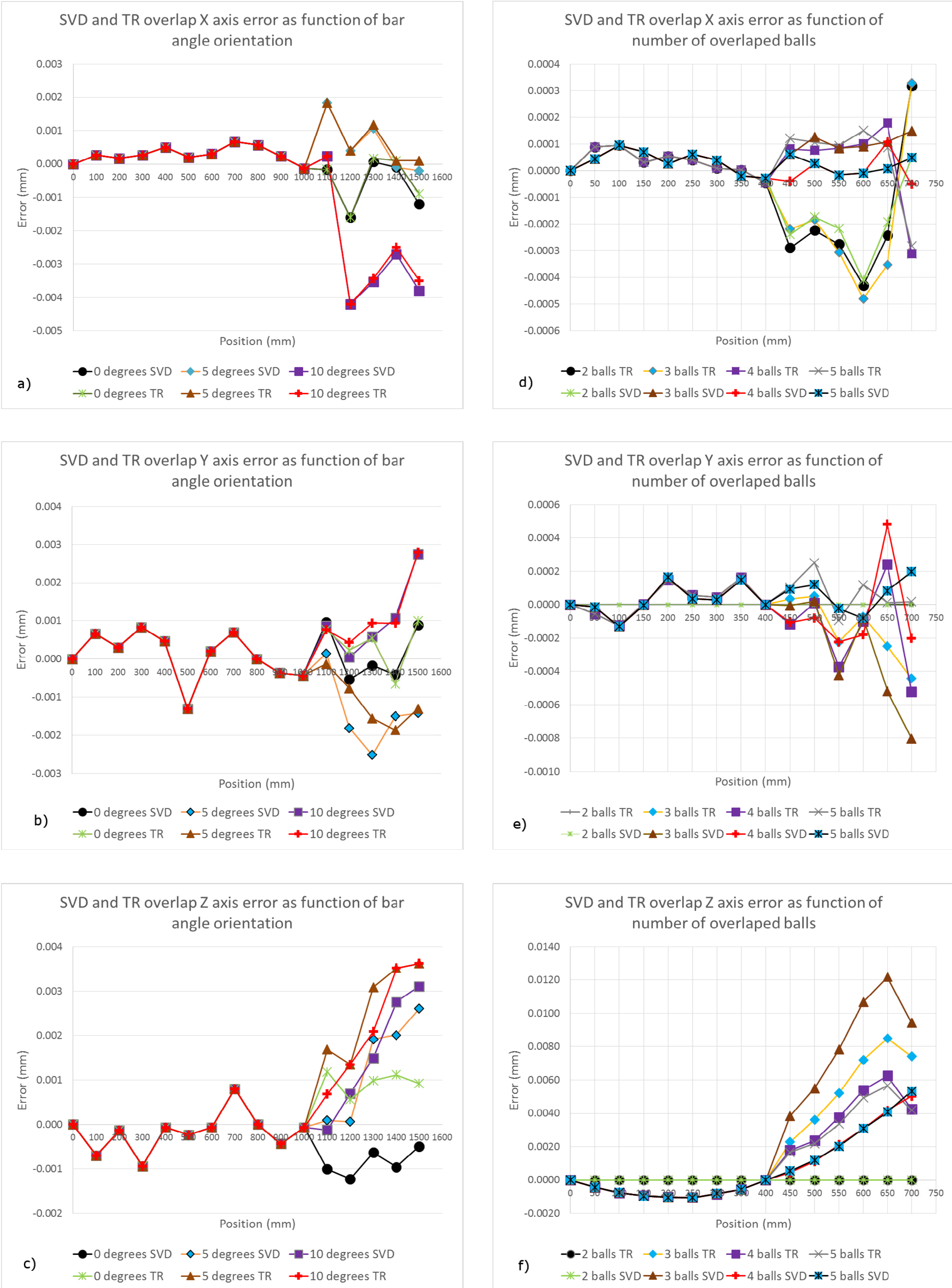

Figure 6. Overlap error during large ball bar calibration. (a)–(c) Overlap error as function of angle orientation of the ball bar. (c)–(e) Overlap error as a function of the number of balls used for the overlap.

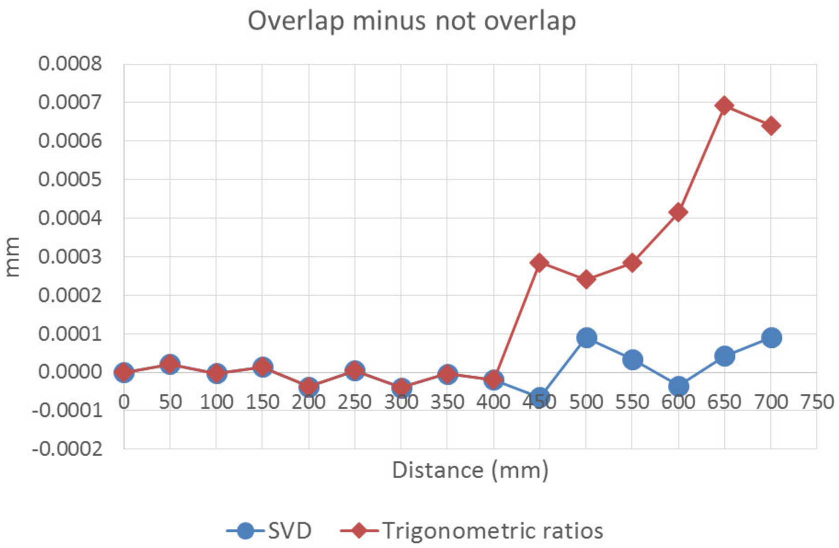

Figure 7. Deviations of a ball bar distance calibration without overlap and with overlap.

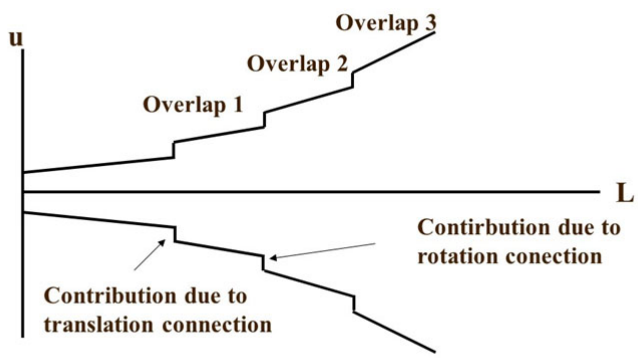

Figure 8. Measurement uncertainty coming from the overlap of geometric elements.

The overlap method is realised on the CMM using segments that are 400 mm long. To cover the range of the ball bar, we must overlap the last three balls of the bar from 300 mm to 400 mm, with the first three balls from the rest of the bar from 450 mm to 550 mm.

Figure 7 shows that overlap starts at 400 mm, from where there are differences in the distance against the bar measured without using overlap. Besides, we can see that the SVD method has less error than the trigonometric ratios method. The magnitude of the errors using both ways is good enough for calibration of ball bars used to evaluate large CMM. But, for the performance evaluation of high accuracy CMM, only the SVD method is appropriate, taking into account an increment of 0.0001 mm for the uncertainty of ball bar calibration coming from the overlap.

7. Discussion

If the right conditions for the CMM and the environment are available, then the overlap method is efficient to calibrate large ball bars and evaluate large CMMs.

The temperature is a factor that influences the errors in the performance evaluation of CMS, as well as in the calibration of large dimensional artifacts. Documentary standards such as ISO/TS 23165:2006 [20] mention that performance evaluation should be done in the actual conditions of CMM, that is, if the CMM's software has the option for temperature correction at 20 °C then these corrections may or may not be activated. In the case of overlap, it is recommended to activate these compensations and bring all measurements to 20 °C using the CMS measurement software. If they are not activated, the uncertainty reported in equation (17) should be equal to zero since no temperature compensation is performed. The above has the consequence that the error evaluated during the performance evaluation may be larger. In the case of the calibration of large one-dimensional artifacts, temperature compensation at 20 °C must be done with the CMS measurement software or offline with the equation for temperature correction showed in ISO/TS 23165:2006 [20].

We must consider how to make the overlap as accurate as possible inside the line contained in the measurement volume of the CMS. In lines parallel to the CMS's axes, which is what we show in the results, there is no difficulty. However, in lines located inside the volume, there could be more uncertainty due to the connection between elements to carry out the overlap.

For the calibration of large ball bars, the overlap method assumes that the line of the CMS does not have significant geometry errors that could affect the roto-translation of elements. Figure 3 shows that spheres from 3, 4, 5 and 6 from the left are the elements used for overlap (white spheres for the first segment position, and gray spheres in the second position). Note that we measure white and gray spheres in different places inside the range of the CMS. Then, the geometrical errors in this line of the CMS will affect the calibration of the ball bar using overlap. If we have a high accuracy CMS, then its geometric errors are negligible in the volume where the bar moves manually. The movements will not affect the calibration because sphere positions 3, 4, 5 and 6 of the bar remain the same if environmental conditions are good enough.

For the performance evaluation of CMS, the overlap method assumes that the errors in a line inside the volume of overlap do not affect the translation of elements. Figure 2 shows clearly that the manual movement of the bar will not result in a perfect evaluation line. If we consider that the errors of CMS do not change significantly within a specific volume of the CMS, then the performance evaluation is done correctly only with translation of errors. Even though it is not the same measuring line, the errors of the CMS do not change abruptly inside the small volume generated by the manual movements of the bar.

How precisely the overlap between elements (movement of the bar) has to be performed depends on how accurate the CMS is for the calibration case. For the case of performance evaluation, this depend on the mapping errors of the CMS. From experience, we can say that the manual bar repositioning when calibrating or evaluating can be a maximum of 10 mm in the translation of the axes and a maximum of five degrees in the rotation of any of the axes. However, overlap precision will depend on the CMS for which the calibration or evaluation is carried out. For example, it may depend on the step defined in the error compensation map of the CMS. However, having a coordinate system, the manual repositioning of the bar can be achieved in no more than 10 mm and no more than five degrees. Figures 6(a)–(c) show the influence of a wrong bar orientation along the line where we are doing the calibration.

Concerning the fixturing of ball bars of large dimensions, it is essential that we support the bar on the same points during all overlaps (preferably Bessel or Airy locations). Otherwise, there will be errors from the elastic deformation of the measurand. For CMM with a movable table or supported by pneumatic suspension, it will be necessary to support the device in a beam or jig sufficiently rigid to avoid oscillations in the critical overlaps to maintain the same support.

Increasing the number of geometric elements in the overlap interval will ensure a better overlap, but also the evaluation or calibration time will grow considerably. Figures 6(c)–(e) shows that the error in the calibration of a ball bar decreases as the number of geometrical elements in the overlap area increases. Four features are generally good enough to do the overlap with the SVD or TR method. But, in case there is no space or time, only two elements could be enough, but in this case, TR is the best method.

Figure 8 is a schematic showing the idea of increase in measurement uncertainty for the evaluation of CMS and the calibration of artifacts. For example, each time we have an overlap, there will be a contribution to the uncertainty from the translation connection (vertical increase) and rotation connection (slope increase). If we use more elements in the overlap interval, the contributions to the uncertainty due to the connection decreases and the vertical jump and slope increment will be smaller in each overlap.

8. Conclusions

The results obtained show that we can use the overlap method for the performance evaluation of any CMS and the calibration of large ball bars using a CMM. Another essential factor is that even when the number of overlaps has no limits, the test uncertainty increases with each of these. Therefore, it will be necessary to reach a consensus between the number of overlaps to be made and the measurement uncertainty required for the test. As future work, we will evaluate how the overlap method works with artifacts that do not have balls inline (non-one-dimensional artifacts). We have already found that the TR method does not work with non-one-dimensional artifacts, but we believe that the SVD method should work with these artifacts.

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