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# Volumetric error measurement and compensation of three-axis machine tools based on laser bidirectional sequential step diagonal measuring method

**Huimin Li[1,3], Peng Zhang[1], Ming Deng[1], Sitong Xiang[2], Zhengchun Du[1,3] and Jianguo Yang[1]**

[1] State Key Laboratory of Mechanical System and Vibration, School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China

[2] Faculty of Mechanical Engineering and Mechanics, Ningbo University, Ningbo 315211, People's Republic of China

E-mail: lillianhmlee@sjtu.edu.cn and zcdu@sjtu.edu.cn

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**Abstract**

This paper proposes an approach to the measurement, modeling, and compensation of the volumetric errors for three-axis machine tools, using laser bidirectional sequential step diagonal measurement. Disparate measuring tracks are specially arranged for forward and backward paths, and the conforming error decoupling method is demonstrated. By decoupling, all the geometric errors which can result in volumetric errors are distinguished simultaneously, including translational and angular errors. In comparison with the traditional unidirectional measuring method, the proposed bidirectional method distinguishes more geometric errors (the angular errors) by measuring the same four body diagonals. Uncertainty analysis based on the Monte Carlo simulation was conducted, followed by experimental validation on a vertical machining center. The result of the experiment indicates that the diagonal systematic deviation of positioning is reduced by 90% after volumetric compensation.

Keywords:volumetric error, bidirectional laser sequential step diagonal measurement, machine tool error compensation

(Some figures may appear in colour only in the online journal)

## 1. Introduction

Three-axis machine tools, such as vertical machine centers, horizontal machine centers, and gantry machine tools, are widely used and contain 21 geometric errors [1] in total. It is important to improve the precision of machine tools and reduce the machining error [2] for industry development. All the errors in machine tools exert an integrated influence on the orientation and position errors of the tool tip relative to the workpiece [3], which can influence the performance of machine tools. In three-axis machine tools, only the position errors of the tool tip relative to the workpiece, called the volumetric errors, can be compensated. Consequently, the measurement, modeling, and compensation of the volumetric errors are of great importance in enhancing the accuracy of machine tools.

In the compensation of the volumetric errors of machine tools, there are three steps in the engineering procedure [3]: the kinematics modeling, the measurement, the modeling of volumetric errors, and their compensation during the positioning of the machine along the tool path. The kinematics model can be obtained by using the homogeneous transformation matrix (HTM) [1], the screw theory [4], the exponential model [5], or the differentiable manifold-based method [6]. The various

[3] Author to whom any correspondence should be addressed.

 

**Table 1.** Twenty-one geometric errors of three-axis machine tools.

| | $X$-axis | $Y$-axis | $Z$-axis |
|---|---|---|---|
| Positioning error | $\delta_{xx}$ | $\delta_{yy}$ | $\delta_{zz}$ |
| Straightness error | $\delta_{yx}$ | $\delta_{xy}$ | $\delta_{xz}$ |
| | $\delta_{zx}$ | $\delta_{zy}$ | $\delta_{yz}$ |
| Angular error | $\varepsilon_{xx}$ | $\varepsilon_{xy}$ | $\varepsilon_{xz}$ |
| | $\varepsilon_{yx}$ | $\varepsilon_{yy}$ | $\varepsilon_{yz}$ |
| | $\varepsilon_{zx}$ | $\varepsilon_{zy}$ | $\varepsilon_{zz}$ |
| Squareness error | $S_{xy}$ | Between $X$-axis and $Y$-axis | |
| | $S_{yz}$ | Between $Y$-axis and $Z$-axis | |
| | $S_{xz}$ | Between $X$-axis and $Z$-axis | |

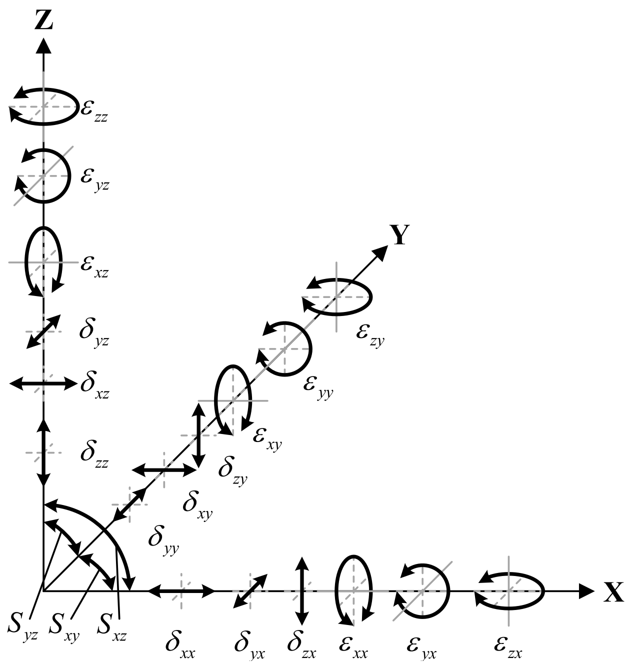


**Figure 1.** Twenty-one geometric errors of three-axis machine tools.

kinematics modeling methods can derive the same results. Li [7] modeled the volumetric errors using the HTM with small angle approximation, and the result is adopted in this paper. Error compensation aims to create an artificial error to eliminate the original error, and the technique has been validated as an effective and cost-efficient method to enhance machine accuracy [8]. The most challenging problem is the measurement and corresponding modeling of volumetric errors [13].

Geometric errors can be obtained by direct or indirect measurement. A direct measuring method means the measurement of one single geometric error for a single machine axis without the engagement of other axes [9]. Direct measurement can be achieved by [9]: material-base methods using artifacts; gravity-based methods using the gravitational field; laser-based methods using the laser's linear propagation and its wavelength as a reference. The laser interferometer, a kind of laser-based method, is the most widely used method to measure geometric errors of translational axes in the direct measurement of machine tools. With the use of a

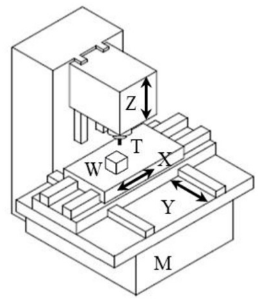


**Figure 2.** Structure of vertical machining centers.

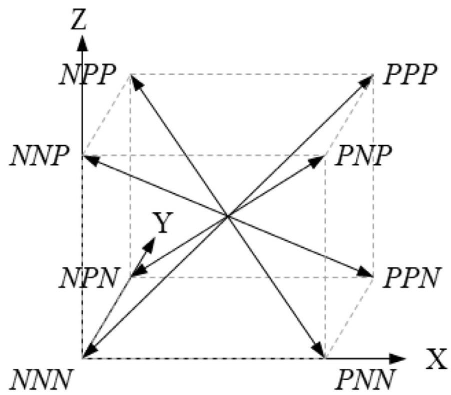


**Figure 3.** Four body diagonals in the cuboidal workspace.

laser interferometer, all the geometric errors apart from three roll errors can be measured directly. Nevertheless, for volumetric error compensation, the efficiency issue of direct measurement can be critical [10]. Indirect measurement focuses on the superposed errors of these single errors, which can increase efficiency. Geometric errors of translational axes can be measured indirectly by circular test [11], laser interferometer [12, 13], specific artifacts measurement [14, 15], passive links [16], tracking interferometer [17, 18], etc. In the indirect measurement of geometric errors of translational axes, the laser interferometer is also widely used by designing particular paths, such as the 9-line test, 12-line test [19], 15-line test [19], 22-line test [20], etc. These indirect measuring methods use the corresponding identification method to decouple the error items from the errors measured along the particularly designed lines. The setup process for each line is extremely time-consuming, typically dozens of hours. To be more efficient, ISO230-6 [12] introduced the body diagonal displacement tests to evaluate the volumetric performance of machine tools. But the error items cannot be decoupled and distinguished for the data deficiency.

To trace the error sourced, Wang [13] proposed a laser vector measurement technique, in which three translational axes move in sequence rather than simultaneously along each body diagonal. Hence, 12 translational geometric errors of three-axis are identified. However, in 2003, Chapman [21] argued limitations in the results produced by diagonal-based measurements and vector or step diagonal methods due to the misalignment errors of the mirror. Svoboda [22] tested on vertical machining centers, and stated that if one of the machine tool axes has a large linear displacement error, this error will

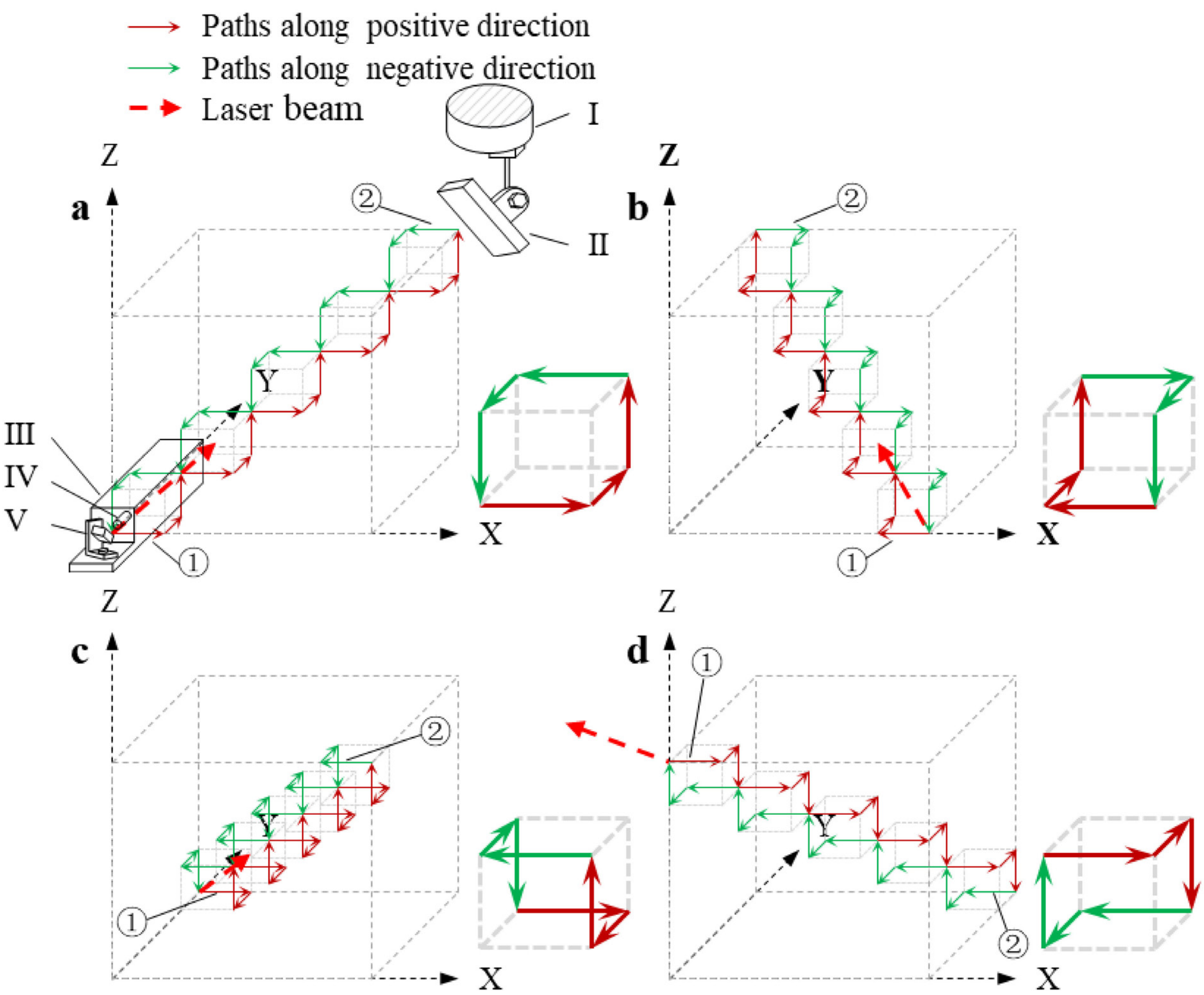


**Figure 4.** The paths for (a) *PPP*, (b) *NPP*, (c) *PNP*, (d) *PPN* body diagonal of the laser bidirectional sequential step diagonal measuring method. (I. spindle, II. flat-mirror, III. laser head, IV. optical adapter, V. diagonal steering mirror. ① The first step of the forward path, ② the first step of the backward path.)

not be correctly identified by the sequential diagonal measurement method. Ibaraki [23, 24] proposed a new formulation of the step-diagonal measurement, in order to accurately identify volumetric errors even with the existence of setup errors. Bui [25] calculated volumetric errors by measuring the displacements of three face step-diagonals and three additional linear positions. However, all angular errors are neglected in these methods. Soons [26] formulated his effect on the step diagonal measurement and clarified that they may cause a significant identification error. He also suggested a formulation to separately identify angular errors from step diagonal tests. However, he only presented its mathematical formulation without discussing its validity in a practical industrial environment. Ibaraki [24] also stated that angular errors of each axis are a potentially critical error factor for step diagonal measurements. To further increase the accuracy of the step diagonal measurement method, this paper develops a laser bidirectional sequential step diagonal measurement that is capable of distinguishing all additional angular errors which may significantly affect the measurement accuracy of volumetric errors, followed by an application of the volumetric error compensation of this method for the three-axis machining center.

This paper is organized as follows: the volumetric errors are demonstrated in section 2. It explains why not all 21 geometric errors but only some of them can bring about the volumetric errors. Then, to measure and distinguish the geometric errors which can result in the volumetric errors, disparate measuring tracks are specially arranged for forward and backward paths, and the conforming error decoupling method is demonstrated in section 3. Uncertainty analysis of the proposed method is presented using the Monte Carlo simulation. And the geometric errors are consistent with those from direct measurements by the laser interferometer, which certified the proposed strategies as feasible. Afterwards, the compensation experiments of volumetric errors are conducted on a vertical machining center in section 4. Compared with the unidirectional method, the proposed method is of higher accuracy. Section 5 concludes, summarizing the effectiveness of the proposed method and its practical application in error measurement and compensation for three-axis machine tools.

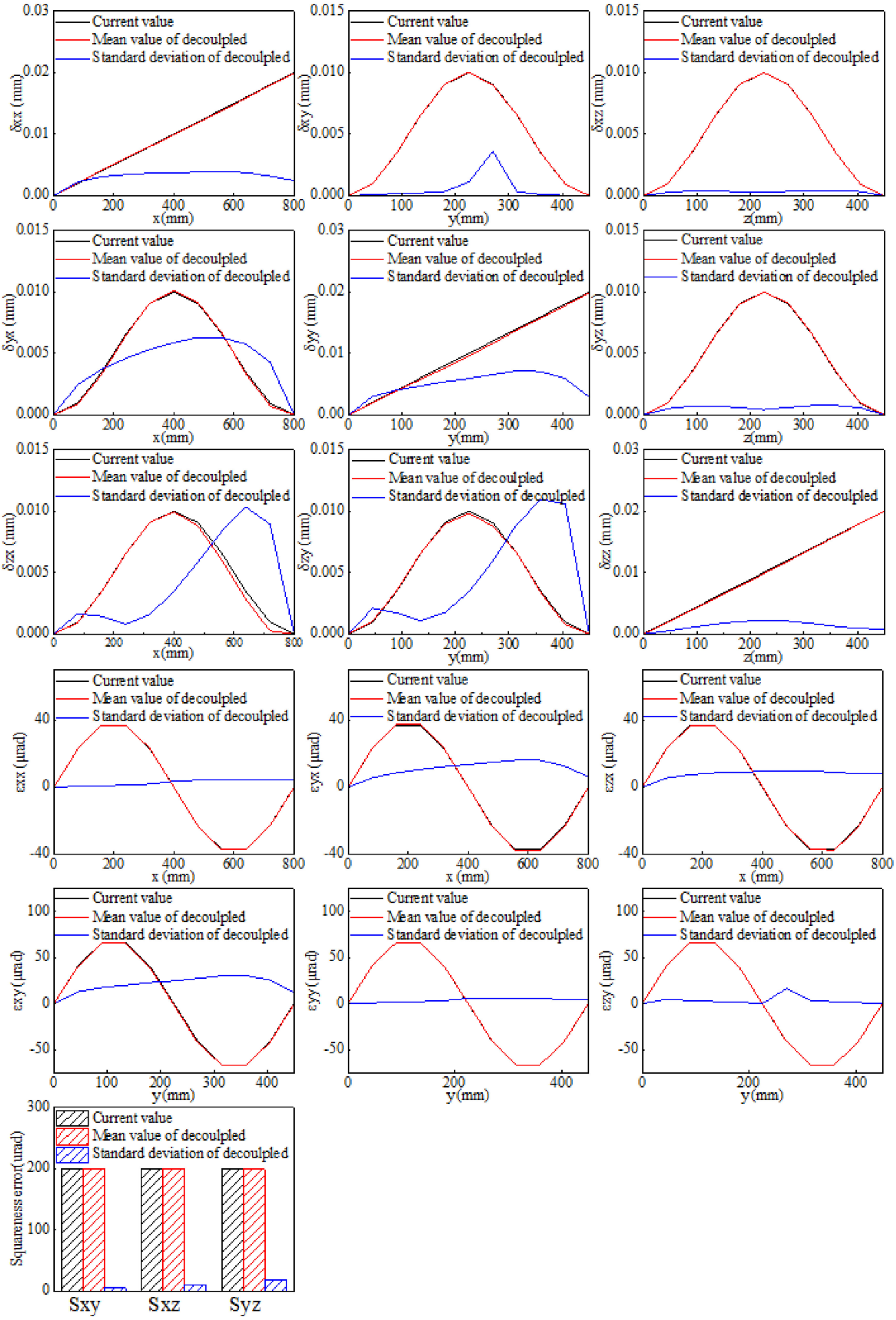


**Figure 5.** Result of Monte Carlo simulation of the proposed method.

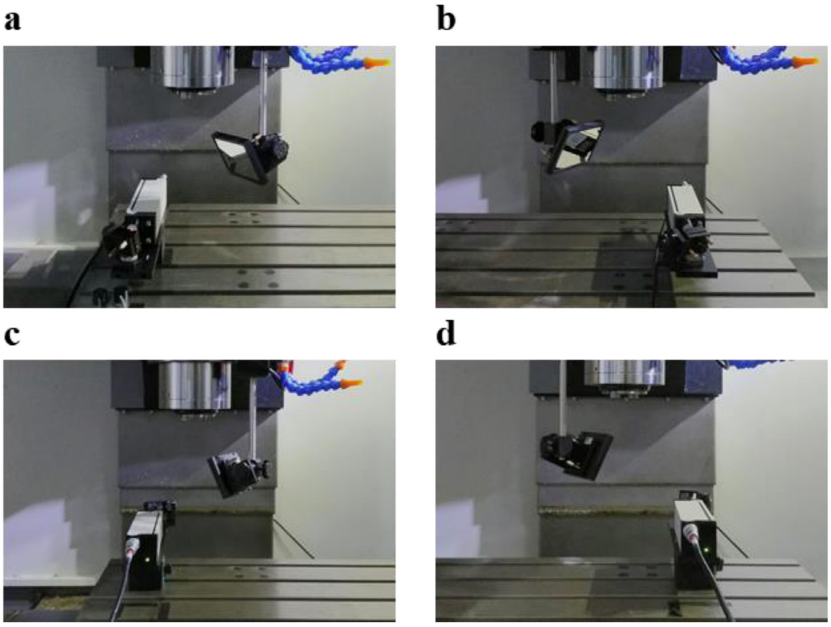


**Figure 6.** Setups of laser measurement for (a) *PPP*, (b) *NPP*, (c) *PNP*, (d) *PPN* body diagonals.

**Table 2.** Detailed values of the experiment parameters.

| Parameters | Values |
|---|---|
| $X_0$ | 800 mm |
| $Y_0$ | 450 mm |
| $Z_0$ | 450 mm |
| $N$ | 10 |
| Air temperature | 20 °C–22 °C |
| Air pressure | 100–101 kPa |
| Humidity | 60% |
| Measurement runs | 5 |
| Feed rate | 4000 mm min$^{-1}$ |

## 2. Modeling of volumetric errors

The 21 geometric errors of three-axis machine tools are classified in table 1, and the illustrations of these errors are shown in figure 1. Each linear axis has six geometric errors, namely the positioning error, straightness errors (horizontal straightness error and vertical straightness error), and angular errors (roll error, pitch error, and yaw error). And there is one squareness error between every two linear axes.

The vertical machining center is taken as an example to clarify the proposed method. And the vertical machining center is simplified as a machine tool with three translational axes, as shown in figure 2. The intention of kinematic modeling is to forecast the relative errors between the tool tip clamped on the spindle and the workpiece on the table while the translational drives move within the working area of the machine tools [3]. Li [7] has modeled the volumetric errors of the vertical machining centers by the HTM with small angle approximation, and the volumetric errors of the vertical machining centers with the movement $(x, y, z)$ can be derived as

$$\begin{cases} \Delta x = -\delta_{xx} - \delta_{xy} + \delta_{xz} - y\varepsilon_{zx} + z\varepsilon_{yx} - y\varepsilon_{zy} + z\varepsilon_{yy} + yS_{xy} - zS_{xz} \\ \Delta y = -\delta_{yx} - \delta_{yy} + \delta_{yz} + x\varepsilon_{zx} + z\varepsilon_{xx} + z\varepsilon_{xy} - zS_{yz} \\ \Delta z = -\delta_{zx} - \delta_{zy} + \delta_{zz} + x\varepsilon_{yx} + y\varepsilon_{xx} + y\varepsilon_{xy} \end{cases} . \tag{1}$$

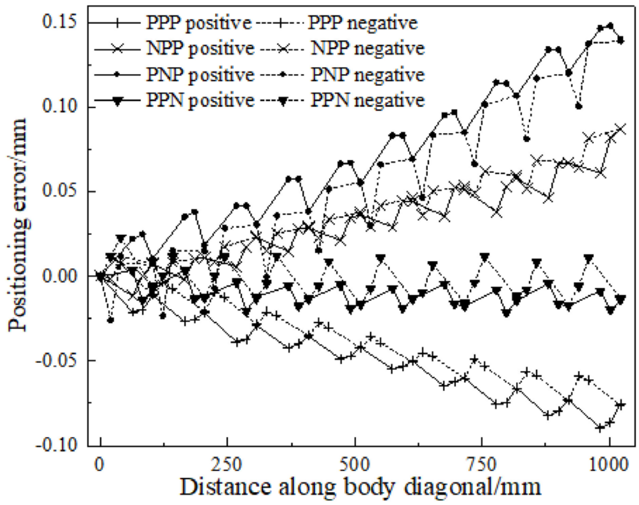


**Figure 7.** The measuring experiment results along four body diagonals.

The volumetric errors of other kinds of three-axis machine tools can be modeled similarly by HTM. According to equation (1), not all 21 geometric errors but only 18 of them can bring about the volumetric errors of vertical machining centers. All the positioning errors, the straightness errors, the squareness errors and some of the angular errors can bring about the volumetric errors. The remaining angular errors can cause no position errors but orientation errors of the tool tip relative to the workpiece, namely they have no influence upon the volumetric errors.

## 3. Measurement and identification of geometric errors

As derived in section 2, not all, but the particular 18 geometric errors, can bring about the volumetric errors of vertical

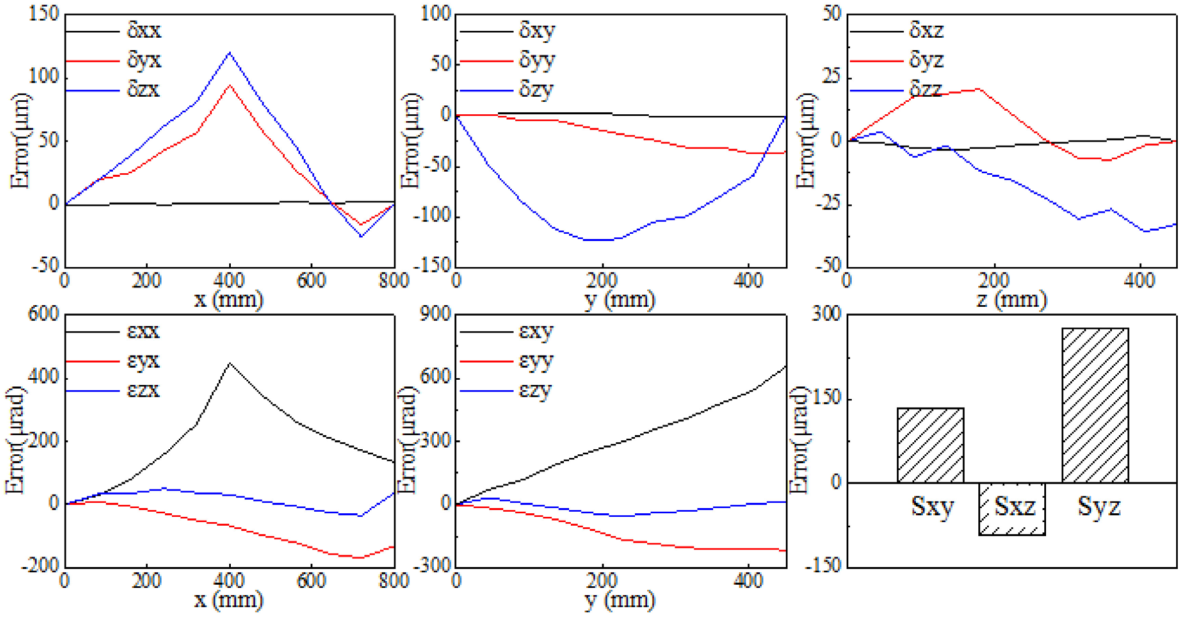


**Figure 8.** Decoupling results of the validation experiment of laser bidirectional sequential step diagonal measurement.

machining centers. The question of how to measure these errors effectively and efficiently is a critical problem for error compensation applications. ISO230-6 [12] introduced the body diagonal displacement tests. It is effective and efficient to evaluate the volumetric performance of machine tools. But the geometric errors cannot be decoupled and distinguished for the data deficiency. As introduced in section 1, the angular errors are considered ignorable in previous research, which results in inaccuracy. How to take the angular errors, which can bring about the volumetric errors, into consideration is the following question. This paper develops the laser sequential step diagonal measurement by proposing a laser bidirectional sequential step diagonal measurement, as follows, to distinguish all additional angular errors which can result in volumetric errors.

### *3.1. Measurement of geometric errors*

Four body diagonals exist in the cuboidal workspace of the typical three-axis machine tools, shown in figure 3. Taking the direction into consideration, the four are named *PPP* (*NNN*), *NPP* (*PNN*), *PNP* (*NPN*), *PPN* (*NNP*). The letters mean the direction of the motion of the *X*-axis, *Y*-axis, and *Z*-axis, successively. *P* denotes positive, and *N* denotes negative. Each diagonal has two names, as shown in parentheses, because of the existence of the backward direction.

The measuring tracks of the laser bidirectional sequential step diagonal measuring method are shown in figure 4. The laser starts from the laser head with the optical adapter and the diagonal steering mirror, aiming at the flat mirror fixed on the spindle. The sketch of setups is shown in figure 4(a). Taking the *PPP* body diagonal (shown in figure 4(a)) as an example, the laser bidirectional step diagonal measuring procedure is as below.

First, for the forward path:

(1) An increment $X_\Delta$ in the *X* direction, then pause and collect data;
(2) An increment $Y_\Delta$ in the *Y* direction, then pause and collect data;
(3) An increment $Z_\Delta$ in the *Z* direction, then pause and collect data;
(4) Repeat (1) to (3) *N* times to the end of the forward path; Then, for the backward path:
(5) An equidistant decrement $-X_\Delta$ in the *X* direction, then pause and collect data;
(6) An equidistant decrement $-Y_\Delta$ in the *Y* direction, then pause and collect data;
(7) An equidistant decrement $-Z_\Delta$ in the *Z* direction, then pause and collect data;
(8) Repeat (5)–(7) *N* times to the end of the backward path.

The end of the backward path theoretically is equal to the start of the forward path. And the measuring tracks of the *NPP*, *PNP* and *PPN* body diagonal is shown in figures 4(b)–(d) successively.

### *3.2. Decoupling method of geometric errors*

The incremental constant $X_\Delta$, $Y_\Delta$ and $Z_\Delta$ mentioned in section 3.1 can be expressed as

$$\begin{cases} X_\Delta = \frac{X_0}{N} \\ Y_\Delta = \frac{Y_0}{N} \\ Z_\Delta = \frac{Z_0}{N} \end{cases} \tag{2}$$

where $X_0$, $Y_0$, $Z_0$, and $N$ donate the max measuring length in the *X*, *Y*, *Z* direction and the repeat times for each path successively. The ideal length of body diagonals in the cuboidal workspace can be derived as

$$R_0 = \sqrt{X_0^2 + Y_0^2 + Z_0^2}. \tag{3}$$

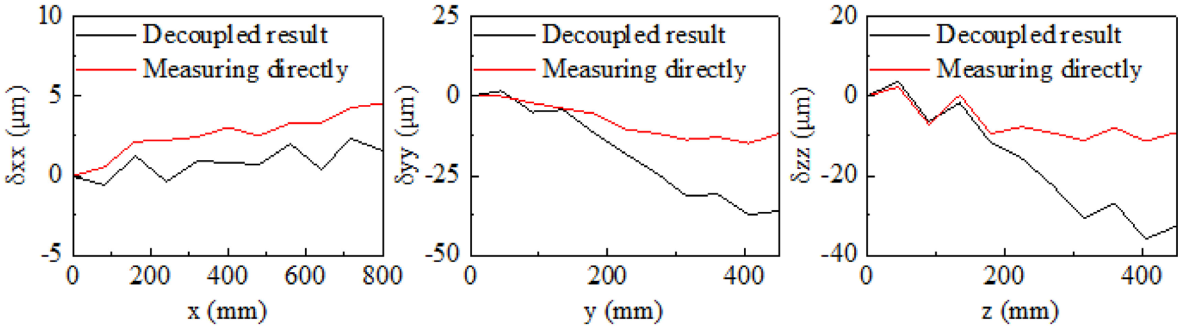


**Figure 9.** Three compound error items measured by two different methods.

Positioning errors along each body diagonal are the projections of the volumetric errors on each diagonal direction, which can be expressed as

$$\begin{cases} \Delta R_{PPP}(x,y,z) = \frac{X_0}{R_0}\Delta x(x,y,z) + \frac{Y_0}{R_0}\Delta y(x,y,z) + \frac{Z_0}{R_0}\Delta z(x,y,z) \\ \Delta R_{NPP}(x,y,z) = -\frac{X_0}{R_0}\Delta x(x,y,z) + \frac{Y_0}{R_0}\Delta y(x,y,z) + \frac{Z_0}{R_0}\Delta z(x,y,z) \\ \Delta R_{PNP}(x,y,z) = \frac{X_0}{R_0}\Delta x(x,y,z) - \frac{Y_0}{R_0}\Delta y(x,y,z) + \frac{Z_0}{R_0}\Delta z(x,y,z) \\ \Delta R_{PPN}(x,y,z) = \frac{X_0}{R_0}\Delta x(x,y,z) + \frac{Y_0}{R_0}\Delta y(x,y,z) - \frac{Z_0}{R_0}\Delta z(x,y,z) \end{cases}. \tag{4}$$

While measuring along each body diagonal, one point is regard as the start point with no error. For the convenience of the measurement, $(0,0,0)$, $(X_0,0,0)$, $(0,Y_0,0)$, and $(0,0,Z_0)$ are chosen as the start points of *PPP*, *NPP*, *PNP*, and *PPN* body diagonals, correspondingly. Consequently, with backlash already compensated, the errors measured along each body diagonal can be derived as

$$\begin{cases} \Delta\tilde{R}_{PPP}(x,y,z) = \Delta R_{PPP}(x,y,z) - \Delta R_{PPP}(0,0,0) \\ \Delta\tilde{R}_{NPP}(x,y,z) = \Delta R_{NPP}(x,y,z) - \Delta R_{NPP}(X_0,0,0) \\ \Delta\tilde{R}_{PNP}(x,y,z) = \Delta R_{PNP}(x,y,z) - \Delta R_{PNP}(0,Y_0,0) \\ \Delta\tilde{R}_{PPN}(x,y,z) = \Delta R_{PPN}(x,y,z) - \Delta R_{PPN}(0,0,Z_0) \end{cases}. \tag{5}$$

Substituting equations (1) and (4) into (5), the geometric errors can be transformed as

$$\mathbf{Tn} = \tilde{\mathbf{T}}n - \mathbf{P}_0 = \mathbf{b} \tag{6}$$

where

$$\tilde{\mathbf{T}} = \frac{1}{R_0}\begin{bmatrix} -X_0 & -X_0 & X_0 & -Y_0 & -Y_0 & Y_0 & -Z_0 & -Z_0 & Z_0 & zY_0+yZ_0 & zY_0+yZ_0 & zX_0+xZ_0 & zX_0 & -yX_0+xY_0 & -yX_0 & yX_0 & -zX_0 & -zY_0 \\ X_0 & X_0 & -X_0 & -Y_0 & -Y_0 & Y_0 & -Z_0 & -Z_0 & Z_0 & zY_0+yZ_0 & zY_0+yZ_0 & -zX_0+xZ_0 & -zX_0 & yX_0+xY_0 & yX_0 & -yX_0 & zX_0 & -zY_0 \\ -X_0 & -X_0 & X_0 & Y_0 & Y_0 & -Y_0 & -Z_0 & -Z_0 & Z_0 & -zY_0+yZ_0 & -zY_0+yZ_0 & zX_0+xZ_0 & zX_0 & -yX_0-xY_0 & -yX_0 & yX_0 & -zX_0 & zY_0 \\ -X_0 & -X_0 & X_0 & -Y_0 & -Y_0 & Y_0 & Z_0 & Z_0 & -Z_0 & zY_0-yZ_0 & zY_0-yZ_0 & zX_0-xZ_0 & zX_0 & -yX_0+xY_0 & -yX_0 & yX_0 & -zX_0 & -zY_0 \end{bmatrix} \tag{7}$$

$$\mathbf{n} = \begin{bmatrix} \delta_{xx}(x) & \delta_{xy}(y) & \delta_{xz}(z) & \delta_{yx}(x) & \delta_{yy}(y) & \delta_{yz}(z) & \delta_{zx}(x) & \delta_{zy}(y) & \delta_{zz}(z) & \varepsilon_{xx}(x) & \varepsilon_{xy}(y) & \varepsilon_{yx}(x) & \varepsilon_{yy}(y) & \varepsilon_{zx}(x) & \varepsilon_{zy}(y) & S_{xy} & S_{xz} & S_{yz} \end{bmatrix}^T \tag{8}$$

$$\mathbf{P}_0 = \begin{bmatrix} \Delta R_{PPP}(0,0,0) \\ \Delta R_{NPP}(X_0,0,0) \\ \Delta R_{PNP}(0,Y_0,0) \\ \Delta R_{PPN}(0,0,Z_0) \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} \Delta\tilde{R}_{PPP}(x,y,z) \\ \Delta\tilde{R}_{NPP}(x,y,z) \\ \Delta\tilde{R}_{PNP}(x,y,z) \\ \Delta\tilde{R}_{PPN}(x,y,z) \end{bmatrix}. \tag{9}$$

Matrix $\mathbf{T}$ is not square, $\mathbf{T}^T$ is defined as the transposition matrix of $\mathbf{T}$, which interchanges the row and column index for each element. The minimum-norm-residual solution $\mathbf{n}$ to equation (6) can be simply calculated by

$$\mathbf{n} = \left(\mathbf{T}^T\mathbf{T}\right)^{-1}\mathbf{T}^T\mathbf{b}. \tag{10}$$

Therefore, the 18 geometric errors which can result in volumetric errors of vertical machining centers can be obtained. Hence, the objective of measuring these particular geometric errors effectively and efficiently can be successfully achieved by the laser bidirectional sequential step diagonal measuring method with the above decoupling method.

### 3.3. Uncertainty analysis

When there is laser displacement, the error measured has a random error. Its standard deviation is estimated by the uncertainty analysis for laser interferometer measurement. The influence on the standard deviation of each identified geometric error should be evaluated. Consequently, the uncertainty analysis is conducted by using the Monte Carlo simulation.

The laser interferometer used is the LDDM MCV-5000 (Optodyne Inc). The user guide lists the sources of errors in displacement measurement as: (1) measurement of atmospheric pressure, air temperature and humidity to determine the speed of light, (2) measurement of temperature measurement,

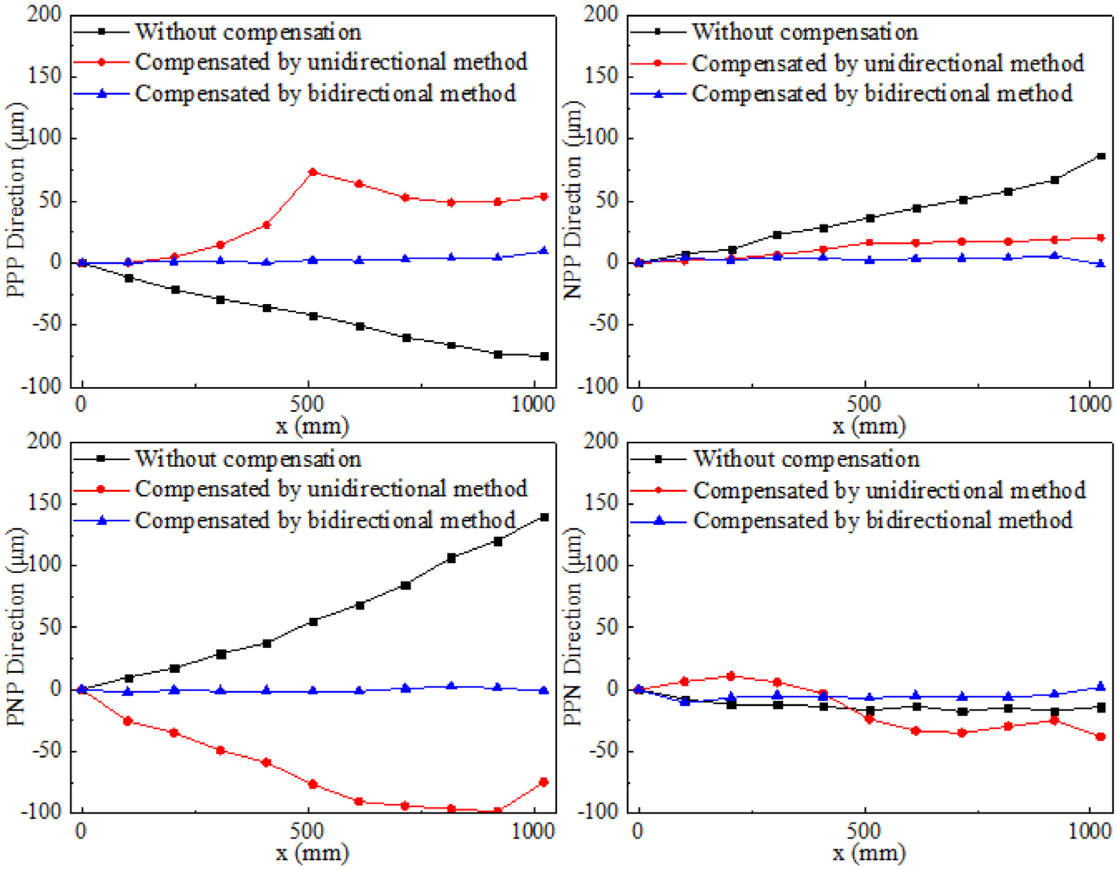


**Figure 10.** The compensation experiment results along the four body diagonals.

(3) misalignment between machine travel and laser measurement axis which causes a cosine error, and (4) mechanical vibration and stress of the retroreflector and the laser head. The user guide also states that there are some additional error sources for the sequential step diagonal measurement: (1) laser beam alignment error when the laser beam direction and the diagonal direction are not exactly parallel, which causes a cosine error, (2) flat-mirror alignment error due to the lateral motion when the flat-mirror target is not exactly perpendicular to the laser beam, and (3) error due to machine angular motion since the intersection of the laser beam and the flat-mirror may not be the center of rotation of the machine angular motion. In the diagonal measurement, an optical adapter (LD-69) is attached to the front of the laser head to gather the laser beam. The light aperture of the optical adapter is less than 1 mm. If the alignment error is not small enough, the laser beam cannot get back to the laser head while moving along the sequential steps. It is announced in the user guide that with automatic temperature and pressure compensation, an accuracy of $\pm 1$ ppm could be achieved.

During the Monte Carlo simulation, ten poses are chosen in each direction. The positions chosen for each axis are 0–800 mm (*X*), 0–450 mm (*Y*), 0–450 mm (*Z*). All 18 geometric errors described in equation (1) have been selected as parameters to be identified with the proposed paths. All other errors can disturb this identification. Except the repeatability of the vertical machining center, all errors are systematic. The current value of each error at a certain axis position is given in figure 5 (black). The accuracy of the measurement is set to $\pm 1$ ppm, as analyzed above. With the Monte Carlo simulation, the accuracy after calibration can be calculated. The mean value of the decoupling result is shown in figure 5 (red), and the standard deviation shown in figure 5 (blue).

The mean value of the decoupling result coincides with the current value. The max deviation of positioning errors and straightness errors is 0.71 μm, and the max deviation of angular errors and squareness errors is 1.77 μrad. The standard deviation is relatively large for three straightness errors (namely $\delta_{yx}$, $\delta_{zx}$, and $\delta_{zy}$). The standard deviation of other geometric errors performs much better. The three straightness errors which have a relatively larger standard deviation also perform better at the beginning. With the position of each axis getting further away, the angular errors which reflect the volumetric errors by multiplying the position of the corresponding axis plays a main role in determining the volumetric errors. Consequently, the three straightness errors with relatively larger standard deviation with the position of each axis getting further is acceptable in error prediction and compensation.

### 3.4. Verification experiment

In order to validate the measurement and the decoupling method proposed in sections 3.1 and 3.2, verification experiments are conducted on a VCM 850E (SMTCL) three-axis vertical machining center with the control system FANUC 0i CNC. The experiments are conducted with the proposed measuring method, shown in figure 4. The laser interferometer used is an LDDM MCV-5000 (Optodyne Inc). The setup of the experiments is shown in figure 6. The detailed values of the experiment parameters are shown in table 2, and the results are shown in figure 7.

Substituting the experimental data from figure 7 into equation (10), the 18 geometric errors can be distinguished simultaneously. The decoupled results are shown in figure 8.

To verify the feasibility of the proposed measuring method, positioning errors of three translational axes are measured directly as ISO 230-2 [27] recommends. The contrast results are shown in figure 9.

The deviation between two different measuring methods may be partly caused by interaction among different kinds of geometric errors. Li [7] has demonstrated that the measuring results are different while the positioning error of one translational axis is measured at different places of another axis, which is induced by the Abbe's error. And the Abbe's error is induced by angular errors with an offset. The proposed decoupling method in this paper has taken the effect of angular errors into account. Moreover, this method may sacrifice little linear positioning accuracy of each axis for higher volumetric accuracy of the whole working volume.

The whole measurement takes less than four hours to obtain all the error items by adopting the proposed laser bidirectional sequential step diagonal measurement. It may take several days by measuring them separately in a traditional way. Moreover, all the geometric errors, which can bring about the volumetric errors, can be identified by the proposed measuring method. In contrast, only 12 geometric errors can be identified by traditional laser sequential step diagonal measurement. Therefore, compared with traditional laser sequential step diagonal measurement, the proposed bidirectional method is more efficient and saves time, with more geometric errors identified, namely the extra angular errors.

## 4. Compensation of volumetric errors

The compensation experiments are conducted on the aforementioned VCM 850E (SMTCL) three-axis vertical machining center with the control system FANUC 0i CNC. The setup of the experiments is shown in figure 6, using LDDM MCV-5000 (Optodyne Inc.) with the same mirror suites as mentioned above.

Firstly, a group of laser diagonal measuring tests without error compensation are conducted along the four body diagonals, as shown in figure 3, as ISO 230-6 [12] recommends. The experiment results of the original errors without compensation are shown in figure 10. Then, another group of laser diagonal measuring tests with error compensation are conducted similarly. The decoupling results of the experiment in section 4 are used in this section. The error compensation system connects the computerized numerical control system through the ethernet. The mechanical coordinates are read out of the PLC module through the ethernet, the error compensation system calculates the error compensation values by substituting the corresponding values of the identified error items shown in figure 8 into equation (1), and then the compensation values are written into the PLC module. The experiment results after compensation are shown in figure 10 as well.

As ISO 230-6 [12] recommends, the diagonal systematic deviation of positioning, which indicates the maximum bidirectional systematic deviation of the positioning of the four body diagonals, is used to evaluate the performance of the vertical machining centers. Without compensation, the maximum bidirectional systematic deviations of the positioning of *PPP*, *NPP*, *PNP,* and *PPN* are 75.12 μm, 86.84 μm, 139.91 μm and 17.62 μm, respectively. Therefore, the diagonal systematic deviation of positioning without compensation is 139.91 μm. After compensation by the proposed method, the maximum bidirectional systematic deviations of the positioning of *PPP*, *NPP*, *PNP,* and *PPN* are 9.46 μm, 7.05 μm, 4.92 μm and 12.90 μm, respectively. Hence, the diagonal systematic deviation of positioning with compensation is 12.90 μm. The diagonal systematic deviation of positioning is reduced by 90%, from 139.91 μm to 12.90 μm. The volumetric accuracy of this vertical machining center is greatly improved.

## 5. Conclusion

It is critical that volumetric errors are measured efficiently. This paper proposes the laser bidirectional sequential step diagonal measurement and leads to the following conclusions.

(1) With the volumetric errors modeled based on the HTM method, it is proved that only part of the geometric errors can bring about volumetric errors.
(2) While measuring, disparate measuring tracks are specially arranged for forward and backward paths, and the conforming error decoupling method is demonstrated. Based on the measuring and decoupling method, the 14 compound error items, reflecting all the geometric errors which can result in volumetric errors, can be distinguished simultaneously.
(3) A validation experiment is conducted using the proposed measuring method and 18 geometric errors are obtained using the decoupling method. Decoupled geometric errors are consistent with those from direct measurements by the laser interferometer, which certified the proposed strategies as feasible. In comparison with the traditional laser sequential step diagonal measuring method, the proposed bidirectional method has a higher efficiency and more geometric errors (angular errors) are distinguished by measuring the same four body diagonals, which can be used to predict and compensate the volumetric errors.
(4) The compensation based on the laser bidirectional step diagonal measurement is demonstrated on a three-axis vertical machining center, and the diagonal systematic deviation of positioning is reduced by 90%, from 139.91 μm to 12.90 μm. The accuracy of the vertical machining centers is greatly improved.

The proposed method can measure the volumetric error effectively and efficiently and has a practical application in error compensation. It can be used for all kinds of three-axis machine tools and the three translational axes of five-axis machine tools as well. Future study may focus on matching the proposed method with other direct measurement, reducing or eliminating the sacrifice of positioning accuracy, and analyzing the influence of installation errors.

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## ORCID iDs

Sitong Xiang https://orcid.org/0000-0002-4434-2948
Zhengchun Du https://orcid.org/0000-0002-3341-7916

## References

[1] Okafor A C and Ertekin Y M 2000 Derivation of machine tool error models and error compensation procedure for three axes vertical machining center using rigid body kinematics *Int. J. Mach. Tools Manuf.* **40** 1199–213
[2] Feng W, Li Z, Gu Q and Yang J 2015 Thermally induced positioning error modelling and compensation based on thermal characteristic analysis *Int. J. Mach. Tools Manuf.* **93** 26–36
[3] Xiang S and Altintas Y 2016 Modeling and compensation of volumetric errors for five-axis machine tools *Int. J. Mach. Tools Manuf.* **101** 65–78
[4] Yang J and Altintas Y 2013 Generalized kinematics of five-axis serial machines with non-singular tool path generation *Int. J. Mach. Tools Manuf.* **75** 119–32
[5] Fu G, Fu J, Xu Y and Chen Z 2014 Product of exponential model for geometric error integration of multi-axis machine tools *Int. J. Adv. Manuf. Technol.* **71** 1653–67
[6] Tian W, Gao W, Zhang D and Huang T 2014 A general approach for error modeling of machine tools *Int. J. Mach. Tools Manuf.* **79** 17–23
[7] Li H, Xiang S, Deng M, Zhu M, Du Z and Yang J 2018 Measuring and modeling of volumetric errors for vertical machining centers based on bi-directional laser sequential step diagonal measurement *ASME 2018 13th Int. Manufacturing Science and Engineering Conf.* (https://doi.org/10.1115/MSEC2018-6579)
[8] Yang J, Yuan J and Ni J 1999 Thermal error mode analysis and robust modeling for error compensation on a CNC turning center *Int. J. Mach. Tools Manuf.* **39** 1367–81
[9] Schwenke H, Knapp W, Haitjema H, Weckenmann A, Schmitt R and Delbressine F 2008 Geometric error measurement and compensation of machines—an update *CIRP Ann.—Manuf. Technol.* **57** 660–75
[10] Ibaraki S and Knapp W 2013 Indirect measurement of volumetric accuracy for three-axis and five-axis machine tools: a review *Int. J. Autom. Technol.* **6** 110–24
[11] ISO 230-4 2005 *Test Code for Machine Tools Part 4: Circular Tests for Numerically Controlled Machine Tools* (International Organization for Standardization)
[12] ISO 230-6 2002 *Test Code for Machine Tools Part 6: Determination of Positioning Accuracy on Body and Face Diagonals (Diagonal Displacement Tests)* (International Organization for Standardization)
[13] Wang Charles 2000 Laser vector measurement technique for the determination and compensation of volumetric positioning errors. Part I: basic theory *Rev. Sci. Instrum.* **71** 3933
[14] Cauchick-Miguel P, Kinga T and Davis J 1996 CMM verification: a survey *Measurement* **17** 1–16
[15] Bernhard B 2007 Improving geometric calibration methods for multi-axis machining centers by examining error interdependencies effects *PhD Thesis* Institute for Machine Tools and Manufacturing (IWF), ETH Zurich (https://doi.org/10.3929/ethz-a-005424013)
[16] Saiki T 2008 Development of measurement for motion accuracy of 5 axis NC machine tool *Int. J. Autom. Technol.* **2** 111–8
[17] Aguado S, Samper D, Santolaria J and Aguilar J J 2012 Identification strategy of error parameter in volumetric error compensation of machine tool based on laser tracker measurements *Int. J. Mach. Tools Manuf.* **53** 160–9
[18] Ibaraki S, Sato G and Takeuchi K 2014 Open-loop tracking interferometer for machine tool volumetric error measurement—two-dimensional case *Precision Eng.* **38** 666–72
[19] Chen G, Yuan J and Ni J 2001 A displacement measurement approach for machine geometric error assessment *Int. J. Mach. Tools Manuf.* **41** 149–61
[20] Chen J S, Kou T W and Chiou S H 1999 Geometric error calibration of multi-axis machines using an auto-alignment laser interferometer *Precision Eng.* **23** 243–52
[21] Chapman M A V 2003 Limitations of laser diagonal measurements *Precision Eng.* **27** 401–6
[22] Svoboda O 2006 Testing the diagonal measuring technique *Precision Eng.* **30** 132–44
[23] Ibaraki S, Hata T and Matsubara A 2009 A new formulation of laser step-diagonal measurement—two-dimensional case *Precision Eng.* **33** 56–64
[24] Ibaraki S and Hata T 2010 A new formulation of laser step diagonal measurement—three-dimensional case *Precision Eng.* **34** 516–25
[25] Bui C B, Hwang J, Lee C H and Park C H 2012 Three-face step-diagonal measurement method for the estimation of volumetric positioning errors in a 3D workspace *Int. J. Mach. Tools Manuf.* **60** 40–3
[26] Soons J A 2005 Analysis of the step-diagonal test *Laser Metrol. Mach. Perform.* **7** 126–37 (https://www.nist.gov/publications/analysis-step-diagonal-test)
[27] ISO 230-2 2014 *Test Code for Machine Tools Part 2: Determination of Accuracy and Repeatability of Positioning of Numerically Controlled Axes* (International Organization for Standardization)