Post-cracking Poisson Ratio of Concrete in Steel-Concrete-Steel Panels Subjected to Biaxial Tension Compression

Manish Prasad[1] and Song XiaoBing[2,3]

1 Master's student, Shanghai Jiao Tong University, China
2 Asst. Professor, Shanghai Jiao Tong University, China
e-mail: xbsong@sjtu.edu.cn

Abstract. Consideration of post-cracking Poisson ratio of concrete is found significant in predicting the behavior of Steel-Concrete-Steel (SCS) panels under biaxial tension compression. Concrete reinforced with face steel plates was found to be under confinement which deviates the established nature of the post-cracking Poisson ratio. In order to account for such confinement, an extension to previously proposed expression for the Poisson effect in reinforced concrete (RC) is presented in this paper to be used for the analysis of SCS in which the reinforcing elements, steel plates, provide confinement in the principle tensile direction. A tri-linear model is proposed for the Poisson ratio and then compared with the experimental results.

1. Introduction

An SCS panel is a composite in which core concrete is reinforced with two face steel plates connected to each other with ties and to the core concrete with shear studs at regular intervals. SCS is being used for different purposes such as nuclear power plants, off-shore structures, high rise buildings and others because of its improved structural capacity and modular usability in construction.

An accurate and efficient analysis of SCS panels demands for a precise modeling of the steel plates and the core concrete. Steel plate models have been almost perfected in past researches but concrete, however, due to its discontinuity after cracking, lacks appropriate 2D models to be used for panels. Different orthotropic models have been presented in past for cracked concrete but most of them were formulate for RC panels which involve rebar reinforcement. Vecchio's [1] model for concrete considered the strength degradation after cracking, strength enhancement due to biaxial or triaxial compression, and concrete lateral expansion. Soften Membrane Model (SMM) [2] by Zhu and Hsu incorporates softening and lateral expansion of concrete.

Even though these models incorporate concrete expansion and confinement effects, an explicit expression for position ratio has not been put forward. Average Poisson ratio for cracked concrete, as evidenced by Zhu and Hsu [3], can exceed the maximum Poisson   ratio of 0.5 for continuous material and can be as large as 1.9. They gave an expression for Zhu/Hsu ratio (Eq.(2)), for average Poisson ratio in cracked concrete as a function of principle tensile strain ε_1 and its behavior is dependent on the yielding of the steel bar. Poisson ratio being a property of concrete should not be dependent of any other material but itself. Also, Zhu/Hsu ratio starts increasing from 0.2 even before cracking, which might be acceptable in RC but in SCS a constant value of Poisson ration 0.2 before cracking should be considered.

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[3] Author to whom any correspondence should be addressed.

Vecchio's [1] model does account for both Poisson ratio and the confinement but the value of Poisson ratio is limited to 0.5 as given by Eq.(3).

$$v_{12} = \begin{cases} 0.2 + 850\varepsilon_1, & \varepsilon_1 \leq \varepsilon_y \\ 1.9, & \varepsilon_1 > \varepsilon_y \end{cases} \tag{1}$$

$$v_{12} = \begin{cases} 0.2 + 850\varepsilon_1, & \varepsilon_1 \leq \varepsilon_y \\ 1.9, & \varepsilon_1 > \varepsilon_y \end{cases} \tag{2}$$

$$v_{12} = \begin{cases} v_o, & 0 > \varepsilon_{c2} > \dfrac{\varepsilon_p}{2} \\ v_o \left[1 + 1.5 \left(\dfrac{2\varepsilon_{c2}}{\varepsilon_p} - 1 \right)^2 \right] \not> 0.5, & \dfrac{\varepsilon_p}{2} > \varepsilon_{c2} \end{cases} \tag{3}$$

Here, v_{12} is the average Poisson ratio caused by the expansion of concrete in direction-1 due to the stress in direction-2, v_o is the initial value of the Poisson ratio, ε_1 is the strain in principal tensile direction, ε_y is the yielding strain of the rebar reinforcement, ε_{c2} is the strain in principal compressive direction and ε_p is the peak compressive strain.

In this paper, the expression for Zhu/Hsu ratio will be modified to incorporate the observed confinement in concrete from the Huang's et al. [4] experimental program of SCS panels. Thus, modified expression for the average Poisson ratio will be used to analyze Huang's et al. [4] panels and compared with the experimental results.

2. Experimental Work

Table 1. Test Specimen Properties and Principle Variables [4].

Specimen	Concrete	Steel plate		Steel plate ratio (ρt)	Bs/ts	Load increment ratio ($\Delta C/\Delta T$)	Load modes
	fcu (MPa)	ts (mm)	fy (MPa)				
S3-10	42	2.95	310	1.14%	25.4	-1/0	U.C.
S6-10	41	6	275	2.31%	12.5	-1/0	
S3-51	42	2.95	310	1.14%	25.4	-5.2/1	P.S.
S5-41	41	4.75	300	1.83%	15.8	-3.7/1	
S6-41	41	6	275	2.31%	12.5	-3.5/1	
S6-81	41	6	275	2.31%	12.5	-8/1	
S5-11	41	4.75	300	1.83%	15.8	-1/1	S- P.S.-C.
S6-21	41	6	275	2.31%	12.5	-1.8/1	
S6-01	41	6	275	2.31%	12.5	0/1	U.T.

U.C.—Uniaxial compression S-P.
S.-C.—Sequential loading.
P.S.—Proportional loading in stepped increment.
U.T. —Uniaxial Tension.

Nine SCS panels with properties as shown in Table 1 were tested with the testing apparatus as shown in Figure 3. To obtain the behavior of panels under different loading conditions, a proportional loading with wide range of compression to tension ratio (C/T) were adopted. Different loading modes are shown

in Figure 2 For uniform transfer of load from the loading device to the specimen, each specimen was designed to have compression and tension load transfer zones as shown in Figure 1.

3. Equilibrium and Compatibility Equations

For SCS panels [4], tensile load was applied in direction-1 and compressive load was applied in direction-2 as shown in the Figure 3 Applied load directions were the principle directions, so the equilibrium equations can be written as Eq.(4) and Eq.(5), where σ_1 , σ_1^c and $\rho\sigma_1^s$ are equivalent

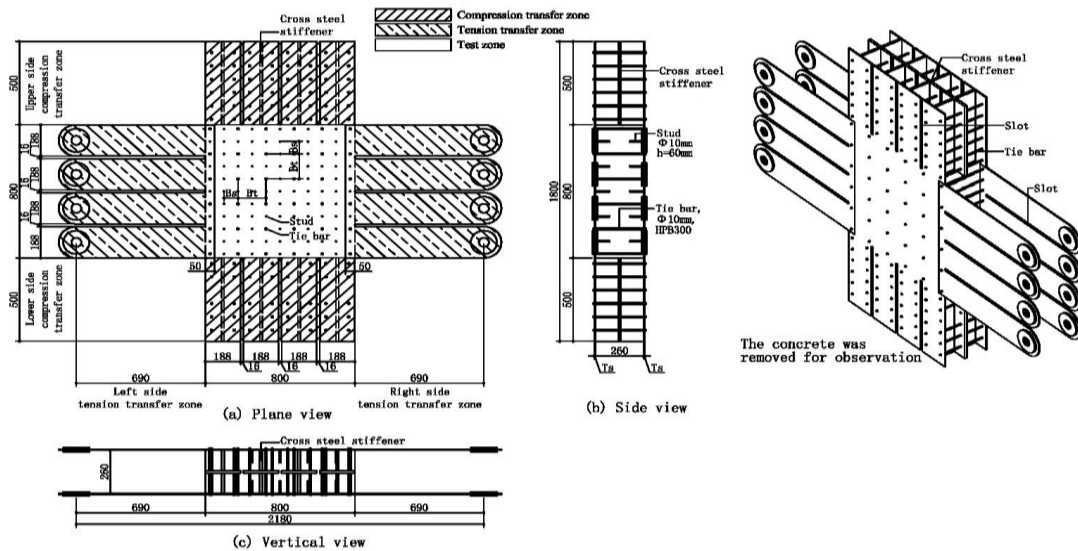**Figure 1.** Test Specimen Properties [4].

stresses in direction-1 on the panel, concrete and steel plate respectively. Similarly, σ_2, σ_2^c and $\rho\sigma_2^s$ are equivalent stresses in direction-2 on the panel, concrete and steel plate respectively. Equivalent stress is the stress on the element equivalent to the stress on concrete element of same thickness. ρ is the reinforcement ratio which is the ratio of steel plates thickness $2t_s$ to the concrete thickness t_c. Since the applied load directions coincides with the principal directions, compatibility equations can be written as Eq.(6) and Eq.(7), where ε_1 , ε_1^c and ε_1^s are the average strains in direction-1 of the panel, concrete and steel plate respectively. ε_2 , ε_2^c and ε_2^s are the average strains in direction-2 of the panel, concrete and steel plate respectively. Average strain is the total strain summed over several cracks.

$$\sigma_1 = \sigma_1^c + \rho\sigma_1^s \tag{4}$$

$$\sigma_2 = \sigma_2^c + \rho\sigma_2^s \tag{5}$$

$$\varepsilon_1 = \varepsilon_1^c = \varepsilon_1^s \tag{6}$$

$$\varepsilon_2 = \varepsilon_2^c = \varepsilon_2^s \tag{7}$$

4. Stress-strain Relationship of Concrete

Constitutive law of concrete in compression as proposed by Belarbi and Hsu [5] is adopted and modified based on the Huang's et al. [4] experimental data. From the analysis of experimental data, it was observed that a confining phenomenon exists along with the softening phenomenon in concrete due to the difference in Poisson ratio of steel plate and that of concrete. A modified stress strain relationship of

concrete is proposed as Eq(8), where ζ is the combined stress softening and confining parameter and ξ is the combined strain softening and confining parameter.
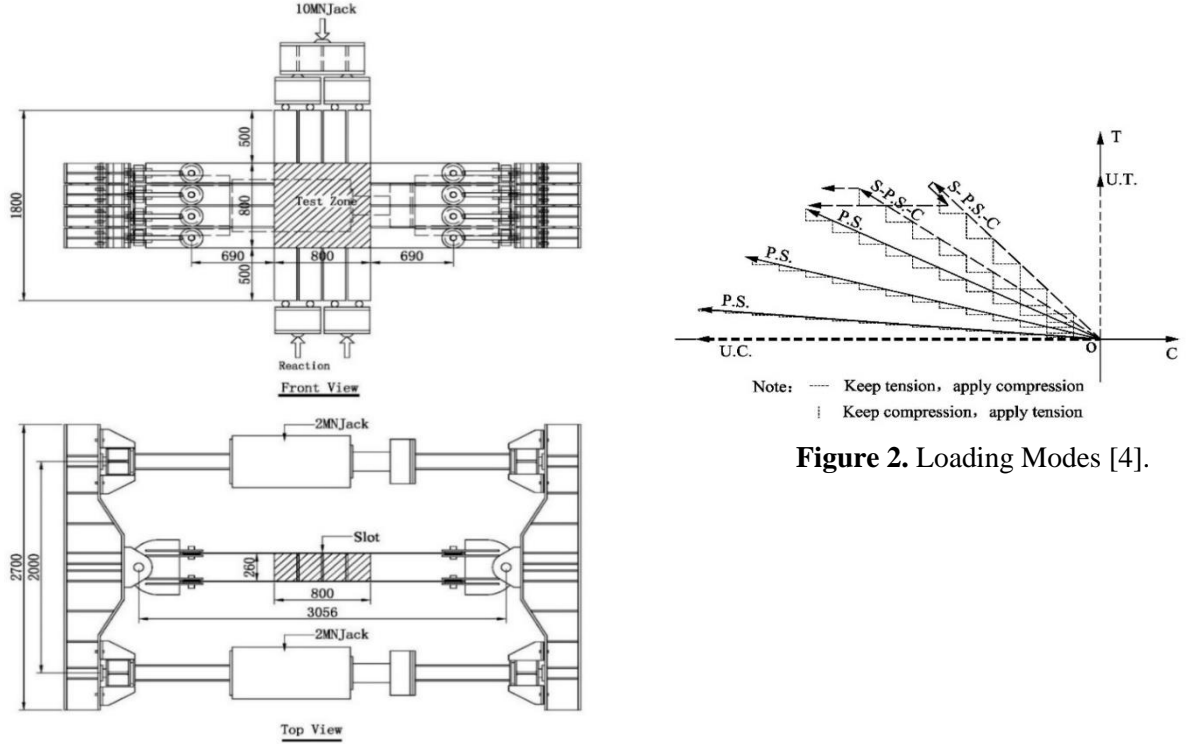

Figure 2. Loading Modes [4].

Figure 3. Load Setup and Testing Apparatus [4].

$$\sigma_2^c = \zeta f_c' \left[2\frac{\bar{\varepsilon}_2^c}{\xi\varepsilon_o} - \left(\frac{\bar{\varepsilon}_2^c}{\xi\varepsilon_o}\right)^2 \right] \tag{8}$$

$$\zeta = \left(1 - 12\frac{\sigma_C^1}{f_C'}\right)\left(\frac{1}{\sqrt{1+200\varepsilon_1}}\right) \tag{9}$$

$$\xi = 2\zeta \tag{10}$$

Here, σ_2^c is compressive stress in principal direction-2, f_c' is compressive strength of the concrete, $\bar{\varepsilon}_2^c$ is the uniaxial compressive strain in principal direction-2, and ε_1 is the tensile strain in principle direction-1. Stress strain relationship of concrete in tension as proposed by Belarbi and Hsu [6] and Pang and Hsu [7] is adopted and modified to incorporate the observed confinement. A modified relationship is presented in Eq.(11).

$$d\sigma_1^c = E_c' d\bar{\varepsilon}_1^c \tag{11}$$

$$\text{where } E_c' = 4f_{cr}\frac{\varepsilon_{cr}^{0.4}}{\varepsilon_1^{1.4}}$$

Here, σ_1^c is tensile stress in principal direction-1, f_{cr} is cracking stress the concrete, $\bar{\varepsilon}_1^c$ is the uniaxial tensile strain in principal direction-1, and ε_{cr} is cracking tensile strain.

5. Stress-strain Relationship of Steel Plate

Before yielding a linear relationship (Eq.(12)) and after yielding Prandtl-Reuss model [8] for plane stress case (Eq.(13)) is adopted.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \end{Bmatrix} = \left\{ \frac{E}{1-v^2} \begin{bmatrix} 1 & v \\ v & 1 \end{bmatrix} \right\} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix} \tag{12}$$

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \end{Bmatrix} = \left\{ \frac{E}{1-v^2} \begin{bmatrix} 1 & v \\ v & 1 \end{bmatrix} - \frac{1}{s} \begin{bmatrix} t_1^2 & t_1 t_2 \\ t_1 t_2 & t_2^2 \end{bmatrix} \right\} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \end{Bmatrix} \tag{13}$$

Where, $t_1 = \frac{E}{1-v^2}(s_1 + vs_2)$, $t_2 = \frac{E}{1-v^2}(s_2 + vs_1)$, $s = s_1 t_1 + s_2 t_2$, v is the Poisson ratio of the steel plate, and s_1, s_2 are the deviatoric stresses.

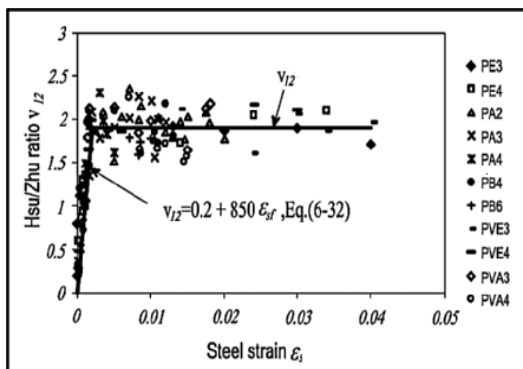## 6. Modification of the Zhu/Hsu ratio

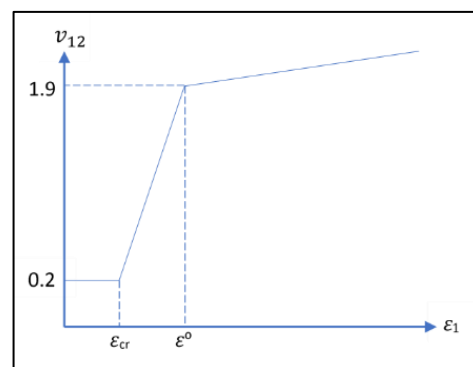**Figure 4.** Zhu/Hsu ratio [3].

Figure 5. Modified Zhu/Hsu ratio.

Poisson ratio of concrete after cracking can be measured as Zhu/Hsu ratio as shown in Figure 4 Eq(2) suggested for Hsu/Hsu ratio was formulated from an extensive test program of 12 full-scale reinforced concrete panels. The expression for Zhu/Hsu ratio was in fact obtained for RC panel but was used for concrete in SMM model because of the fact that measured Zhu/Hsu ratios were not influenced by the reinforcement ratio. It is, however, apparent that if the Zhu/Hsu ratio is measured for plain concrete, it would be influenced by the presence of reinforcement. Moreover, the suggested expression for Zhu/Hsu ratio is dependent on the yielding property of the reinforcement. This conclusion makes it difficult for the suggested expression for Zhu/Hsu ratio to be implemented for concrete when it is reinforced with any other material whose properties are not similar with steel rebar. Therefore, the expression might need some modification when implemented for panels with different kind of reinforcement.

In modeling of the concrete for SCS panel, Poisson effect has a significant influence. Reinforcing steel plate has a Poisson ratio of 0.3 before yielding and about 0.5 after yielding. Concrete has a Poisson ratio of 0.2 before cracking and the average Poisson ratio or the Zhu/Hsu ratio after cracking keeps increasing even beyond 0.5. Due to this difference in the Poisson ratios, the expansion in the principal direction-2 due to the applied stress in principle direction-1 is different for concrete and steel plate. Thus, with compatibility condition imposed, concrete is under tension when Poisson ratio of concrete is smaller than that of steel plate, and is under compression when the Poisson ratio of concrete is larger than that of steel plate.

Behavior of concrete is very sensitive to the Poisson ratio in SCS panel, both before and after cracking. Before cracking, concrete is isotropic and the Poisson ratio should be constant, however, Eq.(2) shows that it begins increasing even before cracking. Even though the change in Poisson ratio is small before cracking, as given by Eq.(2), if it exceeds the value 0.3(Poisson ratio of Steel plate) before yielding, the stress condition of concrete might change as explained in the earlier section. Therefore, it

is essential for the concrete model for SCS to have a Poisson ratio of 0.2 before cracking. Thus, the first line of the trilinear model, as shown in Figure 5, proposed for the modified Zhu/Hsu ratio represents a constant value of 0.2 until the concrete cracks.

After cracking, the Zhu/Hsu ratio increases rapidly from a value of 0.2 to a relatively large value of 1.9. This rapid increase to a value of 1.9 might have caused because of the brittle nature of the concrete and the width of continuously forming cracks in the concrete. Zhu and Hsu [3] proposed a steep slope of 850 for v_{12} versus ε_1 curve. This slope was defined by and the value of v_{12} near yielding strain of the reinforcement as observed from the experimental work [3]. The slope for plain concrete, however, could have been underestimated because of the bond strength provided by the reinforcement which tends to restrict the cracking. So, if it were for plain concrete, a steeper slope could have been more plausible. Based on this assumption and analysis of the Huang's et al. experimental data [4] , a slope of 1889 for v_{12} versus ε_1 curve is suggested as shown in Figure 5 by the second line. This line was assumed to continue till it reaches a value of 1.9 similar as in Zhu/Hsu ratio.

A constant value of 1.9 for the average Poisson ratio was proposed followed by the yielding of the reinforcement. Not only relating the property of the concrete to the property of reinforcement but also a constant value of the average Poisson ratio, when not implemented for RC panels, seems a bit inappropriate. The behavior of the average Poisson ratio reaching a value of 1.9, and after that can be explained without relating it to the yielding of reinforcement. The increasing nature of v_{12} can be related with the formation of new cracks after each loading step. Formation of the cracks and its severity can be measured as a damage parameter ε_1 and when concrete reaches a certain damage level ε^o, the formation of new cracks in SCS panels can be assumed to slow down. Hence, it can be represented with a gentle slope as shown in Figure 5 by the third line instead of a horizontal line as in Zhu/Hsu ratio.
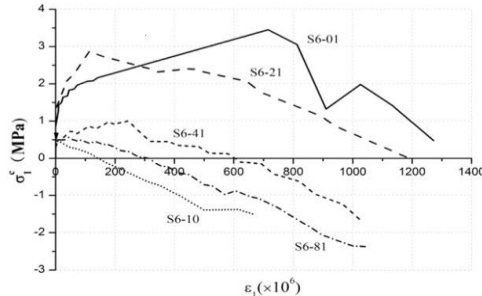

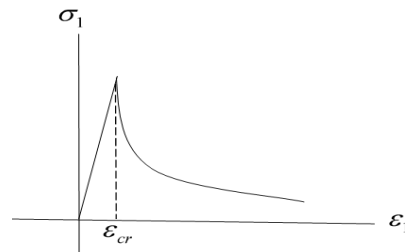**Figure 6.** Averaged concrete tensile stress-strain relationship in SCS (before the yielding of steel plate) [4].

Figure 7. Tensile Stress-Strain Relationship of Concrete [7] [2].

$$v_{12} = \frac{\Delta\varepsilon_1}{\Delta\varepsilon_2} = \frac{\Delta\sigma_1}{\overline{E_1}\Delta\varepsilon_2} \tag{14}$$

An increasing v_{12} after it reaches a value of 1.9 for concrete in SCS, unlike what suggested by Zhu and Hsu, can be further supported by the Eq(14). A relatively constant v_{12} in later stage of the Zhu and Hsu's experiment might have been observed because of the tensile nature of concrete in absence of confinement as shown in the Figure 7, in which $\Delta\sigma_1$ gets smaller with increasing applied load. However, in Huang's et al. experiment, $\Delta\sigma_1$ seems to be relatively constant even near the later stages of loading as shown in the Figure 6. Average Poisson ratio of panels S3 5.2-1 and S4.5 1-1from Huang's et al.

experiment is show in Figure 8. It can be seen from the figures that the average Poisson ratio of the panels kept increasing throughout the loading history. As the Poisson ratio of steel plate does not increase beyond 0.5, the resulting average Poisson ratio of the panel should be because of the concrete property. Therefore, based on these observations and the data of Huang's et al. experiment, a modified equation for Zhu/Hsu ratio is suggest for confined concrete in SCS panels as Eq.(15). Conventional values for v_{21} as in Eq.(16) is used. With v_{12} and v_{21} , uniaxial strains can be calculated using Eq.(17) and Eq.(18).
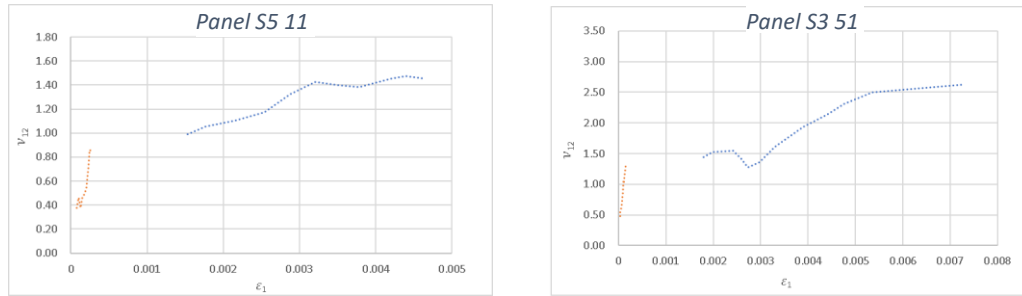

Figure 8. Average Poisson Ratio of the Panels [4].

$$v_{12} = \begin{cases} 0.2 & \varepsilon_1 \le \varepsilon_{cr} \\ 0.2 + 1889\left(\varepsilon_1 - \varepsilon_{cr}\right) & \varepsilon_{cr} < \varepsilon_1 \le \varepsilon^o \\ 1.9 + 80\left(\varepsilon_1 - \varepsilon^o\right) & \varepsilon_1 > \varepsilon^o \end{cases} \quad (15)$$

$$v_{21} = \begin{cases} 0.2, & \varepsilon_1 \le \varepsilon_{cr} \\ 0, & \varepsilon_1 > \varepsilon_{cr} \end{cases} \quad (16)$$

$$\bar{\varepsilon}_1^c = \frac{\varepsilon_1}{1 - v_{12}v_{21}} + \frac{v_{12}\varepsilon_2}{1 - v_{12}v_{21}} \quad (17)$$

$$\bar{\varepsilon}_2^c = \frac{\varepsilon_2}{1 - v_{12}v_{21}} + \frac{v_{21}\varepsilon_1}{1 - v_{12}v_{21}} \quad (18)$$

7. Application of the Modified Zhu/Hsu ratio and Results

The modified Eq.(15) is implemented along with the equilibrium, compatibility and constitutive equations of concrete and steel plate to calculate the response of SCS panels of Huang's et al. [4] experiment. An iterative procedure is adopted to solve for the unknowns. Load deformation curve obtain was plotted against the experimental results in Figure 9 which shows a very good agreement between two data.

8. Conclusion

Post cracking Poisson effect in concrete suggested as Zhu/Hsu ratio by Zhu and Hsu [3] was modified based on the experimental data of nine SCS panels to be used for the concrete model. The use of thus modified Zhu/Hsu ratio in the analysis of SCS panels not only increased the result accuracy but also increased the calculation efficiency. Incorporation of modified Zhu/Hsu ratio also led to a better understanding of the confinement developed in the concrete in SCS panels. Moreover, this study helped us to understand that the relationship developed for the Poisson effect from the test program of RC other than steel rebars.
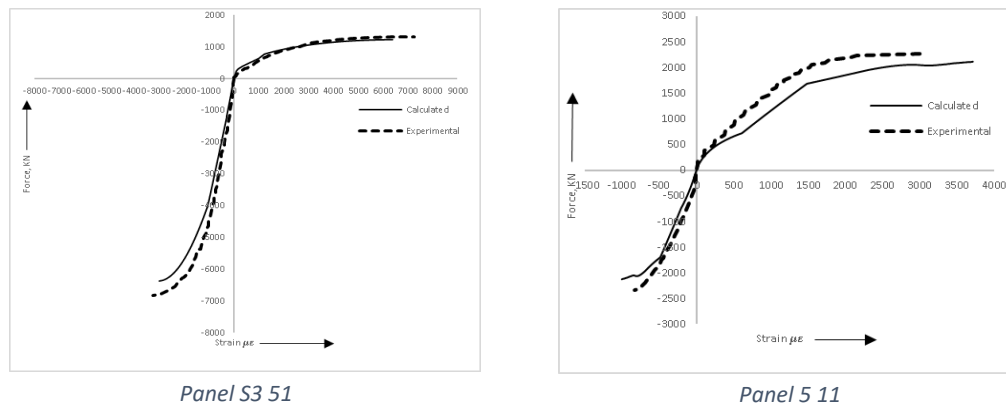
Panel S3 51   *Panel 5 11*

Figure 9. Calculated Vs. Experimental results with modified Zhu/Hsu ratio.

Acknowledgements

References

[1]   F. Vecchio, "Finite Element Modeling of Concrete Expansion and Confinement," *Journal of Structural Engineering,* pp. 2390-2406, 1992.

[2]   T. Zhu and R. Hsu, "Softened Membrane Model for Reinforced Concrete Elements in Shear," *ACI Structural JournalACI Structural Journal,* vol. 99, no. 4, pp. 460-496, 2002.

[3]   R. Zhu and T. Hsu, "Poisson Effect in Reinforced Concrete Membrane Elements," *ACI Structural Journal,* pp. 631-640, 2002.

[4]   C. Huang, S. J. Chen, Y. Leng, C. Qian and X. B. Song, "Experimental research on steel-concrete-steel sandwich panels," *Journal of Constructional Steel Research,* p. In progress, 2019.

[5]   T. Hsu and A. Belarbi, "Constitutive Laws of Softened Concrete in Biaxial Tension-Compression," *American Concrete Institute- Structural Journal,* pp. 562-572, 1995.

[6]   A. Belarbi and T. Hsu, "Conctitutive laws of Concrete in Tension and Reinforcing Bars Stiffened by Concrete," *ACI Srtuctural Journal,* pp. 465-474, 1994.

[7]   X. Pang and T. Hsu, "Behaviour of Reinforce Concrete Menbrane Elements in Shear," *ACI Structural Journal,* pp. 665-679, 1995.

[8]   W. Chen, Constitutive Equations for Engineering Materials, vol. 1, New York: John Wiley & Sons, Inc, 1994.