

Effect of End-to-End Connectors in Elastic GFRP Gridshell

Kedar Baral², Bin Cheng^{1,2*}, Sheng Xiang² and Suraksha Sharma²

¹State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, Shanghai 200240, China.

²Department of Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, China.

*Corresponding author's e-mail: cheng_bin@sjtu.edu.cn

Abstract. The presence of end-to-end connectors for lengthening the glass fiber reinforced polymer (GFRP) pipes in elastic gridshell causes imperfections in the structure. Such geometrical imperfections alter the structural properties of members and affect the overall performance of the structure. Hence, for a comprehensive analysis of such structures, the aforementioned imperfections need to be acknowledged. In this study, an equivalent rigid part (ERP) for the connector was proposed for taking into account the effect of connectors in the structural analysis. The applicability of ERP for predicting the deformation of structural members was first verified by bending beam models. A typical GFRP elastic gridshell was further utilized for investigating the effects of connectors (with two different arrangement) in the shape and stress of the structure. Results show that proposed ERP is practicable for simplifying the end-to-end connectors in the structural analysis model. Significant differences in results were found between the analysis with and without the consideration of end-to-end connectors, and therefore the effect of connectors should be considered during structural analysis of elastic gridshell.

1. Introduction

Elastic gridshells are long-span shell-like[1] structures which derives its stiffness from its double curvature[2]. They have simple construction techniques and employ the principle of active bending[3]. To achieve the target shape, regular flat quadrangular grids which are arranged in superimposed layers[4] are elastically deformed.

Elastic gridshell came into existence when Frei Otto, Edmund Happold and W. I. Liddell achieved famous Mannheim Multihalle in 1975[5]. This structure is an inspiration in both form and aesthetic to modern gridshell like Weald and Downland gridshell. Traditionally wooden laths were used in elastic gridshells because of their high elastic limit strain[6]. However, in recent decades, the attention of the scientific community has shifted towards the use of new innovative materials that possess better mechanical properties than wood, have simpler connection systems, and are economical. GFRP, as suggested by Ashby, has higher elastic limit strain and Young's modulus compared to wood[6-8]. Moreover, industrial production of GFRP ensures more reliability on mechanical properties[9].

Realizing the benefits of GFRP, Navier laboratory constructed a GFRP elastic gridshell in 2011 for Solidays[7] and Ephemeral Cathedral of Créteil in 2013[10]. In both gridshells, nodes are formed with the help of commercially available swivel couplers applied at the member intersections[7, 10]. On the other hand, gridshell members are lengthened by using proper end-to-end connection system. The connection



system must transfer normal stress effectively and also have comparable bending stiffness as that of the connecting members to confirm the continuity of the global shape[11]. Thus, an specific end-to-end connector should be employed in the gridshells for member lengthening[12]. In the Solidays gridshell, two swivel couplers are used as end-to-end connectors to connect members and observations showed that such treatment does not provide sufficient stiffness[7]. In Ephemeral Cathedral gridshell, a specially designed end-to-end connection system consisting of two steel tube and a linked threaded rod is adopted[13].

The aforementioned connectors provide geometrical imperfection in structural members. They might affect the global shape and have negative impact on the structure's performance. Therefore, such imperfections need to be integrated for predicting the response of the gridshell. Even though the advancement in finite element analysis and numerical modelling have aided comprehensive structural analysis of structures, integrating such geometric imperfections are still very complicated. In addition, limited research has been conducted focusing on the effect of connectors in gridshell. Therefore, in this study, efforts have been made to understand the impact of the presence of end-to-end connectors in gridshell by evaluating the equivalent rigid part (ERP) for the connector. The ERP is then included as a connector in the finite element model of the gridshell for analysis. Moreover, connectors are placed in two different ways in the gridshell to investigate the effect of the arrangement of connectors in the structure.

In this paper, finite element method is described in Section 2 where ERP is worked out and validated. In addition, a typical gridshell is briefly described and analyzed in this section. Similarly, in Section 3, results and discussions are mentioned, followed by Section 4 where few conclusions are drawn.

2. Finite Element Model

The finite element analysis was performed in two stages. In the first stage, ERP was worked out and validated as mentioned in Section 2.1.2. Moreover, the flexural behavior of the GFRP pipe was analyzed for three cases: with connector, without connector and ERP. To represent connector, respective sectional bending stiffness values were assigned throughout the length of the connector. Similarly, in the second stage, the effect of presence of the connectors in the gridshell was investigated by comparative analysis. The analysis was performed by comparing the gridshell, with and without connectors.

To model the GFRP pipes in ABAQUS, deformable wire feature and thin-walled pipe section were chosen. Moreover, to simulate the swivel connector at grid nodes, a hinge connector in an interaction module was used. Geometric non-linear solver of ABAQUS was employed so that large deformation behaviors during the erection can be accurately analyzed. The element type chosen for the analysis of the structure was B32, which is a Timoshenko beam element that considers all the axial forces, shear forces and bending moments. To incorporate end-to-end connectors in the gridshell, the ERP was used. The ERP derived in Section 2.1.1 was placed in all the locations of the end-to-end connectors in the gridshell for considering the effects of such connectors.

2.1 Equivalent rigid part

2.1.1 Formula derivation. This section aims to calculate the ERP for the connector. The ERP provides a simple technique to integrate the connectors in the gridshell. For simplification, effect of bolts and tube tolerance were not considered.

Figure 1 depicts the symmetric half portion of the member with connector and ERP. Figure 1(a) shows the connection system similar to Ephemeral Cathedral gridshell[11]. The connection system consists of steel sleeve of diameter (d_4) 20 mm and a steel tube having external diameter (d_3) 48 mm with a wall thickness of 3 mm. Similarly, the ERP along with the member is showcased in Figure 1(b). The GFRP pipe has an external diameter (d_2) of 42 mm, and a wall thickness of 3.5 mm.

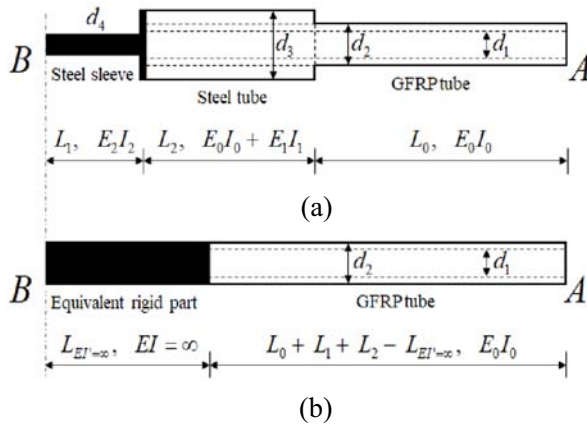


Figure 1. Connector assembly and the corresponding ERP.

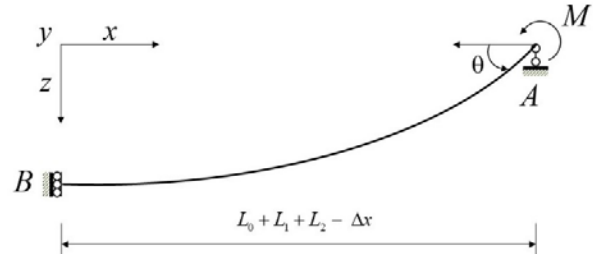


Figure 2. Deflection of member BA under external loading.

Here, EI denotes bending stiffness and L_0 , L_1 , L_2 , and $L_{EI'=\infty}$ are the length of GFRP pipe, steel sleeve, steel tube, and ERP, respectively. Similarly, E_0 and $E_1=E_2$ denotes Young's modulus of GFRP and steel, respectively and ERP has infinite bending stiffness. The second moment of inertia (I_x) of a pipe or tube is given by $\frac{\pi(d_e^4 - d_i^4)}{64}$ while that of rod is given by $\frac{\pi d^4}{64}$ where, d_e is the external diameter, d_i is internal diameter of the tube, and d is diameter of the rod.

Similarly, Figure 2 depicts the bending of a member BA under pure bending, M . End A is roller supported while another end B is collar supported. In order to calculate $L_{EI'=\infty}$, it is assumed that the slope at end A due to pure bending, M , must be equal in both cases. To calculate the slope at the end A, widely used moment area theorem is used. In the figure, Δx is the horizontal translation of the end A.

Equating slope at end A for both cases, Equation (1) was derived, which provides the length of the ERP of the connector.

$$L_{EI'=\infty} = (1 - \mu)L_1 + \left(\frac{1}{1 + \beta} \right) L_2 \quad (1)$$

$$\text{where } \beta = \frac{E_0 I_0}{E_1 I_1} = \frac{E_0 (d_2^4 - d_1^4)}{E_1 (d_3^4 - d_2^4)}, \text{ and } \mu = \frac{E_0}{E_1} \left(\frac{d_2^4 - d_1^4}{d_4^4} \right).$$

2.1.2 Validation of ERP. Figure 3 depicts the bending of a 9 m long GFRP pipe under different circumstances. Figure 3(a) (or Case A) represents a pipe deformed under the influence of gravity load alone, while Figure 3(b) (or Case B) depicts buckling of a GFRP pipe by displacing support A by 4 m. Case B is the typical case of large curvature bending.

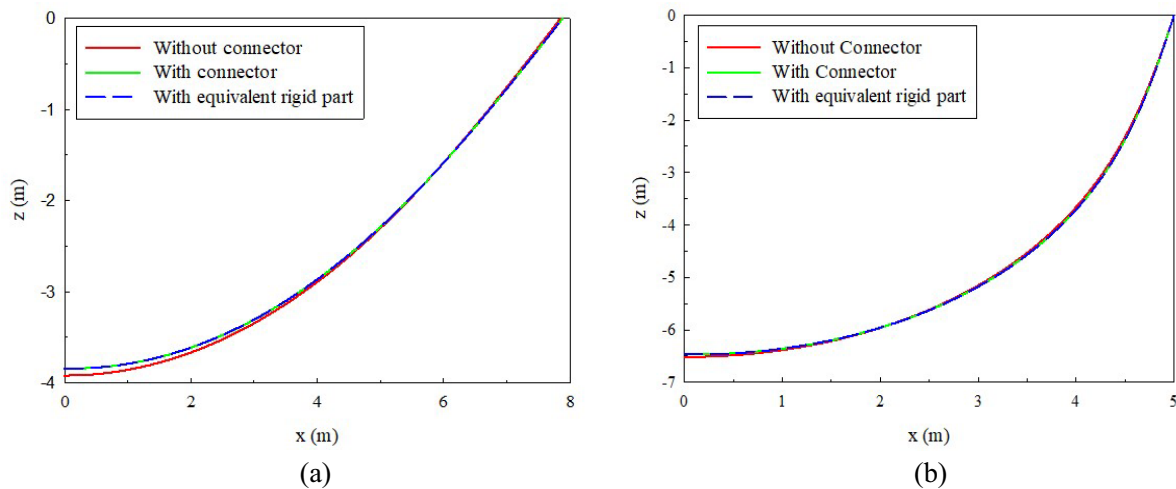


Figure 3. Deflection of gridshell member due to (a) gravity load (b) translation of support A ($\Delta x = 4$ m).

The coinciding graphs (with connector and the ERP) from Figure 3(a) and Figure 3(b) justifies the convenient use of ERP instead of using a connector for finite element analysis. The deflections and stresses developed in pipe sections are recorded in the Table 1. The difference of the deflection at end B (crown) is found to be minute. Also in both load cases, the stresses developed in the gridshell member, with connector and ERP, are almost equivalent. This suggests that the ERP can predict the influence of the connector in the structure with great precision. Furthermore, Figure 3(b) ensures that even at larger curvature, ERP has the capability to measure the influence of connector in a member. Moreover, the effect of connector in the member was also analyzed. From comparative study of Table 1, it can be observed that the presence of connector has minor but noticeable effect on flexural behavior of the member having smaller curvature. Meanwhile, for larger curvature member, the effect is comparatively larger.

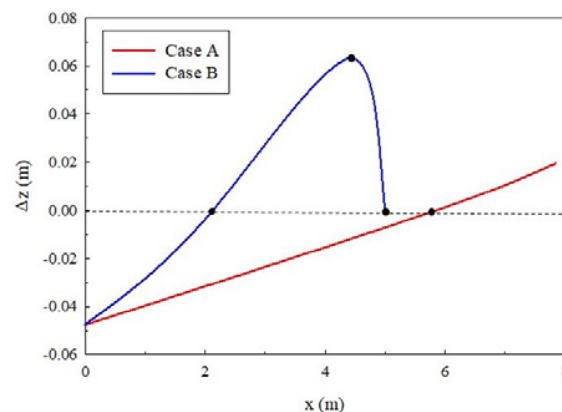


Figure 4. Difference in deflection.

Additionally, it can be noticed from Figure 4 that, for Case A, the presence of the connector has caused a small but noticeable decrease in vertical deflection at crown and a linear decrement away from the crown but it again starts increasing from inflection point highlighted by black dot. While in Case B, the connector has caused a significant flattening of the curve. In this case, two inflection points, one near to the middle and other near to the end, have appeared in the member. Overall, the result suggests that the adoption of

ERP can accurately predict the effects of the end-to-end connectors in a lengthened member with large deformation.

Table 1. Comparisons of deflection and stress for two load cases.

	Deflection at B, m			Max. von Mises Stress, MPa		
	Without connector	With connector	With ERP	Without connector	With connector	With ERP
Case A	-3.920	-3.844	-3.843	63.000	63.350	63.360
Case B	-6.516	-6.470	-6.467	131.110	133.830	133.810

2.2 Elastic gridshell

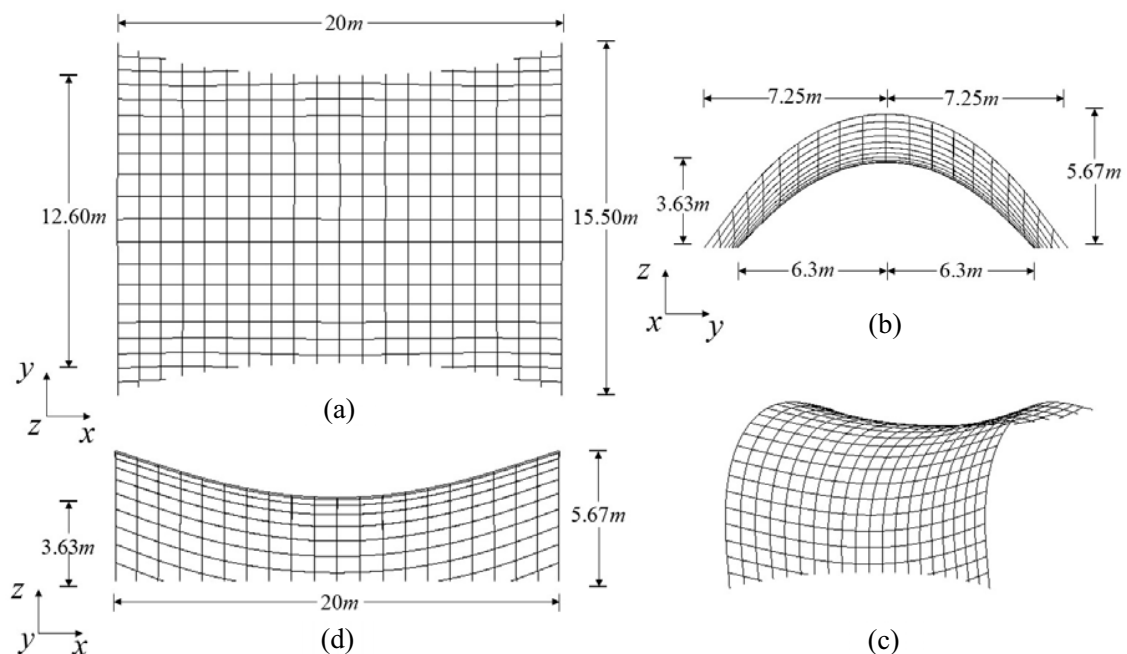


Figure 5. Gridshell (a) Plan view (b) Side view (c) Elevation view (d) Perspective view.

Figure 5 illustrates a simple elastic gridshell of grid size of 1m with all the corresponding views and dimensions considered in the study. Similarly, Figure 6 demonstrates two schemes of the connector arrangements taken into account. The connectors were applied considering the length of the member to be limited to 9 m. For Arrangement I as shown in Figure 6(a), the connectors were symmetrically placed in the plane grid. For y-directional members, the connectors are arranged at the waist of the structure, i.e., nearly at a distance of $L/4$ from the member ends. Connectors in the crown regions, i.e., $L/2$ of the members are avoided intentionally. On the other hand, for Arrangement II as shown in Figure 6(b), connectors were placed at all the possible locations of the members. The arrangement of the connectors is centrosymmetric so that the placement of the connectors have no or minor effect on the prescribed shape of the structure. It should be noted that, for the y-directional members of Arrangement II, some connectors are placed at the crown region of the structure where the stress is high during deformation of the grids.

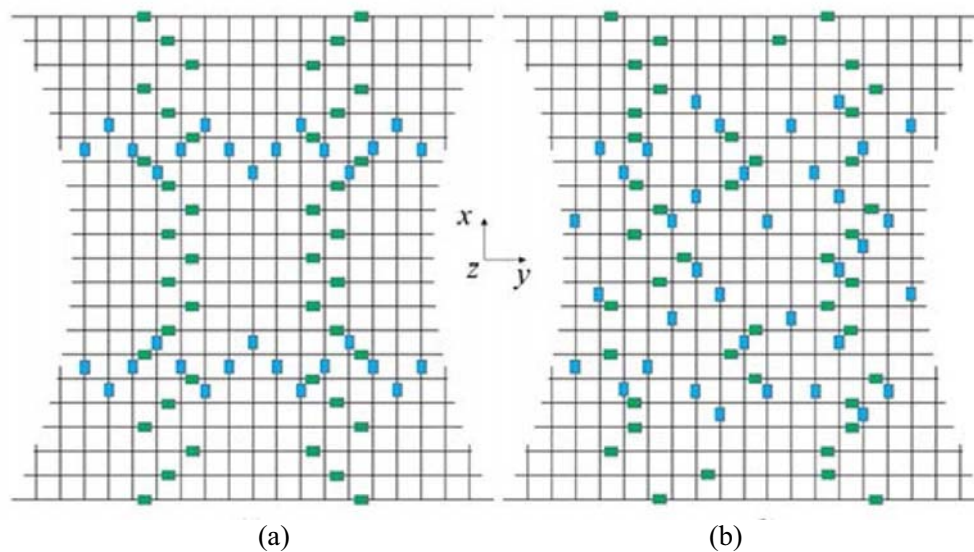


Figure 6. Flat grids with connectors (a) Arrangement I (b) Arrangement II

3. Results and Discussions

Figure 7 and Figure 8 represent the stress distribution in gridshell members and deformation of the gridshell after erection. Figure 7(a) and Figure 8(a) depict the stress in pipe sections and deformation for the gridshell without connectors. Similarly, Figure 7(b) and Figure 8(b) are for gridshell with connectors employing Arrangement I while Figure 7(c) and Figure 8(c) considers proposed Arrangement II.

Figure 7 indicates a significant rise in the stress level due to the presence of connectors in member sections. The maximum stress developed in the member sections without connectors was found to be 160.89 MPa. Whereas, analysis of sections with connectors in gridshell showed an evident increase in stress level in few portions, specifically in the crown region as highlighted in Figure 7(b) and (c). The stress in the highlighted regions of the gridshell with connectors exceeded the estimated maximal stress (160.89 MPa) from gridshell without connectors. These regions are susceptible to failure and member breakage may occur due to underestimation of the sectional stress. Figure 7(b) shows that 11 members were over-stressed for Arrangement I while Figure 7(c) shows that 12 members were over-stressed for Arrangement II. Moreover, Figure 7(b) shows that the over-stressed regions are localized in the central regions of the structure. It can also be observed from Figure 7(b) and (c), more beam sections in the structure with connectors employing Arrangement I exceeded the stress (160.89 MPa) compared to the structure with connectors using Arrangement II. But the intensity of stress was higher in the latter case. It can also be noticed that the arrangement of the connectors caused significant rise in the stress, which cannot be neglected in the design. For Arrangement I, the stress rose by 6.51 MPa, whereas for Arrangement II, the stress rose by 20.55 MPa. A surplus stress of 14.04 MPa was caused due to the poor arrangement of the connectors (i.e., the Arrangement II) in the structure. In the gridshell considered in this research, y-directional members had large curvature compared to the x-directional members. Therefore, presence of connectors in middle part of the y-directional members in Arrangement II caused significant rise in maximal stress in member sections.

Similarly, it is apparent from Figure 8 that presence of connectors does not cause any significant change in the global shape whatsoever. The distribution of deformation is almost similar in all the cases and same in case of both type of connector arrangements. An increase in maximum deformation of 0.01 m was caused by the presence of connectors which is negligible compared to the global dimension of the structure.

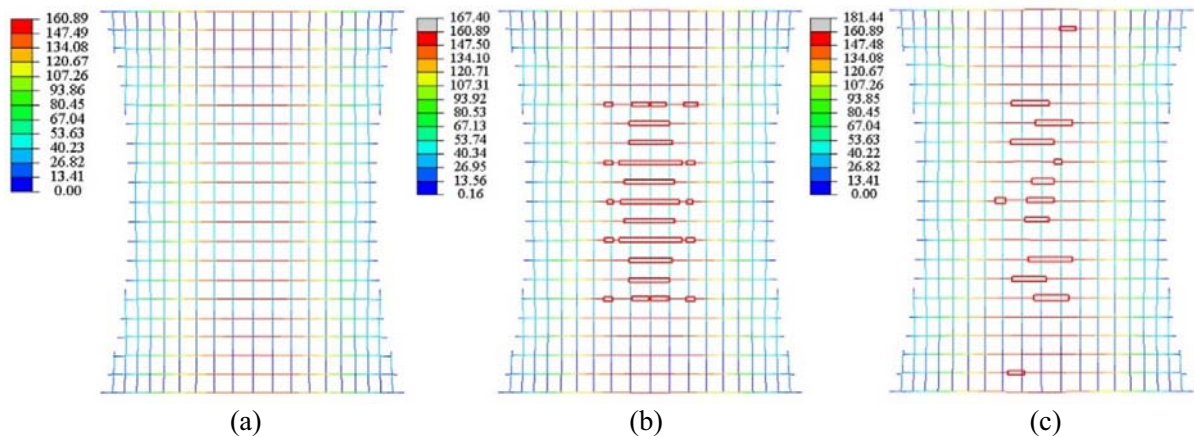


Figure 7. Stress after erection (a) Without connector (b) Arrangement I (c) Arrangement II (MPa).

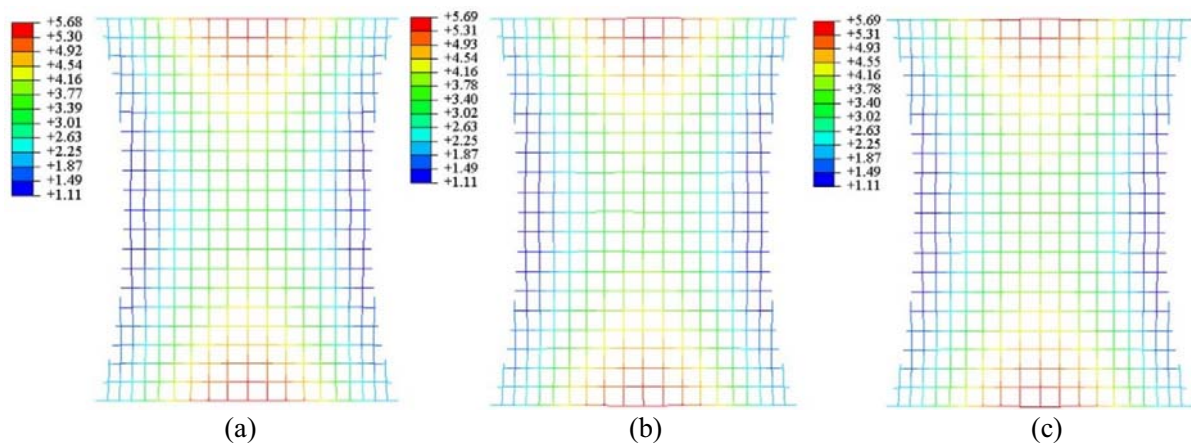


Figure 8. Deformation after erection (a) Without connector (b) Arrangement I (c) Arrangement II (m).

4. Conclusion

A concept of equivalent rigid part (ERP) for simplifying the modeling of end-to-end connector in the structural analysis of GFRP elastic gridshell is proposed in this paper. The ERP is further adopted for evaluating the effects of the connectors in the GFRP elastic gridshell by utilizing a typical structure, where the deformations and stresses are considered. The major conclusions drawn from the study are as follows:

- It is evident from the study that the ERP can predict the effect of connector to a greater precision.
- The effect of connector in flexure of a gridshell member is found to be minimal. Moreover, no significant effect on global shape was observed. Therefore, the flexibility of the member shall be restored as long as possible to attain the prescribed form.
- The presence of connector increased the member stress significantly. Hence, including the connectors while designing gridshell structures, aids to predict the true member stress and prevents failure of structural members before attaining the form.
- It is observed that the connector arrangement has significant effect on the member stress. Therefore, extra attention must be given to the proper arrangement of the connectors in the gridshell while designing and analyzing the structure.

Therefore, for designing GFRP elastic gridshell, where the end-to-end connectors are involved, it is suggested to incorporate the connectors in the analysis for better prediction of structural behaviors. For this purpose, the presented concept of ERP could be an option for simplification instead of connectors in the whole model for analysis.

References:

- [1] Paoli C 2007 *Past and Future of Grid Shell Structures* [Masters](Massachusetts Institute of Technology)
- [2] Peloux LD, Baverel O, Caron J-F and Tayeb F 2013 From shape to shell a design tool to materialize freeform shapes using gridshell structures *Des. Model. Symp. (Berlin)*
- [3] Lienhard J, Alpermann H, Gengnagel C and Knippers J 2013 Active Bending, a Review on Structures where Bending is Used as a Self-Formation Process *Int. J. Sp. Struct.* **28** 187-96
- [4] Hernández EL, Baverel O and Gengnagel C 2013 On the Design and Construction of Elastic Gridshells with Irregular Meshes *Int. J. Sp. Struct.* **28** 161-74
- [5] Happold E and Liddell WI 1975 Timber lattice roof for the Mannheim Bundesgartenschau *Struct. Eng.* **53**
- [6] Tayeb F, Lefevre B, Baverel O, Caron J-F and Peloux Ld 2015 Design and realisation of composite gridshell structures *J. Int. Assoc. Shell Spat. Struct.* **56** 49-59
- [7] Baverel O, Caron J-F, Tayeb F and Peloux LD 2012 Gridshells in composite materials: Construction of a 300 m2 forum for the solidays' festival in Paris *Struct. Eng. Int.* **22** 408-14
- [8] Hernández EL, Gengnagel C, Sechelmann S and Rörig T 2011 On the materiality and structural behaviour of highly-elastic gridshell structures *Computational Design Modelling* ed Gengnagel C, Kilian A, *et al.* (Berlin: Springer) p 123-35
- [9] Douthe C, Baverel O and Caron J-F 2006 Form-finding of a grid shell in composite materials *J. Int. Assoc. Shell Spat. Struct.* **47** 53-62
- [10] Du Peloux L, Tayeb F, Caron J-F and Baverel O 2015 The Ephemeral Cathedral of Créteil: a 350m2 lightweight gridshell structure made of 2 kilometers of GFRP tubes *CIGOS 2015: "Innovations in Construction", Proc. of CIGOS 2015 (Chanchan, France)*
- [11] Du Peloux L, Tayeb F, Baverel O and Caron J-F 2016 Construction of a large composite gridshell structure: a lightweight structure made with pultruded glass fibre reinforced polymer tubes *Struct. Eng. Int.* **26** 160-7
- [12] Quinn G and Gengnagel C 2014 A review of elastic grid shells, their erection methods and the potential use of pneumatic formwork *Mobile and Rapidly Assembled Struct. IV* **136** 129
- [13] THINKSHELL 2014 *Temporary Cathedral in Paris 2013* [20-Aug-2019] Available from: <http://thinkshell.fr/gridshell-cathedral-2013/>