Roughened bed stilling basin and its hydraulic jump characteristics

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Abstract: The roughened-bed stilling basin, serving as one new type of energy dissipator, has been analyzed both theoretically and experimentally in the literature. This study deduces the calculation formula of the length of the curve C_0 between the flow-depth contraction section (*i.e.* h_c) and the section just before the hydraulic jump (*i.e.* h_1) when the same elevation is employed for both the bottom of roughened-bed stilling basin and the river-bed surface in the downstream. Further, by approximately assuming the equation of the resistance force exerted on the flow and caused by the bed roughness, this study obtains the expression of the ratio between the flow depth before the hydraulic jump and that behind the hydraulic jump, namely h_1/h_2. With the aid of the experimental results in the existing literature, it can be concluded that, when compared with the traditional smooth-bed hydraulic jump, the sequent depth and the length of the rolling hydraulic jump can be reduced by about 20~30% and 20~50%, respectively, in the condition of the chopping roughness riverbed. Additionally, by combining with the hydraulic model tests, this study indicates that adding the bed-roughness can eliminate the secondary hydraulic jump, improve the flow regime and (under the circumstance of the sill-roughness stilling basin) decrease the length of the stilling basin up to about 40%~50%.
Keywords: Stilling Basin; Hydraulic Jump; Hydraulic Characteristics; Roughened Bed; Sequent Depth; Energy Dissipation

1. Introduction

As for medium or low head weir flow and orifice outflow, the hydraulic jump energy dissipation is frequently used with the aid of the stilling basin [1-4]. Usually, the conjugate depth h_c'' of the flow depth in the contraction section (*i.e.* h_c) is greater than the flow depth of the channel in the downstream (*i.e.* h_t, corresponding to the discharge considered). Therefore, the bench-type or sill-type stilling basin is often constructed for the purpose of increasing the water depth up to approximate $1.05h_c''$ within the stilling basin and further avoiding the occurrence of the repelled downstream hydraulic jump [5-8].

As shown in *Figure* 1, the total length of the repelled downstream hydraulic jump includes two parts: 1).The length of the curve C_0 (*i.e.* l_{C0}) stretching between the contraction section and the cross-section just in front of the hydraulic jump. 2). The length of the rolling hydraulic jump (*i.e.* l_{rj}) spanning between the cross-section just before the hydraulic jump (whose water depth is denoted by h_1) and that just behind the hydraulic jump (whose water depth is expressed by h_2). Obviously, if the value of l_{C0} is very large, it is not economical to construct such a long stilling basin for the repelled downstream hydraulic jump. Thus, in practical engineering it is common that the channel bottom or the river bed is intentionally roughened (*viz.* increasing the friction drag) in order to reduce the values of l_{C0} and l_{rj} in order to keep the length of the stilling basin within the acceptable range [1, 5, 9, 10].

The influence of different roughness degrees on the ratio h_2/h_1 and the roller length l_{rj} for the hydraulic jumps on corrugated beds has been studied experimentally in *Ref.* [1], whose results indicate that the increase of the roughness can bring down the aforementioned two values. Besides, from the water-surface equation deduced in this paper, it is clear that the increase of the roughness can also lead to the reduction of the length of the curve C_0.

Combining with the model test of a practical engineering, this study qualitatively compares the hydraulic characteristics of the roughened bed stilling basin with that of the traditional stilling basin. It can be concluded that the addition of the bed roughness can result in a shorter length of the stilling basin, a smooth flow regime and a satisfactory energy dissipation effect.

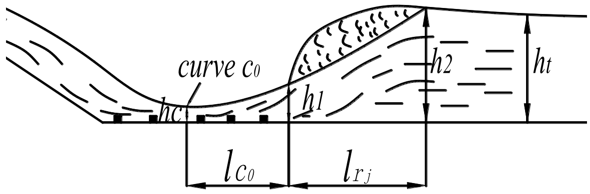

Figure 1. The schematic diagram of the repelled downstream hydraulic jump

2. Hydraulic design of the roughened bed stilling basin

2.1. The designing thought of the stilling basin

The designing thought of the stilling basin is described as following: After obtaining the contraction water depth h_c in the downstream of outlet structures, to compare the downstream channel flow depth h_t and the conjugate water depth h_c''. If $h_t \le h_c''$, either the repelled downstream hydraulic jump or the critical hydraulic jump happens, and it is necessary to construct a stilling basin for the energy dissipation. However, when $h_t > h_c''$, the submerged hydraulic jump takes place, and the downstream water will be pushed towards the upstream under the effect of the hydrodynamic pressure. In practical engineering, under the circumstance of $h_t \le h_c''$, the stilling basin is usually constructed to guarantee a smooth connection between the sequent depth h_2 and the downstream flow depth h_t [9-10].

As displayed in *Figure* 1, the downstream channel flow depth h_t and the Froude number F_r are required in the course of designing the roughened bed stilling basin. The sequent depth h_2 corresponding to h_1 can be calculated by:

$$h_2 = \frac{h_1}{2}\left(\sqrt{1+8\left(F_{r1}\right)^2}-1\right) \tag{1}$$

2.2 The length of the curve C_0

Assuming that the breadth of channel (*i.e. b*) is much larger than the depth (*i.e. h*), the describing equation of the water surface curve is:

$$\frac{dh}{dl} = \frac{i-i_f}{1-F_r^2} \quad [9] \tag{2}$$

where l is the length along the flow direction, i is the base slope and i_f is the slope of friction.

$$i_f = \frac{n^2q^2}{h^{10/3}} \tag{3}$$

$$F_r^2 = \frac{h_{cr}^3}{h^3} \tag{4}$$

$$q^2 = h_{cr}^3 \cdot g \tag{5}$$

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where n is the coefficient of roughness, q is the discharge per unit width and h_{cr} is the critical depth. For a smooth-bed stilling basin with i=0, substituting *equations* (3)~(5) into *equation* (2) , and further defining:

$$Y=\frac{h}{h_{cr}} \quad , \quad L=\frac{l}{h_{cr}} \tag{6}$$

Equation (2) switches to:

$$dL=-\frac{1}{K_n}(Y^{\frac{10}{3}}-Y^{\frac{1}{3}})dY \tag{7}$$

$$K_n=\frac{n^2 g}{h_{cr}^{\frac{1}{3}}} \tag{8}$$

Integrating *equation* (2) along the l_{C0} line, we obtain:

$$K_n L_{C_0}=(\frac{3}{4}Y^{\frac{4}{3}}-\frac{3}{13}Y^{\frac{13}{3}})\Big|_{Y_c}^{Y_1} \tag{9}$$

$$l_{C_0}=L_{C_0}\cdot h_{cr}=\frac{h_{cr}}{K_n}(\frac{3}{4}Y^{\frac{4}{3}}-\frac{3}{13}Y^{\frac{13}{3}})\Big|_{Y_c}^{Y_1} \tag{10}$$

Along the whole curve C_0, the value of Y is smaller than 1. Consequently,

$$l_{C_0}<\frac{0.5192h_{cr}}{K_n} \tag{11}$$

Considering that K_n is directly proportional to n^2, l_{C_0} is inversely proportional to n^2, which means that increasing n can decrease the length of the curve C_0 dramatically. In order to obtain the value of Y_c, it is necessary to compute the value of h_c, which can be expressed by the following empirical equation [11]:

$$h_c=E(\frac{1}{3}-\frac{2}{3}\sin(\frac{\pi}{6}-\frac{1}{3}\cos^{-1}(1-13.5\beta))) \tag{12}$$

where

$$\beta=\frac{q^2}{2gE^3} \tag{13}$$

and E is the head above the bottom of the pool.

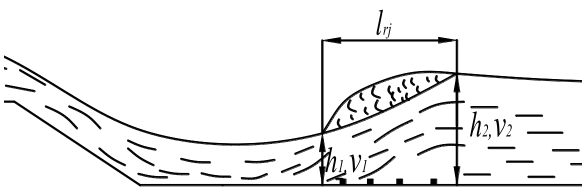

Figure 2. The schematic diagram of the rolling hydraulic jump on roughened bed

2.3 The sequent depth ratio and the length of the rolling hydraulic jump on roughened bed

The rolling hydraulic jump on roughened bed is demonstrated in *Figure* 2, and the corresponding momentum governing equation is:

$$\frac{\rho gbh_1^2}{2}-D-\frac{\rho gbh_2^2}{2}=\rho qb(v_2-v_1) \tag{14}$$

where D is the roughness resistance exerted on the water flow within the rolling jump region, which can be approximately defined as (from *Ref.* [1]):

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$$D = \frac{C_D \rho g b h_1^2}{2} \tag{15}$$

where C_D is the corresponding drag coefficient and b is the breadth of the river bed.
Letting:

$$\eta = \frac{h_2}{h_1} \tag{16}$$

Substituting *equation* (15) into *equation* (14) and using *equation* (16), we can obtain:

$$\eta^3 - A\eta + B = 0 \tag{17}$$

$$A = 1 + 2F_{r1}^2 - C_D \tag{18}$$

$$B = 2F_{r1}^2 \tag{19}$$

$$F_{r1} = \frac{v_1}{\sqrt{gh_1}} \tag{20}$$

$$\eta = \frac{2\sqrt{A}}{\sqrt{3}} \cos\left(\frac{1}{3}\cos^{-1}(-1.5\sqrt{3}M)\right) \tag{21}$$

$$M = \frac{B}{A^{3/2}} \le 0.385 \tag{22}$$

Only a small number of studies have been carried out for the hydraulic jump on roughened bed in the literature. The diagrammatic sketch of the roughened-bed configuration employed in *Ref.* [1] is illustrated in *Figure* 3 and the experimental results are listed in *Table* 1.

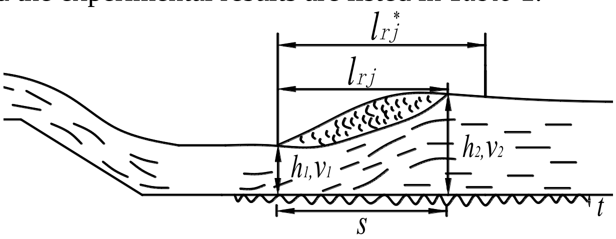

Figure 3. The diagrammatic sketch of the roughened bed adopted in *Ref.* [1]

Table 1. Characteristics of the hydraulic jump on the roughened-bed (b= 0.446 m)

$t(mm)$	$s(mm)$	$h_1(mm)$	$v_1(m/s)$	F_{r1}	$h_2(m)$	η	$L_{rj}(m)$	h_2^*	$L_{rj}^*(m)$	h_2/h_2^*	L_{rj}/L_{rj}^*	C_D
13	68	25.4	2	4	0.104	4.09	0.31	0.132	0.5	0.79	0.62	8.45
13	68	25.4	2.5	5	0.128	5.04	0.41	0.167	0.69	0.77	0.59	15.68
13	68	25.4	3	6	0.145	5.71	0.48	0.203	0.88	0.71	0.55	27.79
13	68	25.4	3.49	7	0.188	7.4	0.61	0.239	1.06	0.79	0.58	31
13	68	50.8	2.82	4	0.21	4.13	0.75	0.263	0.96	0.8	0.78	8.19
13	68	50.8	4.07	5.8	0.31	6.1	1.22	0.39	1.59	0.79	0.77	20.04
22	68	50.8	2.82	4	0.21	4.13	0.61	0.263	0.96	0.8	0.64	8.19
22	68	50.8	4.07	5.8	0.31	6.1	1.02	0.39	1.59	0.79	0.64	20.04

h_2^* and L_{rj}^* in *Table* 1 are the sequent depth of the traditional smooth-bed hydraulic jump and the length of its rolling jump region, respectively. Their values can be calculated by *equation* (23) and *equation* (24) [12].

$$h_2^* = \frac{h_1}{2}\left(\sqrt{1 + 8F_{r1}^2} - 1\right) \tag{23}$$

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$$L_{rj}^{*\,[12]} = \begin{cases} 8(F_{r1}-1.5)h_1\ , & 2.5 < F_{r1} < 8 \\ (160\tanh(\dfrac{F_{r1}}{20})-12)h_1, & F_{r1} < 15, \dfrac{h_1}{b} < 0.1 \\ (100\tanh(\dfrac{F_{r1}}{12.5})-12)h_1, & F_{r1} < 15, 0.1 < \dfrac{h_1}{b} < 0.7 \end{cases} \tag{24}$$

It can be concluded from *Table* 1 that the value of h_2/h_2^* (on the chopping roughness bed shown in *Figure* 3) lies in the range of 0.71~0.80, which means h_2 has been shortened by approximately 20~30% relative to h_2^*. Besides, L_{rj}/L_{rj}^* is equal to about 0.52~0.78, which indicates the length of the rolling jump region can be reduced by approximately 20~50% for the roughened bed. Technical indexes make it clear that it is potential to put the roughened-bed stilling basin into engineering use. The drag coefficient in *Table* 1 is calculated with the aid of the experimental η, F_{r1} and *equations.* (17)~(19). In *Ref.* [1], the C_D value for the chopping roughness bed is summarized as:

$$C_D = (F_{r1}-1)^2 \tag{25}$$

Table 1 also indicates that, when the height of chopping roughness t increases from 13*mm* to 22*mm*, C_D and h_2/h_2^* stay almost the same while L_{rj}/L_{rj}^* gets much smaller. Data in *Table* 1 have provided the influence degree of the roughened bed on the characteristics of the hydraulic jump.

Figure 4. The secondary hydraulic jump generated after the end of the traditional smooth-bed stilling basin.

Figure 5. Comparison of the hydraulic jump characteristics between the roughened-bed stilling basin and the traditional smooth-bed stilling basin.

3. Application in practical engineering projects

Combining with a practical engineering, the stilling basins with and without artificial roughness are tested by the hydraulic model to examine the effect of the artificial roughness on the flow characteristics of the hydraulic jump. The scale of length is 1/70. By setting five different roughness degrees within the stilling basin, the length of the hydraulic jump can be significantly reduced.

By comparing *Figure* 4 and *Figure* 5, it can be concluded that the secondary hydraulic jump may be produced after the end of the traditional smooth-bed stilling basin, which is not safe in practical engineering. Extending the length of the stilling basin is an alternative but expensive way. Therefore, adding the roughness degree of the bed may be a better approach. From the *Figure* 4 and *Figure* 5, it is obvious that the addition of the bed roughness can reduce the flow depth at the end of the still basin, keep the original length of stilling basin unchanged and at the same time keep the flow regime smooth.

In view of the fact that the chopping-roughness stilling basin base slab is difficult to construct in practical engineering projects, this study will further investigate the hydraulic-jump characteristics on the sill-roughness stilling basin base slab, which is relatively more feasible to construct in reality.

3.1 The drag coefficient C_D and the roughness coefficient n on sill-roughened bed

Figure 6 illustrates the schematic of the test case adopted to measure the roughness coefficient of the sill-roughness bed. The height of the sill-roughness structure is either 50/3*mm* or 62/3*mm*, and the streamwise length and the interval distance of this structure is 10*mm* and 83.33*mm* respectively. The width of the water channel is equal to 0.567*m*, and the flow discharge per unit width, denoted by q, can be calculated by u_1, defined as the flow velocity at the upstream section of the riverbed part considered (*i.e.* l=0.426*m*). The experimental results, such as the water depth at different sections (*i.e.* h_1 and h_2), the equivalent roughness coefficient n and the equivalent drag coefficient C_D are given by *Table* 2.

In *Table* 2, the following parameters are defined: $Y_1 = \frac{h_1}{h_{cr}}$, $Y_2 = \frac{h_2}{h_{cr}}$, $L = \frac{l}{h_{cr}}$, and

$$n = \frac{h_{cr}^{\frac{1}{3}}((\frac{3}{4}Y_2^{\frac{4}{3}} - \frac{3}{13}Y_2^{\frac{13}{3}}) - (\frac{3}{4}Y_1^{\frac{4}{3}} - \frac{3}{13}Y_1^{\frac{13}{3}}))^{\frac{1}{2}}}{gL} \tag{27}$$

$$C_D = \frac{(1+2F_{r1}^2)Y - Y^3 - 2F_{r1}^2}{Y}, \quad Y = \frac{Y_2}{Y_1} \tag{28}$$

It can be concluded that, for all the test cases considered in this study, the values of n and C_D don't experience significant variation when the height of the sill-roughness structure is changed from 50/3*mm* to 62/3*mm*. A similar formulation (to *equation* (25)) can be deduced from *Table* 2:

$$C_D = 1.18(F_{r1} - 1)^2 \tag{29}$$

More experimental tests are necessary in order to further calibrate the above C_D equation.

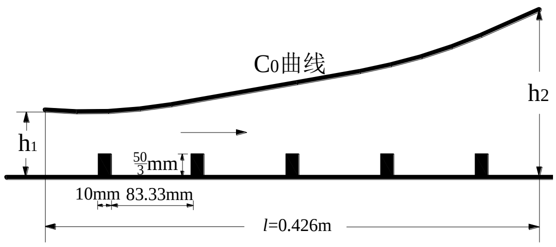


Figure 6. Schematic of the test case adopted to measure the roughness coefficient of the sill-roughness bed.

Table 2. The experimental results of the sill- roughness bed test case

t(mm)	*s(mm)*	$h_1(m)$	$h_2(m)$	$v_1(m/s)$	Fr_1	$h_{cr}(m)$	Y_1	Y_2	L	Y	C_D	n
16.67	83.33	0.118	0.150	1.58	1.47	0.153	0.774	0.978	2.785	1.265	0.31	0.061
16.67	83.33	0.156	0.178	1.80	1.46	0.200	0.778	0.887	2.128	1.141	0.22	0.061
16.67	83.33	0.146	0.165	1.92	1.61	0.200	0.729	0.822	2.128	1.128	0.31	0.063
20.67	83.33	0.114	0.142	1.58	1.49	0.149	0.766	0.952	2.855	1.242	0.32	0.061
20.67	83.33	0.115	0.143	1.60	1.50	0.151	0.763	0.946	2.826	1.241	0.34	0.062
20.67	83.33	0.160	0.183	1.79	1.43	0.203	0.787	0.899	2.095	1.142	0.21	0.061
20.67	83.33	0.154	0.178	1.86	1.51	0.203	0.759	0.874	2.095	1.151	0.28	0.066

3.2 Comparison of the energy dissipation characteristics between two types of stilling basin

With the purpose of making the description more convenient and more distinct, the following engineering project is analyzed: the elevation of the weir crest of the overflow dam is 99.8*m*, the check reservoir water level is 115.0*m*, the corresponding flow discharge per unit width in the stilling basin is q=106m^3/s, the water level in the downstream river channel is 34*m*, the downstream river-bed elevation is 10*m*, as displayed by *Figure* 7.

3.2.1 Design of the traditional stilling basin

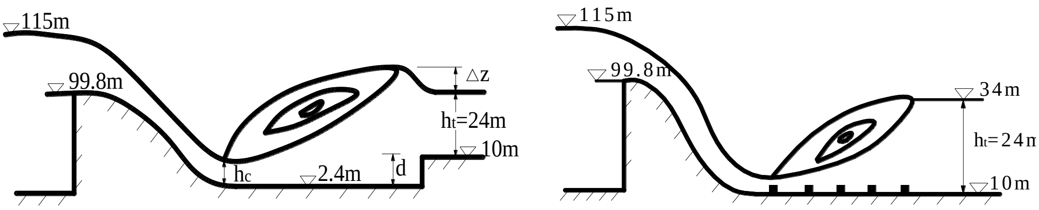
In practical engineering projects, the trial and iterative method is utilized, in other words, a depth of the stilling basin (denoted by *d*) is firstly assumed, and then we examine that whether this depth can result in a slightly submerged hydraulic jump within the stilling pool. If the aforementioned requirement can be met, the assumed *d* is the actual depth needed. Otherwise, a iteration procedure is adopted to obtain the correct depth of the stilling basin.

Supposing *d*=7.6*m*, it can be deduced that the total head above the stilling pool base slab is equal to *E*=115-2.4=112.6*m*. With the aid of *equatoin* (4), *equation* (12) and *equation* (23), it can be acquired that h_c=2.275*m*, F_{rc}=9.863, $h_c^{'}$=30.615*m*. Considering that

$$\Delta z = \frac{q^2}{2g}\left(\frac{1}{(\varphi\times h_t)^2}-\frac{1}{(\sigma_j\times h_c'')^2}\right) \tag{30}$$

where $\varphi = 0.95$ is the velocity coefficient of the broad crested weir, and $\sigma_j = 1.05$ is the submer-gence factor of the hydraulic jump occurring within the stilling pool. Consequently, it can be obtained from *equation* (30) that $\Delta z = 0.548m$. Seeing that:

$$d = \sigma_j h_c'' - h_t - \Delta z \tag{31}$$



(*a*) the traditional stilling basin (*b*) the sill-roughness stilling basin

Figure 7 Schematic of the design of the traditional stilling basin and the sill-roughness stilling basin

Substituting $h_c^{'}$, h_t and Δz, we can get the computed depth of the stilling basin $d^{'}$=7.598*m*, which means that the initial assumption of *d*=7.6*m* can meet the design criterion (*i.e.* $d \approx d^{'}$). Then, the length of the traditional stilling basin can be calculated according to the following equation [13, 14].

$$l_s = 6.0(h_c'' - h_c) \approx 170.04m \tag{32}$$

3.2.2 Design of the sill-roughness stilling basin

As illustrated by *Figure* 7(*b*), for the sill-roughness stilling basin, *d*=0 and h_c=2.362*m* (ignoring the loss of water head and calculated from *equation* (12)). At this time, the water depth in the contraction section is actually the water depth just in front of the hydraulic jump (*i.e.* h_1=h_c), and the water depth in the downstream river channel is in fact the sequent depth of the hydraulic jump (h_2=h_t). Therefore, the corresponding sequent depth ratio is:

$$\eta = \frac{h_t}{h_c} = 10.161, \quad F_{r1} = F_{rc} = 9.323 \tag{33}$$

Substituting *equation* (33) into *equation* (17), we can get the roughness drag coefficient C_D=54.487.

Referring to *equation* (25) and *equation* (29), for this project it can be presumed that:

$$C_D = \alpha(F_{r1}-1)^2 \tag{34}$$

Therefore, $\alpha = \frac{C_D}{(F_{r1}-1)^2}$=0.787. Taking into account that α=1.0 for the chopping roughness bed In *Ref.* [1] (*i.e. Table* 1 in this study) and α=1.18 for the sill-roughness river bed (*i.e. Table* 2 and *Figure* 6 in this study), it can be inferred that it is not difficult to construct the roughened river-bed with α=0.787 in practical engineering projects.

When the critical hydraulic jump happens (*i.e.* α=0.787), in view of that the shortening coefficient (defined as $\xi = L_{rj}/L_{rj}^*$ here) of the roughed-bed stilling basin is equal to about 0.52~0.78 (known from *Table* 1), for the sake of conservation, taking ξ=0.70 in this study. Consequently, the length of the roughened stilling basin is:

$$l_s' = 6.0(h_c'' - h_c)\xi = 90.88m \approx 53\% \cdot l_s \tag{35}$$

Obviously, by roughening the river bed, the length of the stilling basin can be reduced as much as about 50% (*i.e.* $l_s'/l_s = \frac{90.88}{170.04} \approx 53\%$) when compared with the traditional smooth-bed stilling pool.

Being different from the design of the traditional stilling basin, in this situation the sequent depth h_2

and the corresponding Froude number Fr_2 are known in advance, but the water depth and the Froude number just in front of the hydraulic jump (*i.e.* h_1 and Fr_1) need to be determined. Hence, when solving the sequent depth ratio η, *equation* (21) is not able to be directly applied because the unknown quantity Fr_1 is included in the equation of C_D, as shown by *equation* 25, *equation* 28, *equation* 29 and *equation* 34. Considering that:

$$\eta^3 = \frac{F_{r1}^2}{F_{r2}^2} \tag{36}$$

The hydraulic jump equation can be rewritten as:

$$\frac{F_{r1}^2}{F_{r2}^2} - (1+2F_{r1}^2 - \alpha(F_{r1}-1)^2)(\frac{F_{r1}}{F_{r2}})^{\frac{2}{3}} + 2F_{r1}^2 = 0 \tag{37}$$

When the repelled downstream hydraulic jump occurs (*i.e.* α <0.787, taking α =0.6 as an example), due to Fr_2=0.288, it can be deduced that Fr_1=7.78. Thus, the corresponding $h_1 = (\frac{q^2}{gF_{r1}^2})^{\frac{1}{3}}$ =2.665*m*, and the critical water depth h_{cr} =10.460*m*. In addition, $Y_c = \frac{2.362}{10.46} =$ 0.2258, $Y_1 = \frac{2.665}{10.46}$ =0.2548, and consequently the length of the curve stretching between the contraction section and the cross-section just in front of the hydraulic jump (*i.e.* l_{C_0}) can be obtained by *equation* (10):

$$l_{C_0} = L_{C_0} \cdot h_{cr} = \frac{h_{cr}}{K_n}(\frac{3}{4}Y^{\frac{4}{3}} - \frac{3}{13}Y^{\frac{13}{3}})\Big|_{Y_c}^{Y_1} = \frac{0.0414}{n^2} \tag{38}$$

It can be presumed that *n*=0.06 from *Table* 2, Thus:

$$l_{C_0} = \frac{0.0414}{n^2} = 11.49m,$$

$$l'_{rj} = 6.0(h''_c - h_c) = 89.61m,$$

$$l''_s = l_{C_0} + l'_{rj} = 101.10m \approx 59.5\% \cdot l_s \tag{39}$$

It can be concluded from the above discussion that, when the roughness scale of the downstream river-bed is large enough, the critical hydraulic jump can be formed, and the length of the roughened stilling basin is about 53% of that of the traditional stilling pool. However, when the roughness scale of the downstream river-bed is not large enough, the repelled downstream hydraulic jump develops, and the length of the roughened stilling basin is about 60% of that of the traditional stilling basin.

3.2.3 Comparison of the energy dissipation ways

The two stilling basins (demonstrated by *Figure* 7(*a*) and *Figure* 7(*b*)) share the same working condition, namely the equivalent reservoir water level, the same flow discharge per unit width, the identical downstream river-bed elevation and the equivalent water level of the downstream river channel. Therefore, their energy dissipation ratios are identical, although their energy dissipation ways are not exactly same. *Figure* 8 illustrates the specific energy at different typical sections of the stilling basin shown by *Figure* 7. In *Figure* 8, the point *A* stands for the contraction section (namely the section just in front of the hydraulic jump) in *Figure* 7(*a*), and its water depth and specific energy are expressed by $h_{c,A}$ and E_{SA}, respectively. The section just after the hydraulic jump lies in the point A'', the conjugate water depth is equal to $h_{c,A}'' = h_t + d + \Delta z$, and the corresponding specific energy is expressed by $E_{SA''}$. Therefore, the energy dissipation ratio is equal to $E_{SA} - E_{SA''}$ for the traditional stilling basin displayed by *Figure* 7(*a*). Due to the fact that the elevation of the stilling pool base slab in *Figure* 7(*b*) is *d*(*m*) higher than that in *Figure* 7(*a*), the specific energy at the contraction section of the flow in *Figure* 7(*b*) (*i.e.* the point *B*) is $E_{SB} = E_{SA} - d$, and the corresponding contracted water depth is represented by $h_{c,B}$. If the critical hydraulic jump occurs in *Figure* 7(*b*), the contraction section *B* is corresponding to the section just before the hydraulic jump, and the section just behind the hydraulic jump is denoted by the point B'' (the relevant sequent depth is $h_{c,B}'' = h_t$). Thus, the energy dissipation ratio is equal to $E_{SB} - E_{SB''}$ in *Figure* 7(*b*) when the critical hydraulic jump happens. In view of $E_{SB} = E_{SA} - d$ and $E_{SB''} = E_{SA''} - d$, it can be concluded that $E_{SA} - E_{SA''} = E_{SB} - E_{SB''}$, which means that the energy dissipation ratios are equivalent for the aforementioned two stilling basins.

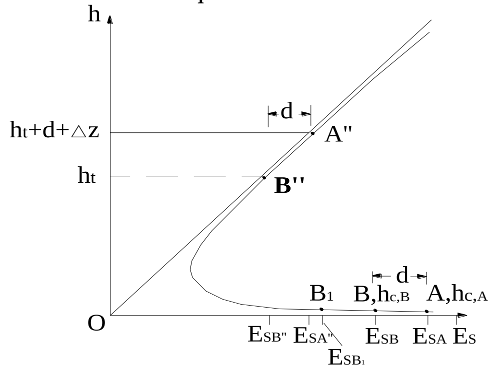


Figure 8. The specific energy at different typical sections of the stilling basin shown by *Figure* 7

If the repelled downstream hydraulic jump occurs in *Figure* 7(*b*) and the point B_1 (in *Figure* 8) indicates the section just in front of the hydraulic jump, BB_1 is corresponding to the curve C_0 in *Figure* 1 and the section just behind the hydraulic jump is still located at B'' (thus $h_t = h_{B1}''$). At this time, the energy dissipation consists of two parts. The first part is the energy dissipation along the curve C_0, being equal to $E_{SB} - E_{SB_1}$. The second part is the energy dissipation through the hydraulic jump, denoted by $E_{SB_1} - E_{SB''}$. At this moment, the water-depth difference caused by the hydraulic jump has been reduced to $h_t - h_{B1}$.

When the water outflow from the stilling basin in *Figure* 7(*a*), its specific energy will be transformed from the point A'' to the point B'' in *Figure* 8. During this process, a portion of potential energy will be transformed into kinetic energy once again, giving rise to additional disturbance of the water flow. As a consequence, although the nominal energy dissipation ratios are identical for *Figure* 7(a) and *Figure* 7(b), the former's real energy dissipation ratio will be smaller that that of the latter, especially for the low water-head outlet structure the aforementioned discrepancy will be even more significant [15, 16].

4. Conclusion

Compared to the traditional smooth-bed stilling basin, there is a higher ratio of damping on the roughened bed stilling basin. With the aid of the theoretical analysis and the hydraulic model experiments, it has been proved that the addition of the chopping roughness on the riverbed can decrease the sequent depth (*i.e.* h_2/h_2^* <0.71~0.80) and reduce the length of the rolling hydraulic jump region (*i.e.* L_{rj}/L_{rj}^* <0.52~0.78) and that of the curve C_0 (*i.e.* l_{C_0} is inversely proportional to n^2). Besides, it has been shown that the roughened bed stilling basin can result in a better flow regime between the sequent depth h_2 and the downstream water depth h_t (*e.g.* eliminating the secondary hydraulic jump). In addition, this study indicates that adding the sill-roughness structure on the downstream river-bed can decrease the length of the stilling basin up to about 40%~50% when compared with the smooth-bed stilling basin. These conclusions are of important reference value for the design of the flood-discharge energy-dissipation stilling basin in practical engineering projects.

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