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Low velocity impact analysis of high-order rectangular FG-CNTRC plates using the weak form QEM

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Abstract. The functionally graded carbon nanotubes reinforced composites (FG-CNTRC) plates have excellent properties and broad application prospects. In the analysis of low velocity impact response of functionally graded carbon nanotubes reinforced composite plates, innovative numerical formulations are urgently needed. As a new high-order numerical method, the weak form quadrature element method (QEM) has its unique advantages and has been widely used in the engineering structural field. In this paper, the nonlinear Hertzian contact law is employed to describe the impact process between a rigid impactor and the rectangular FG-CNTRC plate, that was modelled with the Reddy's high-order shear deformation theory. By using the weak form QEM, the calculation model for the impact system is established, and the numerical solution is carried out with the help of Newmark-β step-by-step time domain integration method and *Newton-Raphson* iteration technology. The effectiveness and accuracy of the weak form QEM formulation were verified through the comparison with the existing literature and the relevant low velocity impact properties of FG-CNTRC plate are explored..

1. Introduction

Functional gradient carbon nanotube reinforced composite (FG-CNTRC) is a new type of advanced composite material using carbon nanotubes distributed in the matrix as reinforcements[1]. In order to give full play to the material advantages of carbon nanotubes and effectively improve the overall macro mechanical properties and designability of the composite, Shen Huishen first applied the concept of functional gradient to carbon nanotubes reinforced composite in 2009, and made a nonlinear bending analysis of the composite plate with gradient arrangement of carbon nanotubes reinforcement in the polyvinylidene chloride (PmPv) matrix[1]. Many scholars have done a lot of research on the bending, buckling and free vibration analyses of the composite beam, plate and shell structure, FG-CNTRC shows excellent thermal, mechanical and electrical properties, which indicates that it will have a broad application prospect in aerospace, energy engineering, bioengineering, military protection and other fields.

Generally speaking, when the duration of impact process is large enough to ignore the influence of strain rate and stress wave propagation of the impacted structural material, it can be called "low velocity impact". FG-CNTRC plate, as a structural component, is inevitably impacted by other small objects at low speed in the process of processing and utilizing. Especially when such components are directly used for shielding and protection purposes. Therefore, it is necessary to analyze the low-velocity impact dynamic response of this kind of plate.

Compared with a large number of research results in the field of low-velocity impact of traditional fiber-reinforced composite plates, the research work on low-velocity impact of FG-CNTRC plate structure is very limited. In the existing research, various kinds of analytical or numerical methods are used to obtained the solutions. Low-velocity impact is a time-dependent dynamic response process. To analyze this problem, we need to solve the original continuous problem discretely in both space and time domain. For the discrete solution of space domain, the displacement type control differential equation derived from Reddy's high-order shear deformation theory[2] needs to satisfy C^1 continuous condition on the element boundary, which increases the computational complexity of traditional finite element method. For example, the two-dimensional four node finite element rectangular element proposed by Ebrahimi et al.[3] requires 15 degrees of freedom for each node, which urges many researchers to adopt other numerical formulations in space domain, including the traditional Navier series method[4], the Rayleigh Ritz method[5,6], the new quadratic perturbation method[7], the differential quadrature method and some meshless method[8]. These methods have their advantages, but they also have shortcomings. As Navier series method, quadratic perturbation method is generally only suitable for plate structures with specific geometric features and boundary conditions; meshless method has strong adaptability to different geometric features and boundary conditions, but its preprocessing process is complex, and the boundary conditions are not easy to be directly applied[9]. The research on the low-velocity impact response of FG-CNTRC plate structure is not enough, and the demand for new and efficient numerical methods is still urgent.

Zhong Hongzhi[10] proposed a new numerical method in 2007, weak form quadrature element method (QEM), which has been widely used and developed in the field of engineering structure analyses. The weak form QEM is a new numerical method based on the weak form (control function) of the differential equation of the problem to be solved. It uses a unified element node to calculate the integral and differential in the control function of the problem. It has many advantages, such as accurate and efficient, direct implementation steps, simple and intuitive pre- and post- processing, etc. The application of this method shows its attractive advantages in dealing with complex geometry, inhomogeneous materials and nonlinear problems compared with the traditional finite element method. The calculation and analysis of linear elastic problems of structures by the method of quadrature element is mature, and the research focus is gradually turning to the field of quasi-static and dynamic nonlinear analysis.

In this paper, the nonlinear Hertzian contact law is employed to describe the impact process between a rigid impactor and the rectangular FG-CNTRC plate. Two different models for the impact system are considered. In model I, the impactor and FG-CNTRC rectangular plate are considered respectively, and the nonlinear impact contact force is considered as the external load; in model II, the impactor and FG-CNTRC rectangular plate are considered as the overall system, and the impact contact force is viewed as the internal force. By using the weak form quadrature element method, the calculation model of the system is established, and the solution is carried out by using the Newmark-β step-by-step time domain integration method and *Newton-Raphson* iteration technology. The effectiveness and accuracy of the weak form QEM formulation were verified through the comparison with the existing literature and the relevant low velocity impact properties of FG-CNTRC plate are explored.

2. Theoretical formulation

2.1. Material model

In this paper, FG-X rectangular plate adopts FG-X distribution and establishes coordinate system, as shown in the Figure1.

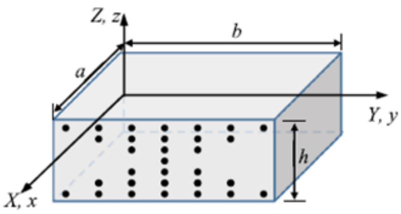

Figure 1. FG-X distribution of FG-CNTRC rectangular plate.

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In this paper, we use the generalized mixed law model[1] to describe the FG-CNTRC plate. According to this model, Young's modulus and shear modulus of FG-CNTRC plate can be expressed as follows:

$$E_{11} = \eta_1 V_{CNT} E_{11}^{CNT} + V_m E^m,\ \eta_2 / E_{22} = V_{CNT} / E_{22}^{CNT} + V_m / E^m,$$
$$\eta_3 / G_{12} = V_{CNT} / G_{12}^{CNT} + V_m / G^m,\ \nu_{12} = V_{CNT}^* \nu_{12}^{CNT} + (1 - V_{CNT}^*)\nu^m. \tag{1}$$

In Eq. (1), E_{11}^{CNT} , E_{22}^{CNT} and G_{12}^{CNT} are the Young's moduli and shear modulus of the CNT respectively. The polymer matrix is assumed to be elastic and isotropic, with Young's modulus E^m and shear modulus G^m . Three efficiency parameters, η_1 , η_2 and η_3 , are introduced in order to consider the scale-dependent effect of the CNTs. The three efficiency parameters were determined by matching the Young's moduli and shear modulus of CNTRC observed from the molecular dynamics simulations with the numerical results from the extended rule of mixture[1]. Moreover, ν_{12} , ν_{12}^{CNT} and ν^m are the Poisson's ratio of the FG-CNTRC plates, CNT and matrix respectively..

2.2. Reddy's high-order plate theory

The Reddy's high-order shear deformation theory[2] is adopted to model the displacement field of the FG-CNTRC plate as follows:

$$d(x,y,z,t) = \begin{bmatrix} u + z\phi - (4/3h^2)z^3(\phi + w_{,x}) \\ v + z\varphi - (4/3h^2)z^3(\varphi + w_{,y}) \\ w \end{bmatrix} \tag{2}$$

where u , v and w are the displacement components of an arbitrary material point (x , y , z) on the neutral surface of the composite plate along the X , Y and Z coordinate directions at the time t ; ϕ and -φ are the rotations of the normal to the neutral surface about the Y and X coordinate axis, respectively. In addition, $w_{,x}$ and $w_{,y}$ denote the first order derivatives of w with respect to the X , Y coordinate.

Thus, for small strains and moderate rotations problems as the paper concerned, the non-zero strains associated with the displacement field (Eq. (2)) can be expressed as

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yy} & \gamma_{xy} & \gamma_{yz} & \gamma_{xz} \end{bmatrix}^T = \bar{\varepsilon} + \tilde{\varepsilon} \tag{3}$$

where $\bar{\varepsilon}$ and $\tilde{\varepsilon}$ are the linear strain term and the nonlinear strain term respectively.

$$\tilde{\varepsilon} = \frac{1}{2}\begin{bmatrix} w_{,x}^2 & w_{,y}^2 & 2\, w_{,x} w_{,y} & 0 & 0 \end{bmatrix} \tag{4}$$

The strains in Eq. (4) are also known as von Kármán strains.

2.3. Impact contact law

The indentation $\alpha(t)$ is related to the impact load with the help of nonlinear Hertzian contact law [9],

$$F(t) = K_c \alpha(t)^p \tag{5}$$

$\alpha(t)$ can also be expressed as

$$\alpha(t) = s(t) - w(t) \tag{6}$$

Where, $s(t)$ is the displacement at the tip of the impactor, $w(t)$ is the displacement at the top of the plate.

2.4. The weak form quadrature element formulation

Considering the dynamic response of an FG-CNTRC plate, it is assumed that the solutions for the kinematic and stress variables at all time steps, from time t_0 to time t_n , have been solved, and that the solution for time t_{n+1} can be achieved employing the principle of virtual displacements,

$$\delta W_{\text{int}}^{t_{n+1}} = \delta W_{ext}^{t_{n+1}} \tag{7}$$

where $\delta W_{\text{int}}^{t_{n+1}}$ and $\delta W_{ext}^{t_{n+1}}$ are the internal and external virtual work expression at time t_{n+1} respectively. As for the Total Lagrange Formulation (T.L.), all variables in Eq. (7) are referred to the initial undeformed configuration, i.e. $t = t_0$, of the FG-CNTRC plate. Eq. (7) can be expressed as

$$\int_V \Delta\bar{\varepsilon}^T E \delta\Delta\bar{\varepsilon} dV + \int_V \delta\Delta\tilde{\varepsilon}^{T\ t_n}\sigma dV = \delta^{t_{n+1}} W_{ext} - \int_V \delta\Delta\bar{\varepsilon}^{T\ t_n}\sigma dV \tag{8}$$

First of all, similar to the finite element method, the weak form quadrature element method divides the whole physical domain of the problem to be solved into several continuous sub regions. In each sub region, the control equation is transformed from physical domain to unit square standard field by geometric mapping for the convenient of numerical integration and differential utilizing. After determine the number of integration points in the sub area, according to the Gauss-Lobatto integral rule[11], the integral term in Eq. (8) is expressed with the function value and their differential term at the numerical integral point. Take the one-dimension problem as example, the Gauss-Lobatto numerical integration formulation whose standard domain is the closed interval of [-1,1] is expressed as:

$$\int_{-1}^{1} f(\xi) d\xi = \sum_{i=1}^{N} W_i f(\xi_i) \tag{9}$$

where, ξ_i is the coordinate of the integral point i in the sub region, N is the total number of the integral points, and W_i is the integral coefficient at ξ_i.

Furthermore, by using the differential quadrature method (DQM) and generalized differential quadrature method (GDQM), the differential terms contained in the above equations are further expressed as function values at differential points which were the same with the numerical integral point. Similarly, take the one-dimension problem as example, The differential quadrature method is as follows:

$$\left.\frac{\partial^m f(\xi)}{\partial \xi^m}\right|_{\xi=\xi_i} = \sum_{i=1}^{N} h_i^{(m)} f(\xi_i) \quad (i = 1, 2, \cdots, N) \tag{10}$$

where $h_i^{(m)}$ is weighting coefficient and $f(\xi_i)$ is the function value at the point ξ_i.

After the above process, the Eq. (8) finally becomes the summation expression of the function values of the integral point. Next, using the transformation matrix, the expression in the standard domain is transformed into the expression in the physical domain, and the global incremental equation is obtained.

$$\boldsymbol{M}\,{}^{t_{n+1}}\ddot{\boldsymbol{d}} + \boldsymbol{K}\Delta\boldsymbol{d} = {}^{t_{n+1}}\boldsymbol{F}_{\text{ext}} - {}^{t_n}\boldsymbol{F}_{\text{int}} \tag{11}$$

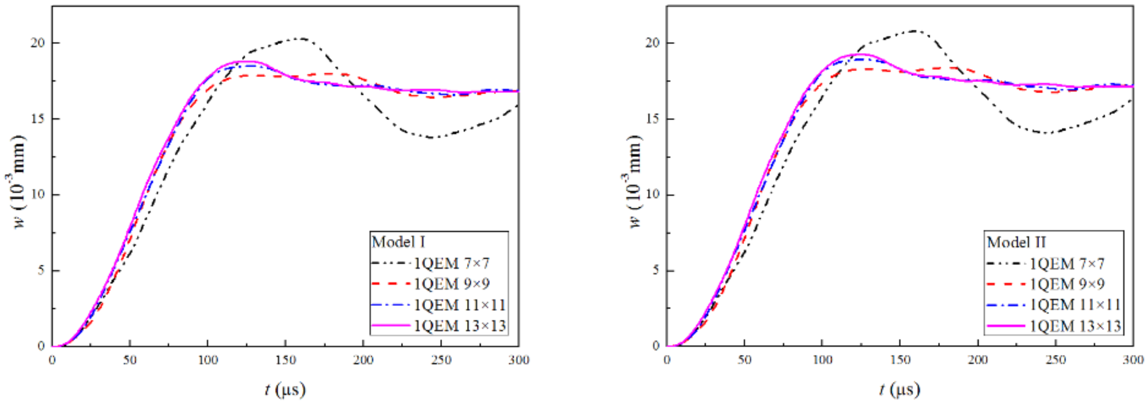
Among them, $\boldsymbol{M}$ is the overall mass matrix, $\boldsymbol{K}$ is the overall stiffness matrix, $\ddot{\boldsymbol{d}}$ is the node acceleration vector, $\boldsymbol{F}_{\text{ext}}$ is the external force vector, $\boldsymbol{F}_{\text{int}}$ is the internal force vector.

Then, solve the above equation. Since the impact load is a variable vs time, it needs to be solved by the famous constant acceleration method (Newmark-β type time integration method). What must be done is to divide the whole process into multiple time steps. Because of the geometrical nonlinearity of the plate, the stiffness matrix $\boldsymbol{K}$ is a function of the displacement vector $\boldsymbol{d}$, so it is necessary to use *Newton-Raphson* method to iterate in each load step until the equation meets the required convergence criteria. Simultaneously, using Newmark-β time integration method, the acceleration and velocity are obtained immediately.

3. Numerical analysis and discussion

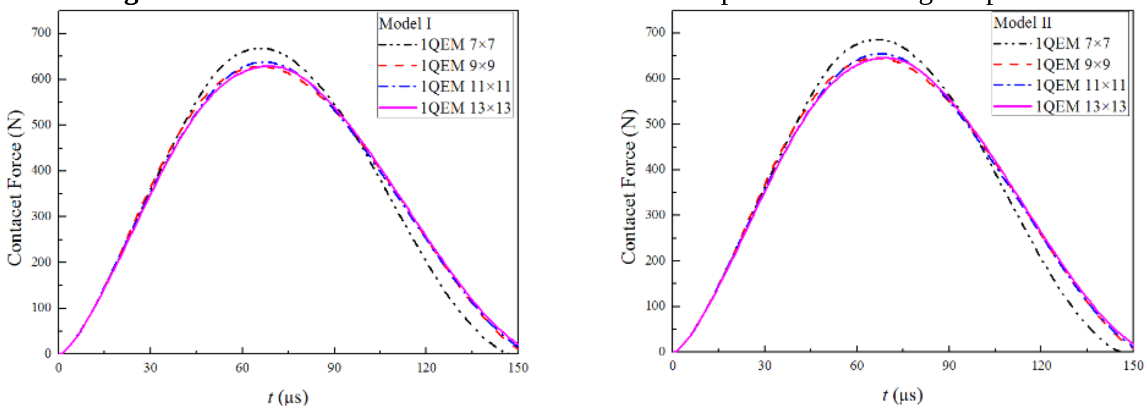
In this paper, a numerical example in the literatures[9,12] is selected and compared with the results in the literature to verify the quadrature element model of the low-speed impact response of the FG-CNTRC rectangular plate. In the calculation example selected in this paper, the relevant parameters are: the density of metal spherical impactor is 7960kg/m^3, the radius is 10mm, and the impact speed is 1.0 m/s. In the FG-CNTRC rectangular plate, the arrangement of carbon nanotubes is FG-X distribution. The length and width of the rectangular plate are both 0.5m, and the width thickness ratio is 30. The impact position of the impactor is the center point of the plate.

When the QEM is used for calculation, the FG-CNTRC rectangular plate is divided into one quadrature element. Four different schemes of integration points are adopted in the quadrature element, namely 7×7,9×9,11×11,13×13. As mentioned before, for the impact load in the research system, two different models are used to deal with it, and the response under the two models is obtained respectively, so as to further prove the effectiveness of the calculation model. The analysis results are as follows:



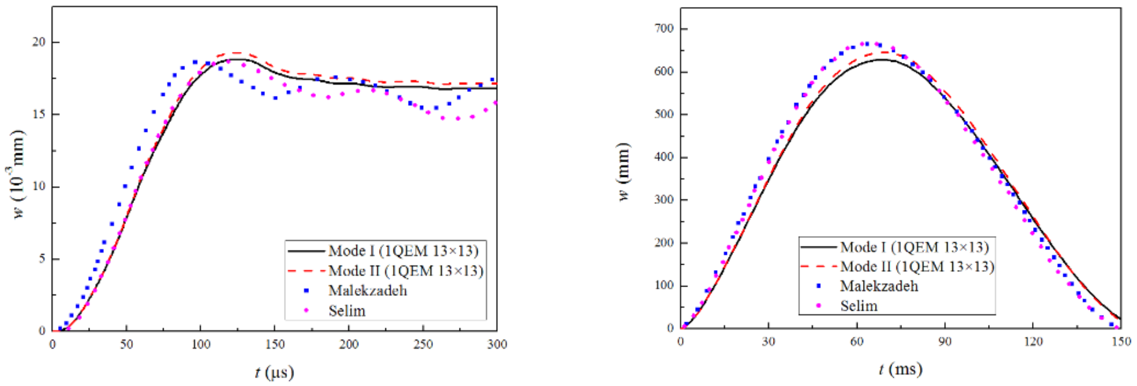
(a) Model I (b) Model II

Figure 2. The central deflection histories of the clamped FG-X rectangular plates.



(a) Model I (b) Model II

Figure 3. The contact force histories of the clamped FG-X rectangular plates.



(a) The central deflection (b) The contact force

Figure 4. The comparison of the numerical results from the two models and related literatures.

The time history curves of deflection and impact contact force calculated by the two models are given in Figure 2-4 respectively. The results show that: (1) with the increase of the number of numerical integration points, the calculation values of deflection and impact contact force improve rapidly; in order to get the ideal calculation results, more than 11×11 numerical integration points are needed. (2) The calculation results in this paper are in good agreement with those in references[9,12]. The calculated values of deflection and impact contact force in model II are slightly higher than those in model I, and the relative difference between the two maximum values is not more than 3%.

4. Conclusion

In this paper, the problem of single low-velocity impact at the center of FG-CNTRC rectangular plate is studied by using the weak form quadrature element method. Through the research of this paper:

- The weak form quadrature element model of dynamic response of FG-CNTRC rectangular plate under single low-velocity impact load is established.
- The efficiency and applicability of the weak form quadrature element method in the geometric nonlinear problems of FG-CNTRC rectangular plates and the load nonlinear problems of impact are verified.
- It provides a new theoretical calculation method and reference data for the research and application of FG-CNTRC plate in impact dynamic response.
- It provides a solid theoretical basis for the study of FG-CNTRC plates with different shapes, continuous impact and eccentric impact.

5. References

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