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A regenerative chatter observer analysis for micro-milling

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Abstract. The regenerative chatter in micro-milling that is employed to produce small products with complex 3D geometries and high precision is gradually getting attention from researchers in the field of manufacturing, mechatronics and control for its severe damage to the cutting tool and micro-milling stability. To facilitate developing observer-based control strategy for suppressing the regenerative chatter, the integrating an adopted observer proposed for nonlinear systems with time delay and a practical regenerative chatter model for micro-milling is presented in this paper. And the observer analysis is shown. Simulation results are given to validate the integration scheme.

1. Introduction

It is well-established in literature that the regenerative chatter known as self-excited vibrations in micro-milling can lead to unexpected surface finish, rapid tool wear, and severely unstable cutting process. For increasing chatter-free material removal rate to improve cutting precision, cutting efficiency and cutting quality, various modelling methods and control strategies have been developed for suppressing or controlling regenerative chatter in micro-milling.

There are three important control strategies for suppressing regenerative chatter which include passive chatter suppression techniques that are related to choosing preferred cutting conditions (e.g., spindle speed,depths of cut, and tooth path) and applying dynamic vibration absorbers in cutting process, active chatter control methods that are based on using active actuators and structural optimization for designing optimized machine tool structure forms and cutter geometry (e.g., helix and pitch).

In the pioneering works on chatter suppression in micro-milling, Shakeri and Samani [1] designed two linear and two nonlinear vibration absorbers composed of mass, spring and dashpot elements in two intersecting directions by combing the second-order differential equations for illustrating the dynamics of micro-milling, and the impactions of tool run-out, cutting velocity and micro cutter edge radius. As a result, the effectiveness and the optimized parameters of the absorbers were shown. And it implied that the nonlinear absorbers were inclined to provide more chatter-free zone than the linear ones. Liu et al. [2] completed a study for controlling regenerative chatter in micro-milling by employing a novel simultaneous time-frequency control strategy. And it indicated from the simulations and experimental results that the tool chatter could be negated.To obtain the critical depths of cut and asymptotic spindle speed for regenerative chatter stability lobes in micro-milling with process damping effect, an improved mathematical model was formulated by Wang et al. [3]. According to the proposed mathematical model validated by comparison with prior works and experimental results, unstable chatter can be avoided by specifying the depths of cut and spindle speed for micro-milling. The active vibration control system integrating the novel design of spindle with piezoelectric stack actuators and the dynamic control strategy based on H2 optimal control scheme has been investigated by Aggogeri et al. [4], ensuring an appreciable decrement of the un-stability of chatter in micro-milling. The structural optimization of a 3-

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axis miniaturized machine tool for micro-milling was presented by [5], which offered a promising methodology for chatter isolation. The dynamic surface controller with prescribed performance has been derived for controlling regenerative chatter in micro-milling by adopting the active control scheme with piezoelectric actuators as control elements in [6]. And the accomplished proof and simulation results indicated the boundness of all signals of the controlled system.

During the past several years, there have been several formulas developed for demonstrating the dynamics of regenerative chatter in micro-milling. An analytical regenerative chatter model based on velocity and chip load dependent cutting coefficients was established for depicting the regenerative chatter system with one degree of freedom in high-speed micromilling of Ti6Al4V in [7], wherein the experiments were carried out to validate the effectiveness of the established chatter model. Another work [8] on modeling of regenerative chatter in high-speed micromilling of Ti6Al4V incorporated the regenerative chatter system with two degrees-of-freedom and the cutting forces with segmented velocity–chip load dependent cutting coefficients for describing the dynamics of chatter, and employed the experimental results to illustrate the effectiveness of the created model. A characteristic equation for predicating the stability of dynamic chatter system in high speed micromilling of Ti6Al4V was evolved by including the effects of instantaneous chip thickness with dynamic run-out in [9], time delay of regenerative chatter, frequency response functions at tool tip, and immersion dependent directional cutting coefficient matrix. The results pointed out that the proposed model yield reasonably good estimates of the regenerative chatter stability, suggesting it can be extended for the dynamic modeling of chatter. Jin and Altintas [10] presented a practical equation for depicting the dynamic regenerative chatter system in micro-milling with process damping effect depended on the velocity of chatter vibration.

The innovative works introduced above can be utilized to develop dynamic models and control schemes for the regenerative chatter in micro-milling. There are also a number of outstanding research works on the prediction and analysis of chatter stability in micro-milling, facilitating the chatter suppression. While the significant contributions of the works mentioned above focused on the modeling, suppressing, predicting and analyzing of the chatter, developing observers that can be applied to develop controllers for chatter suppression should be payed attention to.

This paper will adopt the observer for nonlinear systems with time delay effect from the work by Ghanes et al. [11] and integrate it with the dynamic regenerative chatter system constructed by Singh et al. [7] for micro-milling, expecting to devote to the perspective of exploiting observer-based control approach for the chatter suppression.

This paper is organized as follows for presenting the integration scheme. The regenerative chatter system demonstration, the observer derivation and the observation error analysis are presented in Section 2. And the simulation results that illustrate the performances of the integration scheme are described in Section 3. The paper is concluded with a brief summary for the presented integration scheme and a perspective for possible observer-based controller developments in Section 4.

2. Observer analysis for regenerative chatter system in micro-milling

2.1 Dynamics depiction of regenerative chatter system

The following mass-spring-damper system in [7] is borrowed to describe the dynamics of regenerative chatter vibration which is assumed to be the micro-tool vibration with the single degree of freedom in Y-direction as shown in Figure 1.

$$\left.\begin{array}{l} \ddot{y}(t)+2\zeta\omega_n\dot{y}(t)+\omega_n^2 y(t)=\frac{\omega_n^2}{k_y}\sum_{j=1}^{N}F_{yj} \\ F_{yj}=F_{tj}\sin\phi_j-F_{rj}\cos\phi_j \\ F_{tj}=K_t a h(\phi_j) \\ F_{rj}=K_r F_{tj} \\ h(\phi_j)=[y(t)-y(t-\tau)]\cos(\phi_j) \\ \phi_j(t)=\Omega t+(j-1)\phi_p \end{array}\right\} \qquad (1)$$

where y is the micro-tool vibration in Y-direction with $\dot{y}$ and $\ddot{y}$ as velocity and acceleration, N is the tooth number of the micro-mill, ζ, ω_n and k_y are the damping ratio, natural frequency and stiffness of the chatter system, F_{yj} is the cutting force caused by the jth tooth in Y-direction, F_{tj} and F_{rj} are the tangential force and radial force caused by the jth tooth, $\mathrm{K_t}$ and $\mathrm{K_r}$ are the tangential and radial cutting force coefficients, ϕ_j is the immersion angle, $h(\phi_j)$ is the dynamic chip load at ϕ_j, a is the axial depth of cut, Ω is the spindle speed, $\phi_p = 2\pi/N$ is the pitch angle of the micro-tool, t is the time parameter, τ is the time delay between two successive cuttings.

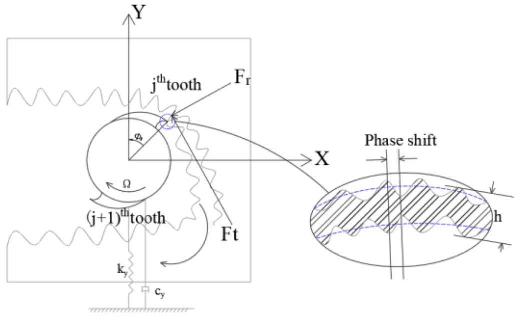

Figure 1. Lumped mass-spring-damper depiction of micro-milling [7]

The system (1) can be rewritten as

$$\Sigma: \begin{cases} \dot{y}(t) = Ay(t) + \Psi(y(t), y_\tau) \\ Y(t) = Cy(t) \end{cases} \tag{2}$$

with $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, $y_1 = \mathrm{y}$, $y_2 = \dot{\mathrm{y}}$, $y_\tau = \begin{bmatrix} y_{1,\tau} \\ y_{2,\tau} \end{bmatrix}$, $y_{1,\tau} = y_1(t-\tau)$, $y_{2,\tau} = y_2(t-\tau)$, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $C = [1 \quad 0]$, $\Psi(y(t), y_\tau) = \begin{bmatrix} 0 \\ \Psi_2(y_1, y_{1,\tau}, y_2) \end{bmatrix}$, $\Psi_2(\mathrm{y_1}, \mathrm{y_{1,\tau}}, \mathrm{y_2}) = -2\xi \mathrm{w_n} y_2 - \mathrm{w_n^2} y_1 + \frac{\mathrm{w_n^2}}{\mathrm{k_y}} \sum_{\mathrm{j=1}}^{\mathrm{N}} \mathrm{F_{yj}}$.

2.2 Observer design

The observer which is adopted from [11] to estimate the chatter vibration for the system (2) when it is local or globally Lipschitz on $y \in R^n$ with regard to y and y_τ is expressed as

$$\Sigma_\Lambda: \begin{cases} \dot{\hat{y}}(t) = A\hat{y}(t) + \Psi(\hat{y}(t), \hat{y}_\tau) - \theta \Delta_\theta^{-1} S^{-1} C^T \{\hat{Y}(t) - Y(t)\} \\ \hat{Y}(t) = C\hat{y}(t) \end{cases} \tag{3}$$

where $\Delta_\theta = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\theta} \end{bmatrix}$ with the design parameter $\theta > 1$, and S is the unique symmetric positive definite solution of the matrix equation

$$S + A^T S + SA - C^T C = 0 \tag{4}$$

The observation error is defined as

$$\varepsilon = \hat{y} - y \tag{5}$$

with

$$\dot{\varepsilon} = \{A - \theta \Delta_\theta^{-1} S^{-1} C^T C\}\varepsilon + \Psi(\hat{y}, \hat{y}_\tau) - \Psi(y, y_\tau) \tag{6}$$

(5) can be transformed into the following equation

$$e = \Delta_\theta \varepsilon \tag{7}$$

Considering the forms of A ,C and Δ_θ ,

$$\left. \begin{aligned} \Delta_\theta A \Delta_\theta^{-1} &= \theta A \\ C &= C\Delta_\theta^{-1} \\ \Delta_\theta \theta \Delta_\theta^{-1} &= \theta \end{aligned} \right\} \tag{8}$$

are obtained. Combing (6), (8) and

$$\left. \begin{aligned} \dot{e} &= \Delta_\theta \dot{\varepsilon} \\ \Delta_\theta^{-1} e &= \varepsilon \end{aligned} \right\} \tag{9}$$

yields the following expression

$$\dot{e} = \theta\{A - S^{-1} C^T C\}e + \Delta_\theta \Psi(\hat{y}, \hat{y}_\tau) - \Delta_\theta \Psi(y, y_\tau) \tag{10}$$

The Lyapunov–Krasovskii candidate functional is chosen as

$$V(e) = e^T S e + \int_{t-\tau}^{t} exp^{-\frac{\alpha}{2\tau}(t-\sigma)} e^T(\sigma)\, e(\sigma) d\sigma \tag{11}$$

where $\alpha > 0$ is a design parameter. Considering the time derivative of (11) , the definition of S, (4) , and (10), the following equation holds

$$\dot{V}(e) + \frac{\alpha}{2\tau}V(e) = -(\theta - \frac{\alpha}{2\tau})e^T Se - \theta e^T C^T Ce + e^T e - exp^{-\frac{\alpha}{2}} e_\tau^T e_\tau + 2e^T S\Delta_\theta\{\Psi(\hat{y},\hat{y}_\tau) - \Psi(y,y_\tau)\} \tag{12}$$

with $e(t-\tau) = e_\tau$. According to the following formulas

$$\left.\begin{array}{l} \|\Delta_\theta\{\Psi(\hat{y},\hat{y}_\tau) - \Psi(y,y_\tau)\}\| \le \kappa\|\Delta_\theta(\hat{y}-y)\| + \kappa\|\Delta_\theta(\hat{y}_\tau - y_\tau)\| \le \kappa\|e\| + \kappa\|e_\tau\| \\ \|(Se)^T\| \le \|e\|\|S\|_2 \le \lambda_2\|e\| \\ 2e^T S\Delta_\theta\{\Psi(\hat{y},\hat{y}_\tau) - \Psi(y,y_\tau)\} \le 2\lambda_2\kappa\|e\|^2 + 2\lambda_2\kappa\|e\|\|e_\tau\| \\ 2\lambda_2\kappa\|e\|\|e_\tau\| \le \lambda_2\kappa(\|e\|^2 + \|e_\tau\|^2) \\ \lambda_1 e^T e \le e^T Se \le \lambda_2 e^T e \end{array}\right\} \tag{13}$$

where λ_1 , λ_2 and $\|S\|_2$ are the minimum, maximum eigenvalues and 2-norm matrix of S, κ is a Lipschitz constant. (12) can be written as

$$\dot{V}(e) + \frac{\alpha}{2\tau}V(e) \le -[\lambda_1\left(\theta - \frac{\alpha}{2\tau}\right) - 1 - 3\lambda_2\kappa]\|e\|^2 - \left[exp^{-\frac{\alpha}{2}} - \lambda_2\kappa\right]\|e_\tau\|^2 \tag{14}$$

with $\left(\theta - \frac{\alpha}{2\tau}\right) > 0$.

Selecting θ and α satisfying $[\lambda_1\left(\theta - \frac{\alpha}{2\tau}\right) - 1 - 3\lambda_2\kappa] > 0$, then

$$\left.\begin{array}{l} \dot{V}(e) + \frac{\alpha}{2\tau}V(e) \le 0 \\ V(e) \le exp^{-\frac{\alpha}{2\tau}t} V(e_0) \end{array}\right\} \tag{15}$$

can be achieved.

2.3 Observation error analysis

Based on Lemma1 in [7], the following inequality can be proved

$$\left.\begin{array}{l} \int_{t-\tau}^{t} exp^{-\frac{\alpha}{2\tau}(t-\sigma)} e^T(\sigma)e(\sigma)d\sigma < \delta_M(\alpha,\tau) \max\limits_{s\in[t-\tau,t]} \|e(s)\|^2 \\ \delta_M(\alpha,\tau) = \frac{2\tau}{\alpha}\left(1 - exp^{-\frac{\alpha}{2}}\right) \end{array}\right\} \tag{16}$$

Then

$$\lambda_1\|e(t)\|^2 \le V\big(e(t)\big) \le \delta_M^*(\alpha,\tau) \max_{s\in[t-\tau,t]} \|e(s)\|^2 \tag{17}$$

$$\delta_M^*(\alpha,\tau) = \lambda_2 + \delta_M(\alpha,\tau) \tag{18}$$

are obtained. Consequently, the above inequality and (15) yield

$$\left.\begin{array}{l} \|e(t)\| \le K(\alpha,\tau) exp^{-\frac{\alpha}{4\tau}t} \max\limits_{s\in[-\tau,0]} \|e(s)\| \\ K(\alpha,\tau) = (\frac{\delta_M^*(\alpha,\tau)}{\lambda_1})^{1/2} \end{array}\right\} \tag{19}$$

which can be reformed as

$$\left.\begin{array}{l} \|e(t)\| \le K(\alpha,\tau)\, exp^{-\frac{\alpha}{4\tau}T_0}\, \delta_1 \le \delta_2, \forall t \ge T_0 \\ \max\limits_{s\in[-\tau,0]} \|e(s)\| \le \delta_1 \end{array}\right\} \tag{20}$$

with δ_1 and δ_2 are any possible positive constant. And the inequality

$$\|\varepsilon(t)\| \le \theta\, K(\alpha,\tau)\, exp^{-\frac{\alpha}{4\tau}t} \max_{s\in[-\tau,0]} \|\varepsilon(s)\| \tag{21}$$

can be derived. Then, the observation error is bounded can be guaranteed for the condition when the system (2) is local or globally Lipschitz on $y \in R^n$ with regard to y and y_τ.

3. Numerical simulations

The parameters for depicting the dynamic regenerative chatter are chosen as $N = 2$, tool diameter $T_d = 600\mu m$, $K_t = 40$, $K_r = 28$, $\zeta = 0.07$, $w_n = 240HZ$, $k_y = 0.06N/m$. The condition 1 for the chatter system and the observer system are $\Omega = 5\times10^5 rad/min$, $a = 20\mu m$, $\mathrm{y}(0) = 2\times10^{-6}\mathrm{m}$, $\dot{\mathrm{y}}(0) = 1.5\times10^{-5}\mathrm{m/s}$, $\hat{\mathrm{y}}(0) = 1.9\times10^{-6}\mathrm{m}$, $\dot{\hat{\mathrm{y}}}(0) = 1.5\times10^{-5}\mathrm{m/s}$ and $\theta = 7\times10^6$, for which the simulation results are shown in Figure 2 and Figure 3. The simulations were realized by MATLAB/Simulink.

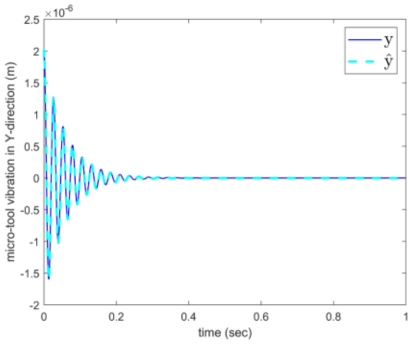

Figure 2. y and $\hat{y}$ for initial condition 1

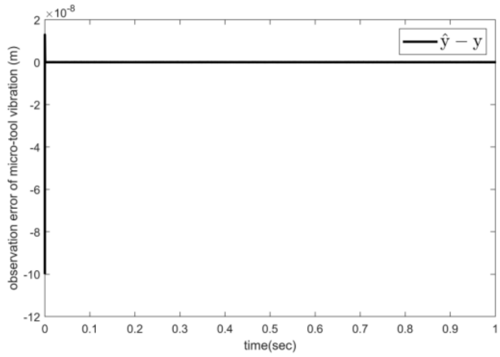

Figure 3. Observation error of micro-tool vibration for initial condition 1

The simulation results are displayed in Figure 4 and Figure 5 for the condition 2, where $a = 150\mu m$, $\Omega = 5 \times 10^3 rad/min$, $y(0) = 2.5 \times 10^{-6}$m and the other parameters are the same as that of the condition 1, and it is assumed that if y is larger than $1 \times 10^{-3} m$ which is the maximum tool vibration allowed during cutting, the cutting procedure will be terminated. For the condition 3 with the same parameters of the condition 2 except for $\hat{y}(0) = 2.5 \times 10^{-6}$m, the simulation result for observation error of micro-tool vibration is presented in Figure 6.

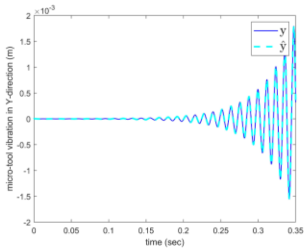

Figure 4. y and $\hat{y}$ for condition 2

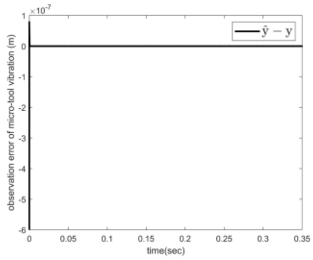

Figure 5. Observation error of micro-tool vibration for condition 2

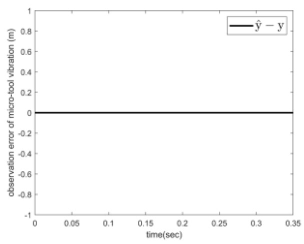

Figure 6. Observation error of micro-tool vibration for condition 3

It can be found from Figure 2- Figure 6 that the observer can lead to good estimate of chatter vibrations for different cutting conditions. Figure 5 and Figure 6 indicate the observation error of micro-tool vibration is affected by the initial value of approximated tool vibration.

4. Conclusions

In this paper, an integration scheme based on an observer for nonlinear systems with time delay and a regenerative chatter model for micro-milling was presented. And the observer analysis was provided. The simulation results showed the designed parameters can be used to estimate the chatter vibration when it satisfies the local or global Lipschitz conditions, showing possible applications in observer-based control strategy for suppressing regenerative chatter in micro-milling.

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