A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

March 2020

EPL, **129** (2020) 57001 www.epljournal.org
doi: 10.1209/0295-5075/129/57001

Thermal relaxation of magnons and phonons near resonance points in magnetic insulators

Q. XI[1], B. LIU[1], Z. SHI[2(a)], T. NAKAYAMA[1,3], J. ZHOU[1(b)] and B. LI[4]

[1] *Center for Phononics and Thermal Energy Science, China-EU Joint Lab for Nanophononics, School of Physics Science and Engineering, Tongji University - Shanghai 200092, China*
[2] *Shanghai Key Laboratory of Special Artificial Microstructure Materials and Technology, School of Physics Science and Engineering, Tongji University - Shanghai 200092, China*
[3] *Hokkaido University - Sapporo, Hokkaido 060-0826, Japan*
[4] *Paul M. Rady Department of Mechanical Engineering and Department of Physics, University of Colorado at Boulder - Boulder, CO 80309, USA*

received 27 January 2020; accepted in final form 17 March 2020
published online 30 March 2020

PACS 75.30.Ds – Spin waves
PACS 72.15.Lh – Relaxation times and mean free paths
PACS 05.30.Jp – Boson systems

Abstract – We theoretically investigate the energy relaxation rate of magnons and phonons near the resonance points to clarify the underlying mechanism of heat transport in ferromagnetic materials. We find that the simple two-temperature model is valid for the one-phonon/one-magnon process, as the rate of energy exchange between magnons and phonons is proportional to the temperature difference between them, and it is independent of temperature in the high-temperature limit. We also find that the magnon-phonon relaxation time due to the one-phonon/one-magnon interaction could be reduced to 1.48 μs at the resonance point by applying an external magnetic field. It means that the resonance effect plays a significant role in enhancing the total magnon-phonon energy exchange rate, apart from the higher-order interaction processes.

The heat transport in insulators is generally dominated by phonons which are the quanta of lattice vibrations. In magnetic insulators, the spin waves (magnons) could also act as heat carriers [1,2], as observed for yttrium iron garnet (YIG) [3], Nd_2CuO_4 [4], $RbMnF_3$ [5], and MnF_2 [6]. The notable contribution of magnons to thermal conductivity has also been discovered in the spin ladder compound $(Sr,Ca,La)_{14}Cu_{24}O_{41}$ [1,7,8]. In addition, the characteristics of heat transport due to magnons have been employed to probe spin excitations in InGaAs quantum dot system [9]. In recent years, the heat current due to spin excitations has also stimulated the field of spin caloritronics [10], resulting in the recent discovery of spin Seebeck effect [11], and spin Peltier effect [12].

Intuitively, the total thermal conductivity (κ_T) of magnetic insulators could be evaluated by a simple sum of the lattice thermal conductivity (κ_p) and magnon thermal conductivity (κ_m) contributions: $\kappa_T = \kappa_p + \kappa_m$. However, magnons will be scattered by phonons and vice versa, thus the interaction between phonons and magnons becomes relevant in determining the total thermal conductivity of magnetic materials. An effective strategy to evaluate the heat transport through both phonons and magnons, together with their interaction, is the two-temperature model [13,14]. It was first proposed by Sanders and Walton for the coupled magnon-phonon mode diffusion in the ferrimagnet YIG and the antiferromagnet MnF_2 [6]. In their paper, phonons and magnons are assumed to be excited to their equilibrium states with different effective temperatures, with the local energy exchange rate between these two carriers proportional to their temperature difference. They found that thermal conductivity is not determined only by κ_m and κ_p but also by the magnon-phonon relaxation time τ_{mp} [6]. Recently, Chen *et al.* have generalized a two-temperature model including the effect of the concurrent magnetization flow by assuming a constant magnon-phonon relaxation time [15]. Schreier *et al.* evaluated τ_{mp} through $\tau_{mp} \approx \frac{\hbar}{\alpha_G k_B T}$, where α_G is the Gilbert damping parameter [16]. However, there has

(a)E-mail: shizhong@tongji.edu.cn
(b)E-mail: zhoujunzhou@tongji.edu.cn

been no theoretical work to check the validity of the introduction of τ_{mp} as a temperature difference relaxation time which is different from the transport lifetime [17]. In addition, the exact relaxation time [16,18], together with the temperature and magnetic field dependence of τ_{mp} are also important.

We take YIG as a prototype for studying the magnon properties in ferromagnetism [19,20]. Sato [21] suggested that the thermal conductivity due to magnon proportional to T^2 could be greater than that of phonons to T^3 below 1 K by assuming that the mean free paths of phonons and magnons are limited by the boundary condition and comparable in magnitude. Later on, Daugless [22] experimentally observed a large decrease of thermal conductivity in YIG when applying a magnetic field of 2 T at 0.5 K. To explore the field and temperature dependence of thermal transport properties, many theoretical and experimental researches have been carried out. Those have revealed that the magnetoelastic coupling between phonons and magnons plays a significant role for governing the total thermal conductivity [14,23,24]. The strength of the coupling reaches a peak at the resonance condition, where the one-phonon/one-magnon process with the phonon and magnon of same frequency and wave vector becomes relevant [25,26].

In this letter, we focus our attention on the resonance behavior of the one-phonon/one-magnon interaction. We derive a simple formula for the rate of energy exchange between magnons and phonons in analogy with the two-temperature model for electron-phonon system [27]. We present the physical conditions to set up the two-temperature model based on the one-phonon/one-magnon interaction process. The field dependence of the temperature relaxation rate and the relaxation time corresponding to the one-phonon/one-magnon interaction is also obtained.

The magnetic Hamiltonian of YIG consists of dipolar, exchange interactions between spins, and the Zeeman splitting due to the external magnet field (H) along the z-direction [28]:

$$H_{\text{mag}} = \frac{\mu_0 (g\mu_{\text{B}})^2}{2}\sum_{i\neq j}\frac{|\boldsymbol{r}_{ij}|^2\boldsymbol{S}_i\cdot\boldsymbol{S}_j - 3(\boldsymbol{r}_{ij}\cdot\boldsymbol{S}_i)(\boldsymbol{r}_{ij}\cdot\boldsymbol{S}_j)}{|\boldsymbol{r}_{ij}|^5} - J\sum_{i\neq j}\boldsymbol{S}_i\cdot\boldsymbol{S}_j - g\mu_B H\sum_i S_i^z, \tag{1}$$

where μ_0 is the vacuum permeability, μ_B is the Bohr magneton, g is the g-factor, J is the exchange integral. The spin $\boldsymbol{S}_i = \boldsymbol{S}(\boldsymbol{r}_i)$ locates on the site $\boldsymbol{r}_i$ with $S = |S_i| = a_0^3 M_s/g\mu_B$, where a_0 is the unit cell lattice constant, M_s is the saturation magnetization density, and $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$. By employing the Holstein-Primakoff transformation, the quantized Hamiltonian for spin excitations could be expressed as [28]

$$H_{\text{mag}} = \sum_{\boldsymbol{k}} A_{\boldsymbol{k}} a_{\boldsymbol{k}}^\dagger a_{\boldsymbol{k}} + \frac{1}{2}(B_{\boldsymbol{k}} a_{-\boldsymbol{k}} a_{\boldsymbol{k}} + B_{\boldsymbol{k}}^* a_{\boldsymbol{k}}^\dagger a_{-\boldsymbol{k}}^\dagger), \tag{2}$$

with

$$\begin{aligned}\frac{A_{\boldsymbol{k}}}{\hbar} &= D\mathcal{F}_{\boldsymbol{k}} + \gamma\mu_0 H + \frac{\gamma\mu_0 M_s \sin^2\theta_{\boldsymbol{k}}}{2},\\ \frac{B_{\boldsymbol{k}}}{\hbar} &= \frac{\gamma\mu_0 M_s\sin^2\theta_{\boldsymbol{k}}}{2} e^{-2i\phi_{\boldsymbol{k}}},\end{aligned} \tag{3}$$

where $a_{\boldsymbol{k}}^\dagger (a_{\boldsymbol{k}})$ is the magnon creation (annihilation) operators with wave vector $\boldsymbol{k}$, $D = 2SJa_0^2$ the exchange stiffness, $\gamma = g\mu_B/\hbar$ the gyromagnetic ratio, $\theta_{\boldsymbol{k}} = \arccos(k_z/k)$ the polar angle between wave vector $\boldsymbol{k}$ and the magnetic field along the z-direction, and ϕ the azimuthal angle of $\boldsymbol{k}$ in the xy-plane. In the long-wavelength limit, the form factor $\mathcal{F}_{\boldsymbol{k}} \approx k^2$ [28]. Equation (2) could be diagonalized by using the Bogoliubov transformation

$$\begin{bmatrix} a_{\boldsymbol{k}} \\ a_{-\boldsymbol{k}}^\dagger\end{bmatrix} = \begin{bmatrix} u_{\boldsymbol{k}} & -v_{\boldsymbol{k}} \\ -v_{\boldsymbol{k}}^* & u_{\boldsymbol{k}}\end{bmatrix}\begin{bmatrix}\alpha_{\boldsymbol{k}} \\ \alpha_{-\boldsymbol{k}}^\dagger\end{bmatrix}, \tag{4}$$

with

$$u_{\boldsymbol{k}} = \sqrt{\frac{A_{\boldsymbol{k}} + \hbar\omega_{\boldsymbol{k}}}{2\hbar\omega_{\boldsymbol{k}}}},\quad v_{\boldsymbol{k}} = \sqrt{\frac{A_{\boldsymbol{k}} - \hbar\omega_{\boldsymbol{k}}}{2\hbar\omega_{\boldsymbol{k}}}} e^{2i\phi_{\boldsymbol{k}}}. \tag{5}$$

The Hamiltonian is then simplified to

$$H_{\text{mag}} = \sum_{\boldsymbol{k}} \hbar\omega_{\boldsymbol{k}}\alpha_{\boldsymbol{k}}^\dagger\alpha_{\boldsymbol{k}}, \tag{6}$$

and the dispersion relation for bulk magnons in the long-wavelength limit is

$$\omega_{\boldsymbol{k}} = \sqrt{Dk^2 + \gamma\mu_0 H}\sqrt{Dk^2 + \gamma\mu_0(H + M_s\sin^2\theta_{\boldsymbol{k}})}. \tag{7}$$

The Hamiltonian for one-phonon/one-magnon interaction process has also been derived by Flebus *et al.* [28]:

$$\begin{aligned} H_{\text{int}} &= \hbar n B_\perp\left(\frac{\gamma\hbar^2}{4M_s\bar{\rho}}\right)^{\frac{1}{2}}\sum_{\boldsymbol{k},\lambda} k\omega_{\boldsymbol{k}\lambda}^{-\frac{1}{2}} e^{-i\phi} a_{\boldsymbol{k}}(c_{-\boldsymbol{k}\lambda} + c_{\boldsymbol{k}\lambda}^\dagger)\\ &\quad\times(-i\delta_{\lambda1}\cos2\theta_{\boldsymbol{k}} + i\delta_{\lambda2}\cos\theta_{\boldsymbol{k}} - \delta_{\lambda3}\sin2\theta_{\boldsymbol{k}}) + \text{H.c.},\end{aligned} \tag{8}$$

where $n = 1/a_0^3$ is the number density of the magnetic particles in the system, $B_\perp$ the magnetoelastic constants, $\bar{\rho}$ the average mass density, $c_{\boldsymbol{k}\lambda}^\dagger(c_{\boldsymbol{k}\lambda})$ the phonon creation (annihilation) operators with wave vector $\boldsymbol{k}$. δ is the Kronecker delta, and $\lambda = 1, 2$ labels the transverse acoustic (TA) phonon, $\lambda = 3$ labels the longitudinal acoustic (LA) phonon. Under Debye approximation, the phonon dispersion relation is $\omega_{\boldsymbol{k}\lambda} = C_\lambda|\boldsymbol{k}|$. Following the procedure of Bogoliubov transformation, eq. (8) could be expressed in terms of the magnon quasiparticles operators $\alpha_{\boldsymbol{k}}^\dagger (\alpha_{\boldsymbol{k}})$:

$$\begin{aligned} H_{\text{int}} &= \hbar n B_\perp\left(\frac{\gamma\hbar^2}{4M_s\bar{\rho}}\right)^{\frac{1}{2}}\sum_{\boldsymbol{k},\lambda} k\omega_{\boldsymbol{k}\lambda}^{-\frac{1}{2}} e^{-i\phi}(u_{\boldsymbol{k}}\alpha_{\boldsymbol{k}} - v_{\boldsymbol{k}}\alpha_{-\boldsymbol{k}}^\dagger)\\ &\quad\times(c_{-\boldsymbol{k}\lambda} + c_{\boldsymbol{k}\lambda}^\dagger)(-i\delta_{\lambda1}\cos2\theta_{\boldsymbol{k}} + i\delta_{\lambda2}\cos\theta_{\boldsymbol{k}}\\ &\quad-\delta_{\lambda3}\sin2\theta_{\boldsymbol{k}}) + \text{H.c.}\end{aligned} \tag{9}$$

$$\frac{\partial n_{\rm m}(\boldsymbol{k})}{\partial t} = -\frac{2\pi}{\hbar^2}\sum_{\lambda}|M_{\boldsymbol{k},\boldsymbol{k}\lambda}|^2\delta(\omega_{\rm m}-\omega_{\rm p})\times\{n_{\rm m}(\boldsymbol{k})[1+n_{\rm p}(\boldsymbol{k},\lambda)]-[1+n_{\rm m}(\boldsymbol{k})]n_{\rm p}(\boldsymbol{k}\lambda)\}, \tag{10a}$$

$$\frac{\partial n_{\rm p}(\boldsymbol{k}\lambda)}{\partial t} = -\frac{2\pi}{\hbar^2}|M_{\boldsymbol{k},\boldsymbol{k}\lambda}|^2\delta(\omega_{\rm m}-\omega_{\rm p})\times\{n_{\rm p}(\boldsymbol{k}\lambda)[1+n_{\rm m}(\boldsymbol{k})]-[1+n_{\rm p}(\boldsymbol{k}\lambda)]n_{\rm m}(\boldsymbol{k})\}, \tag{10b}$$

$$\frac{\partial E_{\rm m}}{\partial t} = -\frac{\partial E_{\rm p}}{\partial t} = -\frac{2\pi}{\hbar^2}\sum_{\boldsymbol{k}\lambda}\hbar\omega_{\rm m}(\boldsymbol{k})|M_{\boldsymbol{k},\boldsymbol{k}\lambda}|^2\delta(\omega_{\rm m}-\omega_{\rm p})[n_{\rm m}(\boldsymbol{k})-n_{\rm p}(\boldsymbol{k}\lambda)]. \tag{11}$$

$$\begin{aligned} n_{\rm m}(\omega)-n_{\rm p}(\omega) = {}& \frac{z^2e^z}{(e^z-1)^2}\frac{k_B}{\hbar\omega}(T_{\rm m}-T_{\rm p})+\frac{1}{3!}\left[\frac{3e^zz^4}{2(e^z-1)^2}-\frac{3e^{2z}z^5}{(e^z-1)^3}+\frac{3e^zz^5}{2(e^z-1)^2}\right.\\ &\left.+\frac{3e^{3z}z^6}{2(e^z-1)^4}-\frac{3e^{2z}z^6}{2(e^z-1)^3}+\frac{e^zz^6}{4(e^z-1)^2}\right]\left(\frac{k_B}{\hbar\omega}\right)^3(T_{\rm m}-T_{\rm p})^3+\ldots, \end{aligned} \tag{12}$$

The decay rate of the magnon and phonon distribution functions $n_{\rm m}(\boldsymbol{k})$ and $n_{\rm p}(\boldsymbol{k}\lambda)$ are

see eq. (10a) *and eq.* (10b) *above*

where $|M_{\boldsymbol{k},\boldsymbol{k}\lambda}|^2 = \frac{\hbar^4n^2B_\perp^2\gamma}{4M_s\bar{\rho}}k^2\omega_{\boldsymbol{k}\lambda}^{-1}\beta_\lambda$, $\beta_1 = |u_{\boldsymbol{k}}+v_{\boldsymbol{k}}|^2\cdot\cos^2 2\theta_{\boldsymbol{k}}$, $\beta_2 = |u_{\boldsymbol{k}}+v_{\boldsymbol{k}}|^2\cos^2\theta_{\boldsymbol{k}}$, $\beta_3 = |u_{\boldsymbol{k}}-v_{\boldsymbol{k}}|^2\sin^2\cdot 2\theta_{\boldsymbol{k}}$. In this model, we assumed that other collision processes such as phonon-phonon interaction and magnon-magnon interaction are strong enough to keep the local equilibrium, then the distribution functions $n_{\rm m}(\boldsymbol{k})$ and $n_{\rm p}(\boldsymbol{k}\lambda)$ can be replaced by the equilibrium ones $\{\exp[\hbar\omega(\boldsymbol{k})/k_BT_{\rm m}]-1\}^{-1}$ and $\exp[\hbar\omega(\boldsymbol{k}\lambda)/k_BT_{\rm p}]-1\}^{-1}$ where magnon and phonon temperatures are noted as $T_{\rm m}$ and $T_{\rm p}$, respectively.

The energies of magnons and phonons are $E_{\rm m} = \sum_{\boldsymbol{k}}\hbar\omega_{\boldsymbol{k}}n_{\rm m}(\boldsymbol{k})$ and $E_{\rm p} = \sum_{\boldsymbol{k}\lambda}\hbar\omega_{\boldsymbol{k}\lambda}n_{\rm p}(\boldsymbol{k}\lambda)$, thus the changing rate of energy becomes

see eq. (11) *above*

A Taylor expansion of $n_{\rm m}(\omega)-n_{\rm p}(\omega)$ in terms of $T_{\rm m}-T_{\rm p}$ is

see eq. (12) *above*

where $z=\hbar\omega/k_BT$ and $T=(T_{\rm m}+T_{\rm p})/2$. Since the first term in the right-hand side of eq. (12) is much larger than other higher-order terms, only the linear term is enough in the evaluation. Therefore, the energy changing rate is linearly dependent on the temperature difference, and the temperature changing rate becomes

$$\frac{\partial T_{\rm m}}{\partial t} = g_{\rm mp}(T)(T_{\rm p}-T_{\rm m}), \tag{13a}$$

$$\frac{\partial T_{\rm p}}{\partial t} = g_{\rm pm}(T)(T_{\rm m}-T_{\rm p}), \tag{13b}$$

Table 1: Parameters of magnetoelastic coupling in YIG [28].

	Symbol	Value	Unit
Lattice constant	a_0	12.376	Å
Average mass density	$\bar{\rho}$	5.17×10^3	Kg m^{-3}
Gyromagnetic constant	γ	$2\pi\times28$	GHz T^{-1}
Saturation magnetization	μ_0M_s	0.2439	T
Exchange stiffness	D	7.7×10^{-6}	m^2 s^{-1}
Magnetoelastic coupling	$B_\perp$	$2\pi\times1988$	GHz
TA phonon velocity	$c_{1,2}$	3.9×10^3	m s^{-1}
LA phonon velocity	c_3	7.2×10^3	m s^{-1}

with

$$g_{\rm mp}(T) = \frac{\frac{2\pi k_B}{\hbar^2}\sum\limits_{\boldsymbol{k}\lambda}|M_{\boldsymbol{k},\boldsymbol{k}\lambda}|^2\delta(\omega_{\rm m}-\omega_{\rm p})\frac{z^2e^z}{(e^z-1)^2}}{C_{\rm m}(T)}, \tag{14a}$$

$$g_{\rm pm}(T) = \frac{\frac{2\pi k_B}{\hbar^2}\sum\limits_{\boldsymbol{k}\lambda}|M_{\boldsymbol{k},\boldsymbol{k}\lambda}|^2\delta(\omega_{\rm m}-\omega_{\rm p})\frac{z^2e^z}{(e^z-1)^2}}{C_{\rm p}(T)}. \tag{14b}$$

At the same time, the temperature difference between $T_{\rm m}$ and $T_{\rm p}$ will decay exponentially, that

$$\frac{\partial}{\partial t}\Delta T = -\frac{\Delta T}{\tau_{\rm mp}(T)}, \tag{15}$$

where $\Delta T = T_{\rm m}-T_{\rm p}$ and $\tau_{\rm mp}(T) = [g_{\rm mp}(T)+g_{\rm pm}(T)]^{-1}$.

We present a numerical calculation of the relaxation rate $\tau_{\rm mp}^{-1}$ due to magnetoelastic coupling in YIG. The parameters used in our calculation are listed in table 1. Figure 1 shows the temperature dependence of relaxation rate in the absence of magnetic field. $\tau_{\rm mp}^{-1}$ decreases rapidly with the increasing of temperature and saturates above 100 K, which is mainly attributed to the temperature dependence

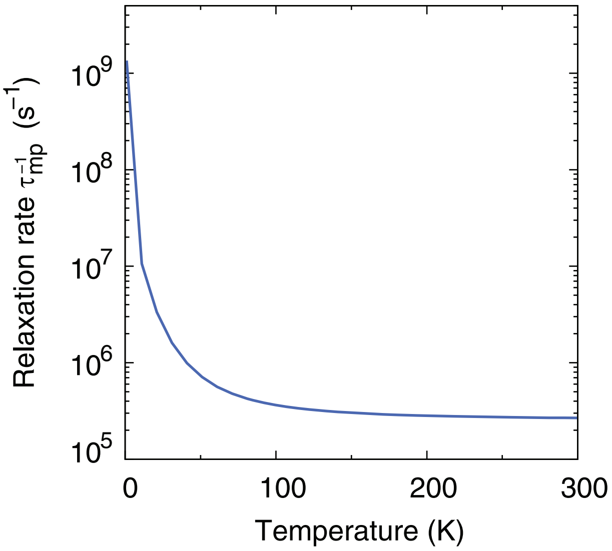

Fig. 1: The magnon-phonon relaxation rate as a function of temperature when $\mu_0 H = 0\,\mathrm{T}$.

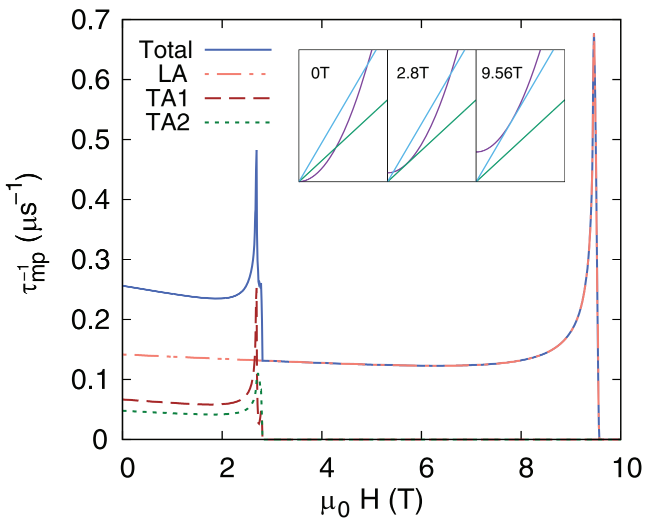

Fig. 2: The magnon-phonon relaxation rate as a function of external magnetic field at high-temperature limit. The inset shows the dispersion relation of acoustic phonons and magnon as an illustration, where $\theta_k = \pi/2$.

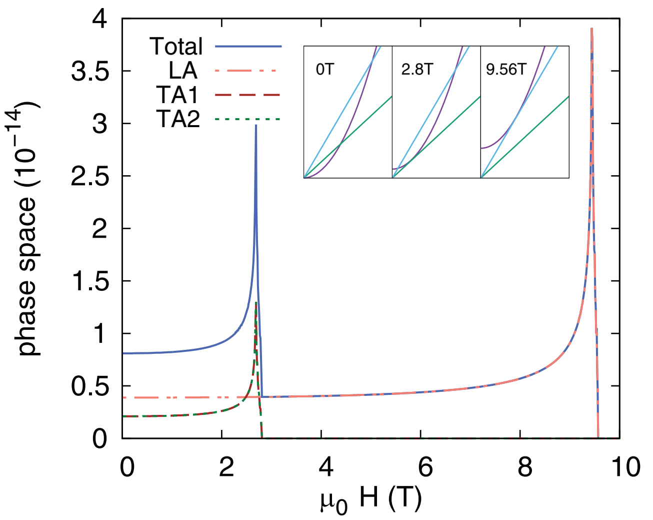

Fig. 3: The phase space as a function of external magnetic field. The inset shows the dispersion relation of acoustic phonons and magnons as an illustration, where $\theta_k = \pi/2$.

of specific heat. The specific heat of both phonon and magnon decreases with the decreasing of temperature, thus magnons and phonons are heated or cooled more efficiently. In the high-temperature limit ($\hbar\omega \ll k_B T$), $z^2 e^z/(e^z-1)^2 \to 1$, $C_\mathrm{m} \to N k_B$ and $C_\mathrm{p} \to 3 N k_B$, eq. (14) could be reduced to

$$g_\mathrm{mp}(T) = \frac{2\pi}{N\hbar^2}\sum_{\boldsymbol{k}\lambda} |M_{\boldsymbol{k},\boldsymbol{k}\lambda}|^2 \delta(\omega_\mathrm{m} - \omega_\mathrm{p}), \tag{16a}$$

$$g_\mathrm{pm}(T) = \frac{2\pi}{3N\hbar^2}\sum_{\boldsymbol{k}\lambda} |M_{\boldsymbol{k},\boldsymbol{k}\lambda}|^2 \delta(\omega_\mathrm{m} - \omega_\mathrm{p}), \tag{16b}$$

and it is obtained that $\tau_\mathrm{mp} \to 0.26\,\mu s^{-1}$ at high-temperature limit.

In the presence of magnetic field, the relaxation rate at high-temperature limit is plotted in fig. 2 as a function of the strength of magnetic field. The magnetic field could effectively shift the dispersion relation to higher energy. Thus the intersects of magnon and phonon dispersions which satisfy the energy conservation vary with the magnetic field. As a result, for individual phonon modes, the relaxation rate firstly decreases with the increasing of magnetic field, and then increases sharply near the critical magnetic field, $\mu_0 H = \frac{(\gamma D \mu_0 M_s \sin^2\theta - c_\lambda^2)^2}{4\gamma D c_\lambda^2}$. The sharp increasing of relaxation rate is attributed to the tangency of phonon dispersion and magnon dispersion, which maximizes the interaction phase space as shown in fig. 3, where the phase space is defined as $P_{mp} = \frac{1}{3N}\sum_{\boldsymbol{k}\lambda}\delta(\omega_\mathrm{m}(\boldsymbol{k}) - \omega_\mathrm{p}(\boldsymbol{k}\lambda))$. It is also noteworthy that one-magnon/one-phonon interaction is forbidden for TA phonon and LA phonon when $\mu_0 H > 2.807\,\mathrm{T}$ and $\mu_0 H > 9.567\,\mathrm{T}$, respectively. The magnetic field dependence of the relaxation rate is consistent with the observed enhancement of magnetic-field–dependent spin Seebeck effect voltage, which also originates from the one-magnon/one-phonon process [20,29].

Considering the scattering rate, magnon-phonon interactions of higher orders are dominant, mainly due to the relatively small phase space of one-phonon/one-magnon process [17], while at the tangency between magnon dispersion and phonon dispersion, the one-phonon/one-magnon process is non-negligible. Moreover, the magnon dispersion will be affected by magnon-magnon interaction at high temperature, so the temperature effect on magnon dispersion should also be considered at high temperature [19,30,31].

In summary, we have theoretically studied the one-phonon/one-magnon interaction in ferrimagnet YIG. It has been obtained that one-phonon/one-magnon interaction is dominant at low temperature below ∼50 K or at the resonant condition. It has also been verified that the two-temperature model is valid for one-phonon/one-magnon interaction. The magnon-phonon relaxation is found to be tunable by the external magnetic field. A maximum relaxation rate is obtained when the tangency of phonon dispersion and magnon dispersion occurs.

* * *

This work was supported by National Key R&D Program of China (No. 2017YFB0406004) and National Natural Science Foundation of China (No. 11890703 and No. 11774259).

REFERENCES

[1] Hess C., Büchner B., Ammerahl U., Colonescu L., Heidrich-Meisner F., Brenig W. and Revcolevschi A., *Phys. Rev. Lett.*, **90** (2003) 197002.

[2] Jin R., Onose Y., Tokura Y., Mandrus D., Dai P. and Sales B., *Phys. Rev. Lett.*, **91** (2003) 146601.

[3] Boona S. R. and Heremans J. P., *Phys. Rev. B*, **90** (2014) 064421.

[4] Li S. Y., Taillefer L., Wang C. H. and Chen X. H., *Phys. Rev. Lett.*, **95** (2005) 156603.

[5] Gustafson J. and Walker C. T., *Phys. Rev. B*, **8** (1973) 3309.

[6] Sanders D. J., *Phys. Rev. B*, **15** (1977) 1489.

[7] Hohensee G. T., Wilson R. B., Feser J. P. and Cahill D. G., *Phys. Rev. B*, **89** (2014) 024422.

[8] Sologubenko A., Gianno K., Ott H., Ammerahl U. and Revcolevschi A., *Phys. Rev. Lett.*, **84** (2000) 2714.

[9] Lorenz T., Hofmann M., Grüninger M., Freimuth A., Uhrig G., Dumm M. and Dressel M., *Nature*, **418** (2002) 614.

[10] Bauer G. E. W., Saitoh E. and van Wees B. J., *Nat. Mater.*, **11** (2012) 391.

[11] Xiao J., Bauer G. E., Uchida K.-c., Saitoh E., Maekawa S. *et al.*, *Phys. Rev. B*, **81** (2010) 214418.

[12] Flipse J., Bakker F., Slachter A., Dejene F. and Van Wees B., *Nat. Nanotechnol.*, **7** (2012) 166.

[13] Agrawal M., Vasyuchka V., Serga A., Karenowska A., Melkov G. and Hillebrands B., *Phys. Rev. Lett.*, **111** (2013) 107204.

[14] An K., Olsson K. S., Weathers A., Sullivan S., Chen X., Li X., Marshall L. G., Ma X., Klimovich N., Zhou J. *et al.*, *Phys. Rev. Lett.*, **117** (2016) 107202.

[15] Liao B., Zhou J. and Chen G., *Phys. Rev. Lett.*, **113** (2014) 025902.

[16] Schreier M., Kamra A., Weiler M., Xiao J., Bauer G. E. W., Gross R. and Goennenwein S. T. B., *Phys. Rev. B*, **88** (2013) 094410.

[17] Streib S., Vidal-Silva N., Shen K. and Bauer G. E. W., *Phys. Rev. B*, **99** (2019) 184442.

[18] Roschewsky N., Schreier M., Kamra A., Schade F., Ganzhorn K., Meyer S., Huebl H., Geprägs S., Gross R. and Goennenwein S. T., *Appl. Phys. Lett.*, **104** (2014) 202410.

[19] Princep A. J., Ewings R. A., Ward S., Tóth S., Dubs C., Prabhakaran D. and Boothroyd A. T., *npj Quantum Mater.*, **2** (2017) 63.

[20] Man H., Shi Z., Xu G., Xu Y., Chen X., Sullivan S., Zhou J., Xia K., Shi J. and Dai P., *Phys. Rev. B*, **96** (2017) 100406.

[21] Sato H., *Prog. Theor. Phys.*, **13** (1955) 119.

[22] Douglass R., *Phys. Rev.*, **129** (1963) 1132.

[23] Bhandari C. and Verma G., *Phys. Rev.*, **152** (1966) 731.

[24] Walton D., Rives J. and Khalid Q., *Phys. Rev. B*, **8** (1973) 1210.

[25] Kittel C., *Phys. Rev.*, **110** (1958) 836.

[26] Weber R., *Phys. Rev.*, **169** (1968) 451.

[27] Allen P. B., *Phys. Rev. Lett.*, **59** (1987) 1460.

[28] Flebus B., Shen K., Kikkawa T., Uchida K.-i., Qiu Z., Saitoh E., Duine R. A. and Bauer G. E., *Phys. Rev. B*, **95** (2017) 144420.

[29] Kikkawa T., Shen K., Flebus B., Duine R. A., Uchida K., Qiu Z., Bauer G. E. W. and Saitoh E., *Phys. Rev. Lett.*, **117** (2016) 207203.

[30] Barker J. and Bauer G. E. W., *Phys. Rev. Lett.*, **117** (2016) 217201.

[31] Shen K., *New J. Phys.*, **20** (2018) 043025.