



PAPER

Gupta-Feynman based Quantum Field Theory of Einstein's Gravity

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Abstract

This paper is an application to Einstein's gravity (EG) of the mathematics developed in (Plastino and Rocca 2018 *J. Phys. Commun.* **2**, 115029). We will quantize EG by appeal to the most general quantization approach, the Schwinger-Feynman variational principle, which is more appropriate and rigorous than the functional integral method, when we are in the presence of derivative couplings. We base our efforts on works by Suraj N. Gupta and Richard P. Feynman so as to undertake the construction of a Quantum Field Theory (QFT) of Einstein Gravity (EG). We explicitly use the Einstein Lagrangian elaborated by Gupta (Gupta, *Proc. Phys. Soc. A*, **65**, 161) but choose a new constraint for the theory that differs from Gupta's one. In this way, we avoid the problem of lack of unitarity for the S matrix that afflicts the procedures of Gupta and Feynman. Simultaneously, we significantly simplify the handling of constraints. This eliminates the need to appeal to ghosts for guaranteeing the unitarity of the theory. Our ensuing approach is obviously non-renormalizable. However, this inconvenience can be overcome by appealing to the mathematical theory developed by (Bollini *et al* *Int. J. of Theor. Phys.* **38**, 2315, Bollini and Rocca *Int. J. of Theor. Phys.* **43**, 1909, Bollini and Rocca *Int. J. of Theor. Phys.* **43**, 59, Bollini *et al*, *Int. J. of Theor. Phys.* **46**, 3030, Plastino and Rocca *J. Phys. Commun.* **2**, 115029). Such developments were founded in the works of Alexander Grothendieck (Grothendieck *Mem. Amer. Math. Soc.* **16**) and in the theory of Ultradistributions of Jose Sebastiao e Silva *Math. Ann.* **136**, 38) (also known as Ultrahyperfunctions). Based on these works, we have constructed a mathematical edifice, in a lapse of about 25 years, that is able to quantize non-renormalizable Field Theories (FT). Here we specialize this mathematical theory to treat the quantum field theory of Einstein's gravity (EG). Because we are using a Gupta-Feynman inspired EG Lagrangian, we are able to evade the intricacies of Yang-Mills theories.

1. Introduction

Quantifying Einstein gravity (EG) is still an open problem, a kind of holy grail for quantum field theory (QFT). The failure of some attempts in this direction have failed because i) they appeal to Rigged Hilbert Space (RHS) with undefined metric, ii) problems of non-unitarity, and also iii) non-renormalizability issues. Here we quantize EG by appeal to the most general quantization approach, the Schwinger-Feynman variational principle, which is more appropriate and rigorous than the functional integral method, when we are in the presence of derivative couplings.

Here we build up an unitary EG's QFT in the wake of related effort by Suraj N. Gupta [1]. We deviate from his work by using a different EG-constraint, facing then a problem similar to that posed by Quantum Electrodynamics (QED). In order to quantize the concomitant non-renormalizable variational problem we appeal to mathematics developed by Bollini *et al* [2–6], based upon the theory of Ultradistributions de J. Sebastiao e Silva (JSS) [7], also known as Ultrahyperfunctions. The above cited mathematics were specifically devised to quantify non-renormalizable field theories during 25 years, culminating in [6]. We consequently face a theory similar to QED, endowed with unitarity at all finite orders in the power expansion in G (gravitation

constant) of the EG Lagrangian. This was attempted without success first by Gupta and then by Feynman, in his celebrated Acta Physica Polonica paper [8].

Mathematically, quantizing a non-renormalizable field theory is tantamount to suitably defining the product of two distributions (a product in a ring with zero-divisors in configuration space), an old problem in functional theory tackled successfully in [2–6].

Remark that, in QFT, the problem of evaluating the product of distributions with coincident point singularities is related to the asymptotic behavior of loop integrals of propagators.

In references [2–5] it was demonstrated that it is possible to define a general convolution between the ultradistributions of JSS [7] (Ultrahyperfunctions). This convolution yields another Ultrahyperfunction. Therefore, we have a product in a ring with zero divisors. Such a ring is the space of distributions of exponential type, or ultradistributions of exponential type, obtained applying the anti-Fourier transform to the space of tempered ultradistributions or ultradistributions of exponential type.

We must clarify at this point that the ultrahyperfunctions are the generalization and extension to the complex plane of the Schwartz tempered distributions and the distributions of exponential type. That is, the tempered distributions and those of exponential type are a subset of the ultrahyperfunctions.

In our work we do not use counter-terms to get rid of infinities, because our convolutions are always finite. *We do not want counter-terms, since a non-renormalizable theory involves an infinite number of them.*

At the same time, we conserve all extant solutions to the problem of running coupling constants and the renormalization group. The convolution, once obtained, converts configuration space into a ring with zero-divisors. In it, one has now defined a product between the ring-elements. Thus, any unitary-causal-Lorentz invariant theory quantized in such a manner becomes predictive. The distinction between renormalizable on non-renormalizable QFT's becomes unnecessary now.

With our convolution, that uses Laurent's expansions in the parameter employed to define it, all finite constants of the convolutions become completely determined, eliminating arbitrary choices of finite constants. This is tantamount to eliminating all finite renormalizations of the theory. The independent term in the Laurent expansion give the convolution value. This translates to configuration space the product-operation in a ring with divisors of zero.

This paper is organized as follows:

- section 2 presents preliminary materials.
- section 3 is devoted to the QFT Lagrangian for EG
- In section 4 we quantize the ensuing theory.
- In section 5 the graviton's self-energy is evaluated up to second order.
- In section 6 we introduce axions into our picture and deal with the axions-gravitons interaction.
- In section 7 we calculate the graviton's self-energy in the presence of axions.
- In section 8 we evaluate, up to second order, the axion's self-energy.
- Finally, in section 9, some conclusions are drawn.

2. Preliminary materials

We appeal here to the most general quantification approach, Schwinger-Feynman variational principle [9], which is able to deal even with high order supersymmetric theories, as exemplified by [10, 11]. Such theories can not be quantized with the usual Dirac-brackets technique.

We introduce the action for a set of fields defined by

$$\mathcal{S}[\sigma(x), \sigma_0, \phi_A(x)] = \int_{\sigma_0}^{\sigma(x)} \mathcal{L}[\phi_A(\xi), \partial_\mu \phi_A(\xi), \xi] d\xi, \quad (2.1)$$

where $\sigma(x)$ if a space-like surface passing through the point x . σ_0 is that surface at the remote past, at which all field variations vanish. The Schwinger-Feynman variational principle dictates that

'Any Hermitian infinitesimal variation $\delta\mathcal{S}$ of the action induces a canonical transformation of the vector space in which the quantum system is defined, and the generator of this transformation is this same operator $\delta\mathcal{S}$ '.

Accordingly, the following equality holds:

$$\delta\phi_A = i[\delta\mathcal{S}, \phi_A]. \quad (2.2)$$

Thus, for a Poincare transformation we have

$$\delta\mathcal{S} = a^\mu \mathcal{P}_\mu + \frac{1}{2} a^{\mu\nu} \mathcal{M}_{\mu\nu}, \quad (2.3)$$

where the field variation is given by

$$\delta\phi_a = a^\mu \hat{P}_\mu \phi_A + \frac{1}{2} a^{\mu\nu} \hat{M}_{\mu\nu} \phi_A. \quad (2.4)$$

From (2.2) one gathers that

$$\partial_\mu \phi_A = i[\mathcal{P}_\mu, \phi_A]. \quad (2.5)$$

Specifically,

$$\partial_0 \phi_A = i[\mathcal{P}_0, \phi_A]. \quad (2.6)$$

This last result will be employed in quantizing EG.

3. The lagrangian of einstein's QFT

Our EG Lagrangian reads [1]

$$\mathcal{L}_G = \frac{1}{\kappa^2} R \sqrt{|g|} - \frac{1}{2} \eta_{\mu\nu} \partial_\alpha h^{\mu\alpha} \partial_\beta h^{\nu\beta}, \quad (3.1)$$

where $\eta^{\mu\nu} = \text{diag}(1, 1, 1, -1)$, $h^{\mu\nu} = \sqrt{|g|} g^{\mu\nu}$. The second term in equation (3.1) fixes the gauge. We apply now the linear approximation

$$h^{\mu\nu} = \eta^{\mu\nu} + \kappa \phi^{\mu\nu}, \quad (3.2)$$

where κ^2 is the gravitation's constant and $\phi^{\mu\nu}$ the graviton field. We write

$$\mathcal{L}_G = \mathcal{L}_L + \mathcal{L}_I, \quad (3.3)$$

where

$$\mathcal{L}_L = -\frac{1}{4} [\partial_\lambda \phi_{\mu\nu} \partial^\lambda \phi^{\mu\nu} - 2 \partial_\alpha \phi_{\mu\beta} \partial^\beta \phi^{\mu\alpha} + 2 \partial^\alpha \phi_{\mu\alpha} \partial_\beta \phi^{\mu\beta}], \quad (3.4)$$

and, up to 2nd order, one has [1]:

$$\mathcal{L}_I = -\frac{1}{2} \kappa \phi^{\mu\nu} \left[\frac{1}{2} \partial_\mu \phi^{\lambda\rho} \partial_\nu \phi_{\lambda\rho} + \partial_\lambda \phi_{\mu\beta} \partial^\beta \phi_\nu^\lambda - \partial_\lambda \phi_{\mu\rho} \partial^\lambda \phi_\nu^\rho \right], \quad (3.5)$$

having made use of the constraint

$$\phi_\mu^\mu = 0. \quad (3.6)$$

This constraint is required in order to satisfy gauge invariance [12] For the graviton we have then

$$\square \phi_{\mu\nu} = 0, \quad (3.7)$$

whose solution is

$$\phi_{\mu\nu} = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \left[\frac{a_{\mu\nu}(\vec{k})}{\sqrt{2k_0}} e^{ik_\mu x^\mu} + \frac{a_{\mu\nu}^+(\vec{k})}{\sqrt{2k_0}} e^{-ik_\mu x^\mu} \right] d^3k, \quad (3.8)$$

with $k_0 = |\vec{k}|$.

4. The quantization of the theory

We need some definitions. The energy-momentum tensor is

$$T_\rho^\lambda = \frac{\partial \mathcal{L}}{\partial \partial^\rho \phi^{\mu\nu}} \partial^\lambda \phi^{\mu\nu} - \delta_\rho^\lambda \mathcal{L}, \quad (4.1)$$

and the time-component of the four-momentum is

$$\mathcal{P}_0 = \int T_0^0 d^3x. \quad (4.2)$$

Using (3.4) we have

$$\begin{aligned} T_0^0 = \frac{1}{4} [& \partial_0 \phi_{\mu\nu} \partial^0 \phi^{\mu\nu} + \partial_j \phi_{\mu\nu} \partial^j \phi^{\mu\nu} - 2 \partial_\alpha \phi_{\mu 0} \partial^0 \phi^{\mu\alpha} - 2 \partial_\alpha \phi_{\mu j} \partial^j \phi^{\mu\alpha} \\ & + 2 \partial_\alpha \phi^{\mu\alpha} \partial_0 \phi_\mu^0 + 2 \partial_\alpha \phi^{\mu\alpha} \partial_j \phi_\mu^j]. \end{aligned} \quad (4.3)$$

Consequently,

$$\mathcal{P}_0 = \frac{1}{4} \int |\vec{k}| [a_{\mu\nu}(\vec{k}) a^{+\mu\nu}(\vec{k}) + a^{+\mu\nu}(\vec{k}) a_{\mu\nu}(\vec{k})] d^3k. \quad (4.4)$$

Appeal to (2.6) leads to

$$\begin{aligned} [\mathcal{P}_0, a_{\mu\nu}(\vec{k})] &= -k_0 a_{\mu\nu}(\vec{k}) \\ [\mathcal{P}_0, a^{+\mu\nu}(\vec{k})] &= k_0 a^{+\mu\nu}(\vec{k}). \end{aligned} \quad (4.5)$$

From the last relation in (4.5) one gathers that

$$|\vec{k}| a^{+\rho\lambda}(\vec{k}') = \frac{1}{2} \int |\vec{k}| [a_{\mu\nu}(\vec{k}), a^{+\rho\lambda}(\vec{k}')] a^{+\mu\nu}(\vec{k}) d^3k. \quad (4.6)$$

The solution of this integral equation is

$$[a_{\mu\nu}(\vec{k}), a^{+\rho\lambda}(\vec{k}')] = [\delta_\mu^\rho \delta_\nu^\lambda + \delta_\nu^\rho \delta_\mu^\lambda] \delta(\vec{k} - \vec{k}'). \quad (4.7)$$

As customary, the physical state $|\psi\rangle$ of the theory is defined via the equation

$$\phi_\mu^\mu |\psi\rangle = 0. \quad (4.8)$$

We use now the usual definition

$$\Delta_{\mu\nu}^{\rho\lambda}(x - y) = \langle 0 | T [\phi_{\mu\nu}(x) \phi^{\rho\lambda}(y)] | 0 \rangle. \quad (4.9)$$

The graviton's propagator then turns out to be

$$\Delta_{\mu\nu}^{\rho\lambda}(x - y) = \frac{i}{(2\pi)^4} (\delta_\mu^\rho \delta_\nu^\lambda + \delta_\nu^\rho \delta_\mu^\lambda) \int \frac{e^{ik_\mu(x^\mu - y^\mu)}}{k^2 - i0} d^4k. \quad (4.10)$$

As a consequence, we can write

$$\mathcal{P}_0 = \frac{1}{4} \int |\vec{k}| [a_{\mu\nu}(\vec{k}) a^{+\mu\nu}(\vec{k}') + a^{+\mu\nu}(\vec{k}') a_{\mu\nu}(\vec{k})] \delta(\vec{k} - \vec{k}') d^3k d^3k', \quad (4.11)$$

or

$$\mathcal{P}_0 = \frac{1}{4} \int |\vec{k}| [2a^{+\mu\nu}(\vec{k}') a_{\mu\nu}(\vec{k}) + \delta(\vec{k} - \vec{k}') a_{\mu\nu}(\vec{k})] \delta(\vec{k} - \vec{k}') d^3k d^3k'. \quad (4.12)$$

Thus, we obtain

$$\mathcal{P}_0 = \frac{1}{2} \int |\vec{k}| a^{+\mu\nu}(\vec{k}) a_{\mu\nu}(\vec{k}) d^3k, \quad (4.13)$$

where we have used the fact that the product of two deltas with the same argument vanishes [2], i.e., $\delta(\vec{k} - \vec{k}') \delta(\vec{k} - \vec{k}') = 0$. This illustrates the fact that using Ultrahyperfunctions is here equivalent to adopting the normal order in the definition of the time-component of the four-momentum

$$\mathcal{P}_0 = \frac{1}{4} \int |\vec{k}| : [a_{\mu\nu}(\vec{k}) a^{+\mu\nu}(\vec{k}) + a^{+\mu\nu}(\vec{k}) a_{\mu\nu}(\vec{k})] : d^3k. \quad (4.14)$$

Now, we must insist on the fact that the physical state should satisfy not only equation (4.8) but also the relation (see [1])

$$\partial_\mu \phi^{\mu\nu} |\psi\rangle = 0. \quad (4.15)$$

The ensuing theory is similar to the QED-one obtained via the quantization approach of Gupta-Bleuler. This implies that the theory is unitary for any finite perturbative order. In this theory only one type of graviton emerges, ϕ^{12} , while in Gupta's approach two kinds of graviton emerge. Obviously, this happens for a non-interacting theory, as remarked by Gupta.

4.1. Undesired effects if one does not use our constraint

If we do NOT use the constraint (4.8), we have

$$\mathcal{P}_0 = \frac{1}{2} \int |\vec{k}| \left[a^{+\mu\nu}(\vec{k}) a_{\mu\nu}(\vec{k}) - \frac{1}{2} a_\mu^{+\mu}(\vec{k}) a_\nu^\nu(\vec{k}) \right] d^3k, \quad (4.16)$$

and, appealing to the Schwinger-Feynman variational principle we find

$$\begin{aligned} & |\vec{k}| a_{\rho\lambda}^+(\vec{k}') \\ &= \frac{1}{2} \int |\vec{k}| \left\{ a^{+\mu\nu}(\vec{k}) [a_{\mu\nu}(\vec{k}), a_{\rho\lambda}^+(\vec{k}')] - \frac{1}{2} a_\mu^{+\mu}(\vec{k}) [a_\nu^\nu(\vec{k}), a_{\rho\lambda}^+(\vec{k}')] \right\} d^3k, \end{aligned} \quad (4.17)$$

whose solution is

$$[a_{\mu\nu}(\vec{k}), a_{\rho\lambda}^+(\vec{k}')] = [\eta_{\mu\rho} \eta_{\nu\lambda} + \eta_{\nu\rho} \eta_{\mu\lambda} - \eta_{\mu\nu} \eta_{\rho\lambda}] \delta(\vec{k} - \vec{k}'). \quad (4.18)$$

The above is the customary graviton's quantification, that leads to a theory whose S matrix is not unitary [1, 8].

5. The self energy of the graviton

To evaluate the graviton's self-energy (SF) we start with the interaction Hamiltonian \mathcal{H}_I . Note that the Lagrangian contains derivative interaction terms.

$$\mathcal{H}_I = \frac{\partial \mathcal{L}_I}{\partial \partial^0 \phi^{\mu\nu}} \partial^0 \phi^{\mu\nu} - \mathcal{L}_I. \quad (5.1)$$

A typical term reads

$$\Sigma_{G\alpha_1\alpha_2\alpha_3\alpha_4}(k) = k_{\alpha_1} k_{\alpha_2} (\rho - i0)^{-1} k_{\alpha_3} k_{\alpha_4} (\rho - i0)^{-1}. \quad (5.2)$$

where $\rho = k_1^2 + k_2^2 + k_3^2 - k_0^2$

In ν dimensions, the Fourier transform of (5.2) is

$$\begin{aligned} & \mathcal{F} \left\{ [k_{\alpha_1} k_{\alpha_2} (\rho - i0)^{-1} k_{\alpha_3} k_{\alpha_4} (\rho - i0)^{-1}]_\nu \right\} \\ &= -\frac{2^{2\nu-2}}{(2\pi)^\nu} \pi^\nu \left[\Gamma\left(\frac{\nu}{2}\right) \right]^2 \eta_{\alpha_1\alpha_2} \eta_{\alpha_3\alpha_4} (x + i0)^{-\nu} \\ &+ \frac{2^{2\nu-1}}{(2\pi)^\nu} \pi^\nu \Gamma\left(\frac{\nu}{2}\right) \Gamma\left(\frac{\nu}{2} + 1\right) (\eta_{\alpha_1\alpha_2} x_{\alpha_3} x_{\alpha_4} + \eta_{\alpha_3\alpha_4} x_{\alpha_1} x_{\alpha_2}) (x + i0)^{-\nu-1} \\ &- \frac{2^{2\nu}}{(2\pi)^\nu} \pi^\nu \left[\Gamma\left(\frac{\nu}{2} + 1\right) \right]^2 x_{\alpha_1} x_{\alpha_2} x_{\alpha_3} x_{\alpha_4} (x + i0)^{-\nu-2} \end{aligned} \quad (5.3)$$

where $x = x_1^2 + x_2^2 + x_3^2 - x_0^2$

Anti-transforming the above equation we have

$$\begin{aligned} & [k_{\alpha_1} k_{\alpha_2} (\rho - i0)^{-1} k_{\alpha_3} k_{\alpha_4} (\rho - i0)^{-1}]_\nu \\ &= i \frac{\pi^{\frac{\nu}{2}}}{4} \frac{\left[\Gamma\left(\frac{\nu}{2} + 1\right) \right]^2}{\Gamma(\nu + 2)} (\eta_{\alpha_1\alpha_2} \eta_{\alpha_3\alpha_4} + \eta_{\alpha_2\alpha_3} \eta_{\alpha_1\alpha_4} + \eta_{\alpha_2\alpha_4} \eta_{\alpha_1\alpha_3}) \Gamma\left(-\frac{\nu}{2}\right) (\rho - i0)^{\frac{\nu}{2}} \\ &+ \left\{ i \frac{\pi^{\frac{\nu}{2}}}{2} \frac{\Gamma\left(\frac{\nu}{2}\right) \Gamma\left(\frac{\nu}{2} + 1\right)}{\Gamma(\nu + 1)} (\eta_{\alpha_1\alpha_2} k_{\alpha_3} k_{\alpha_4} + \eta_{\alpha_3\alpha_4} k_{\alpha_1} k_{\alpha_2}) \right. \\ &- i \frac{\pi^{\frac{\nu}{2}}}{2} \frac{\left[\Gamma\left(\frac{\nu}{2} + 1\right) \right]^2}{\Gamma(\nu + 2)} (\eta_{\alpha_1\alpha_2} k_{\alpha_3} k_{\alpha_4} + \eta_{\alpha_1\alpha_3} k_{\alpha_2} k_{\alpha_4} + \eta_{\alpha_1\alpha_4} k_{\alpha_2} k_{\alpha_3} + \eta_{\alpha_3\alpha_4} k_{\alpha_1} k_{\alpha_2} \\ &+ \eta_{\alpha_2\alpha_3} k_{\alpha_1} k_{\alpha_4} + \eta_{\alpha_2\alpha_4} k_{\alpha_1} k_{\alpha_3}) \left. \right\} \Gamma\left(1 - \frac{\nu}{2}\right) (\rho - i0)^{\frac{\nu}{2}-1} \\ &+ i \pi^{\frac{\nu}{2}} \frac{\left[\Gamma\left(\frac{\nu}{2} + 1\right) \right]^2}{\Gamma(\nu + 2)} k_{\alpha_1} k_{\alpha_2} k_{\alpha_3} k_{\alpha_4} \Gamma\left(2 - \frac{\nu}{2}\right) (\rho - i0)^{\frac{\nu}{2}-2} \end{aligned} \quad (5.4)$$

5.1. Self-energy evaluation for $\nu = 4$

We appeal now to a ν -Laurent expansion and retain there the $\nu-4$ independent term [6]. Thus, we Laurent-expand (5.4) around $\nu = 4$ and find

$$\begin{aligned}
 & [k_{\alpha_1} k_{\alpha_2} (\rho - i0)^{-1} * k_{\alpha_3} k_{\alpha_4} (\rho - i0)^{-1}]_{\nu} \\
 = & i \frac{\pi^2}{\nu - 4} \left\{ \frac{1}{5!} (\eta_{\alpha_1 \alpha_2} \eta_{\alpha_3 \alpha_4} + \eta_{\alpha_2 \alpha_3} \eta_{\alpha_1 \alpha_4} + \eta_{\alpha_2 \alpha_4} \eta_{\alpha_1 \alpha_3}) \rho^2 \right. \\
 & - \left[\frac{2}{4!} (\eta_{\alpha_1 \alpha_2} k_{\alpha_3} k_{\alpha_4} + \eta_{\alpha_3 \alpha_4} k_{\alpha_1} k_{\alpha_2}) \right] \\
 & - \frac{1}{6!} (\eta_{\alpha_1 \alpha_2} k_{\alpha_3} k_{\alpha_4} + \eta_{\alpha_3 \alpha_4} k_{\alpha_1} k_{\alpha_2} + \eta_{\alpha_1 \alpha_3} k_{\alpha_2} k_{\alpha_4} + \eta_{\alpha_1 \alpha_4} k_{\alpha_2} k_{\alpha_3} \\
 & + \eta_{\alpha_2 \alpha_3} k_{\alpha_1} k_{\alpha_4} + \eta_{\alpha_2 \alpha_4} k_{\alpha_1} k_{\alpha_3}) \rho + \frac{8}{5!} k_{\alpha_1} k_{\alpha_2} k_{\alpha_3} k_{\alpha_4} \left. \right\} \\
 & - \frac{i\pi^2}{5!2} (\eta_{\alpha_1 \alpha_2} \eta_{\alpha_3 \alpha_4} + \eta_{\alpha_2 \alpha_3} \eta_{\alpha_1 \alpha_4} + \eta_{\alpha_2 \alpha_4} \eta_{\alpha_1 \alpha_3}) \left[\ln(\rho - i0) + \ln \pi + C - \frac{46}{15} \right] \rho^2 \\
 & + i \frac{\pi^2}{4!} \left\{ \left(\eta_{\alpha_1 \alpha_2} k_{\alpha_3} k_{\alpha_4} + \eta_{\alpha_3 \alpha_4} k_{\alpha_1} k_{\alpha_2} \right) \left[\ln(\rho - i0) + \ln \pi + C - \frac{8}{3} \right] \right. \\
 & - \frac{1}{24} (\eta_{\alpha_1 \alpha_2} k_{\alpha_3} k_{\alpha_4} + \eta_{\alpha_3 \alpha_4} k_{\alpha_1} k_{\alpha_2} + \eta_{\alpha_1 \alpha_3} k_{\alpha_2} k_{\alpha_4} + \eta_{\alpha_1 \alpha_4} k_{\alpha_2} k_{\alpha_3} + \eta_{\alpha_2 \alpha_3} k_{\alpha_1} k_{\alpha_4} \\
 & + \eta_{\alpha_2 \alpha_4} k_{\alpha_1} k_{\alpha_3}) \left[\ln(\rho - i0) + \ln \pi + 2C - \frac{101}{15} \right] \left. \right\} \rho \\
 & - i \frac{\pi^2}{30} k_{\alpha_1} k_{\alpha_2} k_{\alpha_3} k_{\alpha_4} \left[\ln(\rho - i0) + \ln \pi + C - \frac{47}{30} \right] + \sum_{n=1}^{\infty} a_n (\nu - 4)^n \}.
 \end{aligned} \tag{5.5}$$

The exact value of the convolution we are interested in, i.e., the left hand side of (5.5), is given by the independent term in the above expansion, as it is well-known. If the reader is not familiar with this situation, see for instance [6]. We reach

$$\begin{aligned}
 & \Sigma_{G\alpha_1 \alpha_2 \alpha_3 \alpha_4}(k) = k_{\alpha_1} k_{\alpha_2} (\rho - i0)^{-1} * k_{\alpha_3} k_{\alpha_4} (\rho - i0)^{-1} \\
 = & - \frac{i\pi^2}{5!2} (\eta_{\alpha_1 \alpha_2} \eta_{\alpha_3 \alpha_4} + \eta_{\alpha_2 \alpha_3} \eta_{\alpha_1 \alpha_4} + \eta_{\alpha_2 \alpha_4} \eta_{\alpha_1 \alpha_3}) \left[\ln(\rho - i0) + \ln \pi + C - \frac{46}{15} \right] \rho^2 \\
 & - i \frac{\pi^2}{4!} \left\{ (\eta_{\alpha_1 \alpha_2} k_{\alpha_3} k_{\alpha_4} + \eta_{\alpha_3 \alpha_4} k_{\alpha_1} k_{\alpha_2}) \left[\ln(\rho - i0) + \ln \pi + C - \frac{8}{3} \right] \right. \\
 & - \frac{1}{24} (\eta_{\alpha_1 \alpha_2} k_{\alpha_3} k_{\alpha_4} + \eta_{\alpha_3 \alpha_4} k_{\alpha_1} k_{\alpha_2} + \eta_{\alpha_1 \alpha_3} k_{\alpha_2} k_{\alpha_4} + \eta_{\alpha_1 \alpha_4} k_{\alpha_2} k_{\alpha_3} + \eta_{\alpha_2 \alpha_3} k_{\alpha_1} k_{\alpha_4} \\
 & + \eta_{\alpha_2 \alpha_4} k_{\alpha_1} k_{\alpha_3}) \left[\ln(\rho - i0) + \ln \pi + 2C - \frac{101}{15} \right] \left. \right\} \rho \\
 & - i \frac{\pi^2}{30} k_{\alpha_1} k_{\alpha_2} k_{\alpha_3} k_{\alpha_4} \left[\ln(\rho - i0) + \ln \pi + C - \frac{47}{30} \right] \}.
 \end{aligned} \tag{5.6}$$

We have to deal with 1296 diagrams of this kind.

6. Including axions into the picture

Axions are hypothetical elementary particles postulated by the Peccei-Quinn theory in 1977 to tackle the strong CP problem in quantum chromodynamics. If they exist and have low enough mass (within a certain range), they could be of interest as possible components of cold dark matter [13].

We include now a massive scalar field (axions) interacting with the graviton. The Lagrangian becomes

$$\mathcal{L}_{GM} = \frac{1}{\kappa^2} R \sqrt{|g|} - \frac{1}{2} \eta_{\mu\nu} \partial_\alpha h^{\mu\alpha} \partial_\beta h^{\nu\beta} - \frac{1}{2} [h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2]. \tag{6.1}$$

We can now recast the Lagrangian in the fashion

$$\mathcal{L}_{GM} = \mathcal{L}_L + \mathcal{L}_I + \mathcal{L}_{LM} + \mathcal{L}_{IM}, \tag{6.2}$$

where

$$\mathcal{L}_{LM} = -\frac{1}{2} [\partial_\mu \phi \partial^\mu \phi + m^2 \phi^2], \tag{6.3}$$

so that \mathcal{L}_{IM} becomes the Lagrangian for the axion-graviton action

$$\mathcal{L}_{IM} = -\frac{1}{2}\kappa\phi^{\mu\nu}\partial_\mu\phi\partial_\nu\phi. \quad (6.4)$$

The new term in the interaction Hamiltonian is

$$\mathcal{H}_{IM} = \frac{\partial\mathcal{L}_{IM}}{\partial\partial^0\phi}\partial^0\phi - \mathcal{L}_{IM}. \quad (6.5)$$

7. The complete self energy of the graviton

The presence of axions generates a new contribution to the graviton's self energy

$$\Sigma_{GM\mu\nu s}(k) = k_\mu k_r(\rho + m^2 - i0)^{-1}k_\nu k_s(\rho + m^2 - i0)^{-1}. \quad (7.1)$$

So as to compute it we appeal to the usual ν dimensional integral together with the Feynman-parameters denoted by the letter x . After a Wick rotation we obtain

$$[k_\mu k_r(\rho + m^2 - i0)^{-1}k_\nu k_s(\rho + m^2 - i0)^{-1}]_\nu = i \int_0^1 \int \frac{k_\mu k_r(p_\nu - k_\nu)(p_s - k_s)}{[(k - px)^2 + a]^2} d^\nu k dx, \quad (7.2)$$

where

$$a = p^2 x - p^2 x^2 + m^2. \quad (7.3)$$

After the variables-change $u = k - px$ we find

$$[k_\mu k_r(\rho + m^2 - i0)^{-1}k_\nu k_s(\rho + m^2 - i0)^{-1}]_\nu = i \int_0^1 \int \frac{f(u, x, \mu, r, \nu, s)}{(u^2 + a)^2} d^\nu u dx, \quad (7.4)$$

where

$$\begin{aligned} f(u, x, \mu, r, \nu, s) = & u_\mu u_r p_\nu p_s (1-x)^2 + u_\mu u_r u_\nu u_s - u_\mu u_s p_r p_\nu x(1-x) \\ & - u_\mu u_\nu p_r p_s x(1-x) - u_r u_s p_\mu p_\nu x(1-x) - u_r u_\nu p_\mu p_s x(1-x) \\ & + p_\mu p_r p_\nu p_s x^2 (1-x)^2 + u_\nu u_s p_\mu p_r x^2. \end{aligned} \quad (7.5)$$

After evaluation of the pertinent integrals we arrive at

$$\begin{aligned} & [k_\mu k_r(\rho + m^2 - i0)^{-1}k_\nu k_s(\rho + m^2 - i0)^{-1}]_\nu \\ & = i \frac{(\eta_{\mu r} k_\nu k_s + \eta_{\nu s} k_\mu k_r) m^{\nu-2} \pi^{\frac{\nu}{2}}}{8} \Gamma\left(1 - \frac{\nu}{2}\right) \\ & \times \left[F\left(1, 1 - \frac{\nu}{2}, \frac{3}{2}; -\frac{\rho}{4m^2}\right) + \frac{1}{3} F\left(1, 1 - \frac{\nu}{2}, \frac{5}{2}; -\frac{\rho}{4m^2}\right) \right] \\ & + i(\eta_{\mu r} \eta_{\nu s} + \eta_{\mu \nu} \eta_{rs} + \eta_{\mu s} \eta_{rv}) \frac{\pi^{\frac{\nu}{2}} m^\nu}{4} \Gamma\left(\frac{\nu}{2}\right) \Gamma\left(-\frac{\nu}{2}\right) F\left(1, -\frac{\nu}{2}, \frac{3}{2}; -\frac{\rho}{4m^2}\right) \\ & - i(\eta_{\mu s} k_r k_\nu + \eta_{\mu \nu} k_r k_s + \eta_{rs} k_\mu k_\nu + \eta_{rv} k_\mu k_s) \frac{m^{\nu-2} \pi^{\frac{\nu}{2}}}{48} \\ & \times \Gamma\left(1 - \frac{\nu}{2}\right) F\left(2, 1 - \frac{\nu}{2}, \frac{5}{2}; -\frac{\rho}{4m^2}\right) \\ & + i k_\mu k_r k_\nu k_s \frac{m^{\nu-4} \pi^{\frac{\nu}{2}}}{12} \Gamma\left(2 - \frac{\nu}{2}\right) F\left(2, 2 - \frac{\nu}{2}, \frac{5}{2}; -\frac{\rho}{4m^2}\right). \end{aligned} \quad (7.6)$$

7.1. Self-energy evaluation for $\nu = 4$

We need again a Laurent's expansion and face

$$\begin{aligned}
& [k_\mu k_r (\rho + m^2 - i0)^{-1} k_\nu k_s (\rho + m^2 - i0)^{-1}]_\nu \\
&= -i \frac{\pi^2}{\nu - 4} \left\{ m^2 (\eta_{\mu r} k_\nu k_s + \eta_{\nu s} k_\mu k_r) \left[\frac{1}{3} + \frac{1}{5} \frac{\rho}{4m^2} \right] \right. \\
&\quad - 2m^4 (\eta_{\mu r} \eta_{\nu s} + \eta_{\mu \nu} \eta_{rs} + \eta_{\mu s} \eta_{rv}) \\
&\quad \times \left[\frac{1}{8} + \frac{1}{6} \frac{\rho}{4m^2} + \frac{1}{15} \left(\frac{\rho}{4m^2} \right)^2 \right] \\
&\quad - \frac{m^2}{4m^2 + k^2 - i0} (\eta_{\mu s} k_r k_\nu + \eta_{\mu \nu} k_r k_s + \eta_{rs} k_\mu k_\nu + \eta_{rv} k_\mu k_s) \\
&\quad \times \frac{k^2 - m^2}{12} + \frac{m^2}{4} + \frac{k^2 - m^2}{30} \frac{\rho}{4m^2} - \frac{1}{6} k_\mu k_r k_\nu k_s \Big\} \\
&\quad + i \frac{m^2 \pi^2}{2} (\eta_{\mu r} k_\nu k_s + \eta_{\nu s} k_\mu k_r) \\
&\quad \times \left[\frac{1}{3} (\ln m^2 + \ln \pi + C - 1) + \frac{1}{5} \frac{\rho}{4m^2} (\ln m^2 + \ln \pi + C) \right] \\
&\quad + i \frac{m^2 \pi^2}{30} (\eta_{\mu r} k_\nu k_s + \eta_{\nu s} k_\mu k_r) \frac{\rho}{4m^2} \\
&\quad \times \left[F\left(1, 1, \frac{7}{2}; -\frac{\rho}{4m^2}\right) + \frac{1}{7} F\left(1, 1, \frac{9}{2}; -\frac{\rho}{4m^2}\right) \right] \\
&\quad + -i 2\pi^2 m^4 (\eta_{\mu r} \eta_{\nu s} + \eta_{\mu \nu} \eta_{rs} + \eta_{\mu s} \eta_{rv}) \\
&\times \left\{ \left[\frac{1}{8} - \frac{1}{6} \frac{\rho}{4m^2} - \frac{1}{15} \left(\frac{\rho}{4m^2} \right)^2 + \right] \right. \\
&\quad \times (\ln m^2 + \ln \pi + 1) - \frac{1}{2} \left[\frac{3}{32} - \frac{1}{3} \left(\frac{\rho}{4m^2} \right) \right] \Big\} \\
&\quad - i \frac{2\pi^2 m^4}{105} (\eta_{\mu r} \eta_{\nu s} + \eta_{\mu \nu} \eta_{rs} + \eta_{\mu s} \eta_{rv}) \left(\frac{\rho}{4m^2} \right)^3 F\left(1, 1, \frac{9}{2}; -\frac{\rho}{4m^2}\right) \\
&\quad - i \frac{\pi^2 m^2 (k^2 - m^2)}{12(4m^2 + k^2 - i0)} (\eta_{\mu s} k_r k_\nu + \eta_{\mu \nu} k_r k_s + \eta_{rs} k_\mu k_\nu + \eta_{rv} k_\mu k_s) \\
&\quad \times \left[\frac{1}{2} \left(\ln m^2 + \ln \pi + C - \frac{1}{4} \right) + \frac{1}{5} (\ln m^2 + \ln \pi + C) \frac{k^2}{4m^2} \right] \\
&\quad - i \frac{\pi^2 m^2}{8(4m^2 + k^2 - i0)} (\eta_{\mu s} k_r k_\nu + \eta_{\mu \nu} k_r k_s + \eta_{rs} k_\mu k_\nu + \eta_{rv} k_\mu k_s) \\
&\times m^2 \left[\left(\ln m^2 + \ln \pi + C - \frac{1}{4} \right) + \frac{k^2}{6} + \frac{k^2}{15} \frac{k^2}{4m^2} \right] \\
&\quad - i \frac{\pi^2 m^2}{10} (\eta_{\mu s} k_r k_\nu + \eta_{\mu \nu} k_r k_s + \eta_{rs} k_\mu k_\nu + \eta_{rv} k_\mu k_s) \\
&\quad \times \frac{k^2 - m^2}{21(4m^2 + k^2 - i0)} F\left(1, 1, \frac{9}{2}; -\frac{\rho}{4m^2}\right) \left(\frac{k^2}{4m^2} \right)^2 \\
&\quad - i \frac{\pi^2}{12} k_\mu k_r k_\nu k_s \left[(\ln m^2 + \ln \pi) + \frac{k^2}{4m^2 + k^2 - i0} \right] \\
&\quad - i \frac{\pi^2 m^2}{30} k_\mu k_r k_\nu k_s \frac{k^2 - m^2}{4m^2 + k^2 - i0} \frac{k^2}{4m^2} F\left(1, 1, \frac{7}{2}; -\frac{k^2}{4m^2}\right) \\
&\quad + \sum_{n=0}^{\infty} a_n (\nu - 4)^n. \tag{7.7}
\end{aligned}$$

Again, the exact result for our four-dimensional convolution becomes

$$\begin{aligned}
 \Sigma_{GM\mu\nu rs}(k) &= k_\mu k_r (\rho + m^2 - i0)^{-1*} k_\nu k_s (\rho + m^2 - i0)^{-1} \\
 &= i \frac{m^2 \pi^2}{2} (\eta_{\mu r} k_\nu k_s + \eta_{\nu s} k_\mu k_r) \\
 &\times \left[\frac{1}{3} (\ln m^2 + \ln \pi + C - 1) + \frac{1}{5} \frac{\rho}{4m^2} (\ln m^2 + \ln \pi + C) \right] \\
 &+ i \frac{m^2 \pi^2}{30} (\eta_{\mu r} k_\nu k_s + \eta_{\nu s} k_\mu k_r) \frac{\rho}{4m^2} \\
 &\times \left[F\left(1, 1, \frac{7}{2}; -\frac{\rho}{4m^2}\right) + \frac{1}{7} F\left(1, 1, \frac{9}{2}; -\frac{\rho}{4m^2}\right) \right] \\
 &+ -i 2\pi^2 m^4 (\eta_{\mu r} \eta_{\nu s} + \eta_{\mu \nu} \eta_{rs} + \eta_{\mu s} \eta_{\nu r}) \\
 &\times \left\{ \left[\frac{1}{8} - \frac{1}{6} \frac{\rho}{4m^2} - \frac{1}{15} \left(\frac{\rho}{4m^2}\right)^2 + \right] \right. \\
 &\times (\ln m^2 + \ln \pi + 1) - \frac{1}{2} \left[\frac{3}{32} - \frac{1}{3} \left(\frac{\rho}{4m^2}\right) \right] \left. \right\} \\
 &+ i \frac{2\pi^2 m^4}{105} (\eta_{\mu r} \eta_{\nu s} + \eta_{\mu \nu} \eta_{rs} + \eta_{\mu s} \eta_{\nu r}) \left(\frac{\rho}{4m^2}\right)^3 F\left(1, 1, \frac{9}{2}; -\frac{\rho}{4m^2}\right) \\
 &- i \frac{\pi^2 m^2 (k^2 - m^2)}{12(4m^2 + k^2 - i0)} (\eta_{\mu s} k_r k_\nu + \eta_{\mu \nu} k_r k_s + \eta_{rs} k_\mu k_\nu + \eta_{rv} k_\mu k_s) \\
 &\times \left[\frac{1}{2} \left(\ln m^2 + \ln \pi + C - \frac{1}{4} \right) + \frac{1}{5} (\ln m^2 + \ln \pi + C) \frac{k^2}{4m^2} \right] \\
 &- i \frac{\pi^2 m^2}{8(4m^2 + k^2 - i0)} (\eta_{\mu s} k_r k_\nu + \eta_{\mu \nu} k_r k_s + \eta_{rs} k_\mu k_\nu + \eta_{rv} k_\mu k_s) \\
 &\times m^2 \left[\left(\ln m^2 + \ln \pi + C - \frac{1}{4} \right) + \frac{k^2}{6} + \frac{k^2}{15} \frac{k^2}{4m^2} \right] \\
 &- i \frac{\pi^2 m^2}{10} (\eta_{\mu s} k_r k_\nu + \eta_{\mu \nu} k_r k_s + \eta_{rs} k_\mu k_\nu + \eta_{rv} k_\mu k_s) \\
 &\times \frac{k^2 - m^2}{21(4m^2 + k^2 - i0)} F\left(1, 1, \frac{9}{2}; -\frac{\rho}{4m^2}\right) \left(\frac{k^2}{4m^2}\right)^2 \\
 &- i \frac{\pi^2}{12} k_\mu k_r k_\nu k_s \left[(\ln m^2 + \ln \pi) - \frac{k^2}{4m^2 + k^2 - i0} \right] \\
 &- i \frac{\pi^2 m^2}{30} k_\mu k_r k_\nu k_s \frac{k^2 - m^2}{4m^2 + k^2 - i0} \frac{k^2}{4m^2} F\left(1, 1, \frac{7}{2}; -\frac{k^2}{4m^2}\right)
 \end{aligned} \tag{7.8}$$

We have to deal with 9 diagrams of this kind.

Accordingly, our desired self-energy total is a combination of $\Sigma_{G\alpha_1\alpha_2\alpha_3\alpha_4}(k)$ and $\Sigma_{GM\alpha_1\alpha_2\alpha_3\alpha_4}(k)$.

8. Self energy of the axion

Here the self-energy is

$$\Sigma^{\mu s}(k) = (\eta^{\mu r} \eta^{\nu s} + \eta^{\mu s} \eta^{\nu r}) k_\nu k_r (\rho + m^2 - i0)^{-1*} (\rho - i0)^{-1}. \tag{8.1}$$

In ν dimensions one has

$$[k_\nu k_r (\rho + m^2 - i0)^{-1*} (\rho - i0)^{-1}]_\nu = \int \frac{k_\nu k_r}{(k^2 + m^2 - i0)[(p - k)^2 - i0]} d^\nu k. \tag{8.2}$$

With the Feynman parameters used above we obtain

$$[k_\nu k_r (\rho + m^2 - i0)^{-1*} (\rho - i0)^{-1}]_\nu = i \int_0^1 \int \frac{k_\nu k_r}{[(k - px)^2 + a]^2} d^\nu k dx, \tag{8.3}$$

where

$$a = (p^2 + m^2)x - p^2 x^2. \tag{8.4}$$

We evaluate the integral (8.3) and find

$$\begin{aligned}
 & [k_\nu k_r (\rho + m^2 - i0)^{-1*} (\rho - i0)^{-1}]_\nu \\
 &= i \frac{\eta_{\nu r} m^{\nu-2} \pi^{\frac{\nu}{2}}}{\nu} \Gamma\left(1 - \frac{\nu}{2}\right) F\left(1, 1 - \frac{\nu}{2}, \frac{\nu}{2} + 1; -\frac{\rho}{m^2}\right) \\
 &+ \frac{2ik_\nu k_r m^{\nu-4} \pi^{\frac{\nu}{2}}}{\nu + 2} \Gamma\left(2 - \frac{\nu}{2}\right) F\left(1, 2 - \frac{\nu}{2}, \frac{\nu}{2} + 2; -\frac{\rho}{m^2}\right).
 \end{aligned} \tag{8.5}$$

8.1. Self-energy evaluation for $\nu = 4$

Once again, we Laurent-expand, this time (8.5) around $\nu = 4$, encountering

$$\begin{aligned}
 & [k_\nu k_r (\rho + m^2 - i0)^{-1*} (\rho - i0)^{-1}]_\nu \\
 &= i\pi^2 \left\{ \frac{1}{\nu - 4} \left(\frac{\eta_{\nu r} m^2}{2} - 2k_\nu k_r \right) \right. \\
 &+ \frac{\eta_{\nu r} m^2}{4} \left[\left(1 + \frac{1}{3} \frac{\rho}{m^2} \right) \left(\ln m^2 + \ln \pi + C - \frac{1}{2} \right) \right. \\
 &- \left. \left(1 + \frac{1}{9} \frac{\rho}{m^2} \right) \right] - \frac{k_\nu k_r}{3} \left(\ln m^2 + \ln \pi + C - \frac{1}{2} \right) \\
 &+ \frac{1}{4} \left(\frac{\rho}{m^2} \right) \left[\frac{\eta_{\nu r} m^2}{12} \frac{\rho}{m^2} - \frac{k_\nu k_r}{3} \right] F\left(1, 1, 5; -\frac{\rho}{m^2}\right) \\
 &\left. + \sum_{n=1}^{\infty} a_n (\nu - 4)^n \right\}
 \end{aligned} \tag{8.6}$$

The ν -independent term gives the exact convolution result we are looking for:

$$\begin{aligned}
 \Sigma_{\nu r}(k) &= k_\nu k_r (\rho + m^2 - i0)^{-1*} (\rho - i0)^{-1} \\
 &= i\pi^2 \left\{ \frac{\eta_{\nu r} m^2}{4} \left[\left(1 + \frac{1}{3} \frac{\rho}{m^2} \right) \left(\ln m^2 + \ln \pi + C - \frac{1}{2} \right) \right. \right. \\
 &- \left. \left(1 + \frac{1}{9} \frac{\rho}{m^2} \right) \right] - \frac{k_\nu k_r}{3} \left(\ln m^2 + \ln \pi + C - \frac{1}{2} \right) \\
 &\left. + \frac{1}{4} \left(\frac{\rho}{m^2} \right) \left[\frac{\eta_{\nu r} m^2}{12} \frac{\rho}{m^2} - \frac{k_\nu k_r}{3} \right] F\left(1, 1, 5; -\frac{\rho}{m^2}\right) \right\}
 \end{aligned} \tag{8.7}$$

9. Discussion

We have developed above the quantum field theory (QFT) of Einstein's gravity (EG), that is both unitary and finite. Our results critically depend on the use of a rather novel constraint the we introduced in defining the EG-Lagrangian. Laurent expansions were an indispensable tool for us.

In order to quantize the theory we appealed to the variational principle of Schwinger-Feynman's. This process leads to just one graviton type ϕ^{12} .

The underlying mathematics used in this effort has been developed by Bollini *et al* [2–6]. This mathematics is powerful enough so as to be able to quantize non-renormalizable field theories [2–6].

We have evaluated here in finite and exact fashion, for the first time as far as we know, several quantities:

- the graviton's self-energy in the EG-field. This requires full use of the theory of distributions, appealing to the possibility of creating with them a ring with divisors of zero.
- the above self-energy in the added presence of a massive scalar field (axions, for instance). Two types of diagram ensue: the original ones of the pure EG field plus the ones originated by the addition of a scalar field.
- The axion's self-energy.
- Our central results revolve around equations (5.6) and (7.8), corresponding to the graviton's self-energy, without and with the added presence of axions. Also, we give the axion's self-energy.

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