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$\Xi^- t$ quasibound state instead of $\Lambda\Lambda nn$ bound state*

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Abstract: The coupled $\Lambda\Lambda nn-\Xi^- pnn$ system was studied to investigate whether the inclusion of channel coupling is able to bind the $\Lambda\Lambda nn$ system. We use a separable potential three-body model of the coupled $\Lambda\Lambda nn-\Xi^- pnn$ system and a variational four-body calculation with realistic interactions. Our results exclude the possibility of a $\Lambda\Lambda nn$ bound state by a large margin. Instead, we found a $\Xi^- t$ quasibound state above the $\Lambda\Lambda nn$ threshold.

Keywords: baryon-baryon interactions, few-body systems, Faddeev equations

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1 Introduction

The possible existence of bound states of two neutrons and two Λ hyperons is a controversial subject. Recently, Bleser et al. [1] provided a new interpretation of the results of the BNL AGS-E906 experiment to produce and study double hypernuclei through a (K^-,K^+) reaction on ^{9}Be [2]. Following a suggestion made by Avraham Gal, they explored the conjecture that decays of a $^4_{\Lambda\Lambda}$n double hypernucleus may be responsible for some of the observed structures in the correlated $\pi^- - \pi^-$ momenta. However, in a recent calculation using the stochastic variational method in a pionless effective field theory approach [3], it has been concluded that the $\Lambda\Lambda nn$ system is unbound by a large margin. We had previously come to the same conclusion [4] in a study of the uncoupled $\Lambda\Lambda nn$ system using local central Yukawa-type Malfliet-Tjon interactions reproducing the low-energy parameters and phase shifts of the nn system and the latest updates of the $n\Lambda$ and $\Lambda\Lambda$ Nijmegen ESC08c potentials. It is important to note that to create a $\Lambda\Lambda nn$ bound state the four particles must coincide simultaneously because the system does not contain two- or three-body subsystem bound states. Thus, the probability of this event occurring is rather small.

In this study, we take the calculation one step further by considering the coupled $\Lambda\Lambda nn-\Xi^- pnn$ system to investigate whether the inclusion of channel coupling is able to bind the $\Lambda\Lambda nn$ system. If this were not the case, we would study whether there could be a $\Xi^- pnn$ sharp resonance or quasibound state above the $\Lambda\Lambda nn$ threshold. In the $\Lambda\Lambda nn-\Xi^- pnn$ system, the effect of channel coupling arises from the process $\Lambda\Lambda\to\Xi N$ process in the two-body channel $(i,j)=(0,0)$. The channel ΞN can be realized in two ways, $\Xi^0 n$ or $\Xi^- p$; however, if one restricts the calculation to S waves, the subchannel $\Xi^0 n$ cannot contribute, since three neutrons cannot exist with a symmetric space wave function. Thus, only the subchannel $\Xi^- p$ will contribute.[4)]

We present two different approaches. First, we address a three-body model $\Lambda\Lambda(nn)-\Xi^- p(nn)$, where the dineutron (nn) is treated as a particle of isospin 1 and spin 0. All the two-body interactions are assumed to be simple Yamaguchi separable potentials. This allows us to search for solutions in the real axis, bound states, and complex plane, resonances, and quasibound states. Later, we perform a variational four-body calculation with realistic local two-body interactions, which are necessarily restricted to energies in the real axis.

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4) We have explicitly checked that the $\Xi^0 nnn$ system is unbound by a large margin due to the mixed symmetry nature of the spin wave function of the three neutrons, what requires a mixed symmetry radial wave function. We have calculated the binding energy of Λnnn and $\Xi^0 nnn$ states with our variational method, obtaining a result that it is always above threshold. On the other hand, this is reasonable because if the mixed symmetric radial wave function of the three neutrons would not penalize the interaction, the ^{4}H $\equiv pnnn$ would be bound in nature due to the stronger pn interaction. However, no ^{4}H positive parity level has ever been reported.

2 The $\Lambda\Lambda(nn)-\Xi^- p(nn)$ three-body model

In this model, we treat the dineutron (nn) as an elementary particle with mass $m_{(nn)}=2m_n$, isospin 1, and spin 0 with two-body interactions given by Yamaguchi separable potentials [5]. This is based on the model proposed in Ref. [6] to search for resonances of the $\Lambda\Lambda N-\Xi NN$ system. Replacing one of the nucleons in the lower and upper channels by a dineutron, $N\to(nn)$ equations of Ref. [6] are similar to those of this study. The differences originate from the fact that in the $\Lambda\Lambda N-\Xi NN$ system, two of the three particles in the upper channel are identical, while in the $\Lambda\Lambda(nn)-\Xi^- p(nn)$ system, the three particles in the upper channel are different.

2.1 Three-body equations

We take the dineutron (nn) as particle 1. In the lower channel, the two Λ's are particles 2 and 3 while in the upper channel, particles 2 and 3 are the Ξ^- and p, respectively. Following the graphical method of Ref. [7], the equations of the $\Lambda\Lambda(nn)-\Xi^- p(nn)$ system are as follows,

$$
\begin{aligned}
\langle 1|T_1\rangle =& 2\langle 1|t_1^{\Lambda\Lambda}|1\rangle\langle 1|3\rangle G_0(3)\langle 3|T_3\rangle \\
&+\langle 1|t_1^{\Lambda\Lambda-\Xi^- p}|1\rangle\langle 1|2\rangle G_0(2)\langle 2|U_2\rangle \\
&+\langle 1|t_1^{\Lambda\Lambda-\Xi^- p}|1\rangle\langle 1|3\rangle G_0(3)\langle 3|U_3\rangle, \\
\langle 3|T_3\rangle =& -\langle 3|t_3^{(nn)\Lambda}|3\rangle\langle 2|3\rangle G_0(3)\langle 3|T_3\rangle \\
&+\langle 3|t_3^{(nn)\Lambda}|3\rangle\langle 3|1\rangle G_0(1)\langle 1|T_1\rangle, \\
\langle 1|U_1\rangle =& \langle 1|t_1^{\Xi^- p}|1\rangle\langle 1|2\rangle G_0(2)\langle 2|U_2\rangle \\
&+\langle 1|t_1^{\Xi^- p}|1\rangle\langle 1|3\rangle G_0(3)\langle 3|U_3\rangle \\
&+2\langle 1|t_1^{\Xi^- p-\Lambda\Lambda}|1\rangle\langle 1|3\rangle G_0(3)\langle 3|T_3\rangle, \\
\langle 2|U_2\rangle =& \langle 2|t_2^{(nn)p}|2\rangle\langle 2|3\rangle G_0(3)\langle 3|U_3\rangle \\
&+\langle 2|t_2^{(nn)p}|2\rangle\langle 2|1\rangle G_0(1)\langle 1|U_1\rangle, \\
\langle 3|U_3\rangle =& \langle 3|t_3^{(nn)\Xi^-}|3\rangle\langle 3|2\rangle G_0(2)\langle 2|U_2\rangle \\
&+\langle 3|t_3^{(nn)\Xi^-}|3\rangle\langle 3|1\rangle G_0(1)\langle 1|U_1\rangle.
\end{aligned} \tag{1}
$$

For all the uncoupled interactions, we assume separable potentials of the form,

$$V_i^\rho = g_i^\rho\rangle\lambda_i^\rho\langle g_i^\rho, \tag{2}$$

such that the two-body t–matrices are,

$$t_i^\rho = g_i^\rho\rangle\tau_i^\rho\langle g_i^\rho, \tag{3}$$

with

$$\tau_i^\rho = \frac{1}{(\lambda_i^\rho)^{-1}-\langle g_i^\rho|G_0(i)|g_i^\rho\rangle}. \tag{4}$$

In the case of the two-body channel responsible for the channel coupling, $(i,j)=(0,0)$, we use a separable interaction of the form,

$$V_1^{\rho\sigma} = g_1^\rho\rangle\lambda_1^{\rho-\sigma}\langle g_1^\sigma, \tag{5}$$

such that

$$t_1^{\rho-\sigma} = g_1^\rho\rangle\tau_1^{\rho-\sigma}\langle g_1^\sigma, \tag{6}$$

with

$$
\begin{aligned}
\tau_1^{\Lambda\Lambda} =& \frac{-(\lambda_1^{\Lambda\Lambda-\Xi^- p})^2G^{\Xi^- p}-\lambda_1^{\Lambda\Lambda}(1-\lambda_1^{\Xi^- p}G^{\Xi^- p})}{(\lambda_1^{\Lambda\Lambda-\Xi^- p})^2G^{\Lambda\Lambda}G^{\Xi^- p}-(1-\lambda_1^{\Lambda\Lambda}G^{\Lambda\Lambda})(1-\lambda_1^{\Xi^- p}G^{\Xi^- p})}, \\
\tau_1^{p\Xi^-} =& \frac{-(\lambda_1^{\Lambda\Lambda-\Xi^- p})^2G^{\Lambda\Lambda}-\lambda_1^{\Xi^- p}(1-\lambda_1^{\Lambda\Lambda}G^{\Lambda\Lambda})}{(\lambda_1^{\Lambda\Lambda-\Xi^- p})^2G^{\Lambda\Lambda}G^{\Xi^- p}-(1-\lambda_1^{\Lambda\Lambda}G^{\Lambda\Lambda})(1-\lambda_1^{\Xi^- p}G^{\Xi^- p})}, \\
\tau_1^{\Lambda\Lambda-\Xi^- p} =& \tau_1^{\Xi^- p-\Lambda\Lambda} = \frac{-\lambda_1^{\Lambda\Lambda-\Xi^- p}}{(\lambda_1^{\Lambda\Lambda-\Xi^- p})^2G^{\Lambda\Lambda}G^{\Xi^- p}-(1-\lambda_1^{\Lambda\Lambda}G^{\Lambda\Lambda})(1-\lambda_1^{\Xi^- p}G^{\Xi^- p})},
\end{aligned} \tag{7}
$$

and

$$
\begin{aligned}
G^{\Lambda\Lambda} =& \langle g_1^{\Lambda\Lambda}|G_0|g_1^{\Lambda\Lambda}\rangle, \\
G^{\Xi^- p} =& \langle g_1^{\Xi^- p}|G_0|g_1^{\Xi^- p}\rangle,
\end{aligned} \tag{8}
$$

where for simplicity we have redefined $\tau_1^{\Lambda\Lambda}\equiv\tau_1^{\Lambda\Lambda-\Lambda\Lambda}$, $\lambda_1^{\Lambda\Lambda}\equiv\lambda_1^{\Lambda\Lambda-\Lambda\Lambda}$, etc.

Using Eqs. (3) and (6) into the integral equations (1) and introducing the transformations $\langle i|T_i\rangle=\langle i|g_i^{\alpha_i}\rangle\langle i|X_i\rangle$ and $\langle i|U_i\rangle=\langle i|g_i^{\beta_i}\rangle\langle i|Y_i\rangle$, one obtains the one-dimensional integral equations

$$
\begin{aligned}
\langle 1|X_1\rangle =& 2\tau_1^{\Lambda\Lambda}\langle g_1^{\Lambda\Lambda}|1\rangle\langle 1|3\rangle G_0(3)\langle 3|g_3^{(nn)\Lambda}\rangle\langle 3|X_3\rangle \\
&+\tau_1^{\Lambda\Lambda-\Xi^- p}\langle g_1^{\Xi^- p}|1\rangle\langle 1|2\rangle G_0(2)\langle 2|g_2^{(nn)p}\rangle\langle 2|Y_2\rangle \\
&+\tau_1^{\Lambda\Lambda-\Xi^- p}\langle g_1^{\Xi^- p}|1\rangle\langle 1|3\rangle G_0(3)\langle 3|g_3^{(nn)\Xi^-}\rangle\langle 3|Y_3\rangle, \\
\langle 3|X_3\rangle =& -\tau_3^{(nn)\Lambda}\langle g_3^{(nn)\Lambda}|3\rangle\langle 2|3\rangle G_0(3)\langle 3|g_3^{(nn)\Lambda}\rangle\langle 3|X_3\rangle \\
&+\tau_3^{(nn)\Lambda}\langle g_3^{(nn)\Lambda}|3\rangle\langle 3|1\rangle G_0(1)\langle 1|g_1^{\Lambda\Lambda}\rangle\langle 1|X_1\rangle, \\
\langle 1|Y_1\rangle =& \tau_1^{\Xi^- p}\langle g_1^{\Xi^- p}|1\rangle\langle 1|2\rangle G_0(2)\langle 2|g_2^{(nn)p}\rangle\langle 2|Y_2\rangle \\
&+\tau_1^{\Xi^- p}\langle g_1^{\Xi^- p}|1\rangle\langle 1|3\rangle G_0(3)\langle 3|g_3^{(nn)\Xi^-}\rangle\langle 3|Y_3\rangle \\
&+2\tau_1^{\Xi^- p-\Lambda\Lambda}\langle g_1^{\Lambda\Lambda}|1\rangle\langle 1|3\rangle G_0(3)\langle 3|g_3^{(nn)\Lambda}\rangle\langle 3|X_3\rangle,
\end{aligned}
$$

$$\begin{aligned}\langle 2|Y_2\rangle =&\tau_2^{(nn)p}\langle g_2^{(nn)p}|2\rangle\langle 2|3\rangle G_0(3)\langle 3|g_3^{(nn)\Xi^-}\rangle\langle 3|Y_3\rangle\\&+\tau_2^{(nn)p}\langle g_2^{(nn)p}|2\rangle\langle 2|1\rangle G_0(1)\langle 1|g_1^{\Xi^-p}\rangle\langle 1|Y_1\rangle,\\\langle 3|Y_3\rangle =&\tau_3^{(nn)\Xi^-}\langle g_3^{(nn)\Xi^-}|3\rangle\langle 3|2\rangle G_0(2)\langle 2|g_2^{(nn)p}\rangle\langle 2|Y_2\rangle\\&+\tau_3^{(nn)\Xi^-}\langle g_3^{(nn)\Xi^-}|3\rangle\langle 3|1\rangle G_0(1)\langle 1|g_1^{\Xi^-p}\rangle\langle 1|Y_1\rangle.\end{aligned}\tag{9}$$

Eqs. (9) can be extended into the complex energy plane following the method of Ref. [8].

2.2 Two-body inputs

The $\Xi^- t\to\Lambda\Lambda nn$ process occurs with quantum numbers $(I,J)=(1,0)$, such that, since we restrict our calculation to S waves, the contributing two-body channels in our three-body model are: the $(nn)p$ channel $(i,j)=(1/2,1/2)$, the $(nn)\Lambda$ channel $(i,j)=(1,1/2)$, the $(nn)\Xi^-$ channel $(i,j)=(3/2,1/2)$, and the $\Lambda\Lambda-\Xi^- p$ channel $(i,j)=(0,0)$.

We use Yamaguchi form factors for the separable potentials of Eqs. (2) and (5), i.e.,

$$g(p)=\frac{1}{\alpha^2+p^2}.\tag{10}$$

Thus, for each uncoupled two-body channel, we have to fit the two parameters α and λ.

In the case of the $(nn)p$ subsystem with quantum numbers $(i,j)=(1/2,1/2)$, the tritium channel, for a given value of the range α the tritium binding energy (8.48 MeV) determines the strength λ through Eq. (4) as,

$$\lambda=\frac{1}{\langle g|G_0(E_B)|g\rangle},\tag{11}$$

with the value of α determined from the binding energy of ^{4}He (28.2 MeV) through the solution of the three-body system $(nn)pp$. The parameters of this model are given in Table 1.

Table 1. Parameters of different separable potential models for uncoupled partial waves: α (in fm^{-1}) and λ (in fm^{-2}).

Model	Subsystem	(i,j)	α	λ
	$(nn)p$	(1/2,1/2)	1.07	−0.5444
1	$(nn)\Lambda$	(1,1/2)	1.0	−0.1655
	$(nn)\Xi^-$	(3/2,1/2)		−0.2904
2	$(nn)\Lambda$	(1,1/2)	2.0	−1.1560
	$(nn)\Xi^-$	(3/2,1/2)		−1.7719
3	$(nn)\Lambda$	(1,1/2)	3.0	−3.9450
	$(nn)\Xi^-$	(3/2,1/2)		−5.4162

In the case of the $(nn)\Lambda$ subsystem with quantum numbers $(i,j)=(1,1/2)$, we fit the two parameters of the interaction to the ground state and spin-excitation energies of the ${}^4_\Lambda$H hypernucleus. This is considered as a three-body system $(nn)p\Lambda$ with quantum numbers $(I,J)=(1/2,0)$. For the $(nn)p$ subsystem, we use the interaction previously described and for the $p\Lambda$ the separable potentials for $j=0$ and $j=1$, constructed in Ref. [6]. Thus, for a given value of the range α, we fit the strength λ to the binding energy of ${}^4_\Lambda$H (10.52 MeV) [9]. To obtain the range α, we calculate the binding energy of the excited state $(I,J)=(1/2,1)$ (9.43 MeV) [9]. For α = 1, 2, and 3 fm^{-1}, the values 9.93, 9.81, and 9.77 MeV, were obtained, respectively, which are labeled as models 1, 2, and 3 in Table 1. The ${}^4_\Lambda$H spin excitation is difficult to fit, since it depends strongly on the tensor force arising from the transition $\Lambda N-\Sigma N$ [9–12]. Therefore, we did not consider larger values of α.

In the case of the $(nn)\Xi^-$ subsystem with quantum numbers $(i,j)=(3/2,1/2)$, we do not have any experimental information available to calibrate our separable potential model. However, in a couple of recent calculations [13, 14] based in the strangeness –2 Nijmegen ESC08c potential [15], a bound state is predicted with a binding energy of 2.89 MeV below the ΞNN threshold. Thus, we used this result to obtain the strength λ of the separable potential employing Eq. (11) and taking the range α equal to that of the $(nn)\Lambda$ subsystem. We provide in Table 1 the parameters corresponding to different models 1, 2, and 3.

In the case of the coupled $\Lambda\Lambda-\Xi^- p$ subsystem, we first use a recent lattice QCD study by the HAL QCD Collaboration [16] with almost physical quark masses (m_π = 146 MeV and m_K = 525 MeV). In this model, the H dibaryon was calculated through the coupled channel $\Lambda\Lambda-\Xi N$ system, appearing as a very sharp resonance just below the ΞN threshold [16, 17]. We have constructed a model, labeled A, yielding similar $\Lambda\Lambda$ and ΞN phase shifts as those in Ref. [16]. The parameters of this model are listed in Table 2. Furthermore, we also considered the separable potential model of the $\Lambda\Lambda-\Xi N$ system constructed in Ref. [6], which is based in the Nijmegen ESC08c potential [15]. This model is shown in Table 2 as model B. Naturally, in the $\Lambda\Lambda(nn)-\Xi^- p(nn)$ calculation, we use the parameters $\lambda_1^{\Lambda\Lambda-\Xi^- p}=\lambda_1^{\Lambda\Lambda-\Xi N}/\sqrt{2}$ and $\lambda_1^{\Xi^- p}=\lambda_1^{\Xi N}/2$.

Table 2. Parameters of two separable potential models for coupled partial wave $(i,j)=(0,0)$: $\alpha_1^{\Lambda\Lambda}$, $\alpha_1^{\Xi N}$ (in fm^{-1}), $\lambda_1^{\Lambda\Lambda}$, $\lambda_1^{\Xi N}$, and $\lambda_1^{\Lambda\Lambda-\Xi N}$ (in fm^{-2}).

Model	$\alpha_1^{\Lambda\Lambda}$	$\lambda_1^{\Lambda\Lambda}$	$\alpha_1^{\Xi N}$	$\lambda_1^{\Xi N}$	$\lambda_1^{\Lambda\Lambda-\Xi N}$
A	1.3465	−0.1390	1.1460	−0.3867	0.0977
B	1.25	−0.0959	4.287	1.302	1.243

2.3 Results

We show in Table 3 the energy eigenvalues of the two models A−B of the coupled $\Lambda\Lambda-\Xi N$ system and the three models 1−3 of the $(nn)\Lambda$ and $(nn)\Xi^-$ systems. We

Table 3. Energy eigenvalue of the $\Lambda\Lambda(nn)-\Xi^- p(nn)$ system (in MeV) measured with respect to the $\Xi^- pnn$ threshold. Results in parenthesis are those of uncoupled $\Xi^- t$ binding energy.

Model	1	2	3
A	$-12.80-i0.05$ (-12.73)	$-13.46-i0.04$ (-13.37)	$-13.52-i0.04$ (-13.43)
B	$-10.99-i0.06$ (-10.92)	$-11.04-i0.07$ (-10.93)	$-10.89-i0.07$ (-10.77)

also provide in parentheses the energy of the uncoupled $\Xi^- t$ system. This table indicates that the real part of the energy eigenvalue is slightly below the energy of the uncoupled $\Xi^- t$ system, and the imaginary part of the energy eigenvalue is roughly the difference between the uncoupled energy and the real part of the energy eigenvalue. Thus, this state appears as a narrow $\Xi^- t$ quasibound state decaying to $\Lambda\Lambda nn$. The reason for the narrow width of the $\Xi^- t$ state stems from the weakness of the $\Lambda\Lambda-\Xi N$ transition potential [15, 16], that on the other hand is also responsible for the H dibaryon appearing as a very sharp resonance just below the ΞN threshold [17].

Finally, in Table 4 we list the corresponding values of the $\Xi^- t$ scattering lengths of the two models A-B, which may be of use in the calculation of the energy shift of the atomic levels of the $\Xi^- t$ atom.

Table 4. $\Xi^- t$ scattering length (in fm). Results in parentheses depict those of uncoupled $\Xi^- t$ scattering length.

Model	1	2	3
A	$1.286-i0.005$ (1.293)	$1.030-i0.003$ (1.036)	$0.957-i0.003$ (0.963)
B	$1.551-i0.015$ (1.567)	$1.315-i0.016$ (1.339)	$1.268-i0.018$ (1.298)

3 The $\Lambda\Lambda nn$ and $\Xi^- pnn$ four-body problems

3.1 Four-body calculation

The four-body problem has been addressed by means of a generalized Gaussian variational (GGV) method [18, 19]. The nonrelativistic Hamiltonian is given by,

$$H=\sum_{i=1}^{4}\frac{\vec{p}_i^{\,2}}{2m_i}+\sum_{i<j=1}^{4}V_{ij}(\vec{r}_{ij}), \tag{12}$$

where $V(\vec{r}_{ij})$ is a local central two-body potential.

The four-body wave function is taken to be a sum over all allowed channels with well-defined symmetry properties:

$$\psi(\vec{x},\vec{y},\vec{z})=\sum_{\kappa=1}^{s}\chi_\kappa^{SI}R_\kappa(\vec{x},\vec{y},\vec{z}), \tag{13}$$

where s is the number of channels allowed by the Pauli principle. $\vec{x}=\vec{r}_1-\vec{r}_2$, $\vec{y}=\vec{r}_3-\vec{r}_4$, and $\vec{z}=(m_1\vec{r}_1+m_2\vec{r}_2)/(m_1+m_2)-(m_3\vec{r}_3+m_4\vec{r}_4)/(m_3+m_4)$ are the Jacobi coordinates. χ_κ^{SI} are orthonormalized spin-isospin vectors, and $R_\kappa(\vec{x},\vec{y},\vec{z})$ is the radial part of the wave function of the κ^{th} channel. To obtain the appropriate symmetry properties in a configuration space, $R_\kappa(\vec{x},\vec{y},\vec{z})$ is expressed as the sum of four components,

$$R_\kappa(\vec{x},\vec{y},\vec{z})=\sum_{n=1}^{4}w_\kappa^n R_\kappa^n(\vec{x},\vec{y},\vec{z}), \tag{14}$$

where $w_\kappa^n=\pm1$. Finally, each $R_\kappa^n(\vec{x},\vec{y},\vec{z})$ is expanded in terms of N generalized Gaussians

$$R_\kappa^n(\vec{x},\vec{y},\vec{z})=\sum_{i=1}^{N}\alpha_\kappa^i\exp\Big[-a_\kappa^i\vec{x}^2-b_\kappa^i\vec{y}^2-c_\kappa^i\vec{z}^2-d_\kappa^i s_1^n\vec{x}\cdot\vec{y}-e_\kappa^i s_2^n\vec{x}\cdot\vec{z}-f_\kappa^i s_3^n\vec{y}\cdot\vec{z}\Big], \tag{15}$$

where s_i^n are equal to ±1 to guarantee the symmetry properties of the radial wave function and $\alpha_\kappa^i, a_\kappa^i,\cdots,f_\kappa^i$ are the variational parameters. The latter are determined by minimizing the intrinsic energy of the four-body system. We follow closely the developments of Refs. [18, 19], where further technical details can be found about the wave function and the minimization procedure.

The numerical method described in this section has been tested in different few-body calculations in comparison to the hyperspherical harmonic formalism, for example Refs. [19, 20], or the stochastic variational approach of Ref. [21] for some of the results presented in Ref. [22]. As a benchmark calculation to show the capability of the method, we have studied the ^{4}He, a $nnpp$ system with $(I,J)=(0,0)$, using the spin-averaged Malfliet-Tjon (MT-V) potential of Ref. [23]. Results for the $(I,J)=(0,0)$ four-nucleon problem can be found in Table 11.2 of Ref. [21]. This was solved with different numerical methods, obtaining a full converged binding energy of 31.3 MeV.

We have studied the $(I,J)=(0,0)$ $nnpp$ state with the GGV method using the MT-V potential of Ref. [23],

$$V_{ij}(r)=-A\frac{e^{-\mu_A r}}{r}+B\frac{e^{-\mu_B r}}{r}, \tag{16}$$

with parameters: $A=578.09$ MeV, $\mu_A=1.55$ fm^{-1}, $B=1458.05$ MeV, $\mu_B=3.11$ fm^{-1}. As in Ref. [21], we have used $\hbar^2/m_N=41.47$ MeV fm^2. As a pure S wave cal-

Table 5. S wave two-body channels contributing to $nnpp$ system with $(I,J)=(0,0)$

V_{12}	–	V_{34}	V_{13}	–	V_{24}
$nn(i,j)$=(1,0)	–	$pp(i,j)$=(1,0)	$np(i,j)$=(0,1)	–	$np(i,j)$=(0,1)
			$np(i,j)$=(1,0)	–	$np(i,j)$=(1,0)

culation, the different two-body channels contributing to the $(I,J)=(0,0)$ $nnpp$ state are shown in Table 5. With $N=25$ generalized Gaussians in Eq. (15), we obtained a binding energy of 31.2 MeV, which demonstrates the capability of our method and gives confidence in the results. The spin-averaged MT-V potential reproduces the tritium binding energy reasonably well, yielding a result of 8.25 MeV.

3.2 $\Lambda\Lambda nn$ system

The uncoupled $\Lambda\Lambda nn$ system with $(I,J)=(1,0)$ was examined in detail in Ref. [4] using local central Yukawa-type Malfliet-Tjon interactions. In Table 6, we summed up the different two-body channels contributing to the $(I,J)=(1,0)$ $\Lambda\Lambda nn$ state. The parameters of the ΛN and $\Lambda\Lambda$ two-body channels were obtained by fitting the low-energy data and the phase-shifts of each channel, as given in the most recent update of the strangeness -1 [24] and -2 [15] Nijmegen ESC08c potential. The low-energy data and the parameters of these models, together with those of the NN interaction from Ref. [25], are given in Table 7. As can be seen in Fig. 2 of Ref. [4], there is no $\Lambda\Lambda nn$ bound state.

The system hardly gets bound for a reasonable increase of the strength in the $\Lambda\Lambda$ interaction. Although one cannot exclude that the genuine $\Lambda\Lambda$ interaction in dilute states, as the one studied here, could be slightly stronger that the one reported in Ref. [15]. However, one needs a multiplicative factor in the attractive term of Eq. (16) $g_{\Lambda\Lambda}\geq 1.8$ to obtain a bound state. Such modification would destroy the agreement with the Nijmegen ESC08c $\Lambda\Lambda$ phase shifts. This is a very sensitive parameter for the study of double-Λ hypernuclei [26], and this modification would produce an almost $\Lambda\Lambda$-bound state in free space, in particular it would give rise to $a^{\Lambda\Lambda}_{^1S_0}=-29.15$ fm and $r_0{}^{\Lambda\Lambda}_{^1S_0}=1.90$ fm. The four-body system would also become bound by taking a multiplicative factor 1.2 in the NN interaction. However, such a change would make the $^1S_0$ NN potential as strong as the $^3S_1$ [23], and thus the singlet S wave would develop a dineutron bound state, $a^{NN}_{^1S_0}=6.07$ fm and $r_0{}^{NN}_{^1S_0}=1.96$ fm. The situation is slightly different when dealing with the ΛN interaction. We used a common factor $g_{N\Lambda}$ for the attractive part of the two ΛN partial waves, $^1S_0$ and $^3S_1$. The four-body system develops a bound state for $g_{N\Lambda}=1.1$, giving rise to the ΛN low-energy parameters: $a^{\Lambda N}_{^1S_0}=-5.60$ fm, $r_0{}^{\Lambda N}_{^1S_0}=2.88$ fm, $a^{\Lambda N}_{^3S_1}=-2.91$ fm, and $r_0{}^{\Lambda N}_{^3S_1}=2.99$ fm, far from the values constrained by existing experimental data. In particular, these scattering lengths point to the unbound nature of the $\Lambda\Lambda nn$ system based on the hyperon-nucleon interactions derived from chiral effective field theory in Ref. [27], because it is less attractive: $a^{\Lambda p}_{^1S_0}\in[-2.90,-2.91]$ fm and $a^{\Lambda p}_{^3S_1}\in[-1.40,-1.61]$ fm (see Table 1 of Ref. [27]).

It is also worth mentioning that Ref. [28] tackled the same problem by fitting low-energy parameters of older versions of the Nijmegen-RIKEN potential [29, 30] or chiral effective field theory [31, 32], by means of a single Yukawa attractive term or a Morse parametrization. The method, employed to solve the four-body problem is similar to the one we applied in our calculation, thus the results might be directly comparable. Our improved description of the two- and three-body subsystems and the introduction of the repulsive barrier for the $^1S_0$ NN partial wave, relevant for the study of the triton binding energy (see Table 2 of Ref. [33]), leads to a four-body state above threshold, which cannot get bound by a reliable modification of the two-body subsystem interactions. As clearly explained in Ref. [28], the window of Borromean binding is more and more reduced for potentials with harder inner cores.

For the sake of consistency with Sec. 2, we have repeated the calculation using the latest $\Lambda\Lambda$ interaction derived by the HAL QCD Collaboration [16]. The parameters of the $\Lambda\Lambda$ HAL QCD potential are given in the last line of Table 7. Although the $\Lambda\Lambda$ interaction of Ref. [16] is slightly more attractive than that of the Nijmegen ESC08c potential [15], the $\Lambda\Lambda nn$ state remains unbound. The more attractive character of the HAL QCD $\Lambda\Lambda$ interaction can be easily tested by trying to generate a $\Lambda\Lambda nn$ bound state with the multiplicative factor in the attractive term of Eq. (16) of the $\Lambda\Lambda$ interaction. While with the model of Ref. [15] a multiplicative factor $g_{\Lambda\Lambda}=1.8$ is necessary to obtain a bound state, whereas with that of Ref. [16], the bound state is developed for $g_{\Lambda\Lambda}=1.6$.

We also studied the coupled $\Lambda\Lambda nn-\Xi^- pnn$ system, to

Table 6. S wave two-body channels contributing to the $\Lambda\Lambda nn$ system with $(I,J)=(1,0)$

V_{12}	–	V_{34}	V_{13}	–	V_{24}
$nn(i,j)$=(1,0)	–	$\Lambda\Lambda(i,j)$=(0,0)	$n\Lambda(i,j)$=(1/2,0)	–	$n\Lambda(i,j)$=(1/2,0)
			$n\Lambda(i,j)$=(1/2,1)	–	$n\Lambda(i,j)$=(1/2,1)

Table 7. Low-energy parameters and parameters of local central Yukawa-type potentials given by Eq. (16) for NN, ΛN, and $\Lambda\Lambda$ systems contributing to $(I,J)=(1,0)$ $nn\Lambda\Lambda$ state.

	Ref.	(i,j)	A(MeV fm)	μ_A(fm^{-1})	B(MeV fm)	μ_B(fm^{-1})	a(fm)	r_0(fm)
NN	[25]	(1,0)	513.968	1.55	1438.72	3.11	−23.56	2.88
ΛN	[24]	(1/2,0)	416	1.77	1098	3.33	−2.62	3.17
		(1/2,1)	339	1.87	968	3.73	−1.72	3.50
$\Lambda\Lambda$	[15]	(0,0)	121	1.74	926	6.04	−0.85	5.13
	[16]	(0,0)	207.44	1.87	627.6	3.63	−0.62	7.32

check if the coupling to the upper channel $\Xi^- pnn$ could help to generate a $\Lambda\Lambda nn$ bound state. For this purpose, one needs a parametrization of the $\Lambda\Lambda-\Xi N$ transition potential. As has been explained in Sec. 2.2 the HAL QCD Collaboration has recently derived a $\Lambda\Lambda-\Xi N$ transition potential [16] with almost physical quark masses. In their results, the H dibaryon appears as a very sharp resonance just below the ΞN threshold, which indicates a rather weak $\Lambda\Lambda-\Xi N$ transition potential. We have parametrized this interaction by means of a Malfliet-Tjon interaction as in Eq. (16) with parameters: $A=61.66$ MeV, $\mu_A=1.79$ fm^{-1}, $B=227.01$ MeV, $\mu_B=3.25$ fm^{-1}. The details of the ΞN interaction are discussed in the next subsection. The coupled $\Lambda\Lambda nn-\Xi^- pnn$ system is clearly unbound. Thus, it only remains to study the possible existence of a $\Xi^- pnn$ bound state that may decay to $\Lambda\Lambda nn$.

3.3 $\Xi^- pnn$ system

We now study the uncoupled $\Xi^- pnn$ system with quantum numbers $(I,J)=(1,0)$, to look for a possible bound state. This system contains several bound states made of subsets of two- and three-body particles. It contains the deuteron, the tritium, the $(i,j)=(1,1)$ ΞN bound state predicted by the Nijmegen potential [15] with a binding energy of 1.56 MeV, and the $(i,j)=(3/2,1/2)$ ΞNN bound state with a binding energy of 2.89 MeV discussed in Sec. 2.2. If there is a $\Xi^- pnn$ bound state, it would not be stable unless its binding energy exceeds $m_{\Xi^- p}-m_{\Lambda\Lambda}=28.6$ MeV. Otherwise, it would decay to $\Lambda\Lambda nn$. If its binding energy is larger than that of the tritium, it would appear as a $\Xi^- t$ resonance or quasibound state decaying to $\Lambda\Lambda nn$.

To perform this study we need the ΞN in three different partial waves. In Table 8, we show the different two-body channels contributing to the $(I,J)=(1,0)$ $\Xi^- pnn$ state. First, we use the full set of ΞN interactions of the Nijmegen group [15]. As for the case of the two-body channels described in Sec. 3.2, we have constructed the two-body amplitudes for all subsystems entering the four-body problem studied by solving the Lippmann-Schwinger equation of each (i,j) channel,

$$t_{ij}(p,p';e)=V_{ij}(p,p')+\int_0^\infty p''^2\mathrm{d}p''V_{ij}(p,p'')\times\frac{1}{e-p''^2/2\mu}t_{ij}(p'',p';e),\tag{17}$$

where

$$V_{ij}(p,p')=\frac{2}{\pi}\int_0^\infty r^2\mathrm{d}r\, j_0(pr)V_{ij}(r)j_0(p'r),\tag{18}$$

and the two-body potentials consist of an attractive and a repulsive Yukawa term as in Eq. (16). The parameters of the ΞN channels were obtained by fitting the low-energy data as given in the most recent update of the strangeness -2 Nijmegen ESC08c potential [15]. Further, as mentioned above, the HAL QCD Collaboration [16] has recently derived a potential for the $(i,j)=(0,0)$ $\Lambda\Lambda-\Xi N$ channel with almost physical quark masses. Thus, we have performed the calculation with both models for the $(i,j)=(0,0)$ $\Lambda\Lambda-\Xi N$ channel, Nijmegen ESC08c [15] and HAL QCD [16]. The low-energy data and the parameters of the different ΞN interactions are given in Table 9.

With $N=15$ generalized Gaussians in Eq. (15), we have obtained a $\Xi^- pnn$ bound state of 14.43 MeV with the $(i,j)=(0,0)$ HAL QCD interactions and 10.78 MeV with the Nijmegen potentials[1)]. In both cases, the $(I,J)=(1,0)$ $\Xi^- pnn$ state lies below the lowest two-body threshold, $\Xi^- t$. This state would decay to the $\Lambda\Lambda nn$ channel with a very small width, as shown in Sec. 2.3 and Ref. [34]. The results are in close agreement with those obtained with the separable potential three-body model shown in Table 3.

Table 8. S wave two-body channels contributing to $\Xi^- pnn$ system with $(I,J)=(1,0)$

V_{12}	–	V_{34}	V_{13}	–	V_{24}
$nn(i,j)$=(1,0)	–	$p\Xi^-(i,j)$=(0,0)	$np(i,j)$=(1,0)	–	$n\Xi^-(i,j)$=(1,0)
$nn(i,j)$=(1,0)	–	$p\Xi^-(i,j)$=(1,0)	$np(i,j)$=(0,1)	–	$n\Xi^-(i,j)$=(1,1)

1) Note that the mass of ^{4}He changes by 0.24 MeV from $N=15$ to $N=25$, so the result is fully converged.

Table 9. Low-energy parameters and parameters of local central Yukawa-type potentials given by Eq. (16) for ΞN system contributing to the $(I,J)=(1,0)$ $\Xi^- pnn$ state.

	Ref.	(i,j)	A/(MeV fm)	μ_A/fm^{-1}	B/(MeV fm)	μ_B/fm^{-1}	a/fm	r_0/fm
ΞN	[16]	(0,0)	161.38	1.17	197.5	2.18	–	–
	[15]		120	1.30	510	2.30	–	–
	[15]	(1,0)	290	3.05	155	1.60	0.58	−2.52
	[15]	(1,1)	568	4.56	425	6.73	4.91	0.53

In all models, the binding is larger than that of the tritium, and a slightly deeper bound state is obtained when using the HAL QCD interactions for the two-body coupled channel $(i,j)=(0,0)$. By including the Coulomb $\Xi^- p$ potential, the binding energies are increased roughly by 0.75 MeV with the HAL QCD interaction and 0.53 MeV with the Nijmegen potentials, yielding final binding energies of 15.18 MeV and 11.31 MeV, respectively.

4 Outlook

It has been suggested in Ref. [1] that some of the structures observed in the correlated $\pi^- - \pi^-$ momenta by the BNL AGS-E906 experiment [2], aiming to produce and study double hypernuclei through a (K^-,K^+) reaction on ^{9}Be, could result from the decays of a $^{4}_{\Lambda\Lambda}$n double hypernucleus. We have studied the coupled $\Lambda\Lambda nn-\Xi^- pnn$ system to investigate whether the inclusion of channel coupling is able to bind the $\Lambda\Lambda nn$ system. We employed two different approaches. The first one is a separable potential three-body model of the coupled $\Lambda\Lambda nn-\Xi^- pnn$ system, tuned to the known experimental data that allows us to evaluate the $\Xi^- t$ binding energy and its decay width to $\Lambda\Lambda nn$. The second one is a generalized Gaussian variational method based on realistic two-body interactions tuned in known two-, three-, and four-body systems experimental data.

With the available two-body interactions that are adjusted to describe what is known about the two- and three-baryon subsystems, neither a $\Lambda\Lambda nn$ bound state nor a resonance is obtained. However, we have found a $\Xi^- t$ quasibound state with quantum numbers $(I,J)=(1,0)$ above the $\Lambda\Lambda nn$ threshold. The stability of the state is increased by considering the Coulomb potential. The different approaches to the $\Lambda\Lambda-\Xi N$ interaction lead to similar results, and the weakness of the $\Lambda\Lambda-\Xi N$ transition potential explains the narrow width of the $\Xi^- t$ quasibound state. Finally, we calculated the $\Xi^- t$ scattering length, which may be useful in the calculation of the energy shift in the atomic levels of the $\Xi^- t$ atom.

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