PAPER

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Taking the fear out of vector addition by concentrating on the superimposed axes

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Abstract

Introductory physics students often express fears regarding graphical vector addition. To ameliorate student trepidation of possibly making a mistake when moving the second vector to its new position at the head of the first vector before being added, as is most often advised by physics teachers, an alternative method is detailed here. This subtly different method instead concentrates student effort on using ‘sensing personification’ to identify a ‘center of the universe’ position that shifts for each vector by considering the initial axes choice when drawing the first vector, then superimposing a new axes for the second vector onto the same graph at the head of the first vector before drawing in the second vector. The vector is then drawn to-scale onto the new axes. This process then allows students to render the vector correctly without worry of inadvertent error of vector rotation and/or accidental elongation or truncation of the vector. A simple example is given to demonstrate this alternative method and how it also better aligns with analytical vector addition which uses vector equations. Use of sensing personification in vector addition can also set the stage for its use in other physics problems the student will encounter later, such as remembered experiences on amusement park rides.

Keywords: physics teaching, vectors, physics education research

1. Introduction

Introductory physics students often confess fears regarding vector addition. I too felt the fear as a young physics student—as if I were handling a fragile object: a vector! I remember how given a problem with a vector I was instructed to slide the vector over, millimeter-by-millimeter, to align it with another vector, head-to-tail, in the process called vector addition. I was scared! How to move the vector but not change its alignment, not stretch or shorten it? What if I made a mistake and could not slide it over correctly? Whatif I damaged it? Drew it incorrectly? What if I accidently tilted or rotated it? Elongated or truncated it? My hands would shake! I would feel sea-sick!

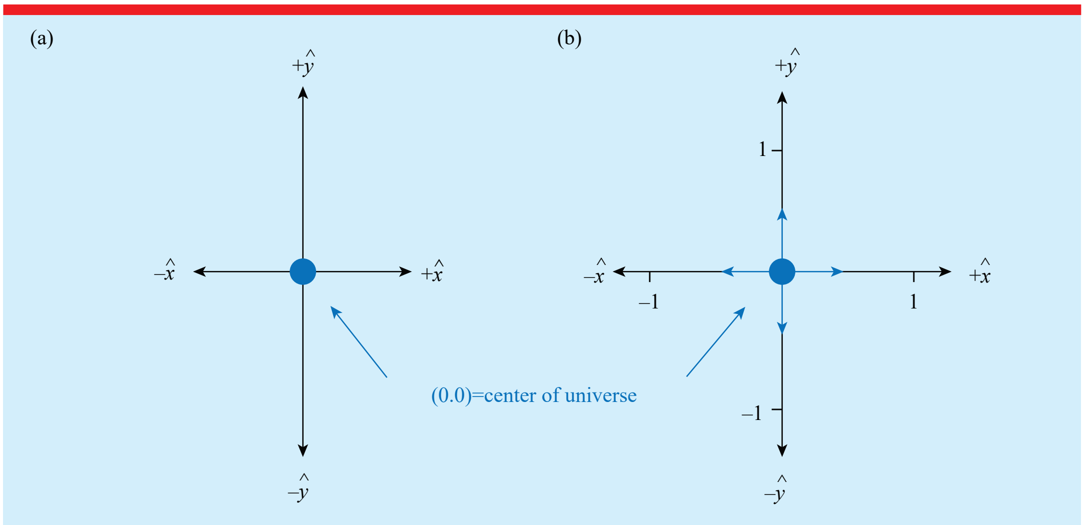

Figure 1. (a) The 'center of the universe' at (0,0) point, (b) *x*-*y* axes emphasized.

Students other than my young self are known to have difficulties, concerns, and mishaps with physics problems involving vector addition [1–5]. Students can have difficulty specifically with vector representations when adding distance vectors [6], considering projectile motion problems [7], and working with ideations of force fields [8]. Adapting good habits of mind can assist in problem solving [9]. Such is suggested here.

Years of teaching experience later, after assisting many students I discovered this helpful new emphasis that helps ground students when handling graphical vector addition problems and reduce their fear of mishandling a vector: instead of concentrating on the vector, this method involves instead concentrating on the axes themselves, with a few reminders to the students. So I am sharing this teaching 'trick'. Distance vectors are used in this example, but the steps are the same for any units on vectors (for example velocity or force) with the idea you always start at some point on the graph and end at another.

2. Methodology and results

Students are often successfully asked to revisit or remember experiences and personify themselves in situations [10, 11]. The first step here involves personifying yourself in the situation by first simply experiencing an x versus y axis system (x-y axes) with a technique that asks one to sense the environment like a detector in the situation, but does not use this personification to endow any object with thoughts, emotions, or goals. Using this 'sensing personification'[11] perspective, imagine yourself at the center of an axes in the place mathematicians call $(x,y) = (0,0)$ or the origin that here will be called the '*center of the universe*'. This is important!

Note that while astronomy tells us there is no center of the universe—that the universe is expanding everywhere—yet from a perspective of relativity you could choose any point as the '*center of the universe*'. Indeed, as you are reading this does it not feel right now like the universe is spread out around you!?! On a map of a park or a mall, the location of a person is often indicated by a coloured dot and the words 'You Are Here' asking you to put yourself mentally on the map and in its perspective. Cell phone map apps use a similar dot. You, for the students, also identify and choose that to be that point and start drawing a vector from your location.

On your graph (0,0) on the x-y axes will be the location of the tail of the first vector. When drawing the vector, mentally move your location to the head of that vector (even if it is not a distance vector); see figure 1.

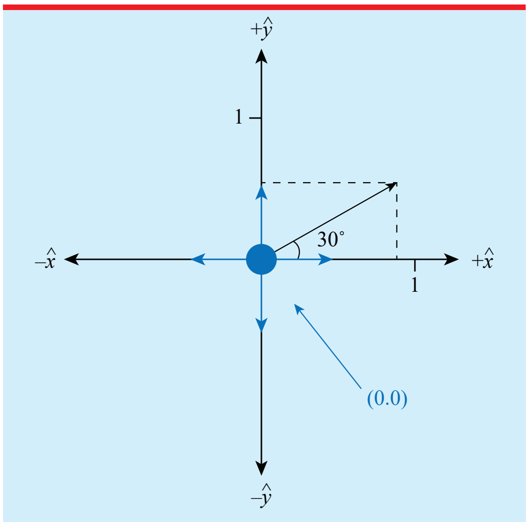

Figure 2. The *x*-*y* axes with a sample vector: vector with a 30 degree angle and hypotenuse a single unit in length (could be centimeters, for example).

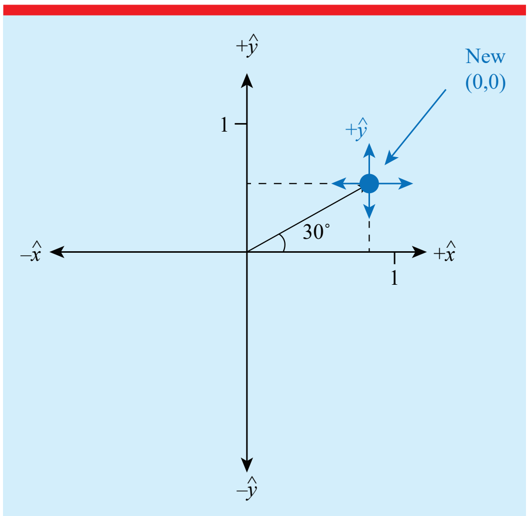

Figure 3. The head of the vector is now declared a new (0,0) emphasizing a new 'center of the universe' with a new *x*-*y* axes.

In the second step you draw the vector in the appropriate direction coming out of the '*center of the universe*' point at (0,0)—either by stepping out along the Cartesian coordinate axes so many units in the *x*-direction and so many in the *y*-direction (no *z*-direction for this example) or, if instead given, using the angle and the radius in polar coordinates. The tail of this vector is located at (0,0) and you draw the arrow head at the new location and the vector arrow shaft out to that position—the usual procedure. So far so good (see figure 2).

Now here is the 'trick': declare the point at the head of the first vector as a new (0,0) and then draw a new coordinate system at the tip of the head of that first vector—in other words, a new 'center of the universe' axes. Why? It makes sense–if you had travelled along the vector, you would be at a new location and this indeed would be a new personal *'center of the universe'* (see figure 3).

The new axes must align with the first axes (no tilting the axes!): the new *x*-axis parallel to the old *x*-axis and the new *y*-axis parallel to the old *y*-axis. The new *x*-*y* axes directions are also appropriately perpendicular to each other (no squished, crooked, or otherwise skewed axes!) and use the same unit distances. This should not be hard as physics students know how to draw parallel and perpendicular lines and check that they are parallel or perpendicular with a ruler but teachers might want to quickly review this in class. The only concern here is the exact location of the new *x*-*y* axes may not land on an integer point, but that is alright. When this graphical method is used it is rendered by drawing the vector making sure that the exact position of the unit markings are consistent so that the new vector has the correct length steps in the *x* and *y* directions if Cartesian coordinates, or correct angle and radius if polar coordinates, consistent with the new *x*-*y* axes. Penciled in ruler markings on the new axes can be used to ensure this. This second vector now starts at the new (0,0) and its head is drawn at the new final location. Again the emphasis is on the new axes, not the movement of the second vector! Once the new axes is drawn, the second vector can be added with ease. More vectors could be added too using this method, but just stick to two vectors for now. As long as a person correctly draws the two separated *x*-*y* axes and inserts the vectors onto those axes then there is much less fear of mis-rendering the vector (see figure 4).

Once the vectors are drawn the resultant is found in the usual way using the original (first vector's) axes. The resultant vector is as if you had come from the initial position, with its tail at

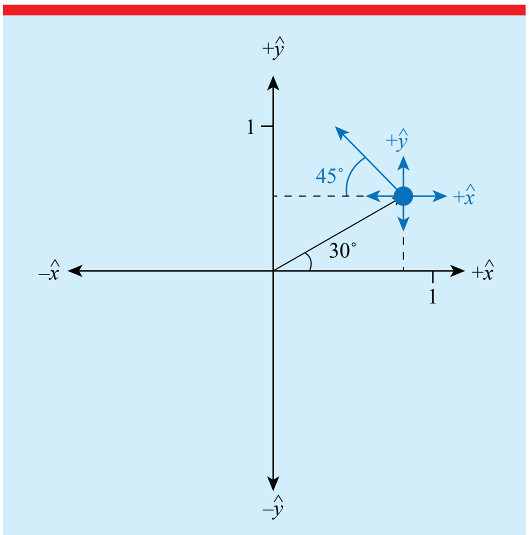

Figure 4. A second 45 degree vector as an example is added starting at the new (0,0) and *x*-*y* axes.

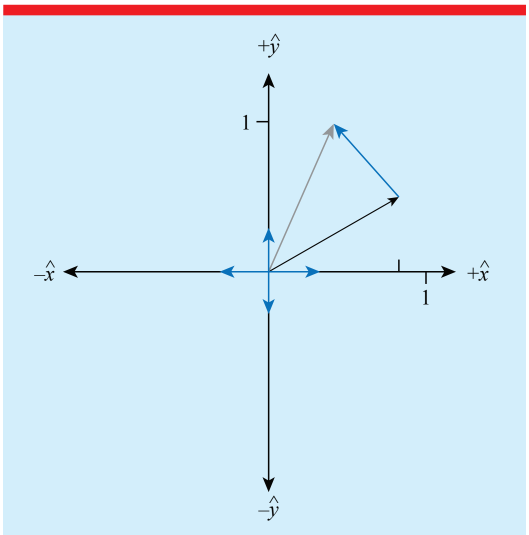

Figure 5. Resultant vector (drawn from the original origin to the head of the second vector) with emphasis on the axes of that final vector.

the original (0,0), to that final position directly '*as the crow flies*' (as you as a bird or even an airplane would have flown directly) instead of arriving there via the two vectors (you imagine walking there along each vector). As usual, you then refer back to the first *x*-*y* axes' unit markings to find the data point at the location of the head of the resultant vector and record the final (x,y) position as the vector components and/or find the angle and radius of that resultant vector. As is traditionally done, ruler and pencil is used to plot the resultant vector with the ruler finding the length against the original axes. A protractor finds the angle using that same original axes. The resultant vector (addition of the two vectors) is indicated in figure 5.

This method also better reflects the process when you 'analytically' add two vectors directly since an individual independent (0,0) starting location is assumed for BOTH vectors (here designated as **A** and **B**). For this example:

$$\mathbf{A} = \;0.866\hat{x} + \;0.500\hat{y}$$

$$+\mathrm{B} = \;-0.500\hat{x} \pm 0.500\hat{y}$$

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$$\mathbf{C} = \;0.366\hat{x} + 1.000\hat{y}$$

The resultant (**C**) is the same as when found graphically as in figure 5. This method can, of course, be extended to adding an unlimited number of vectors.

3. Conclusion

To assist in reducing fear for beginning physics students learning to graphically add vectors it is suggested for teachers to model concentrating on rendering correct axes for the first vector and then superimposing the second vector's axes on the graph at a new (0,0) location (new '*center of the universe*') at the first vector arrow's head, and so on for other added vectors. This example of 'sensing personification' perspective is demonstrated here as an only slightly and subtly different but yet very helpful method for students to more confidently solve graphical vector addition problems, over the traditional suggestion for students to image moving vectors across the page when adding them graphically. This suggested method also aligns with the analytical vector addition process where separate independent axes are used. This too is an opportunity for students to start to get used to thinking in terms of 'sensing personification' in physics, often used in later physics examples such as when students are asked to remember experiences of velocity and acceleration in everyday life such as at amusement parks when on bumper cars, roller coasters or carousels.

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