A Letters Journal Exploring the Frontiers of Physics

March 2020

EPL, **129** (2020) 52001
www.epljournal.org
doi: 10.1209/0295-5075/129/52001

# A collective description of the unusually low ratio $B_{4/2} = B(E2; 4_1^+ \to 2_1^+)/B(E2; 2_1^+ \to 0_1^+)$

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received 7 February 2020; accepted in final form 16 March 2020
published online 1 April 2020

PACS 21.60.Fw – Models based on group theory
PACS 21.60.Ev – Collective models

**Abstract** – The unusually low value of the ratio $B_{4/2} = B(E2; 4_1^+ \to 2_1^+)/B(E2; 2_1^+ \to 0_1^+)$ along the yrast band is described in the interacting boson model-1. Two three-body interactions with $SU(3)$ symmetry are additionally introduced into the usual Hamiltonian describing the transitional behaviors from the spherical vibration to the ellipsoidal rotation. In the $SU(3)$ limit, the energy of the $4^+$ state in the irrep $(2N-8, 4)$ can be lower than the one of the $4^+$ state in the irrep $(2N, 0)$, which causes the anomaly.

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**Introduction.** – The interacting boson model (IBM) presents an algebraic description of the nuclear structure, in which the nucleon pair is regarded as a boson [1]. In the simplest case (IBM-1), proton pair and neutron pair in an even-even nucleus are not distinguished and the total boson number $N$ is conserved. Many collective properties of nuclei can be well described by this model with up to two-body interactions. Experience shows that, if the nucleus moves away from the closed-shell configuration, collectivity emerges and gradually dominates the behaviors of the low-lying levels.

Recently, a cluster of a few extremely neutron-deficient nuclei $^{168}$Os ($N = 8$) [2], $^{166}$W ($N = 9$) [3], $^{172}$Pt ($N = 8$) [4] and $^{170}$Os ($N=9$) [5] are experimentally found to have an unpredictably small ratio of reduced transition probabilities $B_{4/2} = B(E2; 4_1^+ \to 2_1^+)/B(E2; 2_1^+ \to 0_1^+)$. This small ratio has also been found in other nuclei away from closed shell, such as $^{50}$Cr ($N = 3$) [6], $^{48}$Cr ($N = 4$) [6], $^{74}$Zn ($N = 4$) [7], $^{72}$Zn ($N = 5$) [7], $^{114}$Te ($N = 7$) [8,9] and $^{114}$Xe ($N = 7$) [10]. These experimental results are very surprising. In a standard collective model, such as the geometrical collective model or IBM, the $B(E2)$ values increase with spin along the yrast band, so the ratio $B_{4/2}$ is strictly larger than unity. For an ideal rotor, this quantity is 1.43 (Alaga rule), while for a harmonic vibrator, it is 2. Although this value can become a little smaller in the IBM, it still cannot be less than 1. Despite a lot of attempts to calculate the small value of $B_{4/2}$, such as large-scale shell models and state-of-the-art beyond-mean-field models, this anomaly cannot be reproduced so far in a convincing way [5].

One possible explanation for this anomaly is that these neutron-deficient nuclei follow seniority symmetry [11]. This pairing-correlation dominating behavior is usually expected to be found near magic neutron or proton numbers. An interesting study on the transition from non-collective seniority-like excitations to collective modes is performed with $^{206}$Po ($N = 4$) and $^{204}$Po ($N = 5$) recently [12]. In ref. [4] great efforts have been made to explain the $B_{4/2}$ anomaly with a seniority-conserving structure, but there are no definite results for the large model spaces involved. Furthermore, it is shown that the values of $B(E2; 2_1^+ \to 0_1^+)$ of these nuclei can be very well reproduced with existing theories. The value of $B(E2; 2_1^+ \to 0_1^+)$ is a key observable for understanding the emergence of collectivity [12] and the quantum phase transition of the shapes of atomic nuclei [13]. The $B(E2; 2_1^+ \to 0_1^+)$ values of the $^{168}$Os, $^{166}$W, $^{172}$Pt and $^{170}$Os are, respectively, 74(13) W.u., 150(9) W.u., 49(11) W.u. and 97(8) W.u., which are all larger than the value 40.60(20) W.u. of the typical $\gamma$-soft nucleus $^{196}$Pt [14]. These results mean that the $2_1^+$ states of these nuclei should be collective excitations. The experimental discrepancy on the small $B_{4/2}$ is merely determined by the value of the $B(E2; 4_1^+ \to 2_1^+)$ [5], which is nearly 4 times smaller than the theoretical value. If the $0_1^+$ state and the $2_1^+$ state belong to a different collective mode than the $4_1^+$ state, a small ratio $B_{4/2}$ is possible.

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In this paper we insist that the low-lying states of these nuclei are collective, and try to resolve this $B_{4/2}$ puzzle within the framework of the IBM-1. If two three-body interactions, the $SU(3)$ third-order Casimir operator and $[\hat{L}\times\hat{Q}\times\hat{L}]^{(0)}$ ($Q$ is the quadrupole operator in the $SU(3)$ limit and $L$ is the angular momentum operator), are additionally introduced into the conventional Hamiltonian, it is found that this abnormally small ratio $B_{4/2}$ can be successfully reproduced. The two $SU(3)$ three-body symmetry-conserving operators have been discussed in refs. [15–20] as one of the extensions of the IBM-1. We illustrate this result with the nucleus $^{170}$Os for an example. The reasons for this small $B_{4/2}$ are also clarified. In the $SU(3)$ limit, the energy of the $4^+$ state in the irrep $(2N-8,4)$ can be lower than the one of the $4^+$ state in the irrep (2N, 0), which is the cause of the anomaly.

**Hamiltonian and numerical results.** – In the IBM-1, three dynamical symmetry limits exist: the $U(5)$ limit presents quadrupole vibrations around a spherical nucleus, the $SU(3)$ limit describes an ellipsoidal nucleus having rotational structure, and the $O(6)$ limit shows a $\gamma$-soft triaxial rotation. These nuclei having a small $B_{4/2}$ value locate a transitional region from spherical vibration to prolate rotation [2–5], so the $d$-boson number operator in the $U(5)$ limit $\hat{C}_1[U(5)]=\hat{n}_d$, the $SU(3)$ second-order Casimir operator $\hat{C}_2[SU(3)]$ and the rotational invariant operator $\hat{C}_2[SO(3)]=\hat{L}^2$ should be considered in the Hamiltonian. However, the three terms are not enough to reproduce the anomalous result. If we insist that the small $B_{4/2}$ ratio is also a collective phenomenon, the only possible solution is that higher-order interactions should be additionally introduced into the description.

Two three-body interactions conserving $SU(3)$ symmetry are explored in this paper. One is the $SU(3)$ third-order Casimir operator $\hat{C}_3[SU(3)]$, and another is the $O(3)$ scalar shift operator $\hat{\Omega}=[\hat{L}\times\hat{Q}\times\hat{L}]^0$. These terms have been already discussed in detail in refs. [15,16], but the $SU(3)$ third-order invariant term is ignored for actual simulation. In a recent study, it is shown that a combination of the $SU(3)$ second-order and third-order Casimir operators can describe the quantum phase transition from the prolate to the oblate shapes [19], in which analytically solvable description can be obtained.

The Hamiltonian used in this paper is

$$H=\alpha\hat{n}_d+\beta\hat{C}_2[SU(3)]+\gamma\hat{C}_3[SU(3)]+\delta\hat{\Omega}+\zeta\hat{L}^2, \qquad (1)$$

where $\alpha,\beta,\gamma,\delta,\zeta$ are five parameters used for fitting. We take the nucleus $^{170}$Os ($N=9$) for an example [5] to describe the small $B_{4/2}$ value for these nuclei $^{168}$Os, $^{166}$W, $^{172}$Pt and $^{170}$Os have similar level structures and $B(E2)$ features. The numerical results are performed based on our $SU(3)$ basis diagonalization program [21]. The Hamiltonian (1) is diagonalized under the $U(6)\supset SU(3)\supset SO(3)$ basis spanned by $|N(\lambda,\mu)\chi L\rangle$, where $\chi$ is the branching multiplicity occurring in the reduction of $SU(3)\downarrow SO(3)$. The basis vectors are orthonormal, so the eigenstates of (1) can be expressed as

$$|N,L_\xi;\alpha,\beta,\gamma,\delta,\zeta\rangle = \sum_{(\lambda,\mu)\chi} C^{L_\xi}_{(\lambda,\mu)\chi}(\alpha,\beta,\gamma,\delta,\zeta)|N(\lambda,\mu)\chi L\rangle, \qquad (2)$$

where $\xi$ is an additional quantum number distinguishing different eigenstates with the same angular momentum $L$ and $C^{L_\xi}_{(\lambda,\mu)\chi}(\alpha,\beta,\gamma,\delta,\zeta)$ is the corresponding expansion coefficient.

Table 1: The experimental and numerical results for the energy $E_{2_1^+}$ of the first $2^+$ state, the energy $E_{4_1^+}$ of the first $4^+$ state, the energy $E_{6_1^+}$ of the first $6^+$ state, the energy $E_{8_1^+}$ of the first $8^+$ state, the energy $E_{10_1^+}$ of the first $10^+$ state, the reduced transitional probabilities $B(E2;2_1^+\to 0_1^+)$, $B(E2;4_1^+\to 2_1^+)$ and the ratio $B_{4/2}$ for $^{170}$Os. The effective charge $e=0.149\ eb$. The experimental values are adopted from refs. [5,22].

| | Expt. | Present results |
|---|---|---|
| $E_{2_1^+}$(keV) | 286.70(14) | 283.05 |
| $E_{4_1^+}$(keV) | 749.90(20) | 729.79 |
| $E_{6_1^+}$(keV) | 1325.42(24) | 1228.43 |
| $E_{8_1^+}$(keV) | 1945.8(4) | 1946.20 |
| $E_{10_1^+}$(keV) | 2545.2(5) | 3114.18 |
| $B(E2;2_1^+\to 0_1^+)(e^2b^2)$ | $0.54^{+0.05}_{-0.05}$ | 0.540 |
| $B(E2;4_1^+\to 2_1^+)(e^2b^2)$ | $0.21^{+0.07}_{-0.04}$ | 0.204 |
| $B_{4/2}$ | 0.38(11) | 0.378 |

Some key points for fitting the five parameters will be mentioned in the discussion part. In table 1 we list the experimental and numerical results for the energies of the $2_1^+$, $4_1^+$, $6_1^+$, $8_1^+$, $10_1^+$ states and the $B_{4/2}$ value for $^{170}$Os, where $\alpha=302.40$ keV, $\beta=-30.09$ keV, $\gamma=3.79$ keV, $\delta=-10.38$ keV, $\zeta=18.66$ keV. It is shown that the energies of the $2_1^+$, $4_1^+$, $6_1^+$, $8_1^+$, $10_1^+$ states in the yrast band and the unusually small value $B_{4/2}$ can be reproduced very well in the IBM-1 with higher-order interactions.

**Discussions.** – Now we reveal the reasons for this $B_{4/2}$ anomaly. The four parameters $\beta,\gamma,\delta,\zeta$ in front of the four $SU(3)$-conserving operators are fixed, and the parameter $\alpha$ for the $U(5)$ limit operator changes from 0 keV to 600 keV. The evolution of the energies of the $0_1^+$, $2_1^+$, $4_1^+$, $0_2^+$, $2_2^+$ and $4_2^+$ states as a function of $\alpha$ are plotted in fig. 1. In the parameter regions, the Hamiltonian gives a $\gamma$-soft-like spectrum. The energies of the $2_2^+$ state and the $0_2^+$ state are lower than the one of the $4_1^+$ state, which is similar to anharmonic vibrations in a spherical nucleus. The nuclei having low value $B_{4/2}$ are also located in such a $\gamma$-soft area which is adjacent to the spherical nucleus [5]. The fitting results are in line with expectations very well.

The evolution of the $B(E2)$ values of the $B(E2;2_1^+\to 0_1^+)$, $B(E2;4_1^+\to 2_1^+)$ and $B(E2;4_2^+\to 2_1^+)$ are plotted in

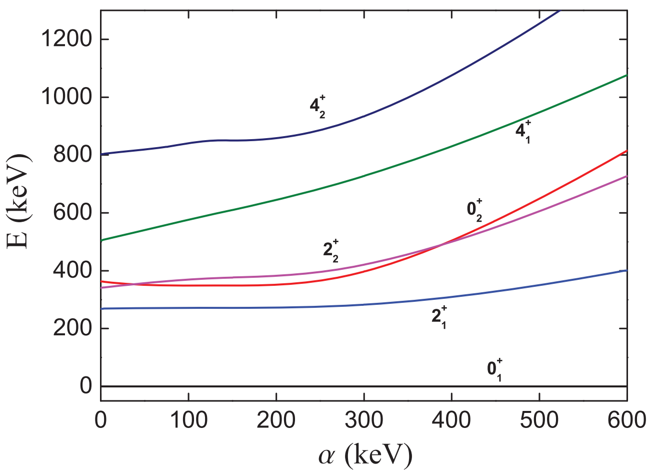


Fig. 1: The evolution of the energies of the $0_1^+$, $2_1^+$, $4_1^+$, $0_2^+$, $2_2^+$ and $4_2^+$ states as a function of $\alpha$ from 0 keV to 600 keV, here $\beta = -30.09$ keV, $\gamma = 3.79$ keV, $\delta = -10.38$ keV, $\zeta = 18.66$ keV.

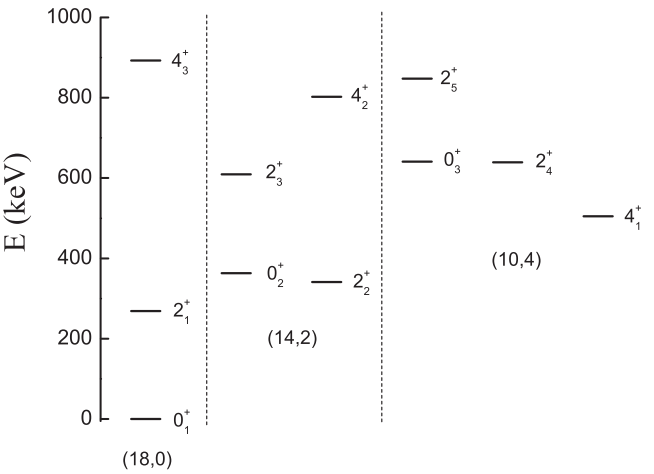


Fig. 3: The spectra of the $SU(3)$ limit with additional two three-body interactions, here $\alpha = 0$ keV, $\beta = -30.09$ keV, $\gamma = 3.79$ keV, $\delta = -10.38$ keV, $\zeta = 18.66$ keV.

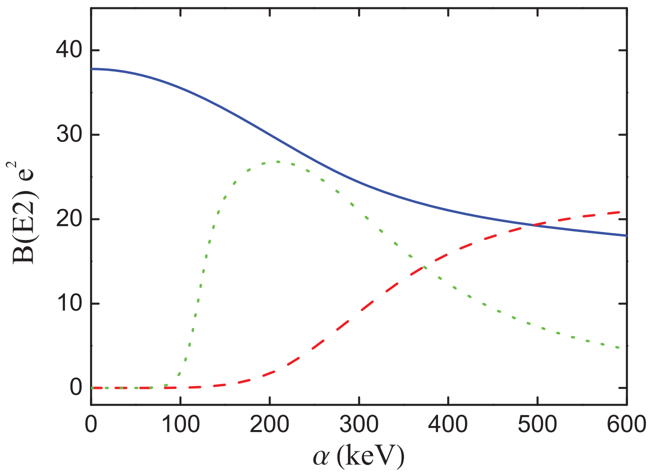


Fig. 2: The evolution of the $B(E2)$ values of the $B(E2; 2_1^+ \to 0_1^+)$ (solid blue line), $B(E2; 4_1^+ \to 2_1^+)$ (dashed red line) and $B(E2; 4_2^+ \to 2_1^+)$ (dotted green line) as a function of $\alpha$ from 0 keV to 600 keV, here $\beta = -30.9$ keV, $\gamma = 3.79$ keV, $\delta = -10.38$ keV, $\zeta = 18.66$ keV.

fig. 2. The $B(E2)$ operator is chosen as

$$\hat{T}(E2) = e\hat{Q}, \tag{3}$$

where $e$ is the boson effective charge, which is usually used in IBM-1 [1,21]. The $B(E2)$ values seem very different from the results obtained from previous $\gamma$-soft spectra from the $U(5)$ limit to the $O(6)$ limit. The values of the $B(E2; 2_1^+ \to 0_1^+)$ are normal (solid blue line). When $\alpha$ becomes larger, the value decreases gradually, which is consistent with our experience. The values ($\alpha \leq 300$ keV) of the $B(E2; 4_1^+ \to 2_1^+)$ (dashed red line) are much smaller than the ones of the $B(E2; 2_1^+ \to 0_1^+)$ which is exactly what we want to get. When $\alpha = 0$, that is in the $SU(3)$ limit, the value of the $B(E2; 4_1^+ \to 2_1^+)$ is exactly zero, which means this transition is forbidden. The behaviors of the values of the $B(E2; 4_2^+ \to 2_1^+)$ are also presented (dotted green line). In fig. 2, it is clear that there is a crossover point for the two evolutional lines of the $B(E2; 4_1^+ \to 2_1^+)$ and $B(E2; 4_2^+ \to 2_1^+)$. This means $4_1^+$ state and $4_2^+$ state are mixed when $\alpha = 302.40$ keV.

When $\alpha = 0$, the left four interactions are $SU(3)$ symmetry conserving, which cannot break the $SU(3)$ symmetry, so the states in this limit can be labeled with the $SU(3)$ irrep $(\lambda, \mu)$. Based on the results in ref. [15], the energies of the low-lying states can be directly obtained. These analytical results are the same as our numerical calculation, which can be easily checked. In fig. 3, the low-lying spectra of the $SU(3)$ limit with additional two three-body interactions are shown. It is easily seen that the energy of the $4^+$ state belonging to the irrep (10, 4) is lower than the one of the $4^+$ state in the irrep (18, 0). Thus the transition between the $4_1^+$ state and $2_1^+$ state is forbidden. The states belonging to different irreps $(\lambda, \mu)$ in the $SU(3)$ second-order and third-order Casimir operators are degenerate. The energies of the $0^+$ states are decided by the two $SU(3)$ Casimir operators. The three-body interaction $[\hat{L} \times \hat{Q} \times \hat{L}]^{(0)}$ and the $\hat{L}^2$ interaction cannot change the energies of the $0^+$ states. Thus the rotational ground-state band for the irrep (18, 0) and other band structures in different irreps are generated by the $[\hat{L} \times \hat{Q} \times \hat{L}]^{(0)}$ and $\hat{L}^2$ interactions.

When the $U(5)$ term is added, fig. 4 predicts the partial low-lying spectra for $^{170}$Os in the IBM-1 with additional two three-body interactions and table 2 presents some predicted partial absolute $B(E2)$ values for $E2$ transitions from $L_i$ state to $L_f$ state. This spectra resemble the ones of a $\gamma$-soft nucleus, but the $B(E2)$ modes are very different. The value of the $B(E2; 0_2^+ \to 2_1^+)$ is very small. Of particular importance, the absolute $BE(2)$ values along the yrast line really decreases with spin as expected, which further verifies the rationality of this theory. Recently the spectra with $\gamma$-soft feature having odd $B(E2)$ values have

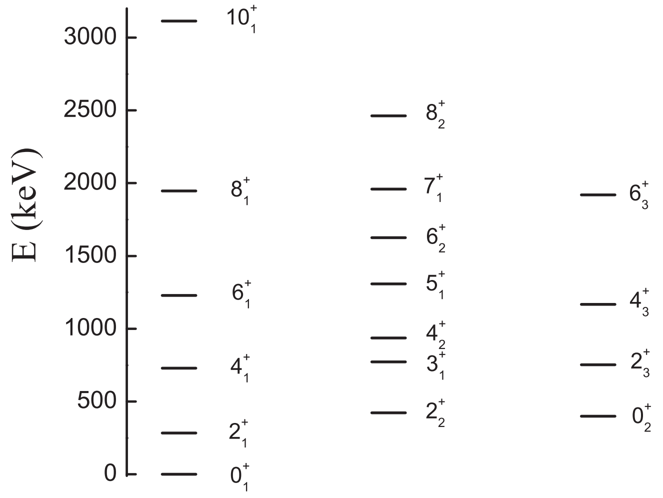


Fig. 4: Predicted partial low-lying spectra for $^{170}$Os in the IBM-1 with additional two three-body interactions, here $\alpha = 302.40\,\text{keV}$, $\beta = -30.09\,\text{keV}$, $\gamma = 3.79\,\text{keV}$, $\delta = -10.38\,\text{keV}$, $\zeta = 18.66\,\text{keV}$.

Table 2: Predicted partial absolute $B(E2)$ values in $e^2b^2$ for $E2$ transitions from $L_i$ state to $L_f$ state for $^{170}$Os when $\alpha = 302.40\,\text{keV}$, $\beta = -30.09\,\text{keV}$, $\gamma = 3.79\,\text{keV}$, $\delta = -10.38\,\text{keV}$, $\zeta = 18.66\,\text{keV}$ with effective charge $e = 0.149\,\text{eb}$.

| $L_i$ | $L_f$ | Present results |
|---|---|---|
| $2_1^+$ | $0_1^+$ | 0.540 |
| $4_1^+$ | $2_1^+$ | 0.204 |
| $6_1^+$ | $4_1^+$ | 0.185 |
| $8_1^+$ | $6_1^+$ | 0.120 |
| $10_1^+$ | $8_1^+$ | 0.065 |
| $2_2^+$ | $2_1^+$ | 0.558 |
| $0_2^+$ | $2_1^+$ | 0.017 |
| $4_2^+$ | $2_1^+$ | 0.462 |

attracted much attention [23]. We expect these results can be verified with future experiments.

The fitting process of the parameters needs further explanation. We start to determine the parameters from the $SU(3)$ second-order Casimir operator and the $\hat{L}^2$ interaction, which shows a typical ellipsoidal rotational spectra [1]. Then the $SU(3)$ third-order Casimir operator is considered [15,19]. This interaction can reduce the energies of the $0^+$ excited states and the rotational bands. Next, the $[\hat{L} \times \hat{Q} \times \hat{L}]^{(0)}$ interaction is added, which can make the energy of the $4^+$ state belonging to the irrep $(2N-8,4)$ lower than the one of the $4^+$ state in the irrep $(2N,0)$. However, it is also noticed that the energies of the $6_1^+$, $8_1^+$, $10_1^+$ states are lower than the one of the $4_1^+$ state. Subsequently, the $U(5)$ interaction is followed. Through observing the $B_{4/2}$ value and the level ratio $E_{4_1^+}/E_{2_1^+}$, the four parameters can be preliminarily confirmed. At last, the $\hat{L}^2$ interaction is again considered to determine the energies of the levels along the yrast band. Final results

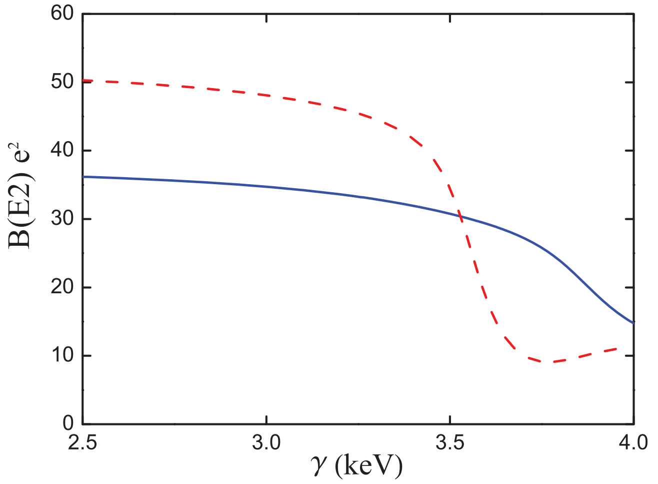


Fig. 5: The evolution of the $B(E2)$ values of the $B(E2; 2_1^+ \to 0_1^+)$ (solid blue line) and $B(E2; 4_1^+ \to 2_1^+)$ (dashed red line) as a function of $\gamma$ from 2.5 keV to 4.0 keV, here $\alpha = 302.40\,\text{keV}$, $\beta = -30.09\,\text{keV}$, $\delta = -10.38\,\text{keV}$, $\zeta = 18.66\,\text{keV}$.

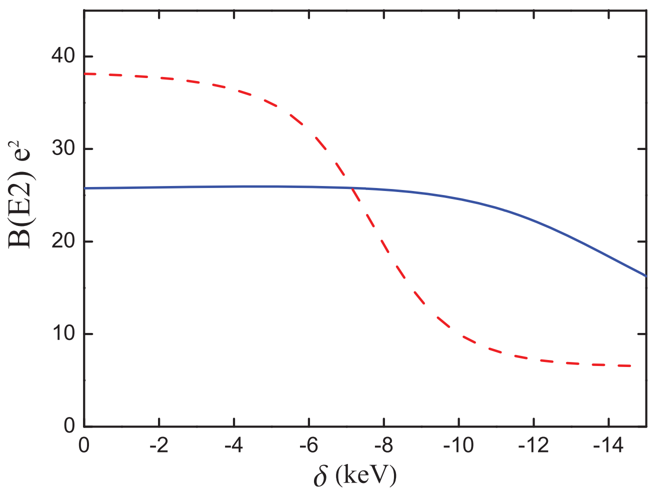


Fig. 6: The evolution of the $B(E2)$ values of the $B(E2; 2_1^+ \to 0_1^+)$ (solid blue line) and $B(E2; 4_1^+ \to 2_1^+)$ (dashed red line) as a function of $\delta$ from 0 keV to $-15$ keV, here $\alpha = 302.40\,\text{keV}$, $\beta = -30.09\,\text{keV}$, $\gamma = 3.79\,\text{keV}$, $\zeta = 18.66\,\text{keV}$.

need to be adjusted and validated repeatedly until they match the experimental data.

The $B_{4/2}$ anomaly occurs in a special parameter region. The evolution of the $B(E2)$ values of the $B(E2; 2_1^+ \to 0_1^+)$ and $B(E2; 4_1^+ \to 2_1^+)$ as a function of $\gamma$ from 2.5 keV to 4 keV are plotted in fig. 5. It can be seen that even the $\gamma$ value becomes a little smaller, such as $\gamma = 3.40\,\text{keV}$, the anomaly disappears. This result may be contrary to our experience for the two three-body interactions are also large. In previous calculation, the higher-order interactions are not so important for the Hamiltonian with up to two-body interactions can also give the main results observed experimentally. Similar situations also hold for the $\delta$ and $\beta$ parameters, see fig. 6 and fig. 7.

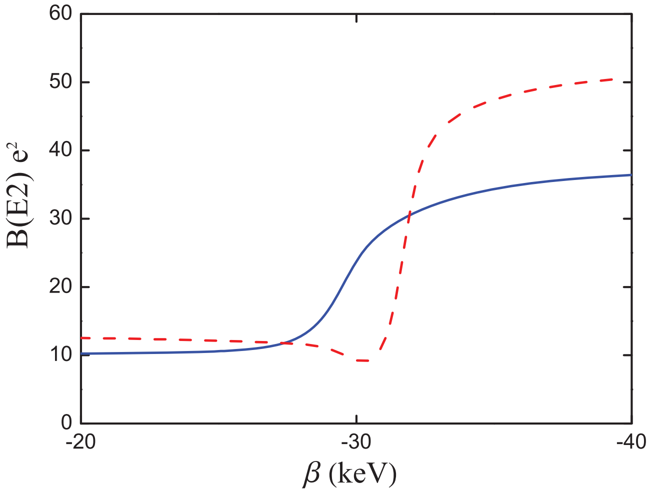


Fig. 7: The evolution of the $B(E2)$ values of the $B(E2; 2_1^+ \to 0_1^+)$ (solid blue line) and $B(E2; 4_1^+ \to 2_1^+)$ (dashed red line) as a function of $\beta$ from $-20\,\text{keV}$ to $-40\,\text{keV}$, here $\alpha = 302.40\,\text{keV}$, $\gamma = 3.79\,\text{keV}$, $\delta = -10.38\,\text{keV}$, $\zeta = 18.66\,\text{keV}$.

We expect the small $B_{4/2}$ value can be also reproduced in the shell model with adding appropriate three-body interactions. The microscopic shell model foundation of the IBM is still under investigation, and the reason of the emergence of the three-body interactions is out of our knowledge. It is essential to investigate the small value $B_{4/2}$ in the IBM-2, in which the proton pair and the neutron pair are treated separately. IBM-2 has a direct correspondence to the shell model. If we can confirm the function of the two three-body interactions in the IBM-2, some clues of the three-body interactions added in the shell model may be obtained. Another method to construct the three-body interactions is to explore the $SU(3)$ shell model [24,25]. Similar to our approach, the low $4_1^+$ state having forbidden transition to the $2_1^+$ state may be generated [20].

It should be noticed that the abruptly decreasing of the $B(E2)$ values can also occur for the $B(E2; 6_1^+ \to 4_1^+)$ or even the $B(E2; 2_1^+ \to 0_1^+)$ for appropriate parameters. The small value $B(E2; 6_1^+ \to 4_1^+)$ compared to the value $B(E2; 4_1^+ \to 2_1^+)$ can be found in $^{72}$Zn ($N = 5$). The previous value is $134^{+57}_{-31}$ e$^2$fm$^4$ and the latter one is $361^{+57}_{-47}$ e$^2$fm$^4$ [7]. For $^{70,72,74}$Zn, our theory may provide a self-consistent result, this will be discussed in future work. An experimental measurement of the value $B(E2; 6_1^+ \to 4_1^+)$ in $^{168}$W ($N = 10$) is highly anticipated [3].

**Conclusions.** – In this paper, the $B(E2)$ anomaly within the yrast band is resolved in the collective framework of the IBM-1. Two $SU(3)$-conserving three-body interactions are additionally introduced into the conventional Hamiltonian describing the transitional behaviors from the spherical vibration to the ellipsoidal rotation. A $\gamma$-soft-like spectra is generated with the small $B_{4/2}$ value. These results are very different from our common experience, just like the experimental anomaly is contrary to our expectation. In the IBM-1, this is the only convincing way to reproduce the anomalous small $B_{4/2}$ value, thus we believe that the three-body interactions may be more important than we thought before, which should be a necessary ingredient to describe the collective nature of nuclei. A complete study in the IBM-2 to discuss the $B_{4/2}$ anomaly will be done in future work, and a similar realization of our work in the shell model, especially the $SU(3)$ shell model is highly expected.

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