Time-reversal asymmetry without local moments via directional scalar spin chirality

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Abstract

Invariably, time-reversal symmetry (TRS) violation in a state of matter is identified with static magnetism in it. Here, a directional scalar spin chiral order (DSSCO) phase is introduced that disobeys this basic principle: it breaks TRS but has no density of static moments. It can be obtained by melting the spin moments in a magnetically ordered phase but retaining residual broken TRS. Orbital moments are then precluded by the spatial symmetries of the spin rotation symmetric state. It is allowed in one, two and three dimensions under different conditions of temperature and disorder. Recently, polar Kerr effect experiments in the mysterious pseudogap phase of the underdoped cuprates hinted at a strange form of broken TRS below a temperature T_K, that exhibits a hysteretic 'memory effect' above T_K and begs reconciliation with nuclear magnetic resonance (which sees no moments), x-ray diffraction (which finds charge ordering tendencies) and the Nernst effect (which detects nematicity). Remarkably, the DSSCO provides a phenomenological route for reconciling all these observations, and it is conceivable that it onsets at the pseudogap temperature $\sim T^*$. A six-spin interaction mediated by enhanced fluctuations of velocity asymmetry between left- and right-movers above the onset of charge ordering in the cuprates is proposed as the driving force behind DSSCO formation. A testable prediction of the existence of the DSSCO in the cuprates is a Kerr signal above T_K triggered and trainable by a current driven along one of the in-plane axes, but not by a current along the other.

Keywords: spin chirality, time reversal symmetry, magnetism, pseudogap, Kerr effect

(Some figures may appear in colour only in the online journal)

1. Introduction

A quantum phase of matter that spontaneously breaks time-reversal symmetry (TRS) invariably develops a finite density of moments. In other words, there exists a set of total angular momentum operators $\{J_i\}$ such that $\langle G|\sum_i J_i|G\rangle$ is extensive in its ground state $|G\rangle$. Common examples contain local spin moments, such as ferromagnets, spin density waves and other spin textures. More complex ones include orbital moments, such as loop current phases [1, 2], anomalous Hall states [3, 4], and various chiral topological phases [5–14]. A property common to all these phases is that TRS is restored as soon as the moments melt. Thus, the phrases 'spontaneous violation of TRS' and the 'formation of local moments' are often used interchangeably. Strictly speaking, though, this synonymy is incorrect because TRS-violation refers to the inequivalence of forward and backward propagation in time without affecting spatial symmetries while local moments also disobey spatial symmetries. A natural question that follows is, can we find a phase of matter that violates TRS but has no moments? Such a phase could be pertinent to a long-standing problem in condensed matter physics—the pseudogap phase of the cuprate high temperature superconductors—which exhibits a Kerr effect [15–18], indicating broken TRS, but shows no signs of magnetism in nuclear magnetic resonance (NMR) experiments [19].

In this work, precisely such a phase of matter is introduced, called the *directional scalar spin chiral spin order* (DSSCO). The DSSCO can be thought of as a state in which classical magnetic order has melted due to quantum, thermal or disorder-driven fluctuations—so spin moments vanish—but TRS-breaking has survived. Moreover, orbital moments involving itinerant particles, if any, are forbidden by the symmetries of the DSSCO. It is captured by an order parameter of the form $\chi \sim \langle \mathbf{S}_1 \cdot \mathbf{S}_2 \times \mathbf{S}_3 \rangle$, where $\mathbf{S}_1, \mathbf{S}_2$ and $\mathbf{S}_3$ are spins on three sites in a straight line. Thus, it is reminiscent of some other phases that involve spin chirality, such as those studied in references [20, 21]. The key difference is that the chirally correlated spins there lie on the vertices of a triangle. Hence, they break enough symmetries to permit a moment perpendicular to its face, even if the moment on each site vanishes. In contrast, the corresponding sites in a DSSCO are collinear, so no such current is possible. The precise conditions in which the DSSCO can form depends sensitively on the dimensionality of space. In particular, it can exist in one dimension (1D) at zero temperature ($T = 0$) in clean systems, in 2D at $T \neq 0$ in clean systems, and in 3D at both $T = 0$ and $T \neq 0$ only in the presence of weak random field disorder. The 3D DSSCO respects spin rotation symmetry (SRS) is respected only on averaging over disorder configurations, and is the one most relevant to the cuprates. Nonetheless, the term DSSCO will be used to denote all the phases based on chiral spin ordering along a preferred direction that break TRS but lack a density of moments.

One of the most enigmatic phases known in condensed matter is the pseudogap phase of the underdoped cuprates. Recently, Kerr effect measurements in this phase showed a signal below a certain temperature dubbed T_K [15, 18, 22, 23], strongly suggesting that TRS is broken below it [24, 25]. However, the symmetries of the phase are distinct from that of an ordinary magnet. Moreover, NMR Knight shifts—usually an excellent probe of magnetic order—have not found any magnetic moments till date [19]. We will see that the Kerr effect in the DSSCO in a clever experimental setup has precisely the same symmetries as that in the cuprates, while the Knight shifts vanish identically. Remarkably, a different scenario in a traditional setup permits a route for reconciling several baffling behaviors experimentally observed in the cuprates: (i) C_4 symmetry breaking above T_K [26] but below the pseudogap temperature T^*, (ii) coincident onsets of the Kerr effect and charge ordering tendencies [27–29], (iii) magnetic moments possibly undetectable by NMR [30], and (iv) a hysteretic 'memory effect' on heating beyond T_K [15]. Moreover, we will show that a particular charge-spin interaction that is likely to be large in the cuprates can induce the DSSCO. The DSSCO, however, does not explain the magnetism predicted by neutron scattering [31–35]. Nonetheless, the phenomenology is rather appealing as it can capture many different experiments in the cuprates.

2. Directional scalar spin chiral order

Let us first sketch the 1D version of the DSSCO. Consider an ordering of classical (large S) spins along a chain as shown in figure 1(a). Here, spins on successive sites are frozen in the

pattern $S_x S_y S_z S_x S_y S_z \ldots$. Such a pattern of magnetic moments obviously violates TRS and SRS; in addition, it also breaks all reflection symmetries. A potential order parameter for it is the pseudoscalar

$$\chi = \frac{1}{L} \sum_x \langle \mathbf{S}(x-1) \cdot \mathbf{S}(x) \times \mathbf{S}(x+1) \rangle \equiv \frac{1}{L} \sum_x \langle \chi(x) \rangle \quad (1)$$

where L is the chain length. Clearly, χ is an Ising order parameter that distinguishes between right-handed ($S_x S_y S_z S_x S_y S_z \ldots$) and left-handed ($S_z S_y S_x S_z S_y S_x \ldots$) sequences of spins. These sequences transform into each other under time-reversal ($\mathbf{S} \to -\mathbf{S}$) or inversion ($x \to -x$). However, χ is invariant under reflection about any plane containing the chain as well as under a global rotation of all the spins, so it does not fully capture the classical order. Let us assume that figure 1(a) depicts the ground state of a classical, local spin Hamiltonian that preserves TRS and SRS and has no disorder. If the spins were quantum objects instead, fluctuations would immediately restore SRS according to the Mermin–Wagner theorem [36]. In contrast, TRS and reflection symmetry are discrete and can hence, remain broken. A closer inspection reveals that the resultant state breaks TRS and inversion symmetry, but is invariant under translation and spin rotation. Therefore, it is faithfully captured by the order parameter χ. This state is defined as the (1D version of) the DSSCO.

How can the DSSCO be extended to higher dimensions? In 2D, SRS can remain broken at zero temperature ($T = 0$), but is restored by thermal fluctuations at any $T \neq 0$ according to the Mermin–Wagner theorem. Thus, the 2D DSSCO is a finite temperature phase and not a true quantum ground state. A straightforward way to obtain it is to couple identical chains ferromagnetically in the transverse directions, as shown in figure 1(b). Both the 1D and the 2D DSSCO are unstable to infinitesimal random field disorder: $H_\mathrm{dis} = \sum_\mathbf{r} \mathbf{h}(\mathbf{r}) \cdot \mathbf{S}(\mathbf{r})$, $\overline{\mathbf{h}(\mathbf{r})} = 0$, $\overline{h_a(\mathbf{r}) h_b(\mathbf{r}')} = h^2 \delta_{ab} \delta(\mathbf{r} - \mathbf{r}')$, $|h| \ll$ all other coupling constants and the overline denotes a configuration average, because χ is an Ising order parameter and $d = 2$ is the lower critical dimension of the random field Ising model [37]. In contrast, such disorder is a prerequisite for the 3D generalization of the DSSCO. This is because, thermal fluctuations cannot restore continuous symmetries in 3D, but quenched weak random fields do, according to the Imry–Ma theorem [38]. Analogous ideas were discussed recently in the context of incommensurate charge density waves (CDWs), that break continuous translational and discrete rotational symmetries, in the pseudogap phase of the cuprates. The analog of the DSSCO there was a *vestigial nematic* phase, in which chemical potential disorder acts as a random field for charge density and restores translational symmetry while rotational symmetry remains broken [39]. The transition temperature for the phase is finite, so the 3D DSSCO is a quantum ground state as well as a $T \neq 0$ phase. The d-dimensional version of the DSSCO is thus naturally captured by the generalization of (1):

$$\chi_d = \frac{1}{L^d} \sum_\mathbf{r} \overline{\langle \mathbf{S}(\mathbf{r} - \widehat{\mathbf{x}}) \cdot \mathbf{S}(\mathbf{r}) \times \mathbf{S}(\mathbf{r} + \widehat{\mathbf{x}}) \rangle} \equiv \frac{1}{L^d} \sum_\mathbf{r} \overline{\langle \chi(\mathbf{r}) \rangle}$$
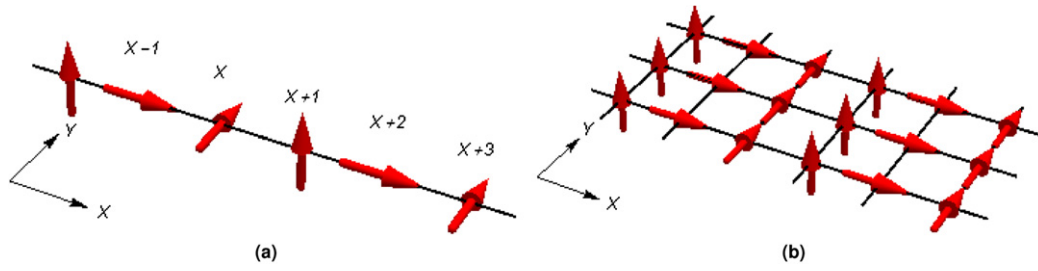$$(2)$$

Figure 1. Classical magnetic orders which form the DSSCO upon melting in 1D (a) and 2D (b). Stacking identical 2D layers gives the precursor to the 3D version of the DSSCO.

Table 1. Dimensions in which the DSSCO can exist under various conditions. T is temperature and 'dirty' refers to weak random field disorder.

	$T = 0$	$T \neq 0$
Clean	1D	2D
Dirty	3D	3D

Table 2. Symmetry properties of the DSSCO, chiral-ordered along x, in various mirror-symmetry breaking fields. M_i denotes $i \rightarrow -i$ reflection and R_i^2 denotes π rotation about the i axis. The CDW is assumed to respect (break) R_x^2 ($M_{x,y,z}$). θ_K is the Kerr angle for reflection off an xy-surface at normal incidence, and KS denotes the NMR Knight shift.

	TRS	M_x	M_y	M_z	R_x^2	R_y^2	R_z^2	θ_K	KS
DSSCO only	×	×	✓	✓	✓	×	×	$= 0$	$= 0$
DSSCO $+ j_x$	×	×	✓	✓	✓	×	×	$= 0$	$= 0$
DSSCO $+ j_y$	×	×	×	✓	×	×	×	$\neq 0$	$\neq 0$
DSSCO $+$ CDW	×	×	×	×	✓	×	×	$\neq 0$	$\neq 0$

χ_d obeys all the symmetries of the underlying lattice except $x \rightarrow -x$ reflection. It is easy to check that translation and reflections symmetries of the lattice prevent equilibrium current loops. Therefore, the DSSCO lacks bulk orbital currents as well as spin moments and consequently, lacks a density of total angular momentum expectation values. The existence conditions of the DSSCO in various dimensions are summarized in table 1, and its symmetries in 3D are listed later in table 2. Section 4 contains a simple toy model that is likely to realize this phase as its ground state.

3. Experimental detection of the DSSCO

Most experiments that probe static TRS breaking, such as NMR and elastic neutron scattering, explicitly measure local moments, so they cannot see the DSSCO. What experiments *can*?

As discussed earlier, spin moments are forbidden in the DSSCO by fundamental properties of continuous symmetries, while mirror symmetries preclude orbital moments involving other mobile degrees of freedom such as itinerant electrons, if present. Unlike spin moments, though, orbital moments only disobey discrete symmetries of the underlying lattice (in addition to TRS). Thus, if sufficient mirror symmetries are broken,

for instance, by applying a suitable electric field or driving a current through the system, current loops will generically form in the system which can then be picked up by standard probes of magnetic order. Explicitly, a straightforward symmetry analysis shows that the electromagnetic response Lagrangian of the DSSCO contains a term $\mathcal{L}_{em} \sim \hat{\mathbf{Q}} \cdot (\mathbf{E} \times \mathbf{B})$ upto coupling constants, where $\hat{\mathbf{Q}} \propto \chi$ is the chiral ordering direction, so an external electric field induces a magnetic response. Thus, a sharp signature of a DSSCO in 2D and 3D will be the appearance of local moments in the presence of electric fields or currents.

In 3D, another common experiment can sense the DSSCO without any other fields for destroying mirror symmetries, namely, the polar Kerr effect: the rotation of the plane of polarization of normally incident linearly polarized light upon reflection. This effect requires vertical mirror symmetries to be absent and, as long as linear response theory applies, also needs broken TRS [24, 25, 40]. In addition, either vertical reflections or time reversal combined with horizontal translations must not be symmetry operations either. Usually, these demands are met by bulk ferromagnetic moments perpendicular to the reflection surface. In such systems, the sign of the effect can be trained by a magnetic field and reverses upon flipping the sample. In contrast, the DSSCO satisfies these conditions if the experiment is performed on a low symmetry surface, such as the $(0kl), k \neq l$ surface of a cubic lattice with chiral ordering of the spins along x. The effect originates from a net magnetic moment on the surface, whose sign is determined by the bulk order parameter and the details of the surface termination[1]. Thus, it cannot be trained by a magnetic field, and has the same sign on opposite surfaces if the terminations are similar. Therefore, it is strikingly different from the Kerr effect in most other systems.

In 1D, alternate ideas are needed to detect the DSSCO because current loops are impossible and Kerr experiments are inapplicable. On the other hand, the excitation spectrum contains gapped states corresponding to domain walls of χ, similar to the domain walls in an Ising antiferromagnet, which are deconfined only in the 1D [41]. A standard technique for probing magnetic domain walls is via inelastic neutron scattering. Neutron spin couples linearly to electron spin, so it creates a fluctuation in the magnetization (magnon) when it scatters off

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[1] Note that the appearance of surface moments does not contradict any of the previous statements about a vanishing density of moments if the sample is thermodynamically thick.

a 1D Ising antiferromagnet. The magnon in turn decays into a pair of domain walls, which leads to the neutron structure factor exhibiting a characteristic '2-particle continuum' rather than sharply defined magnon quasiparticles [42, 43]. Similarly, χ has the same symmetries as an ordinary current and couples linearly to it, so electron diffraction off a 1D DSSCO should show analogous signatures of domain walls in χ. The details of the experiment, though, are beyond the scope of this work.

4. Candidate Hamiltonians

For integer spins per unit cell, the Lieb–Schultz–Mattis theorem places no constraints on the ground state and it is generically gapped. The gap guarantees short range entanglement [44] and therefore amenability to description as a matrix product state. Equivalently, it can be obtained in principle from an Affleck–Kennedy–Lieb–Tasaki type Hamiltonian, which consists of a sum of projection operators onto various spin channels acting on auxiliary spins [45]. However, it is easier and more illuminating to write a classical Hamiltonian that could yield the DSSCO for quantum spins, e.g., $H_{1D} = H_{bi} + H_{f\parallel}$, where

$$H_{bi} = \sum_{i=1}^{2} K_i \sum_x \left[\mathbf{S}(x) \cdot \mathbf{S}(x+i) \right]^2 \qquad (3)$$

$$H_{f\parallel} = -J_\parallel \sum_x \mathbf{S}(x) \cdot \mathbf{S}(x+3) \qquad (4)$$

represent biquadratic and ferromagnetic interactions along the chain, respectively, with $K_{1,2}, J_\parallel > 0$. While ferromagnetic interactions are pervasive in condensed matter literature, repulsive biquadratic interactions have been closely studied in theory and experiment in the context of the bilinear–biquadratic model [46–48]. If the spins are classical (large S), H_{bi} mutually orthogonalizes every set of three consecutive spins along x, and $H_{f\parallel}$ ensures that this arrangement repeats along the chain, thus giving rise to the pattern shown in figure 1(a). For small S, but $> 1/2$, quantum fluctuations partially melt the order and yield the DSSCO. Indeed, recent work involving the author proved the existence of the DSSCO numerically in a spin-1 chain in model related to H_{1D} [49]. In accordance with the Lieb–Schultz–Mattis theorem, the ground state is expected to be gapped for general integer spins and gapless for half-integer spins. Remarkably, the DSSCO would survive the gapless excitations. This is because it is an Ising order that is invariant under spin rotations whereas the gapless excitations can be viewed as space- and time-dependent rotations of the spins.

If $S = 1/2$, $\left[\mathbf{S}(\mathbf{r}) \cdot \mathbf{S}(\mathbf{r}') \right]^2 = \text{const.} - \mathbf{S}(\mathbf{r}) \cdot \mathbf{S}(\mathbf{r}')/2$ and the biquadratic term reduces to exchange; hence, the above procedure does not work and one is forced to start with a Hamiltonian with a six-spin interaction $-g(x - x')\chi(x)\chi(x')$ to induce the DSSCO. For example, the modified $S = 1/2$ Heisenberg model

$$H_{1/2} = \sum_x J\mathbf{S}(x) \cdot \mathbf{S}(x+1) - g(x - x')\chi(x)\chi(x') \qquad (5)$$

is expected to have a DSSCO ground state either when $g(x - x')$ is long-ranged or when $g(x - x')$ is short-ranged but $\gtrsim J$.

In d-dimensions, the discussion before (2) implies that the corresponding Hamiltonian for $S > 1/2$ is $H_{dD} = \tilde{H}_{bi} + \tilde{H}_{f\parallel} + H_{f\perp} + H_{dis}$, where

$$\tilde{H}_{bi} = \sum_{i=1}^{2} K_i \sum_\mathbf{r} \left[\mathbf{S}(\mathbf{r}) \cdot \mathbf{S}(\mathbf{r} + i\hat{\mathbf{x}}) \right]^2 \qquad (6)$$

$$\tilde{H}_{f\parallel} = -J_\parallel \sum_\mathbf{r} \mathbf{S}(\mathbf{r}) \cdot \mathbf{S}(\mathbf{r} + 3\hat{\mathbf{x}}) \qquad (7)$$

$$H_{f\perp} = -\sum_{a=y,z} J_{\perp a} \sum_\mathbf{r} \mathbf{S}(\mathbf{r}) \cdot \mathbf{S}(\mathbf{r} + \hat{\mathbf{a}}) \qquad (8)$$

$$H_{dis} = \begin{cases} 0 & \text{in 1D and 2D} \\ \sum_\mathbf{r} \mathbf{h}(\mathbf{r}) \cdot \mathbf{S}(\mathbf{r}) & \text{in 3D} \end{cases} \qquad (9)$$

with $J_{\perp(y,z)} > 0$ guaranteeing that the spin pattern is identical on all x-directed chains. The appropriate Hamiltonian for $S = 1/2$ is obtained simply by replacing $\tilde{H}_{bi}$ and $\tilde{H}_{f\parallel}$ by $H_{1/2}$, trivially generalized to d-dimensions:

$$\tilde{H}_{1/2} = \sum_\mathbf{r} J\mathbf{S}(\mathbf{r}) \cdot \mathbf{S}(\mathbf{r} + \hat{\mathbf{x}}) - g(\mathbf{r} - \mathbf{r}')\chi(\mathbf{r})\chi(\mathbf{r}') \qquad (10)$$

In the next section, we will elaborate on the phenomenological connection between the cuprates and the DSSCO and describe how (10) naturally arises in their pseudogap phase.

5. Application to the cuprates

Recently, several families of the underdoped cuprates have been found to exhibit a small polar Kerr effect in the pseudogap phase below a temperature T_K [15, 18, 22, 23]. Assuming linear response, the effect indicates broken TRS below T_K [24, 25]. Unlike the effect in ferromagnets and superconducting vortices, but like that in the DSSCO as discussed earlier, its sign cannot be trained by a magnetic field and is the same on opposite surfaces of the sample. These observations imply that the effect does not stem from ordinary ferromagnetic moments normal to the copper oxide planes. NMR experiments support this interpretation, as Knight shift measurements below T_K have set an upper bound on the size of local magnetic moments that is two orders of magnitude lower than that expected from some current proposals of TRS breaking phases [1, 2]. To complicate matters further, the sign of the effect also shows a 'memory effect', i.e., it is unchanged on heating to temperatures well above T_K and cooling back, indicating that some kind of order exists above T_K but does not produce a Kerr effect [15]. Nernst effect data support this hypothesis, as they see the C_4 symmetry of the copper-oxide plane broken down to C_2 above T_K, but below the pseudogap temperature T^*. Various x-ray scattering experiments have detected the onset of incommensurate CDWs at T_K [27–29], which suggests that the phase that forms above T_K breaks only some of the symmetries needed to produce a Kerr effect; the rest are broken by the

CDW. Finally, transmission experiments on thin films indicate that the symmetries that are broken by the CDW are vertical reflections [50]. Below, a phenomenological (but not microscopic) picture involving the DSSCO is presented in which all the above experimental features can be accommodated and which thus, may be relevant to the cuprates.

Suppose the DSSCO forms at a high temperature $T_D > T_K$ with chiral ordering along x, one of the in-plane crystal axes. Since the cuprates are technically 3D, the formation of the DSSCO would naively require random fields according to table 1, which are unlikely to be present in these materials. However, the interlayer couplings are weak in the cuprates, and simply $T > 0$ could suffice for practical purposes. In either case, C_4 symmetry about the z-axis is then broken down to C_2, which would give rise to an anisotropic Nernst effect. However, mirror symmetries about the xy and xz planes, and the absence of static magnetic moments, will suppress a Kerr effect and a Knight shift, respectively. Next, suppose incommensurate CDWs that break all mirror symmetries but respect twofold rotation symmetry about the x or y axis onset at T_K. Such charge orders were discussed recently [51, 52]. Below T_K, a Kerr signal is allowed by symmetry for reflection off the xy plane, and is likely to be small because it relies on the formation of two orders—the DSSCO and the CDW. Moreover, it cannot be trained by a magnetic field and is invariant under flipping the sample. This scenario involving two phase transitions can also capture the memory effect. Specifically, the pattern of mirror symmetry breaking by the CDW is likely determined by lattice defects. These are extremely stable below the melting temperature of the solid, so the sign of the Kerr effect will be the same as long as the $T < T_D$. This scenario requires $T_D \gtrsim 300$K in underdoped YBa$_2$Cu$_3$O$_{6.5}$, the temperature upto which the memory effect has been seen [15]. This is somewhat higher than the temperature below which the Nernst effect saw C_4 symmetry breaking, $T^* \approx 200$–250K [26], but is still within some error bars, so it is not unreasonable to suppose $T_D \approx T^*$. Below T_K, broken TRS and mirror symmetry allow magnetic moments to form. However, these moments will be small, possibly smaller than the NMR resolution, because they depend on two orders. A simple test of the above picture would be a Kerr signal between T_K and T_D triggered by a current along one of the in-plane axes, but not along the other (see figure 2(b)). The signal, moreover, will flip on reversing the current. Table 2 summarizes these symmetry properties and figure 2(a) shows a plausible phase diagram.

Equation (10) is likely to naturally arise in the pseudogap phase as follows. Firstly, nearest-neighbor antiferromagnetic exchange, J, is well-known to be present in these materials. Secondly, at $T \gtrsim T_K$, before the onset of the incommensurate CDW, fluctuations that break reflection symmetries are expected to be strong. In particular, consider the velocity asymmetry $\Delta v_x(\mathbf{r})$ which captures the difference in the velocities of left-movers and right-movers in the x-direction. $\Delta v_x(\mathbf{r})$ is odd under M_x and TRS and $\langle \Delta v_x(\mathbf{r}) \rangle \neq 0$ in the incommensurate CDW phase since TRS is already broken by the DSSCO. Denoting the correlation length which quantifies fluctuations
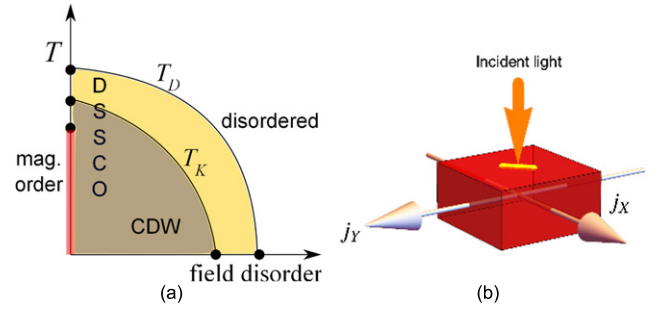

Figure 2. (a) Schematic phase diagram that may be relevant to the cuprates. The DSSCO forms at T_D and coexists with charge order below T_K, the Kerr onset temperature. T_D may be $\approx T^*$; see text for details. At zero disorder, it is not known *a priori* whether T_K is higher or lower than the magnetic ordering temperature. (b) Experimental setup for probing the DSSCO. For chiral ordering along x, $j_y(j_x)$ would (would not) produce a Kerr effect for reflection off the xy-surface.

of Δv_x by ξ_{v_x}, we have $\langle \Delta v_x(0) \Delta v_x(\mathbf{r}) \rangle \sim e^{-r/\xi_{v_x}}$ or equivalently, $\langle \Delta v_x(\mathbf{q}) \Delta v_x(-\mathbf{q}) \rangle \sim 1/(q^2 + \xi_{v_x}^{-2})$. Now, an interaction term allowed by symmetry is

$$H_{cs} = \lambda \sum_{\mathbf{r}} \Delta v_x(\mathbf{r}) \left[\mathbf{S}(\mathbf{r} - \hat{\mathbf{x}}) \cdot \mathbf{S}(\mathbf{r}) \times \mathbf{S}(\mathbf{r} + \hat{\mathbf{x}}) \right] \qquad (11)$$

Integrating out Δv_x fluctuations produces the effective six-spin interaction in (10) with

$$g(\mathbf{r}) \sim \lambda^2 \int \frac{d^2 q}{2\pi} \frac{e^{i\mathbf{q} \cdot \mathbf{r}}}{q^2 + \xi_{v_x}^2} = \frac{\lambda^2}{2\pi} K_0(r/\xi_{v_x}) \qquad (12)$$

where K_0 denotes the zeroth modified Bessel function of the second kind, which diverges as $r/\xi_{v_x} \to 0$ and decays as $e^{-r/\xi_{v_x}}$ for $r \gg \xi_{v_x}$. Since ξ_{v_x} grows and eventually diverges as one approaches the CDW transition, the range of the six-spin interaction grows too, thereby stabilizing the DSSCO. In short, the DSSCO enables Δv_x-fluctuations above an M_x-breaking transition by breaking TRS as well, and the fluctuations return the favor by producing an interaction that stabilizes the DSSCO.

6. Conclusions

In summary, we have proposed the DSSCO as a novel phase of matter that violates TRS but has no density of moments, unlike other TRS-breaking phases known in condensed matter. It appears when a scalar chiral order of spins partially melts, leaving behind residual broken TRS but unbroken continuous SRS. The DSSCO is *allowed* by the basic rules of quantum mechanics, and we have proposed candidate parent Hamiltonians. Whether the ground state of these Hamiltonians is the DSSCO is a crucial but challenging question that is beyond the scope of the current work.

The DSSCO also has crucial implications for cuprate phenomenology. A phenomenological picture, in which the DSSCO coexists with a CDW, can be argued to have many of the features found in Kerr effect, Knight shift, x-ray diffraction and Nernst effect experiments in the pseudogap phase of the underdoped cuprates, and can be tested by looking for a

Kerr signal above T_K on driving a suitable current through the system. We have also argued that the DSSCO results in the cuprates from an effective six-spin interaction mediated by enhanced fluctuations in the velocity asymmetry above the CDW onset. These fluctuations are expected to be enhanced in the region above the mirror-symmetry-breaking transition into the incommensurate CDW. This, in turn, stabilizes the DSSCO in these materials.

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