To the study of vibration of materials with coatings in the presence of defects

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Abstract. In the paper a new method for studying the properties of deformable bodies with coatings is developed. We considered two models of the foundation: a homogeneous elastic layer with clamped lower edge and a two-layer packet containing a flat rigid inclusion located in the plane of the substrate elastic properties. We used the method of eigenfunctions in the study of the problem concerning steady-state oscillations of a deformable material with a cracked coating. For a foundation that does not contain defects, we described a method for the determination of main parameters characterizing the stress-strain state of a structure with a cracked coating composed of two long plates. We considered a continuous coating for the model of defective foundation. In the course of our study we obtained the relations for determining the characteristics of the stress-strain state of coating/substrate systems as well as presented an example of the application of the method for studying edge effects near the junction boundaries of a composite coating.

1. Introduction

The widespread use of composite materials in engineering practice, which often have coatings, requires consideration of the specific features of their mechanical behavior. In the study of the stress-strain state of bodies with composite coatings, the development of mathematical methods that sufficiently describe surface phenomena and edge effects near the junction boundaries of the coating plates and shells is a particularly relevant problem. Nowadays, direct numerical methods are widely used [1–4, etc.], however, their application in case of extended bodies causes difficulties due to the unlimited nature of the region covered by the perturbation.

The paper presents a method for studying the properties of deformable bodies with coatings. We considered two models of the foundation: a homogeneous elastic layer with a clamped lower edge and a two-layer packet containing a flat rigid inclusion located in the plane of the substrate elastic properties differentiation. For a foundation that does not contain defects, we described a method for the determination of main parameters characterizing the stress-strain state of a structure with a cracked coating composed of two long plates. We considered a continuous coating for the model of defective foundation.

Often in various applications the problems associated with the study of a particular mode of oscillation in the system emerge. The purpose of the study is the solution of three-dimensional problems on steady-state oscillations of extended, deformable media with coating. For the steady-state oscillations with a given frequency ω, the dependence on time of all unknown and given functions of the problem is described by a factor $\exp(-\mathrm{i}\omega t)$. Coating elements are modeled by Kirchhoff plates with parameters averaged over thickness [5, 6]. The middle plane of the coating is located in the x_1Ox_2 plane of the Cartesian coordinate system, the Ox_3 axis is directed upwards. Oscillations of Kirchhoff thin plates that are in ideal contact with a deformable foundation are described by a linearized system of differential equations for the displacements of the middle surface [5]. The harmonic load specified on the surface of the coating / foundation system is considered as a source of oscillations. The paper proposes algorithms for solving the problems on elastic bodies with coatings using factorization methods.

2. The problem of oscillations of the cracked coating on an elastic foundation

We are investigating the vibration of a coating / foundation system driven by a localized surface force. The composite coating consists of two plates occupying half-planes with a crack at the interface, along which the Ox_2 axis is directed (figure 1).

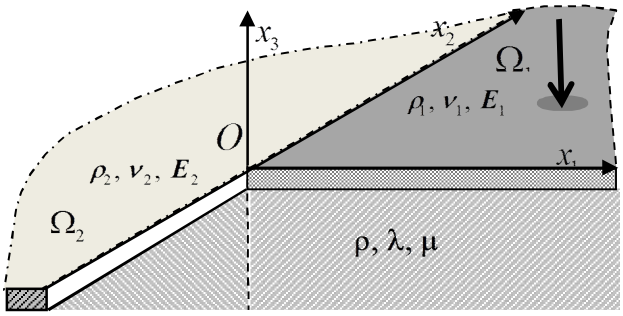

Figure 1. The structure of an elastic medium with a composite coating under the influence of localized surface load

After the separation of the time factor, the equations of plate's displacement [5] take the form

$$\mathbf{R}_j(\partial x_1,\partial x_2)\mathbf{u}_j(x_1,x_2)-\mathbf{E}_j\mathbf{g}_j(x_1,x_2)=\mathbf{b}_j(x_1,x_2),\ x_1\in\Omega_j,\ x_2\in\mathrm{R}. \quad (1)$$

In (1) for the j-th plate, the elements of matrix differential operators $\mathbf{R}_j(\partial x_1,\partial x_2)$ $(j=1,2)$ are given as: $R_{11}^j=\frac{\partial^2}{\partial x_1^2}+\varepsilon_{j1}\frac{\partial^2}{\partial x_2^2}+\varepsilon_{j4}$, $R_{22}^j=\frac{\partial^2}{\partial x_2^2}+\varepsilon_{j1}\frac{\partial^2}{\partial x_1^2}+\varepsilon_{j4}$, $R_{12}^j=R_{21}^j=\varepsilon_{j2}\frac{\partial^2}{\partial x_1\partial x_2}$, $R_{33}^j=\varepsilon_{j3}\left(\frac{\partial^4}{\partial x_1^4}+2\frac{\partial^4}{\partial x_1^2\partial x_2^2}+\frac{\partial^4}{\partial x_2^4}\right)-\varepsilon_{j4}$, $R_{13}^j=R_{23}^j=R_{31}^j=R_{32}^j=0$. Here $\varepsilon_{j1}=\frac{1-\nu_j}{2}$, $\varepsilon_{j2}=\frac{1+\nu_j}{2}$, $\varepsilon_{j3}=\frac{h_j^2}{12}$, $\varepsilon_{j4}=\frac{\omega^2\rho_j(1-\nu_j^2)}{E_j}$, $\varepsilon_{j5}=\frac{1-\nu_j^2}{E_jh_j}$, ρ_j – density, ν_j – Poisson's ratio, E_j – Young's modulus, h_j – thickness, $\mathbf{u}_j=\{u_{j1},u_{j2},u_{j3}\}$ – the displacement amplitude vector of the middle plane, which is a function of coordinates (x_1,x_2); $\mathbf{E}_j=\mathrm{diag}\{-\varepsilon_{j5},-\varepsilon_{j5},\varepsilon_{j5}\}$, $\mathbf{g}_j=\{g_{jk}\}$ describes the effect on the plate from the side of the foundation; $\mathbf{b}_j=-\varepsilon_{j5}\mathbf{t}_j$, $\mathbf{t}_j=\{t_{jk}\}$, $k=\overline{1,3}$ – vector of surface load; $\Omega_1=\{x_1:0<x_1<+\infty\}$, $\Omega_2=\{x_1:-\infty<x_1<0\}$.

On the surface of the foundation (an elastic layer of thickness H with a rigidly fixed lower boundary), the amplitudes of displacements $\mathbf{u}$ and stresses $\mathbf{g}$ are related by equation

$$\mathbf{u}(x_1,x_2,0)=\frac{1}{4\pi^2}\int\limits_{\sigma_1}\int\limits_{\sigma_2}\mathbf{K}(\alpha_1,\alpha_2)\mathbf{G}(\alpha_1,\alpha_2,0)\exp\left(-\mathrm{i}(\alpha_1 x_1+\alpha_2 x_2)\right)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\,, \qquad (2)$$

where $\mathbf{K}=\left\|K_{nm}\right\|_{n,m=1}^{3}$ – the Green matrix of the foundation, which has a clear representation and depends on the elastic parameters of the layer, its thickness and vibration frequency; $\mathbf{G}=\mathrm{V}_2\mathbf{g}$, V_2 – two-dimensional Fourier transform integral operator. The form of Green matrices $\mathbf{K}$ for various elastic media and methods for their construction are presented in [7, etc.]. The estimated steady-state nature of the foundation vibrations requires the fulfillment of conditions ensuring the uniqueness of the solution. In the work we use the principle of limiting absorption [7] to determine the position of the contours σ_1, σ_2 in the complex plane.

The formulation of the problem is complemented by boundary conditions in the area of contact between the components of the composite coating ($x_1=0$, $-\infty<x_2<+\infty$) to describe various types of interaction between them

$$\mathbf{L}_1(\partial x_1,\partial x_2)\mathbf{u}_1(0,x_2)+\mathbf{L}_2(\partial x_1,\partial x_2)\mathbf{u}_2(0,x_2)=\mathbf{f}(x_2), \qquad (3)$$

where the forms of differential operators $\mathbf{L}_j(\partial x_1,\partial x_2)$ $(j=1,2)$ and the function **f** are given.

Full adhesion of the coating to the elastic foundation is considered

$$\mathbf{u}_j(x_1,x_2)=\mathbf{u}(x_1,x_2),\ \mathbf{g}_j(x_1,x_2)=\mathbf{g}(x_1,x_2),\ x_1\in\Omega_j,\ x_2\in\mathrm{R}\,.$$

In the work [8], a method for solving the described problem, based on the transformation of its differential operator, is demonstrated. Such an approach is not the only possible one.

The geometry of the problem and the use of the linear theory of elasticity for the system allows us to use the integral Fourier transform (in relation to variable x_2) for solving the problem, reducing (1) to the system of ordinary differential equations (ODE) for Fourier transform images with the parameter α_2

$$\mathbf{R}_j(\partial x_1,-i\alpha_2)\overline{\mathbf{u}}_j(x_1,\alpha_2)-\mathbf{E}_j\overline{\mathbf{g}}_j(x_1,\alpha_2)=\overline{\mathbf{b}}_j(x_1,\alpha_2),\ x_1\in\Omega_j,\ \alpha_2\in\mathrm{R}\,, \qquad (4)$$

where $j=1,2$.

When constructing general solutions of a system of homogeneous ODEs, one should choose those that are limited in the right $(j=1)$ and left $(j=2)$ half-planes and meet the requirements of the principle of limiting absorption $\overline{\mathbf{v}}_k^{(j)}=\left\{\overline{v}_{km}^{(j)}\right\}$. As a result, having performed the Fourier transform in relation to x_1, we arrive at the following representation for the integral characteristics of the plates displacements

$$\mathbf{U}_j(\alpha_1,\alpha_2)=\left\{\mathbf{R}_j^{-1}(-i\alpha_1,-i\alpha_2)\left[\mathbf{E}_j\mathbf{G}_j(\alpha_1,\alpha_2)+\mathbf{B}_j(\alpha_1,\alpha_2)\right]\right\}^{\pm}+\sum_{k=1}^{4}C_{jk}(\alpha_2)\mathbf{V}_k^{(j)}(\alpha_1,\alpha_2). \qquad (5)$$

Denoting by $\mathrm{V}(\alpha_1)$ the operator the Fourier transform in relation to variable x_1 can be written in the form $\mathbf{V}_k^{(j)}=\mathrm{V}(\alpha_1)\overline{\mathbf{v}}_k^{(j)}(x_1,\alpha_2)$. Here, the superscript of the symbol «$\pm$» corresponds to the value $j=1$ (right plate), the subscript to $j=2$ (left plate) and defines a vector function that is regular above (+) and below (–) the contour σ_1.

The boundary conditions for the joining of the coating and the foundation (3) in two-dimensional Fourier symbols are written as

$$\mathbf{U}(\alpha_1,\alpha_2)=\mathbf{U}_1(\alpha_1,\alpha_2)+\mathbf{U}_2(\alpha_1,\alpha_2),\ \mathbf{G}(\alpha_1,\alpha_2)=\mathbf{G}_1(\alpha_1,\alpha_2)+\mathbf{G}_2(\alpha_1,\alpha_2), \qquad (6)$$

$$\mathbf{G}_j = \mathrm{V}_2\mathbf{g}_j,\ \mathbf{U}_j = \mathrm{V}_2\mathbf{u}_j.$$

From the relations for $\mathbf{U}_j$ (5) and the conditions at the boundary of the coating and the foundation (6), we can obtain the functional-matrix equation with respect to $\mathbf{G}_1(\alpha_1,\alpha_2)\equiv\mathbf{G}_1^+$, $\mathbf{G}_2(\alpha_1,\alpha_2)\equiv\mathbf{G}_2^-$

$$\mathbf{K}_1\mathbf{G}_1^+(\alpha_1,\alpha_2)=\mathbf{K}_2\mathbf{G}_2^-(\alpha_1,\alpha_2)+\sum_{j=1}^{2}\mathbf{R}_j^{-1}(-i\alpha_1,-i\alpha_2)\mathbf{B}_j(\alpha_1,\alpha_2)+$$

$$-\sum_{l=1}^{4}\mathbf{R}_{1l}^{-1}(\alpha_1,\alpha_2)\left[\mathbf{E}_1\mathbf{G}_1^+(q_{1l},\alpha_2)+\mathbf{B}_1(q_{1l},\alpha_2)\right]- \tag{7}$$

$$-\sum_{l=1}^{4}\mathbf{R}_{2l}^{-1}(\alpha_1,\alpha_2)\left[\mathbf{E}_2\mathbf{G}_2^-(q_{2l},\alpha_2)+\mathbf{B}_2(q_{2l},\alpha_2)\right]+\sum_{l=1}^{2}\sum_{k=1}^{4}C_{lk}(\alpha_2)\mathbf{V}_k^{(l)}(\alpha_1,\alpha_2),$$

$\mathbf{R}_{jl}^{-1}(\alpha_1,\alpha_2)=(\alpha_1-q_{jl})^{-1}\operatorname*{Res}_{\eta_1=q_{jl}}\mathbf{R}_j^{-1}(-i\eta_1,-i\alpha_2)$. The roots of the equation $\det\mathbf{R}(-i\alpha_1,-i\alpha_2)_j=0$ positioned above the contour σ_1 (with respect to the variable α_1) are denoted by $q_{1l}(\alpha_2)$, those positioned below the contour σ_1 – $q_{2l}(\alpha_2)$, $\mathbf{K}_j(\alpha_1,\alpha_2)=\pm\left[\mathbf{K}(\alpha_1,\alpha_2)-\mathbf{R}_j^{-1}(-i\alpha_1,-i\alpha_2)\mathbf{E}_j\right]$.

Equation (7) is reduced to the system of loaded Wiener – Hopf equations [9, 10] solved by the factorization method. After substituting the obtained relations for $\mathbf{G}_j$ in (5), the expressions for the Fourier transformants of displacement amplitudes will contain unknown $C_{lk}(\alpha_2)$, $l=1,2$, $k=\overline{1,4}$, determined from the boundary conditions specified at the interface of the plates (3). The inverse Fourier transform is performed numerically using matrix function $\mathbf{K}$ approximation.

The described approach is used for a simplified model. A solution is constructed for the scalar case of vertical oscillations of a plate system on an elastic layer with a clamped lower boundary. A focused source of external influences is described by a function $t_{13}=A\delta(x_1-x_1^0)\exp(-\mathrm{i}\omega t)$, $x_1^0\in\Omega_1$.

Representations (5) for the integral characteristics of the displacements in this case take the form

$$U_{j3}(\alpha_1,\alpha_2)=\frac{\varepsilon_{j5}G_{j3}(\alpha_1,\alpha_2)+B_{j3}(\alpha_1,\alpha_2)}{R_{33}^j}+W_j(\alpha_1,\alpha_2), \tag{8}$$

$$W_j(\alpha_1,\alpha_2)=\frac{\pm iC_{j1}}{\alpha_1\pm iq_{j1}}+\frac{\pm iC_{j2}}{\alpha_1\pm q_{j2}}\mp\frac{1}{2(q_{j1}^2+q_{j2}^2)\varepsilon_{j3}}\left[\frac{\varepsilon_{j5}G_{j3}(\pm q_{j2},\alpha_2)}{q_{j2}(\alpha_1\mp q_{j2})}+\frac{i\varepsilon_{j5}G_{j3}(\pm iq_{j1},\alpha_2)}{q_{j1}(\alpha_1\mp iq_{j1})}+\right.$$

$$\left.+\frac{B_{j3}(\pm q_{j2},\alpha_2)}{q_{j2}(\alpha_1\mp q_{j2})}+\frac{iB_{j3}(\pm iq_{j1},\alpha_2)}{q_{j1}(\alpha_1\mp iq_{j1})}\right].$$

Relations (8) are used for numerical modeling of the system oscillations under the condition of constant properties in the direction of the axis Ox_2 (figures 2, 3).

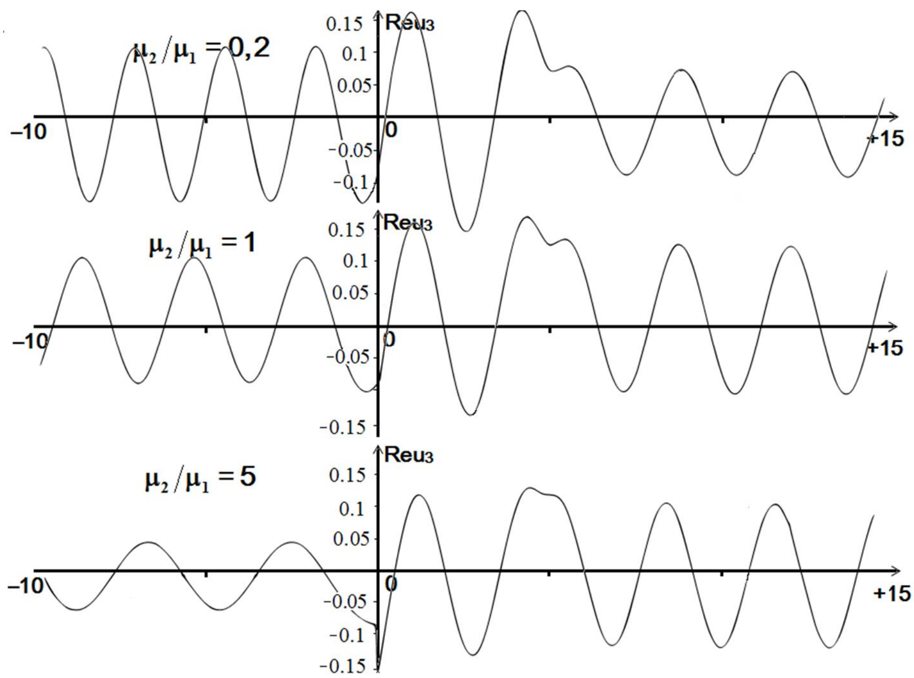

Figure 2. Real parts of the surface displacement amplitudes for more rigid foundation

At the junction of the plates of equal thickness, the bending moments are equal to zero $\left.\frac{\partial^2 u_{3j}}{\partial x_1^2}\right|_{x_1=0}=0,\ j=1,2$, and the «viscous contact» condition is also satisfied $D_1\left.\frac{\partial^3 u_{31}}{\partial x_1^3}\right|_{x_1=0}=-D_2\left.\frac{\partial^3 u_{32}}{\partial x_1^3}\right|_{x_1=0}=\mathrm{i}k\omega\left(u_{32}\left(x_1\right)-u_{31}\left(x_1\right)\right)$, where $D_j=\frac{E_j h_j^{\,2}}{12\left(1-\nu_j^{\,2}\right)}$, $k=0{,}5$. Figure 2 shows the graphs of the real parts of the surface displacement amplitudes for the $\bar{\omega}=2{,}5$ and given model dimensionless characteristics of the base: $\rho=1{,}064$, $\mu=1{,}579$, $\nu=0{,}25$. The frequency is determined by the formula $\bar{\omega}^2=\rho\omega^2 a^2\mu^{-1}$, where ρ is the density of the foundation, μ is the modulus of displacement, a – the characteristic linear dimension, $\bar{x}_0=5$. For plates, the following characteristics are accepted: $\rho_1=\rho_2=1$, $\nu_1=\nu_2=0{,}125$. The foundation material is more rigid than the material of the right plate. Figure 3 corresponds to the case when the substrate material is less rigid than the material of the right plate: $\mu=0{,}67$, $\nu=0{,}125$. The boundary conditions and values of other parameters are as before.

Calculations were carried out for different ratios of plate rigidity: $\mu_2/\mu_1=0{,}2$ at the top, $\mu_2/\mu_1=1$ in the middle, $\mu_2/\mu_1=5$ at the bottom.

The calculations illustrate the dependence of the wave process on the surface of the structure under consideration from the properties of the foundation, parameters of the plates and the nature of their interaction at the interface. Test calculations for the ideal contact of the plates and their properties coinciding with the properties of the substrate, correspond to the surface wave pattern for an elastic strip. In addition, the numerical results are in agreement with those obtained by another method (based on the transformation of the problem differential operator) presented in [8].

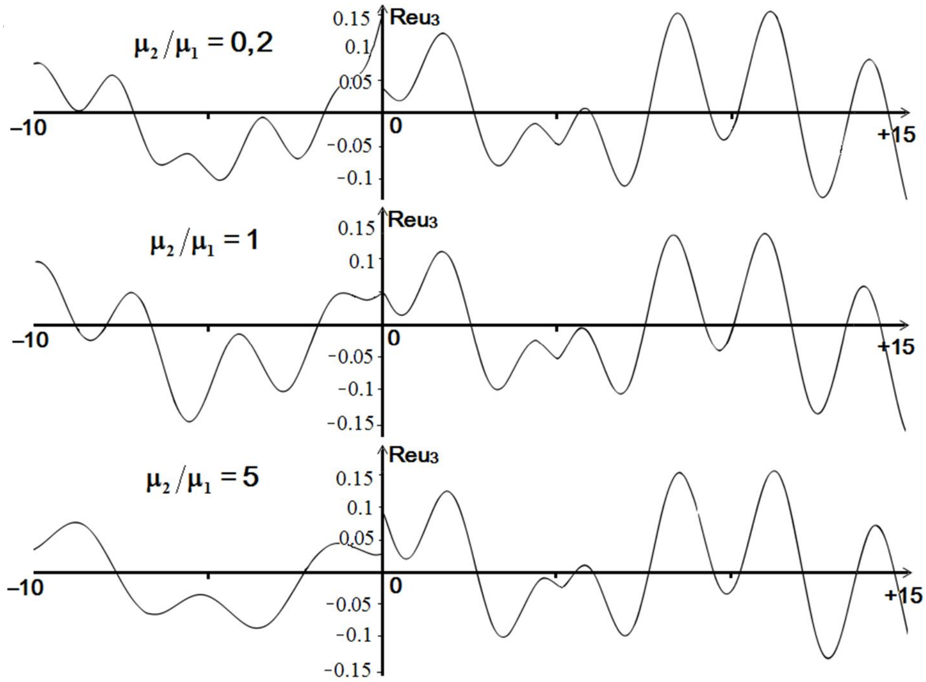

Figure 3. Real parts of the surface displacement amplitudes for less rigid foundation

3. The problem of oscillations of the cracked coating on an elastic foundation

We also consider the problem of steady-state harmonic oscillations of a two-layer packet bounded by planes bounded by the planes $x_3 = 0$, $x_3 = -H$. Layers are considered homogeneous and infinitely extended. The properties of each of the foundation layers are characterized by the elastic constants ν_j, μ_j and material density ρ_j $(j = 1,2)$. The motion of the elastic foundation points is described by the displacement amplitude vector $\mathbf{u}_j = \{u_{jn}\}$ $\left(n = \overline{1,3}\right)$ satisfying the homogeneous system of Lame equations. The lower boundary of the package is rigidly clamped $\mathbf{u}_1 = 0$. In the plane of separation of the layers ($x_3 = -h_1$) there is an inclusion occupying the area Ω_1, stresses $\boldsymbol{\tau}_1^{\pm} = \boldsymbol{\tau}(x_1, x_2, -h_1 \pm 0)$, $\boldsymbol{\tau}_1^{\pm} = \{\tau_{1n}^{\pm}\}$ act on the surfaces, and the displacements are determined by vectors $\mathbf{u}_1^{\pm} = \mathbf{u}(x_1, x_2, -h_1 \pm 0)$. In the inclusion area Ω_1, equal displacements are set on both faces, $x_3 = -h_1$: $\mathbf{u}_1^- = \mathbf{u}_1^+$, $(x_1, x_2) \in \Omega_1$. Outside the inclusion area in the interface plane displacements and stresses are continuous,

$$\boldsymbol{\tau}_1^+ - \boldsymbol{\tau}_1^- = \begin{cases} 0, (x_1, x_2) \notin \Omega_1, \\ \boldsymbol{\tau}_1^*, (x_1, x_2) \in \Omega_1. \end{cases}$$

Here, the stress-jump on the inclusion is indicated by $\boldsymbol{\tau}_1^*$. This type of defect can be attributed to the vibration-strength «viruses» of class 1 [11].

To construct the relations that connect displacements and stresses on the surface of a foundation with a defect, a generalization of the integral approach is used – the differential factorization method [7], which allows to take into account the mutual influence of the physic and mechanical and geometric parameters of the problem. The advantage of this approach is its close connection with the method of integral transformations in structures with parallel flat boundaries. The technique of representing the integral characteristics of displacements and stresses for multilayer media with inclusions and cracks is described in [11, 12, etc.], as a result of its application, we arrive at a system of functional-matrix equations for Fourier transforms of stresses $\mathbf{T}_2^- = \mathrm{V}_2(\alpha_1,\alpha_2)\boldsymbol{\tau}_2^-$, displacements $\mathbf{U}_j^- = \mathrm{V}_2(\alpha_1,\alpha_2)\mathbf{u}_j^-$ and stress-jump in the inclusion area $\mathbf{T}_1^* = \iint\limits_{\Omega_1} \boldsymbol{\tau}_1^*(x_1,x_2)\exp\left(\mathrm{i}\left(\alpha_1 x_1 + \alpha_2 x_2\right)\right)\mathrm{d}x_1\mathrm{d}x_2$:

$$\mathbf{R}_{11}\mathbf{T}_1^* + \mathbf{W}_{11}\mathbf{T}_2^- = \mathbf{U}_1^-,\ x_3 = -h_1,$$
$$\mathbf{R}_{21}\mathbf{T}_1^* + \mathbf{K}_{22}\mathbf{T}_2^- = \mathbf{U}_2^-,\ x_3 = 0.$$

Here, the elements of the matrixes $\mathbf{R}_{11}$, $\mathbf{W}_{11}$, $\mathbf{R}_{21}$, $\mathbf{K}_{22}$ depend on the frequency ω, the geometric and physicomechanical characteristics of the elastic layers and the Fourier transform parameters [13].

The relations for the amplitudes of the surface displacements will take the form

$$\mathbf{u}_2^-(x_1,x_2,0) = \frac{1}{4\pi^2}\int\limits_{\sigma_1}\int\limits_{\sigma_2}\left[\mathbf{R}_{21}\mathbf{T}_1^* + \mathbf{K}_{22}\mathbf{T}_2^-\right]\exp\left(-\mathrm{i}\left(\alpha_1 x_1 + \alpha_2 x_2\right)\right)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 . \tag{9}$$

If the plate-coating is coupled with a deformable foundation, i.e. $\mathbf{u}(x_1,x_2) = \mathbf{u}_2^-(x_1,x_2,0)$, $\mathbf{g}(x_1,x_2) = \boldsymbol{\tau}_2^-(x_1,x_2,0)$, by substituting the integral representation of the displacements of the foundation surface (9) into the equations of motion for the coating, we obtain

$$\frac{1}{4\pi^2}\int\limits_{\sigma_1}\int\limits_{\sigma_2}\left(\mathbf{R}(-i\alpha_1,-i\alpha_2)\mathbf{K}_{22}(\alpha_1,\alpha_2) - \mathbf{E}\right)\mathbf{G}(\alpha_1,\alpha_2)\exp\left(-\mathrm{i}\left(\alpha_1 x_1 + \alpha_2 x_2\right)\right)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 +$$
$$+\frac{1}{4\pi^2}\int\limits_{\sigma_1}\int\limits_{\sigma_2}\mathbf{R}(-\mathrm{i}\alpha_1,-\mathrm{i}\alpha_2)\mathbf{R}_{21}\mathbf{T}_1^*\exp\left(-\mathrm{i}\left(\alpha_1 x_1 + \alpha_2 x_2\right)\right)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 = \mathbf{b}(x_1,x_2).$$

The obtained representations allow us to construct a system of integral equations of the considered problem for $\boldsymbol{\tau}_1^*$ and $\mathbf{g}$

$$K_{11}(\Omega_1)\boldsymbol{\tau}_1^* + K_{12}(\mathrm{R})\mathbf{g} = \mathbf{u}_1,\ x_3 = 0,\ (x_1,x_2)\in\Omega_1;$$
$$K_{21}(\Omega_1)\boldsymbol{\tau}_1^* + K_{22}(\mathrm{R})\mathbf{g} = \mathbf{b},\ x_3 = 0,\ (x_1,x_2)\in\mathrm{R}.$$

Here we use the notation

$$K_{jk}(\Omega)\mathbf{q} = \iint\limits_{\Omega}\mathbf{k}(x_1-\xi_1,x_2-\xi_2)\mathbf{q}(\xi_1,\xi_2)\mathrm{d}\xi_1\mathrm{d}\xi_2,$$
$$\mathbf{k}_{jk}(x_1,x_2) = \frac{1}{4\pi^2}\int\limits_{\sigma_1}\int\limits_{\sigma_2}\mathbf{K}_{jk}(\alpha_1,\alpha_2)\exp\left(-\mathrm{i}\left(\alpha_1 x_1 + \alpha_2 x_2\right)\right)\mathrm{d}\alpha_1\mathrm{d}\alpha_2,\ j,k = 1,2,$$
$$\mathbf{K}_{11} = \mathbf{R}_{11},\ \mathbf{K}_{12} = \mathbf{W}_{11},\ \mathbf{K}_{21} = \mathbf{R}(-\mathrm{i}\alpha_1,-\mathrm{i}\alpha_2)\mathbf{R}_{21},\ \mathbf{K}_{22} = \mathbf{R}(-\mathrm{i}\alpha_1,-\mathrm{i}\alpha_2)\mathbf{K}_{22}(\alpha_1,\alpha_2) - \mathbf{E}.$$

Thus, for a foundation with a rigid inclusion type defect, the application of the differential factorization method algorithm necessitates solving a system of integral equations of the first kind connecting displacements at the interface of the layers and in the contact plane of the coating and the substrate with a jump of stresses in the inclusion area whose solutions for particular cases of defects areas can be constructed using factorization [14] and variational iteration [15] methods.

4. Conclusion

In this paper we considered two models of the base: a homogeneous elastic layer with a clamped lower edge and a two-layer packet containing a flat rigid inclusion in the interface plane. We used the method of eigenfunctions and factorization method in the study of the problem concerning steady-state oscillations of a deformable material with a cracked coating. The proposed approach allows us to study the influence of the properties of the plates and the base, as well as the different contact conditions of the coating elements on the characteristics of the stress-strain state of the system under consideration. For defective base considered a continuous extended coating.

The application of the theory of vibration-strength «viruses» [6] allows us to construct a Green matrix for a substrate with multiple inclusions located in parallel planes. The relevance of the research is determined by the need to build and develop mathematical models to describe wave processes in structures and materials with coatings. Their results can be applied for studying the interaction of geological structures, as well as the processes of vibration of structural elements of engineering constructions.

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