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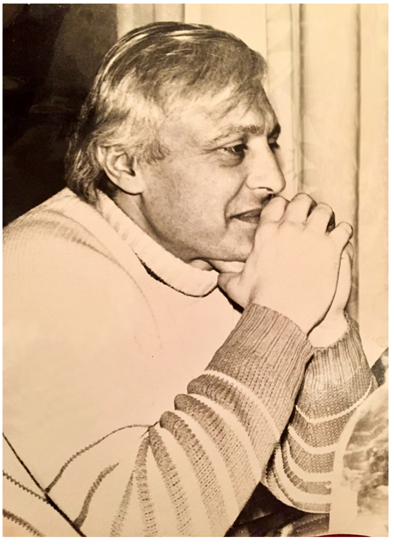
Evgenii Alekseevich Gorin (obituary)



The remarkable Russian mathematician Evgenii Alekseevich Gorin passed away on 4 October 2018.

He was born in Moscow, on 28 January 1936. His father Aleksei Fedorovich Gorin (1903–1953), the eldest son from a poor peasant family with 11 children, was from the village of Verkhnee Khoroshovo in Kolomna Uyezd (a Moscow governorate). At the age of 13 he started working in a factory, developed into a highly skilled locksmith, and then enrolled in and graduated from the Bauman Higher Technical School in Moscow. In his peak years he was a well-known Moscow engineer and the director of several large factories; during World War II he was the head of the important Brake-System Plant.

Gorin's mother, Lidiya Semenovna Gorina (1912–1991), was born in the city of Nikolaev and married his father when she worked at the Brake-System Plant. Before World War II, already a mother of two children, she graduated from the Moscow State Pedagogical Institute (now Pedagogical University) with a diploma in the Russian language and literature, and for a long time she was a director of studies in extended education courses for school teachers.

Evgenii Gorin met his future wife Irina Aleksandrovna Krsovskaya in the Faculty of Mechanics and Mathematics at Moscow State University, where she was one year behind him. Subsequently, she worked for many years in the Department of Applied Mathematics at the Moscow Institute (now University) of Civil Engineering as an associate professor. For almost sixty years she was Gorin's devoted friend and life companion. Their son Andrei graduated from the Moscow Institute of Electronic Engineering (now Moscow Institute of Electronics and Mathematics) with a diploma in computer progamming. He is now a successful businessman. Gorin and his wife had three grandsons, and a great grandson was born shortly before his death.

Gorin went to Moscow School no. 167 on Degtyarnyi Lane (now School no. 2054), where his wonderful school teacher El'frida Moiseevna Abezgauz planted in him an interest in and love of mathematics. As a high-school student, Gorin was successful

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in the Moscow mathematical olympiads for schoolchildren, and after graduating in 1953 he enrolled in Moscow State University.

The students of the year 1953 were the first to study in the Faculty of Mechanics and Mathematics in the newly built Main Building of the university on the Lenin Hills in Moscow. This was the first year, when 450 students enrolled (almost the same number as now), but these were aspiring students from two graduation years. Many think that the new students of that year were exceptionally talented youths who made up one of the best generations the faculty ever had. They included many who would become well-known professors of the faculty, researchers at the Steklov Mathematical Institute, some mathematicians of world renown, directors of research institutes, prominent economists, computer programmers, and winners of state prizes.

Perhaps even more important was that this was the first class selected after the death of the 'best friend of Soviet athletes'.[1] This was a generation of students whose university years coincided with the first years of the period later called 'the thaw', when "…ponderous ice floes began to stir"[2] in the life of our country. As A. N. Kolmogorov wrote about that time, "The main thing was that hope arose in 1953". Gorin turned twenty in 1956, essentially when the 20th congress of the Communist Party of the Soviet Union was held, which set off a fundamental evolution of the society of that period; for instance, the expression *enemy of the people* was officially eliminated from laws and regulations in 1958, the year when Gorin graduated from the university.

Gorin's student years significantly shaped his mode of interrelation with others, with its characteristic features of freedom and openness, warmth, and friendliness.

The development of Gorin into a prominent mathematician took perhaps the first 15 years after graduation. In that period he completed his postgraduate studies and then followed a path from assistant professor to senior researcher in the Department of the Theory of Functions and Functional Analysis at Moscow State University. He wrote important papers and defended his Ph.D. and D.Sc. theses. After turning 30 in 1966, he gave a talk at the International Congress of Mathematicians.

In general, the period between 1958 and 1973 (when I. G. Petrovskii[3] died) is now often called the 'golden age' of Moscow mathematics, and Gorin made a clear contribution to the justification of this assessment. His research seminar "Banach algebras and complex analysis" (which he supervised for many years together with V. Ya. Lin) was one of the visible accumulation points in the general bright picture of the Soviet mathematics of that time. Many of the participants of the seminar (which functioned from 1964 to 1987) developed in its framework into professional mathematicians who became professors at Russian and foreign universities. In total, 28 Ph.D. theses were written and defended with Gorin as advisor.

At the same time Gorin began the huge task he undertook in the Mir publishing house, where he was an editor and translator. Thanks to his selfless and highly professional work, several generations of Soviet mathematicians got access to many

[1]A euphemism for Joseph Stalin.
[2]From a poem by I. Ehrenburg.
[3]The then rector of Moscow State University.

outstanding treatises and textbooks, including Robert Phelps's *Lectures on Choquet's theorem*, Krzysztof Maurin's *Methods of Hilbert spaces*, Helmut Schafer's *Topological vector spaces*, Kenneth Hoffman's *Banach spaces of analytic functions*, Theodore Gamelin's *Uniform algebras*, and Walter Rudin's *Functional analysis*. The importance of this work for the mathematical community cannot be overstated.

The high levels of social optimism marking the 'thaw' period visibly declined during the 1970s, and by the mid-1980s had run out of steam. Along with the general social climate, the situation in science and education and the atmosphere in the Faculty of Mechanics and Mathematics at Moscow State University also changed dramatically. The yearly Voronezh Winter Mathematical Schools were a kind of outlet for Gorin. He took a very active part in organizing and conducting these meetings, which, from about 1970, were a main attraction for hundreds of Soviet mathematicians, from Brest to Chukotka. Gorin supervised the functional analysis section, one of the most busy sections of the Voronezh Schools, where about 40 lectures and short talks were made every year. It so happened that Gorin's birthday fell during the period when the Voronezh Schools were held, and this day was celebrated by the whole school.

For almost a quarter century Gorin worked in the Faculty of Mechanics and Mathematics at Moscow State University, including 15 years as a senior researcher with a D.Sc. degree. In 1987, soon after he had turned 50, Gorin took the difficult decision to move to Moscow State Pedagogical Institute (now University) as a professor in the Department of Mathematical Analysis. Severing ties with Moscow State University, his *alma mater*, was painful. But, as Yu. Trifonov wrote in a slightly different connection, "…this was a quite different time, not even the time that came after, but very, very different…": Gorbachev's *acceleration* was almost over, but *perestroika* had started to rumble and new hopes and illusions arose… .

Despite all these complications Gorin successfully pursued his work in mathematics until he retired in 2016. He established important results in various areas of functional analysis (see [23], [24], [26], [28]–[31], [35], and the details below). At the Pedagogical University he was active in scientific and methodological work, was a scientific advisor of postgraduate students, and in 2011 was awarded the P. S. Novikov prize.

For many years and until recently Gorin was a member of the editorial boards of the journals *Funktsional'nyi Analiz i ego Prilozheniya*[4] and *Russian Journal of Mathematical Physics*. His responsibilities there included a broad spectrum of topics, from measure theory and the geometry of Banach spaces to complex analysis and number theory. He was a member of the Moscow and the American Mathematical Societies and often travelled abroad to give lectures or talks.

He began research work initially with A. B. Shidlovskii as scientific advisor and then with Yu. M. Smirnov, and his first result was connected with Smirnov, who asked whether an arbitrary metric space whose topology has a countable base can be uniformly embedded in a Hilbert space. Gorin showed in [2] that n-dimensional Euclidean space is uniformly homeomorphic to a bounded subset of L_2. It was only in 1969 that P. Enflo answered Smirnov's question in the negative. It turned

[4]Translated as *Functional Analysis and its Applications*.

out that the separable Banach space c_0 cannot be uniformly homeomorphically embedded in L_2. Next, in 1985 I. Aharoni, B. Maurey, and B. S. Mityagin proved that all the spaces l_p with $2 < p < \infty$ have the same property, while for each $p \in [1,2]$ the space l_p has a uniformly homeomorphic embedding in L_2. It happened, however, that Gorin's first published paper [1] was on systems of norms (written together with Mityagin), and it subsequently (by 1970) proved to be useful and even essential for answering a number of questions on bases in spaces of analytic functions (V. P. Zakharyuta) and on equivalence of bases in scales of Hilbert spaces (Mityagin).

After that, Gorin was occupied with functional analysis and differential equations, and for many years he was an active participant of I. M. Gelfand's seminar. As an undergraduate and postgraduate student in the Faculty of Mechanics and Mathematics, he had G. E. Shilov as scientific advisor. On Shilov's advice Gorin wrote one of his most cited papers, "Asymptotic properties of polynomials and algebraic functions of several variables" [3]. He showed there that the Tarski–Seidenberg theory on preservation of semi-analytic sets under polynomial maps can successfully be used in the theory of partial differential equations. With the use of the Tarski–Seidenberg theory one could not only show smoothness but also produce effective bounds for the growth of derivatives of the solution in terms of their order (in the case of one variable, for an analytic function its derivatives at a point can grow like $n!$, while for solutions of the heat equation the time derivatives grow like $(n!)^2$). Another remarkable result in [3] was a description of the equations that are hyperelliptic only in part of the variables. This paper by Gorin was cited by L. Hörmander, V. I. Arnold, and other authors. By the way, Gorin was the first 1958 graduate from the faculty who published a paper in the main section of *Uspekhi Matematicheskikh Nauk*.[5]

In [8], one of his first papers, Gorin gave an algebraic description of partial differential equations with constant coefficients for which the Cauchy problem has a solution in the Lebesgue L_2-space for all initial data in L_2. A typical example here is the equation of sound propagation in a viscous gas. For systems of equations, solving this problem encountered additional difficulties, but in subsequent papers Gorin found broad classes of systems for which the Cauchy problem is solvable. These papers lay at the basis of his Ph.D. thesis, *Partially hypoelliptic partial differential equations with constant coefficients* (1962).

Gorin continued studying differential equations in joint work with V. V. Grushin ([5], [7], [9]). In particular, they proved that if an equation is not hypoelliptic, then the singular set of its fundamental solution is not compact. They also found equations whose solutions all exhibit additional smoothness after differentiation with respect to a distinguished variable. An example in dimension $\geqslant 2$ is given by

$$\frac{\partial}{\partial x_1}\,\Delta u = u,$$

where Δ is the Laplace operator. Combining such facts, they were able to look at the general problem of hypoellipticity from a new point of view.

[5]Translated as *Russian Mathematical Surveys*.

During his postgraduate studies, Gorin also successfully worked in the theory of Banach algebras. After defending his Ph.D. thesis, this became one of his central lines of research. In his first paper [6] published at that time he removed the symmetry condition in the generalization of the classical Stone–Weierstrass theorem proposed by Y. Katznelson; the boundedness of the idempotents in quotient rings of a commutative semisimple normed algebra is a sufficient condition.

Gorin later devoted much attention to the problem of uniform approximation. Together with his graduate student M. I. Karakhanyan he found generalizations, in the context of Banach algebras, of an approximation result due to E. M. Chirka. Then together with M. S. Melnikov and Karakhanyan he proposed new methods for constructing exotic uniform algebras (for instance, analytic algebras such that all points in their maximal ideal spaces are peak points). He developed a theory of relatively maximal subalgebras, gave constructions of non-local derivations and non-local algebras, established far-reaching generalizations of the Fuglede–Putnam theorem on commutators, characterized algebras with biholomorphically homogeneous balls, and described the subalgebras with finite codimension. In his note [18] Gorin gave the first example of an algebra whose group of invertible elements is not separated by characters, which means that the classical result of Gelfand's theory stating that the invertible elements of a commutative semisimple Banach algebra are separated by group characters cannot be extended.

In 1972 Gorin defended his D.Sc. thesis, *Some questions in the theory of commutative Banach algebras and harmonic analysis* (with D. K. Faddeev, D. A. Raikov, and V. I. Arnold as official opponents). It was based on Gorin's papers on algebraic equations with continuous or holomorphic coefficients and their connections with the algebraic theory of braids (written in part with V. Ya. Lin), perhaps his best-known papers to the mathematical community.

The history of the development of this topic is interesting. It is a nice illustration of the mathematical ambiance that existed then in the Faculty of Mechanics and Mathematics. We present the story as told by Gorin himself. It all started with B. M. Levitan's suggestion that the research seminar of Gorin and Lin should listen to a talk of V. V. Zhikov on a generalization of the Bohr–Flanders theory of equations

$$\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_n = 0 \tag{1}$$

whose coefficients a_m are (Bohr) almost periodic functions on the real line $\mathbb{R}$ and whose discriminant d satisfies $|d| \geqslant \text{const} > 0$.

Equation (1) has n continuous solutions λ_k, which are pairwise distinct at each point. As H. Bohr and D. A Flanders showed in their long paper from the 1930s, all the roots are almost periodic. This is a non-trivial result, because in the general case even the equation $\lambda^2 - a = 0$ with almost periodic a has no almost periodic solutions when a has zeros 'at infinity' (although this does not prevent it from having continuous solutions on the whole line).

It was also known that the algebra $\mathrm{AP}(\mathbb{R})$ of Bohr almost periodic functions on the real line is isomorphic to the algebra of continuous functions $C(X)$ on the Bohr compactum X. We recall that X is the connected compact group that is dual to $\mathbb{R}_{\mathrm{d}}$, the real line with the discrete topology. From the theorems of

Brushlinsky–Eilenberg, Bochner, and Bohr–van Kampen we obtain the isomorphisms

$$A^{-1}/\exp A = H^1(X,\mathbb{Z}) = \widehat{X} = \mathbb{R}_{\mathrm{d}}. \tag{2}$$

Here A^{-1} is the group of invertible elements of $A = C(X)$, $H^1(X,\mathbb{Z})$ is the integral Čech cohomology group, and $\widehat{X}$ is the group of one-dimensional characters. By using a result due to H. Cartan and the Arens–Royden theorem the first equality in (2) can be extended to arbitrary commutative Banach algebras provided that X is treated as the maximal ideal space, and the second equality holds even without the assumption of commutativity.

It follows immediately from (2) that the two-term equation $\lambda^n - a = 0$ is solvable in $C(X)$ if the first cohomology group of X is divisible and a is invertible. Then approximating the group X by tori, Gorin and Lin discovered that all equations (1) with $|d| \geqslant \text{const} > 0$ are also solvable in $C(X)$, which simplified the Bohr–Flanders theory. Moreover, it soon became clear that the group X need not be Abelian.

Replacing the coefficients a_m in (1) by independent complex variables $z_1, z_2, \dots, z_n$, we can discuss the polynomial on the left-hand side instead of the equation. Then for z a point in $\mathbb{C}^n$, $d = d(z)$ is a polynomial. Let

$$G_n = \{z \in \mathbb{C}^n \mid d(z) \neq 0\}.$$

After a discussion in Arnold's seminar, G. M. Henkin communicated that Artin's braid group $B(n)$ is isomorphic to the fundamental group of the space G_n of polynomials of degree n with simple zeros, and Arnold gave Gorin and Lin a detailed briefing on the braid group along with references to important papers treating this group. Recall that $B(n)$ has a system of generators $\sigma_1, \sigma_2, \dots, \sigma_{n-1}$ with the relations

$$\sigma_{i+1}\sigma_i\sigma_{i+1} = \sigma_i\sigma_{i+1}\sigma_i, \qquad \sigma_i\sigma_k = \sigma_k\sigma_i \quad \text{for } |i-k| \geqslant 2. \tag{3}$$

It follows from the foregoing that if X is a finite complex, then all polynomials of degree $\leqslant n$ over $C(X)$ with invertible discriminant are reducible if and only if the group of homomorphisms from $\pi_1(X)$ to $B(n)$ is trivial. On the other hand, for two-term polynomials we obtain reducibility by replacing $B(n)$ by $\mathbb{Z}$. Since a braid group has no elements of finite order, the second condition is weaker than the first in general.

It is easy to show that

$$B(n)/B'(n) = \mathbb{Z},$$

where $B'(n)$ is the commutant of the braid group (generated by the elements $\sigma_i\sigma_i^{-1}$). This commutant is the fundamental group of the variety $\{z \in \mathbb{C}^n \mid d(z) = 1\}$. Thus, the solution of the problem required determination of the properties of the group $B'(n)$.

In [12] Gorin and Lin found a finite presentation for $B'(n)$, that is, they defined it as a group with a finite set of generators and relations. It follows from this presentation that $B'(3)$ is a free group of rank 2, $B'(4)$ is a semidirect product of two free groups of rank 2, and $B''(n) = B'(n)$ for $n \geqslant 5$.

Much later Gorin derived the relation

$$\sigma_1^{-1}\sigma_3 = \sigma_2^{-1}\sigma_1^{-1} \cdot [\sigma_1^{-1}\sigma_3, \sigma_1\sigma_2^{-1}] \cdot \sigma_1\sigma_2 \tag{4}$$

for $n \geqslant 4$, which shows directly that $B''(n) = B'(n)$ for $n \geqslant 5$. But of course the presentation mentioned above gives much more.

In the late 1970s Gorin became interested in the Bernstein inequalities in the context of spectral theory. In 1968–1971 F. Bonsall and M. Crabb, A. Browder, V. È. Katznelson, and A. M. Sinclair discovered independently that for each Hermitian element $a \in A$, that is, for elements a of a complex Banach algebra such that $\|\exp(ita)\| = 1$ for all $t \in \mathbb{R}$, we have

$$\|a\| = |a|_A, \tag{5}$$

where $|a|_A$ is the spectral radius. Browder's and Katznelson's proofs were based on Bernstein's classical inequality

$$\|f'\|_\infty \leqslant \sigma\|f\|_\infty \quad \forall\, f \in B_\sigma,$$

where B_σ is the Bernstein space of entire functions of exponential type $\leqslant \sigma$ which are bounded on the real axis, and $\|f\|_\infty$ is the uniform norm of the restriction of f to the real line. We remark that Bernstein's inequality can be treated as the equality (5) for the Hermitian operator $a = -i\,d/dx$ acting in the space B_σ; in fact, in this case $\operatorname{spec}(a) = [-\sigma, \sigma]$.

In 1977 H. König published a paper treating the question of when the property (5) also holds for a function $g(a)$ of a Hermitian element $a \in A$; note that $g(a)$ is not necessarily Hermitian. Starting from that paper, Gorin described in [19] (see also [20] and [21]) the class of bounded functions g in a neighbourhood of the spectrum $\operatorname{spec}(a) \subset \mathbb{R}$ such that

$$\|g(a)\| = |g(a)|_A. \tag{6}$$

His result was stated in terms of positive-definite functions, that is (up to normalization), of the Fourier transforms of probability measures on $\mathbb{R}$.

Let $a \in A$ be a Hermitian element with spectrum $Q = \operatorname{spec}(a) \subset \mathbb{R}$, and let λ_0 be a maximum point of $g\big|_Q$. Let $h\colon Q - \lambda_0 \to \mathbb{R}$ be the function defined by

$$g(\lambda_0)h(\lambda) = g(\lambda + \lambda_0).$$

Then (6) holds if for each such point λ_0 the function h is the trace on a neighbourhood of the set $Q - \lambda_0$ of a positive-definite function on $\mathbb{R}$. On the other hand, if $Q - \lambda_0$ is a set of spectral synthesis (for example, a compact interval), then the condition reduces to $h\big|_{Q-\lambda_0}$ being the trace of a positive-definite function for each maximum point λ_0. This last condition is also necessary.

By selecting suitable a and g we can derive from this many known Bernstein-type inequalities, and in particular, inequalities for fractional powers of the differentiation operator.

In [32] and [27], in generalizing these results to locally compact Abelian groups (for instance, $\mathbb{R}^n$), Gorin introduced the concepts of symbol and universal symbol. A universal symbol is a function on $\mathbb{R}^n$ whose value on an arbitrary family of n commuting Hermitian elements also has norm equal to the spectral radius. In [33] he and his graduate student S. Norvidas gave a criterion for being a universal symbol and described a wide class of universal symbols. This opened a way to deduce new inequalities of Bernstein type.

For many years positive-definite functions were Gorin's favourite tool and object of investigation (see [32] and [34]). For instance, in [28] he used them to obtain an analogue of the asymptotic law of distribution of prime numbers in the case when the integers are replaced by an arbitrary countably generated free Abelian group and its generators play the role of the prime numbers.

In the early 1980s Gorin became interested in generalizations of the uniqueness theorem for Riesz potentials, to the case of an arbitrary Banach space. In [22] and [23] he and his student A. L. Koldobskii discovered a method for solving problems of this type. They introduced the notion of an exceptional exponent of a Banach space E: $\lambda > 0$ is exceptional for E if there exist different measures μ_1 and μ_2 on E such that

$$\int_E \|x-a\|^\lambda \, d\mu_1(x) = \int_E \|x-a\|^\lambda \, d\mu_2(x) < \infty$$

for any $a \in E$.

Using Fourier transforms, they were able to show, for example, that if E is the n-dimensional l_p-space with $p \geqslant 1$, then $\lambda > 0$ is exceptional if and only if λ/p is an integer and one of the following three conditions holds: $\lambda/p < n$, or p is even, or both p and $\lambda/p - n$ are odd. However, the trick with Fourier transforms does not work in the infinite-dimensional case. So they found a new and ingenious approach to convolution equations which is based on Cartan's lemma in potential theory. In particular, for the space L_p the exceptional exponents λ turned out to be those for which λ/p is an integer. For the spaces $C(K)$ of continuous functions on compact spaces uniqueness holds for any $\lambda > 0$. The results in that paper found applications to the description of linear isometries of Banach spaces into Banach spaces (Koldobskii). Gorin returned repeatedly to this problem. For instance, in [35] he generalized the above results to quasi-normed spaces.

Apart from the results already noted, in the last decades (including recent years) Gorin published a number of other remarkable papers on diverse questions in functional analysis: on a generalization of Khinchin's inequality ([24], jointly with S. Yu. Favorov), on a topological characterization of Haar measures on compact groups [26], on regularity of group algebras [29], and on Möbius functions on Abelian semigroups [30]. Also, he proposed and investigated a relative version of Titchmarsh's convolution theorem ([31], jointly with D. V. Treshchev).

Evgenii Gorin was an outstanding scientist, who made significant (and sometimes defining) contributions to the development of several, ostensibly quite different areas of mathematics. Some of his ideas are still in need of further development.

A distinguishing feature was his readiness to discuss mathematical problems, to share his ideas, and to help his colleagues implement their plans. His brilliant wit was always considerate to those in conversation with him. A combination of a sharp and profound mind, rare charm, unselfishness, and kindness attracted many people to him. For many of us our friendship with Gorin and his support in time of need were very important.

Our memory of Evgenii Gorin will remain with us and warm our hearts.

The authors acknowledge the support and helpful comments of all those who read through this text prior to publication, who expressed their solidarity, who shared their memories, and who joined us in their love and respect for Evgenii Alekseevich Gorin, a person whose life, research work, and other activities gave meaning and real content to the expression *mathematical community* by making it a tangible fact of our life: M. L. Agranovsky, T. M. Bandman, V. N. Bergelson, V. M. Buchstaber, V. A. Vassiliev, A. M. Vershik, S. G. Gindikin, E. I. Gordon, V. V. Grushin, V. P. Gurarii, Ya. I. Zhitomirskii, M. I. Zelikin, V. M. Zinde, N. M. Zobin, Yu. S. Ilyashenko, R. S. Ismagilov, Sh. I. Kaliman, A. A. Kirillov, S. V. Kislyakov, V. I. Kononenko, P. A. Kuchment, M. Yu. Lyubich, Yu. I. Lyubich, Yu. I. Manin, G. A. Margulis, A. S. Markus, V. D. Milman, A. A. Mirotin, A. M. Olevskii, G. I. Olshanski, B. T. Polyak, N. Kh. Rozov, V. A. Skvortsov, A. M. Stepin, V. M. Tikhomirov, V. A. Tkachenko, D. V. Treschev, and A. Ya. Helemskii.

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