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Spatial voluntary public goods games with tunable loners' payoff

Kaipeng Hu[1], Hao Guo[2,3], Ran Yang[1] and Lei Shi[1,4(a)]

[1] *School of Statistics and Mathematics, Yunnan University of Finance and Economics - Kunming 650221, China*
[2] *Center for OPtical IMagery Analysis and Learning, Northwestern Polytechnical University - Xi'an, 710072, China*
[3] *School of Mechanical Engineering, Northwestern Polytechnical University - Xi'an, 710072, China*
[4] *School of Statistics and Mathematics, Shanghai Lixin University of Accounting and Finance Shanghai, 201209, China*

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Abstract – Based on the spatial voluntary public goods games, we investigate how loners' income affects the evolution of cooperation on square lattice. In the voluntary model, loners can exit the game by holding a small fixed payoff. By introducing a tunable parameter, we make loners payoff positively related to the synergy factor r. Through *Monte Carlo* simulations, we found that a higher loners' income essentially weakens their own survivability, but strengthens the competitiveness of cooperators. Through the analysis of the evolution process, we clarify the reasons as to how low-income loners totally dominate the population. For other results, we have studied from the perspective of spatial distribution, and found that a higher loners' payoff leads to a more intense strategies transition during the loop dominance process. Further, the result of the strategies transition rate once again shows how the loner's income affects the evolution of cooperation.

Introduction. – Cooperation plays a very important role in the development of human society [1–5]. Especially, the issue of global warming, pollution control, construction of public facilities puts high demands on the cooperation of unrelated individuals. However, in this really common dilemma, cooperative behavior is often difficult to achieve because of the conflict between personal interests and collective interests. Specifically, those who freely ride on the cooperation of others are better than those who cooperate in payoff, and ultimately, defectors will completely conquer the cooperators because of the pursuit of payoff maximization. Thus, how to promote cooperation among individuals to solve social dilemmas has become a challenging problem. It is worth mentioning that the maintenance of cooperation within unrelated individuals is most frequently studied within the evolutionary game theory [6–13]. Particularly, the public goods game (*PGG*) seems to be a favorable tool to solve the tragedy of the commons [14–20].

With the development of the network science, spatial game model as an extension of game theory attracted much attention [21–25]. Based on the seminal work on network reciprocity, fruitful mechanisms have been proposed to solve the social dilemma [26]. For example, both antisocial punishment and pro-social punishment can effectively improve cooperation [27–30], and a reward also has been identified as a possible route to promote cooperation [14,31–33]. What is more, related mechanisms include memory effect [34], preferential selection [35], individual behavior [36], etc.

In addition to what mentioned above, evidences indicate that voluntary participation in such public social dilemmas may provide a way to break away from the deadlock of defection dominant without any other mechanisms and lead to a rock rock-scissors-paper dynamic with cyclic dominance [37]. Not only that, these theoretical results are quickly proved through human experiments [38]. Interestingly, when considering the network reciprocity, the voluntary participation also lead to a persistent willingness to cooperate by cyclic dominance [16]. In the voluntary *PGG* model, both cooperators and defectors are willing to participate in *PGGs*, and cooperators contribute c (cost) to the common pool while defectors contribute nothing. The total contribution is equally distributed between all

(a) Corresponding author.

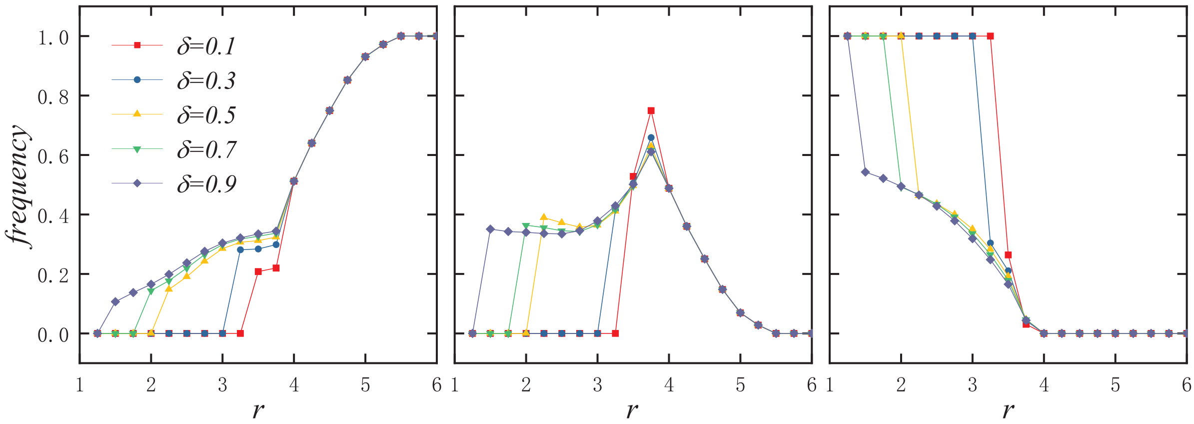

Fig. 1: The density of cooperators (left panel), defectors (middle panel), and loners (right panel) in dependence of the synergy factor r, for different values of δ (see legend). It can be observed that a larger value of δ can promote cooperation when $r < 4$. The results are obtained for $L = 500$, $K = 0.5$.

members (only including cooperators and defectors) of the group after being multiplied by a synergy factor r. And loners can escape the dilemma but they only get a small fixed income σ ($0 < \sigma < r-1$). As mentioned before, these three strategies can lead to a rock-scissors-paper dynamic with cyclic dominance [39,40]. Namely, superabundant cooperators provide a natural environment for the expansion of defectors, the dominant defectors are conductive to the growth of loners, while the excessive loners are easily invaded by cooperators.

Along the seminal research above of voluntary *PGG*, various secondary mechanisms have been proposed to promote cooperation [41–46]. However, most of these related works choose to fix the loner's payoff as a certain value, however, there is still a lack of in-depth study on the differences in the evolution of cooperation brought by the different loner's payoff. Therefore, we designed the model in this paper to systematically explore the impact of loners on cooperative behavior. At the same time, in order to eliminate the inconsistency of the synergy factor r on the individual's competitiveness, we set the loner's payoff as $\delta * (r-1)$, and $0 < \delta < 1$, to satisfy $0 < \sigma < r-1$. It is clear that the value of δ and the synergy factor r together determine the payoff of the loner.

In the following, we will firstly present the evolutionary voluntary *PGG* model in the next section, and then show the numerical simulation results in the third section. Finally, we summarize our conclusions in the last section.

Model. – We consider a spatial voluntary *PGG*, where cooperators, defectors and loners are arranged on a $L \times L$ square lattice and interact with *von Neumann* neighborhood only. Cooperators contribute $c = 1$ to the common pool, while defectors and loners contribute nothing. The sum of the contributions in each group is multiplied by a synergy factor r, then it is distributed equally to cooperators and defectors, while loners can get a small fixed payoff σ but cannot share the benefit of public goods. The payoff of cooperators, defectors, and loners in a given group g can be expressed as

$$\pi_C^g = \left(\frac{n_C^g}{n_C^g + n_D^g}\right) * r - 1, \tag{1}$$

$$\pi_D^g = \left(\frac{n_C^g}{n_C^g + n_D^g}\right) * r, \tag{2}$$

$$\pi_L = \delta * (r-1), \tag{3}$$

where n_C^g and n_D^g denote the number of cooperators and defectors in the group, and $0 < \delta < 1$ in our paper.

Taking the effect of the network into account, the maximum number of participants in a group of *PGG* is $G = 5$, and the maximum number of *PGG* groups that one player can participate in is also $G = 5$. Namely the existence of loners may reduce the number of members of *PGG* and the number of *PGG* groups.

Starting with a random distribution of these three strategies, *Monte Carlo* (*MC*) simulations of the game comprise the following elementary steps. Firstly, a randomly selected player x accumulates payoff P_x by interacting with his $G-1-n_L$ partners as a member of all the *PGG* group he joins or exits the interaction to get a loner's payoff. Next, the randomly selected neighbor player y gets his payoff P_y in the same way. Finally, player y passes his strategy to player x with the probability

$$W = \frac{1}{1 + \exp[(P_x - P_y)/K]}, \tag{4}$$

where $K = 0.5$ indicates a noise factor, which is the interference factor that may occur when an individual updates his strategy. In our simulation, individuals asynchronously update their strategies, namely, everyone has one averaged chance to update his strategy.

The fraction of cooperators, defectors and loners was determined within the last 5000 *MC* steps of overall 50000.

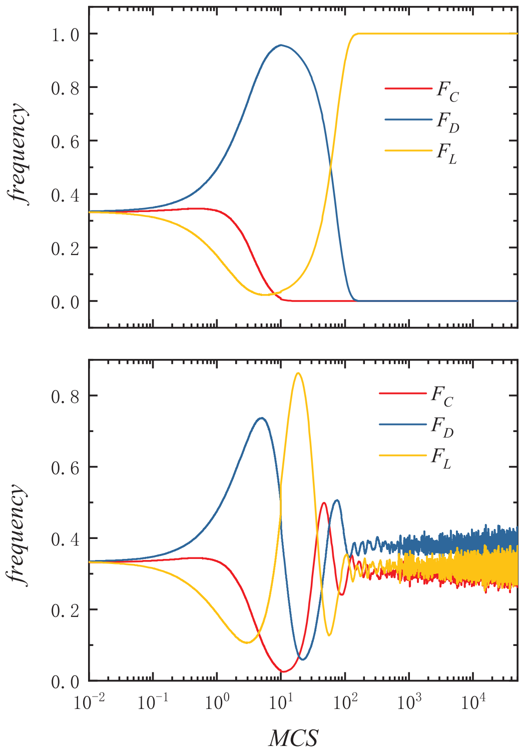

Fig. 2: The time evolution of the density of the three strategies for $\delta = 0.1$ (top panel) and $\delta = 0.9$ (bottom panel). The top panel represents the case of the all-loners phase for a smaller value of δ, and the bottom panel represents the case of the coexistence of the three strategies for larger values of δ. Moreover, in the bottom panel, the three strategies evolve into an active stationary state, this feature is very much the same as the characteristic of the rock-paper-scissors game. The results are obtained for $r = 3$, $L = 500$ and $K = 0.5$.

In order to assure accuracy, all the results were averaged over 20 independent runs for each set of parameter values.

Results. – Next, we mainly explore the impact of the loners' payoff on the evolution of cooperation. From an intuitive perspective, larger values of σ make loners more aggressive in the competing system. Thus, with the increase of δ, there should be more loners in the stationary state. However, how loners affect the system still needs a further analysis of the simulation results.

In fig. 1, from left to right, the stationary frequency of cooperators, defectors, and loners is shown as a function of the synergy factor r for different values of δ. From the left and middle panel, we find that the value of δ can significantly affect the frequency of cooperators and defectors only when $r < 4$. It might seem surprising, but the answer can be quickly found from the right panel. Specifically, when $r \geq 4$, loners lose the ability to survive, and the parameter δ no longer affects the system. Considering the question of the viability of loners, we are more concerned about the situation of $r < 4$.

From the right panel, it can be observed that a smaller value of δ ($\delta = 0.1$) leads to a full dominance of loners for a wide range of r ($r < 3.25$), while a larger value of δ ($\delta \geq 0.3$) induces a coexistence state of the three strategies. That is to say, a lower payoff is better for the competition of loners, which is contrary to our intuitive analysis. What is more, larger values of δ favor the evolution of cooperation. In short, we found that a larger value of δ is conductive to cooperators, but a smaller value of δ is better for loners.

Then, it is necessary to elucidate why a lower payoff is better for the loners' competitive behavior. In fig. 2, we inspect the time courses of strategies from a random distribution for different values of δ ($r = 3$). The top panel indicates the case in which loners finally dominate the system when δ is small ($\delta = 0.1$), while the bottom panel indicates the other case in which cooperators, defectors and loners coexist when δ is large ($\delta = 0.9$). From the top panel, defectors show a strong aggressiveness at the beginning, while both the frequencies of cooperators and loners are faced with a rapid decline. With the early exit of the cooperators, a small number of lucky loners coexist with defectors, and loners subsequently launch an invasion of defectors until the loner's complete victory. From the perspective of payoff, cooperators cannot resist the aggression of defectors, but as free riders, defectors cannot exploit anyone in the absence of cooperators. Loners therefore dominate the population with a smaller value of δ. From the bottom panel, the characteristic of the three-state cycle appears. In detail, when defectors dominate, the invasion of loners will become more intense, when loners are dominant, the offensive of cooperators also becomes more violent, and when cooperators are dominant, defectors become stronger. In other words, cooperator is weaker than defector, defector is weaker than loner, and loner is weaker than cooperator, and those three strategies therefore cyclically dominate.

Due to the relationship of the payoff, these three strategies easily induce spontaneously cyclic dominance. However, the loner's payoff determines the speed at which they invade defectors, and the low income of loners will also lead to the early exit of cooperators, and break the state of cyclic dominance. The exit of cooperators pave the way for the invasion of loners, thus, low payoff is helpful to loner's competition.

As revealed in fig. 1, larger values of δ can significantly promote cooperation. And we find that larger values of δ can easily lead to the three-state cycle from fig. 2. These results imply a potential relationship between the spontaneously emerging cyclic dominance and the evolution of cooperation. Inspired by the relationship, we begin to study the influence of the parameter δ from the perspective of spatial distribution. Figure 3 shows the evolution snapshots of cooperators (blue), defectors (red), loners (yellow) on a square lattice from a random distribution for $\delta = 0.25$ (top row), and $\delta = 0.35$ (bottom row), at 0, 90, 150, 50000 MC steps, respectively. Since the time steps corresponding to the top and bottom panel are the same, we can find some similar phenomena from the top and

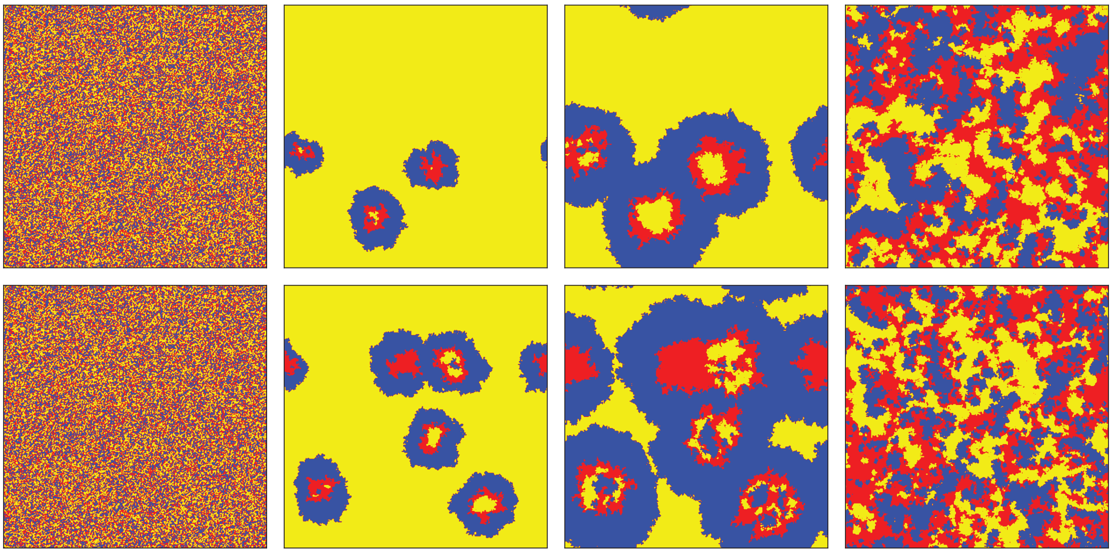

Fig. 3: The evolutionary snapshots of the distribution of cooperators (blue), defectors (red), loner (yellow) on a square lattice at 0, 90, 150, 50000 MC steps from left to right for $\delta = 0.25$ (top panel) and $\delta = 0.35$ (bottom panel). Both in the top and bottom panel, the messy distributed individuals evolve into vortices and propagating waves. However, the value of δ determine the strength of the strategies transition. The results are obtained for $r = 3.2$, $L = 300$ and $K = 0.5$.

bottom panel. From both top and bottom panel, we can see the random initial state spontaneously forming into several small vortices composed of cooperators and defectors distribute in the sea of loners at 90 MC step. As the source of the propagating waves, the vortices further expand and eventually evolved into some spiral clusters. Note that these commonalities are also typical features of strategic loop dominance which can be easily observed in the classic rock-paper-scissors game. The evolutionary snapshots clearly show that the excessive loners support a natural environment for the competition of cooperators, and the gradually growing cooperators provide a great environment for the growth of defectors; afterwards, the superabundant defectors in turn pave the way for loners.

We further analyze the differences between different values of δ. At 90 MC steps, it is very clear that more vortices appear in the system in the bottom panel, and more vortices naturally lead to an explosion of cooperators and defectors, and at 50000 MC steps the spiral clusters in the bottom panel seem smaller. In other words, a more intense cyclic state appears in the system for a larger value of δ, and a stronger strategy conversion crush the clusters into smaller tatter in the stable state. This implies the higher value of δ promotes cooperation by higher intensity strategies transitions.

The previous analysis revealed that the value of δ determines the intensity of cyclic dominance, which in turn affects the evolution of cooperation and defection. However, it is still necessary to get some quantitative results to support these results. In fig. 4, we show the fraction of three strategies as a function of δ in left panel for $r = 3$ (top panel), and 3.5 (bottom panel). From the left panel, we can find that with the increase of δ, the cooperation rate has been improved while defection has been inhibited. In addition, the value of δ hardly influences the dentist of loners especially in the case of $r = 3.5$ (bottom panel). As the cyclic aggression and non-cyclic aggression play different important roles for the evolution of strategies, we therefore present the possible strategies transitions separately in the middle and right panel. Specifically, the middle panel and right panel show the strategies transition rate for cyclic aggression and non-cyclic aggression, respectively. From the middle panel, we can find that a larger value of δ enhances the strength of cyclic invasion. Namely, in each MC step of the stable state, as the value of δ increases, the cyclic strategies transition rate significantly increases. In short, we can find from the middle panel that larger values of δ lead to a stronger cyclic invasion, facilitating the evolution of cooperation. From the right panel, we can find that, with the increase of δ, defectors have more chances of being invaded by cooperators, loners have less chances to become defectors. This explains why a larger value of δ suppresses the evolution of defection.

In fig. 5, we inspect the time courses of strategies from a random distribution for different values of δ when $r = 3$ (left panel) and $r = 3.5$ (right panel). In both panels, we can see that larger values of δ always correspond to smaller amplitudes of the strategies frequency regardless of the value of r. Specifically, larger values of δ lead to a more intensive cycle of three strategies, which also makes the mix of the three strategies more adequate. From the evolutionary perspective, larger values of δ ultimately lead to smaller amplitudes of the strategies frequency.

Conclusion. – In summary, we systematically investigate the impact of the loner's payoff on the evolution of strategies in voluntary public goods game on a square

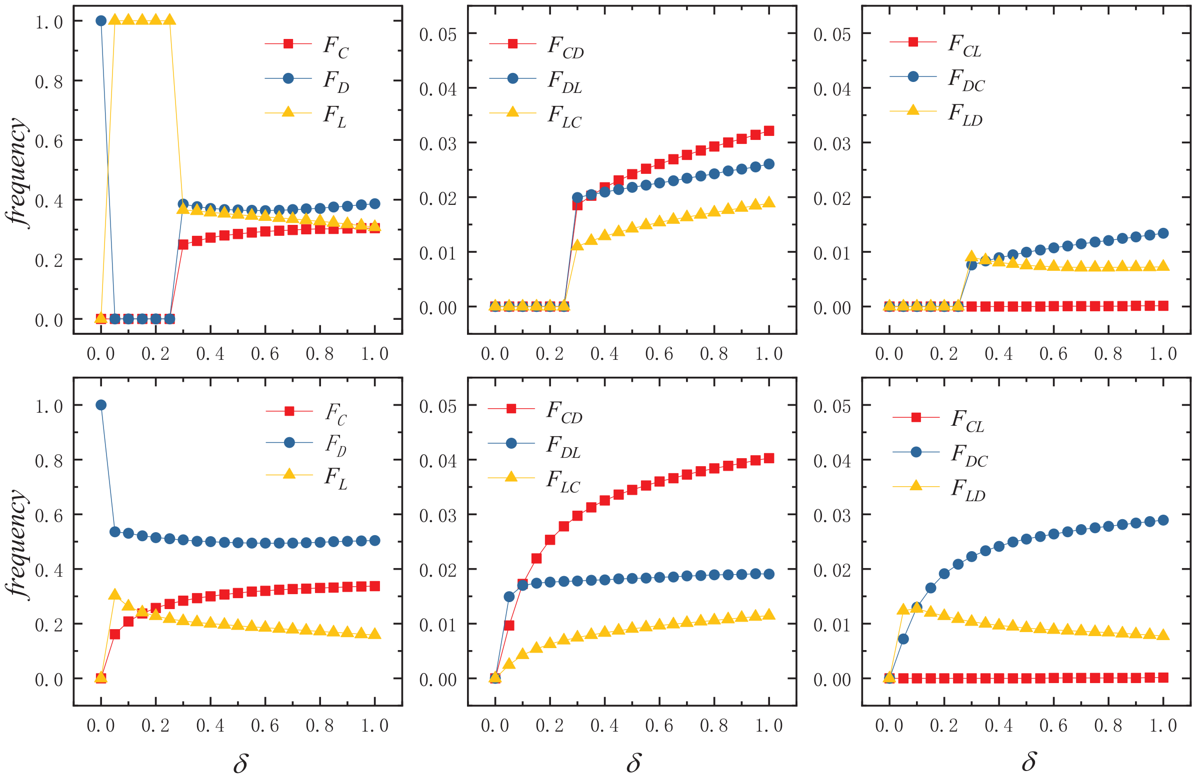

Fig. 4: The left panels represent the frequencies of the three strategies as function of δ. The middle panels are the corresponding strategies transitions of the cyclic invasion, while the right panels are the corresponding strategies transitions of another way of invasion. The top and bottom panels represent the case of the synergy factor $r = 3$ and 3.5, respectively. The results are obtained for $L = 500$, $K = 0.5$.

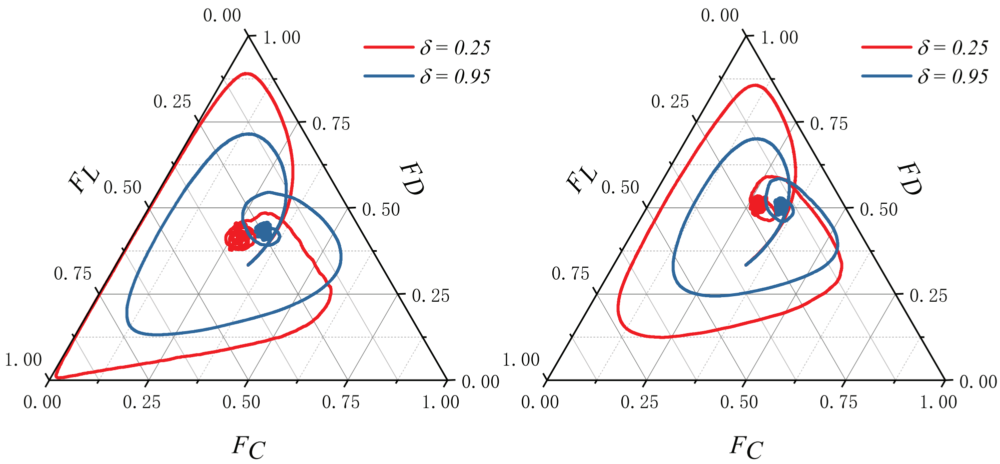

Fig. 5: The time evolution of the density of the three strategies for $r = 3$ (left panel) and $r = 3.5$ (right panel) for different values of δ (see legend). The presented results indicate clearly that larger values of δ correspond to a smaller area of closed orbits regardless of the synergy factor r. The results are obtained for $L = 500$ and $K = 0.5$.

lattice. We analyze the different results caused by the different income of loners. Among them, they mainly include the case where the loner is totally dominant, the three strategies coexist in the system, and there is coexistence of cooperators and defectors. In particular, we have done a relatively detailed analysis of the coexistence of the three strategies, and found some interesting phenomena.

On the one hand, we find that when loners have lower fixed payoff, they have opportunities to fully dominate the population. When the fixed payoff for loners is sufficiently large, the system will enter a cyclic dominance state due to the cycle of these three strategies. On the other hand, we find that a higher loner's payoff is better for the completion of cooperators, but not of loners. In other words, although the larger value of δ makes loners gain more, the benefits of loners are stolen by cooperators. From the perspective of spatial distribution, we find that the promotion of cooperation is related to the intensity of cyclic invasion, and this conclusion is also verified from the quantitative perspective. Namely, when loners have higher returns, the cooperation rate increases with the higher intensity of the cyclic invasion.

Similarly to the seminal work of spatial voluntary *PGG* [16], our research shows that the introduction of loners leads to the cyclic dominance of strategies and promotes substantial levels of cooperation where otherwise defectors dominate. However, through the adjustment of the original model, we find that the loner's income can efficiently affect the evolution of cooperation. Our study further enriches the voluntary participation game theory, and may help us to understand the evolution of cooperation in our human society.

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