

LETTERS TO THE EDITORS

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On the existence of 'supercavity modes' in sub-wavelength dielectric resonators and their relation to bound states in the continuum

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Abstract. It is shown in this letter related to the paper by M V Rybin and M F Limonov [*Physics–Uspekhi* 62 (8) (2019)] that in dielectric sub-wavelength resonators of equal volume but different shape no eigen-modes exist that would be qualitatively different from those in dielectric spheres. In particular, there are no 'supercavity modes' in dielectric cylinders whose Q-factor would exceed that of similar modes in dielectric spheres of the same volume.

Keywords: nanooptics, dielectric nanoresonators, modes, Q-factor

Pluralitas non est ponenda sine necessitate.
Gulielmus Occamus

The existence of high-Q resonance modes in dielectric resonators is long known [1–4], and these modes have many practical applications [5–7]. At the same time, a remarkable phenomenon was recently discovered in optics — the existence of bound states in the continuum (BIC) [8, 9], which is characterized by the complete absence of radiative losses in nontrivial two-dimensional photonic structures and by the exponential decay of fields with distance from them.

In this connection, the authors of paper [10] published in *Physics – Uspekhi* and also of papers [11, 12] reported the existence of earlier unknown high-Q modes in finite-volume subwavelength dielectric cylindrical resonators. They called these modes 'supercavity modes' and related them to BIC. It is unlikely that the choice of a cylindrical resonator for searching for high-Q modes can be called fortunate, because the presence of edges in the general case leads to the additional scattering of fields and a decrease in the Q-factor. It will be shown below that no 'supercavity modes' were discovered in [10–12], but only cylindrical geometries closest to a sphere were found in which mode Q-factors approach (from below) the Q-factor of modes in a sphere.

Note, first of all, that when discussing the eigenstates or eigenmodes of an electromagnetic field in finite volume dielectric resonators I have in mind solutions of Maxwell's equations in the absence of sources. These solutions should exponentially decay at infinity and should be characterized by eigenvalues. It is also quite important that the eigenvalues of the modes are analytic functions of the resonator shape, and this does not allow one to talk about the appearance of new modes upon changing the shape of a subwavelength resonator.

In the case of a circular cylinder, the eigenvalues are the complex frequency, the azimuthal quantum number, and parity along the cylinder axis, while the eigenmodes should be characterized by the distribution of electromagnetic fields exponentially decaying at infinity. It is important to bear in mind that the field modes or states are the intrinsic characteristics of the resonator, and therefore they should not depend on the conditions of resonator excitation by external fields. The particular characteristics of new 'supercavity modes' exponentially decaying at infinity in the format $\{\omega_n = \omega_n' + i\omega_n'', m_n, p_n, \mathbf{E}_n(\mathbf{r})\}$ are not presented in [10–12], which no longer allows one to talk about their existence.

The authors of [10, 11] attempt to prove the existence of earlier unknown 'supercavity modes' indirectly by studying the scattering cross section $\sigma(\omega)$ for plane waves with a certain polarization incident on a dielectric cylinder using the approximation of $\sigma(\omega)$ by the Fano profile with five parameters. Leaving aside question about the accuracy of measuring these five parameters from numerical experiments, the authors [10, 11] reach a conclusion about the existence of 'supercavity' dark modes based on a decrease in the scattering cross section to zero ($q = 0$) with changing problem geometry (the cylinder height). The conclusion about the relation of 'supercavity modes' to BIC is made based on the presence of a symmetric line ($q = \infty$) in the spectrum and the decrease in the parameter γ_0 related by the authors to the Q factor of 'supercavity modes'. However, in this case, such an approach is incorrect, because the decrease in the cross section is caused by the fact that this mode is simply not excited in this configuration. This in no way suggests the disappearance of this mode or the appearance of new modes. Only under specially prepared conditions, the absence of scattering can serve as an evidence of existence of some special effects and existence of bound states in the continuum in particular [8, 9]. It was rigorously shown in [13, 14] that the existence of BIC in three-dimensional finite bodies is impossible, and therefore the use of this concept to analyze modes in a cylinder [10, 11] has no justification.

In fact, the vanishing of the cross section is related to the orthogonality of the excited field and modes under study (TE_{012} in [10, 11] and TM_{111} in [12]) for some geometries of a cylinder at fixed polarization. For other polarizations of the

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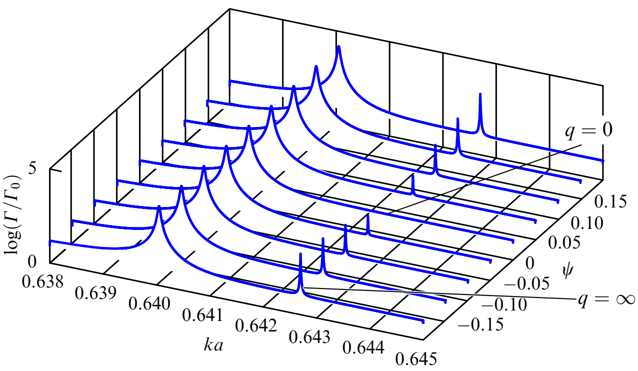

Figure. Radiation power of a dielectric sphere as a function of the size parameter ka and orientation ψ of the exciting dipole for $z = 80$.

exciting field (for example, the azimuthal polarization), the authors of [10, 11] would find no features upon scattering from the same cylinder.

The obvious fact that a change in excitation conditions can reduce the cross section to zero and lead to the ‘vanishing’ of modes is illustrated by the dependence of the normalized radiation power Γ/Γ_0 of a dielectric sphere with radius a on the orientation of the exciting dipole located at a distance r from the center. This dependence is described by the exact analytic expression [15]

$$\frac{\Gamma}{\Gamma_0} = 1 - \frac{3}{2}\,\mathrm{Re}\left\{\cos^2\psi \sum_{n=1}^{\infty} n(n+1)(2n+1)q_n\left(\frac{h_n^{(1)}(z)}{z}\right)^2 + \sin^2\psi \sum_{n=1}^{\infty}\left(n+\frac{1}{2}\right)\left[p_n\left(h_n^{(1)}(z)\right)^2 + q_n\left(\frac{d\left(z h_n^{(1)}(z)\right)}{z\,dz}\right)^2\right]\right\},$$

where ψ is the angle of dipole deviation from the radial direction, $z = kr$, and q_n, and p_n are the Mie coefficients for the TM and TE modes, respectively [15].

The plot presented in the figure shows that, depending on the dipole orientation, various line shapes appear, including the shape corresponding to the decrease in the TE mode intensity to zero ($q = 0$) and the symmetric line shape ($q = \infty$). As a whole, this figure is very similar to Fig. 9a from [10], Fig. 3a from [11], and Fig. 3a from [12]. However, this, of course, does not mean that (as the authors of [10–12] believe) something occurs with modes and they are related to BIC. In fact, modes do not change at all with changing excitation, because they are determined only by the poles of Mie coefficients q_n and p_n. Similar plots take place upon excitation of a sphere by axially symmetric beams.

The absence of any new ‘supercavity modes’ is also demonstrated in Fig. 9 from [12] where the modes have no features in the region of the predicted appearance of ‘supercavity modes’.

The authors of [11] believe that their most important discovery is the demonstration of the fact that the Q-factor of a ‘quasi-BIC-mode’ of a silicon resonator can reach the value $Q_{\mathrm{TE}012} = 200$, which is unachievable for usual modes in a sphere. However, this statement is incorrect: for a mode with a similar spatial structure in a sphere of the same volume (see the structure called the ‘supercavity mode’ in Fig. 1 in [11]), $Q_{\mathrm{TE}301} = 250$, which considerably exceeds the Q-factor of ‘supermodes’.

The increase in the mode Q-factor by the law $\varepsilon^{3.2}$ (claimed in [11] as a sign of ‘supercavity modes’) is typical and well known for modes of dielectric resonators, not with the highest Q-factors, as follows from theoretical papers [2, 4] and is also seen in Fig. 9 in [12]. The TM modes with $m = 0$ [3] (see also blue circles in Fig. 9 in [12]) have the highest Q-factor. According to the exact Mie theory, the Q-factors of similar axially symmetric TM modes in a sphere at high permittivities can be asymptotically described as $Q_1 \approx \varepsilon^{5/2}/2/X_1^3$, $Q_2 \approx 18\varepsilon^{7/2}X_2^5$, $Q_3 \approx \varepsilon^{9/2}2025/2/X_3^7$, etc., where X_n is the root of a spherical Bessel function $j_n(X_n) = 0$. For $\varepsilon = 80$, the Q-factors of these resonances are $Q_1 = 237$, $Q_2 = 13291$, and $Q_3 = 490632$ for $ka = 0.4947$, $ka = 0.644$, and $ka = 0.778$, respectively. The Q-factors of these usual TM modes in a sphere with $n = 2, 3$ for comparable volumes of dielectric resonators significantly exceed Q-factors assigned to ‘supercavity modes’ in a cylinder ($Q_{\mathrm{TM}1.1.1} \approx 10^3$ (see red dots in the green circle in Fig. 9 in [12])), namely, assigned, because not even the mode structure found by the authors themselves contains any ‘supercavity modes’ (see Fig. 9 in [12]) (!), and therefore the prefix ‘super’ can in no way be justified in this case.

Thus, no new ‘supercavity modes’ different from those already known for a sphere have been found in papers [10–12], and their relation to BIC modes has not been shown. Papers [10–12] containing statements and incorrect results concerning ‘superresonance’ quasi-BIC-modes would never see the light of day if the authors, as is understood, preliminarily studied the extensive literature on this subject and followed the known methodological ‘Occam’s razor’ principle, according to which entities should not be multiplied without necessity.

For real new modes to appear, a new resonator physics is required. In particular, fundamentally new modes (compared to modes of standard dielectric resonators) appear if the resonator is made of unusual materials, for example, metamaterials with a negative refractive index [16] or a chiral metamaterial [17]; however, this is another story.

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