# Hubble-induced phase transitions: Walls are not forever

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**Abstract.** The interplay between non-minimally coupled scalar fields and a kinetic-dominated era following the end of inflation triggers the spontaneous symmetry breaking of internal symmetries and the subsequent evolution of the fields towards large expectation values. We present here a detailed analysis of the associated dynamics in quintessential inflation scenarios involving a non-minimally coupled $Z_2$-symmetric spectator field. By analytically following the evolution of the spectator field fluctuations at early times, we characterize the formation of classical, homogeneous and spatially-localized field configurations separated by domain walls. The life expectancy of these dividing barriers is set by the scale of inflation, the non-minimal coupling and self-interactions of the spectator field and potentially, but not necessarily, the duration of the heating stage. For most of the parameter space, the domain walls are doomed to disappear before big bang nucleosynthesis. Potential phenomenological consequences of the scenario are discussed.

# Contents

## 1 Introduction

Spontaneous symmetry breaking is an essential component of modern particle physics theories. Among the different symmetry breaking patterns that could take place in Nature, those involving discrete symmetries are usually understood as problematic in a decelerating Universe [1]. The difficulties are related to the formation of domain walls among causally disconnected regions picking up different degenerate values when these become causally connected. As compared to standard matter or radiation components, the energy of these topological defects redshifts very slowly with the Universe expansion, meaning that, independently of their initial abundance, they will become the dominant energy contribution at sufficiently long times. This unwanted cosmological outcome could be avoided by a suitable choice of initial conditions [2] or if the discrete symmetry is anomalous [3] or explicitly broken by higher-dimensional operators [4], making the domain walls unstable.

In this paper, we study the symmetry breaking dynamics of a $Z_2$-symmetric spectator field non-minimally coupled to gravity within the non-oscillatory quintessential inflation paradigm. The combination of a non-minimal coupling to gravity with the kinetic domination period ubiquitously appearing in these runaway scenarios translates into a Hubble-induced spontaneous symmetry breaking of the $Z_2$ symmetry and the subsequent displacement of the spectator field towards new time-dependent minima of the potential appearing at large field values.[1] The transition between the old and new ground states takes place through a tachyonic or spinodal instability where the long-wavelength fluctuations of the spectator field become significantly enhanced, leading to the formation of homogeneous and localized spatial regions separated by domain wall configurations. The growth of these topological defects will

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[1]We note that similar Hubble-induced mechanisms have been advocated as a solution to the domain wall problem in a supergravity context [5, 6]. We emphasize, however, that, beyond the origin of the Hubble-induced corrections, these scenarios display some conceptual differences with the model under consideration. First, the displacement of the fields towards large field values takes place during inflation, rather than within a post-inflationary kinetic-dominated era. Second, the domain walls in our setting are guaranteed to disappear independently of the heating and thermalization details and without the use of additional fields or symmetry breaking patterns. Indeed, it is precisely the heating stage what triggers their decay. Third, our defects, despite being short-lived, may leave some cosmological imprint in the form of a stochastic gravitational-wave background [7].

stop when the total frequency of fluctuations becomes positive, either because the Universe is heated through the production of relativistic degrees of freedom in an additional sector of the theory or because the self-interactions of the spectator field itself become important. Whatever happens first, the symmetry of the spectator will be *effectively* restored at that time, leading to the rapid annihilation of the domain wall configurations.

This manuscript is organized as follows. After presenting an overview of the model in section 2, we reduce it to a simple tractable form in section 3. The evolution of the non-minimally coupled spectator field in the absence of fluctuations is considered in section 4, leaving for section 5 the more realistic treatment including quantum fluctuations. Following the phase separation process in real time, we will determine the statistical properties of the emerging domains. The decay of these topological defects after symmetry restoration is considered in section 6. Finally, our conclusions are presented in section 7.

## 2   The global picture

Quintessential inflation scenarios [8, 9] are usually formulated in terms of a canonical scalar field subject to a runaway potential, with different potential shapes giving rise to slightly different predictions.[2] In spite of the quantitative differences among these models, the absence of a potential minimum leads, *almost generically*,[3] to the appearance of a kinetic-dominated era soon after the end of inflation. The existence of this unusual expansion epoch may have a strong impact on the dynamics. In particular, the onset of radiation domination does *no* longer require the total depletion of the inflaton condensate since, whatever the efficiency of the heating mechanism at the end of inflation [14, 22, 23], the created particles will always become the dominant energy component at sufficiently long times. The interplay of kinetic domination with non-minimally coupled matter fields is also dramatic since it triggers the spontaneous symmetry breaking of internal symmetries and the subsequent evolution of the fields towards large expectation values. In what follows, we will study this transition in detail. We will focus here on a simple scenario involving a real spectator field $\chi$ in order to expose the essential physics in a clear as possible manner, although some of our considerations may be of more general interest.

The total action of the considered model takes the form

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R + \mathcal{L}_\phi + \mathcal{L}_\chi \right] , \qquad (2.1)$$

with $M_P = 2.44 \times 10^{18} \, \text{GeV}$ the reduced Planck mass and $\mathcal{L}_\phi$ the Lagrangian density of a quintessential inflation field $\phi$ accounting simultaneously for the early- and late-time acceler-ated expansion of the Universe. Although the precise form of the spectator field Lagrangian density $\mathcal{L}_\chi$ will not play a central role in our conclusions, we will assume it to be $Z_2$-symmetric and to involve only dimension-4 operators, namely

$$\mathcal{L}_\chi = -\frac{1}{2} \partial^\mu \chi \partial_\mu \chi - \frac{1}{2} \xi R \chi^2 - \frac{\lambda}{4} \chi^4 , \qquad (2.2)$$

where we have intentionally omitted a potential bare mass contribution that will not play any essential role in the following discussions.

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[2]Several formulations involving non-canonical kinetic terms have been also considered in the literature, see for instance refs. [10–18].

[3]A potential exception are warm quintessential inflation scenarios [19–21], where the radiation domination era starts immediately after inflation.

We are interested in studying the above scenario in a semiclassical test-field approximation in which the backreaction of the spectator field on the spacetime dynamics can be safely neglected. Varying the action (2.1) with respect to $\chi$ and particularizing the result for a *fixed* flat Friedmann-Lemaître-Robertson-Walker metric $g_{\mu\nu} = \mathrm{diag}(-1, a^2(t)\,\delta_{ij})$ with scale factor $a(t)$, we obtain the Klein-Gordon equation

$$\ddot{\chi} + 3H\dot{\chi} - a^{-2}\nabla^2\chi + \xi R\chi + \lambda\chi^3 = 0\,, \tag{2.3}$$

with the dots denoting derivatives with respect to the coordinate time $t$, $H = \dot{a}/a$ the Hubble rate and

$$R = 3(1 - 3w)H^2 \tag{2.4}$$

the Ricci scalar associated to the global equation of state parameter $w$. In order to ensure the consistency of the procedure, we will demand the contribution of the $\chi$ field in eq. (2.4) to stay subdominant with respect to the inflationary counterpart, such that the effective equation of state of the Universe can be well approximated by that of the inflaton condensate. Under this assumption, the mass of the $\chi$ field is positive definite during inflation ($w = -1$, $R = 12H^2$), making the spectator field heavy for $\xi \gtrsim 1/12$ and suppressing the generation of isocurvature perturbations [7, 24]. Note, however, that as soon as the kinetic-dominated epoch starts, it induces a change of sign in the Ricci scalar ($w = 1$, $R = -6H^2$), leading to the spontaneous breaking of the $Z_2$ symmetry and the subsequent evolution of the spectator field towards the new *time-dependent minima* of the potential, located now at large field values

$$|\chi_{\min}| = \left(\frac{6\xi}{\lambda}\right)^{1/2} H\,. \tag{2.5}$$

This Hubble-induced spontaneous symmetry breaking (SSB), initially considered in refs. [25–27] and explored by the present authors in the context of Affleck-Dine baryogenesis [24] and gravitational wave production [7], was recently advocated as a heating mechanism [28].

## 3   Spectator field dynamics

In order to study the dynamics of the spectator field during the symmetry broken phase, we will parametrize the evolution of the scale factor and the Hubble rate as

$$a(t) = a_{\mathrm{kin}}\left[1 + 3H_{\mathrm{kin}}\left(t - t_{\mathrm{kin}}\right)\right]^{1/3}\,, \qquad\qquad H(t) = \frac{H_{\mathrm{kin}}}{1 + 3H_{\mathrm{kin}}(t - t_{\mathrm{kin}})}\,, \tag{3.1}$$

with $a_{\mathrm{kin}}$ and $H_{\mathrm{kin}}$ the corresponding values of these quantities at the onset of kinetic domination, arbitrarily defined at $t = t_{\mathrm{kin}}$. Additionally, we will recast the equations of motion (2.3) in terms of a convenient set of variables

$$\chi \to Y = \frac{a}{a_{\mathrm{kin}}}\frac{\chi}{\chi_*}\,, \qquad\qquad \vec{x} \to \vec{y} \equiv a_{\mathrm{kin}}\chi_*\,\vec{x}\,, \qquad\qquad t \to z \equiv a_{\mathrm{kin}}\chi_*\tau\,, \tag{3.2}$$

with $\chi_* \equiv \sqrt{6\xi}H_{\mathrm{kin}}$ and $\tau \equiv \int dt/a$ the conformal time. In terms of these quantities, the spectator field action $S_\chi = \int d^4x\sqrt{-g}\,\mathcal{L}_\chi$ becomes a "Minkowski-like" action,

$$S_\chi = \int d^3\vec{y}\,dz\left[\frac{1}{2}(Y')^2 - \frac{1}{2}(\nabla Y)^2 - \frac{1}{2}(6\xi - 1)(\mathcal{H}^2 + \mathcal{H}')Y^2 - \frac{\lambda}{4}Y^4\right]\,, \tag{3.3}$$

with the primes denoting derivatives with respect to the dimensionless conformal time $z$ and $\mathcal{H}(z) \equiv a'(z)/a(z) = a(z)H(z)$ the comoving Hubble rate. Taking into account the relations

$$a(z) = a_{\text{kin}} \left(1 + \frac{z}{\nu}\right)^{1/2}, \qquad \mathcal{H}(z) = \frac{1}{2(z+\nu)}, \qquad (3.4)$$

with

$$\nu \equiv \sqrt{\frac{3\xi}{2}} \qquad (3.5)$$

an integration constant ensuring that $z(t_{\text{kin}}) = 0$, the associated Klein-Gordon equation (2.3) can be written as

$$Y'' - \nabla^2 Y - M^2(z)Y + \lambda Y^3 = 0. \qquad (3.6)$$

All the dependence on the non-minimal coupling is encoded in the time-dependent mass term

$$M^2(z) \equiv \frac{\nu^2 - 1/4}{(z+\nu)^2}, \qquad (3.7)$$

which, as expected, vanishes identically in the conformal limit $\xi \to 1/6$ ($\nu \to 1/2$). In the following sections, we will study the equation of motion (3.6) in detail, providing approximate analytical solutions for the evolution of the spectator field $\chi$ during kinetic domination and the fluctuations generated by its dynamics.

## 4 The homogeneous approximation

In order to the facilitate the comparison with previous studies in the literature [25, 27, 28] and highlight the limitations of those treatments, let us start discussing the dynamics of the system in the absence of quantum fluctuations. In this limit, the spectator field is described as a fully coherent state with initial conditions $\chi(t_{\text{kin}})$ and $\dot{\chi}(t_{\text{kin}}) = H_{\text{kin}}\chi(t_{\text{kin}})$ or, equivalently, $Y(0)$ and $dY/dz|_0 = Y(0)/\nu$. The evolution of this condensate during the broken phase can be determined by solving the equation of motion (3.6) in the zero-gradient approximation,

$$Y'' - M^2(z)Y + \lambda Y^3 = 0. \qquad (4.1)$$

As illustrated in figure 1, the Hubble-induced tachyonic instability triggers a rapid growth of the spectator field which is eventually stopped by the cubic self-interaction term. After a transition stage where the effects of the non-minimal coupling become negligible, the field reaches an asymptotic regime where its energy density decays as radiation. A simple but sufficiently accurate description of this evolution can be obtained by matching the growing and oscillatory behaviours at the time

$$\frac{z_{\text{b}}}{\nu} = \left(\frac{8}{\lambda Y^2(0)} \frac{2\nu - 1}{2\nu + 1}\right)^{1/(2\nu+3)} - 1, \qquad (4.2)$$

at which the quadratic and quartic terms in the $Y$-field potential become equal. Neglecting the decaying mode at $z \leq z_b$, we get

$$Y(z) \simeq \begin{cases} \frac{2\nu+1}{4\nu}Y(0)\left(1 + \frac{z}{\nu}\right)^{\nu+1/2} & \text{for} \quad z \leq z_b, \\ Y_{\text{osc}} \, \text{cn}\left[\sqrt{\lambda}\,Y_{\text{osc}}\,(z - z_{\text{osc}}), \frac{1}{\sqrt{2}}\right] & \text{for} \quad z > z_b, \end{cases} \qquad (4.3)$$
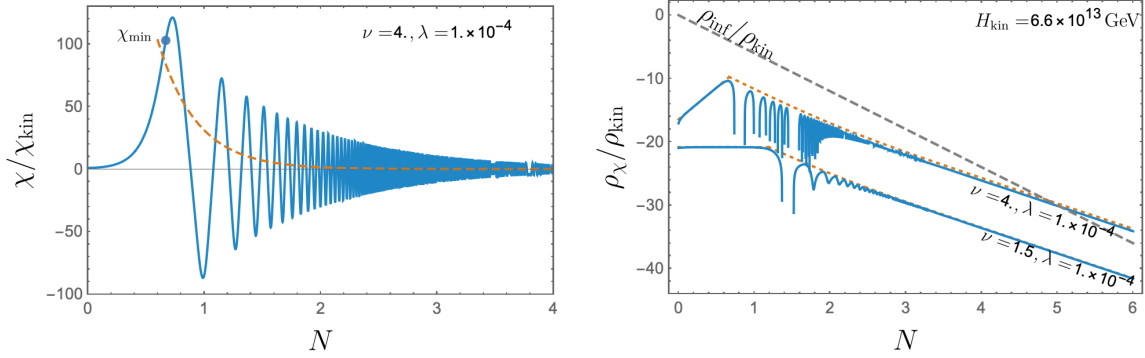
**Figure 1**. (Left) Evolution of the spectator field $\chi$ (solid curve) and the position of the potential minimum (dashed red curve) following from the numerical solution of eq. (3.6) with $H_{\rm kin} = 10^{13}\,{\rm GeV}$ and vanishing initial fluctuations. Here, $N \equiv \ln(a/a_{\rm kin}) = 1/2 \ln(1 + z/\nu)$ denotes the number of $e$-folds after the onset of kinetic domination, with the dot indicating the value at $z_b$, $N(z_b)$. (Right) Evolution of the energy density stored in the spectator field $\chi$ in the absence of quantum fluctuations (solid curve) as compared to the analytical solution (4.8) (dotted red curve) and the background energy density $\rho_\phi$ (dashed gray curve) for different values of the non-minimal coupling $\xi$. All quantities are normalized to the initial energy density. Note that soon after the end of the tachyonic phase the energy density scales as radiation, similar to the result in ref. [28].

with cn denoting the Jacobi elliptic cosine,

$$Y_{\rm osc} = \frac{Y(z_b)}{\sqrt{2}} \left[ 1 + \left( 1 + \frac{2}{\beta^2} \frac{2\nu + 1}{2\nu - 1} \right)^{1/2} \right]^{1/2} \tag{4.4}$$

the amplitude of the field at the time

$$z_{\rm osc} = z_b + \frac{1}{\beta \sqrt{\lambda} Y_{\rm osc}} \arccos\left( \frac{Y(z_b)}{Y_{\rm osc}} \right) \tag{4.5}$$

at which the oscillatory part reaches its maximum ($z_{\rm osc} > z_b$) and $\beta = 2\pi/T_x = 0.8472$ a numerical factor associated with the leading frequency of oscillation in the elliptic cosine series,

$$\mathrm{cn}\left( x, \frac{1}{\sqrt{2}} \right) = \frac{8\pi\sqrt{2}}{T_x} \sum_{n=1}^{\infty} \frac{e^{-\pi(n-1/2)}}{1 + e^{-\pi(2n-1)}} \cos \frac{2\pi(2n-1)x}{T_x} \,, \tag{4.6}$$

with $T_x \approx 7.416$ the oscillation period in $x$ units.

Using the analytical solution (4.3) we can characterize the average evolution of the spectator field energy density

$$\rho_\chi = \rho_Y \mathcal{X} \,, \qquad \rho_Y \equiv \frac{1}{2} Y'^2 - \frac{1}{2}(6\xi - 1)\mathcal{H}^2 Y^2 + \frac{\lambda}{4} Y^4 + \frac{1}{2}(6\xi - 1)\mathcal{H}(Y^2)' \tag{4.7}$$

during the tachyonic ($z \leq z_b$) and oscillatory phases ($z \geq z_b$), namely

$$\bar{\rho}_\chi^{(T)} = c_1 \left( \frac{a_{\rm kin}}{a} \right)^{2(1-2\nu)} \mathcal{H}^4(0) \mathcal{X} \,, \qquad \bar{\rho}_\chi^{(O)} \simeq c_2 \mathcal{H}^4(0) \mathcal{X} \,, \tag{4.8}$$

with

$$c_1 \equiv (2\nu + 1)^3 \nu^2 Y^2(0) \,, \qquad c_2 = \frac{16}{2\nu - 1} \lambda \nu^5 Y^4(z_b) \,, \tag{4.9}$$

and $\mathcal{X} \equiv \chi_*^4 (a_{\mathrm{kin}}/a)^4$ a units' rescaling factor that drops when evaluating physically-relevant dimensionless ratios. Interestingly enough, even though the field $\chi$ is always monotonically increasing during the tachyonic phase, its energy density at $z \leq z_b$ can increase ($\nu > 3/2$), decrease ($1/2 < \nu < 3/2$) or even stay constant if the growth of the scalar field is exactly compensated by the Universe expansion ($\nu = 3/2$). On the other hand, the energy density beyond the transition value $z_b$ decreases as radiation (up to a residual $\mathcal{O}(a^{-6})$ correction associated with the aforementioned transient stage). This scaling agrees with that found in ref. [28] where it was recently shown that the rapid decrease of the effective minimum depth $\Delta = -9\xi^2/\lambda H^4$ towards the origin of the potential precludes the trapping of the scalar field at large field values, allowing it to oscillate around the origin of the potential The number of $e$-folds $N_{\mathrm{osc}} = 1/2 \ln(1 + z_{\mathrm{osc}}/\nu)$ needed for this oscillatory phase to begin can be estimated by combining the scale factor evolution in (3.4) with eqs. (4.2)–(4.5). As shown in figure 2, this quantity can be rather large for part of the parameter space, but, once the oscillation phase is achieved, the spectator field energy density is guaranteed to become dominant over the rapidly decreasing inflaton counterpart, $\rho_\phi \propto a^{-6}$, leading eventually to a radiation dominated epoch [28]. Note that during this period, the scalar field $Y$ oscillates many times per Hubble time, cf. figure 1.

In spite of its simplicity, the assumption of having a homogeneous, fully-coherent and smoothly-evolving scalar field adopted here and in refs. [25, 27, 28] is not physically accurate. On the one hand, the spectator field is not entirely classical and quantum fluctuations around the classical solution (4.3) should be definitely taken into account. On the other hand, due to the exact $Z_2$ symmetry of the original action, it is equally likely for the spectator field to roll down to any of the two minima of the effective potential, meaning that its expectation value should remain zero at all times. This trivial observation has important physical consequences. In particular, although the homogeneous approximation can be useful for estimating the temporal scales in the problem, it fails to describe the formation of topological defects [7], which, as we will see in what follows, it is an essential part of the problem.

## 5 Beyond homogeneity and domain wall formation

Performing a complete analysis of the symmetry breaking dynamics consistently accounting for the evolution of quantum fluctuations is a rather complicated task in an interacting theory like the one under consideration. In spite of this intrinsic limitation, a lot of valuable information can be extracted from the first stages of the evolution, where the self-interactions of the spectator field can be safely neglected. In this limit ($z < z_b$), the action (3.3) becomes essentially quadratic,

$$S_Y \simeq \int d^3\vec{y}\, dz\, \left[\frac{1}{2}(Y')^2 - \frac{1}{2}(\nabla Y)^2 - \frac{1}{2}M^2(z)Y^2\right], \tag{5.1}$$

making the problem exactly solvable. To perform the quantization of the real perturbation $Y(\vec{y}, z)$ we will follow the usual canonical quantization scheme, although path integral techniques could be of course alternatively applied. The Hamiltonian in momentum space,

$$H = \frac{1}{2} \int d^3\kappa\, \left[\Pi_{\vec{\kappa}}(z)\, \Pi_{\vec{\kappa}}^\dagger(z) + \omega_\kappa^2(z)\, Y_{\vec{\kappa}}(z)\, Y_{\vec{\kappa}}^\dagger(z)\right], \tag{5.2}$$

becomes the sum of a continuous set of uncoupled harmonic oscillators with wavenumber

$$\kappa \equiv |\vec{\kappa}| \equiv \frac{|\vec{k}|}{a_{\mathrm{kin}}\chi_*} \tag{5.3}$$
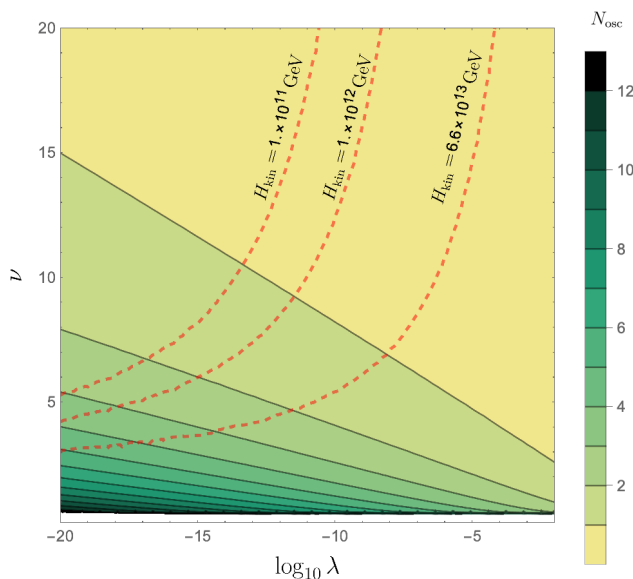
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**Figure 2**. Level curves for the number of $e$-folds $N_{\rm osc} = 1/2 \ln{(1 + z_{\rm osc}/\nu)}$ needed for the oscillatory phase to begin as a function of the model parameters $\nu$ and $\lambda$. The dashed lines correspond to a 10% spectator field correction to the Planck mass for three values of the Hubble rate at the onset of kinetic domination. Note that higher values of this quantity tend to reduce the available parameter space.

and position and momentum operators satisfying the standard equal-time commutation relations $[Y_{\vec{\kappa}}(z), \Pi_{\vec{\kappa}}(z)] = i\delta^3(\vec{\kappa} + \vec{\kappa}')$. A simple inspection of the time-dependent frequency of these operators,

$$\omega_\kappa^2(z) \equiv \kappa^2 - M(z)^2\,, \tag{5.4}$$

reveals that the tachyonic or spinodal instability appearing at the onset of kinetic domination affects *not only* the zero mode of the spectator field but also all *subhorizon* modes with momenta smaller than the Hubble-induced negative mass term $-M^2(z)$, i.e. those within a window $\kappa_{\min}(z) \lesssim \kappa \lesssim \kappa_{\max}(z)$ with

$$\kappa_{\min}(z) = \mathcal{H}(z)\,, \qquad\qquad \kappa_{\max} = (4\nu^2 - 1)^{1/2}\kappa_{\min}\,. \tag{5.5}$$

Although this instability is reminiscent from that appearing in chaotic inflation [29], new inflation [30] or tachyonic preheating scenarios [31–37], there are important differences with these settings:

1. The $z$ variable in our case runs from 0 to $\infty$, as opposed to what happens in a de Sitter stage, where it runs from $-\infty$ to 0. This precludes the use of large argument asymptotic expansions to fix the initial conditions for the problem solutions.

2. Rather than from its own background field configuration, the spectator field takes energy directly from the cosmological expansion induced by the runaway inflaton condensate.

3. The tachyonic instability is limited in time since the modes will eventually leave the unstable band either because of the evanescence of the tachyonic mass $-M^2(z)$ or because the full effective frequency (i.e. the one including also the omitted $\lambda Y^3$ contribution) becomes positive, cf. figure 3.
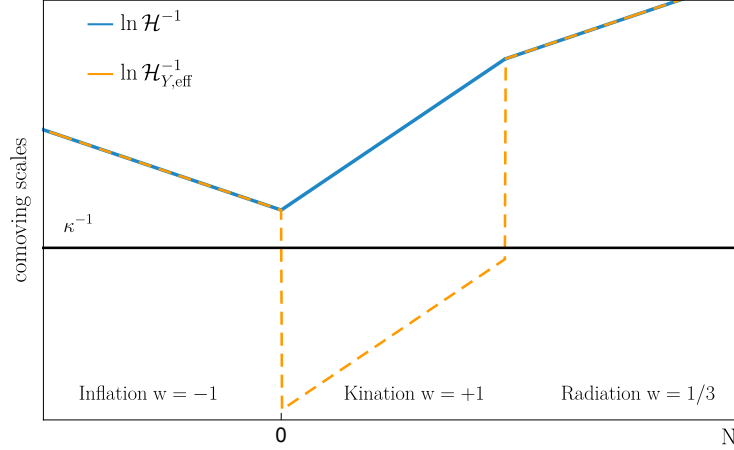
**Figure 3**. Schematic behaviour of "effective horizon radius" of the Y field $\mathcal{H}_{Y,\text{eff}}^{-1}$ (coinciding during kination with the inverse mass $M^{-1}$) and the proper horizon radius $\mathcal{H}^{-1}$ during the different cosmological epochs. The tachyonic instability affects all scales $\kappa^{-1}$ entering the parallelogram in the center of the figure. Note that the separation between $\mathcal{H}_{Y,\text{eff}}^{-1}$ and $\mathcal{H}^{-1}$ during kinetic domination grows with $\nu$, making the integrated contribution of the superhorizon scales reentering the horizon $\mathcal{H}^{-1}$ less and less important as compared with that of modes never amplified by the inflationary dynamics.

The physical content of the theory is encoded, as usual, in the expectation value of field operator products at equal or different spacetime points. For a zero-mean Gaussian field like the one under consideration, the only quantities needed to describe the system are the two-point expectation values

$$\langle v_{\vec{\kappa}}^I(z)\, v_{\vec{\kappa}'}^J(z')\rangle = \Sigma_\kappa^{IJ}(z,z')\, \delta^3(\vec{\kappa}+\vec{\kappa}')\,, \tag{5.6}$$

with $v_{\vec{\kappa}} \equiv (\Pi_{\vec{\kappa}}(z), Y_{\vec{\kappa}}(z))^T$ and the bra-kets referring to a quantum average over the initial state (Heisenberg picture). The correlation matrix $\Sigma_\kappa^{IJ}(z,z')$ in this expression can be expressed in terms of its value at any reference time $z_r$ by taking into account the Heisenberg equations, identical in form to the classical field equations

$$\frac{d}{dz} v_{\vec{\kappa}}(z) \equiv \frac{d}{dz} \begin{pmatrix} \Pi_{\vec{\kappa}}(z) \\ Y_{\vec{\kappa}}(z) \end{pmatrix} = \begin{pmatrix} 0 & -\omega_\kappa^2(z) \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \Pi_{\vec{\kappa}}(z) \\ Y_{\vec{\kappa}}(z) \end{pmatrix}\,. \tag{5.7}$$

Denoting by $\mathbf{M}_\kappa(z)$ the evolution matrix relating the operators at times $z$ and $z_r$ (i.e. $v_{\vec{\kappa}}(z) = \mathbf{M}_\kappa(z)v_{\vec{\kappa}}(z_r)$), we get [37–40]

$$\Sigma_\kappa(z,z') = \mathbf{M}_\kappa(z)\, \Sigma_\kappa(z_r,z_r)\, \mathbf{M}_\kappa^T(z')\,, \quad \mathbf{M}_\kappa(z) = \begin{pmatrix} \sqrt{\frac{2}{\kappa}}\operatorname{Re} g_\kappa(z) & \sqrt{2\kappa}\operatorname{Im} g_\kappa(z) \\ -\sqrt{\frac{2}{\kappa}}\operatorname{Im} f_\kappa(z) & \sqrt{2\kappa}\operatorname{Re} f_\kappa(z) \end{pmatrix}\,, \tag{5.8}$$

with $g_\kappa \equiv i f'_\kappa$ and $f_\kappa$ a solution of the Schrödinger-like differential equation

$$f''_\kappa + \omega_\kappa^2(z)\, f_\kappa = 0\,, \tag{5.9}$$

with initial conditions $f_\kappa(z_r)$ and $f'_\kappa(z_r)$.

The precise form of the initial correlation matrix $\Sigma_\kappa(z_r, z_r)$ depends on the state of fluctuations at the onset of kinetic domination and therefore on the details of the inflation-to-kination transition. Note also that, contrary to what is usually assumed in standard heating scenarios, there is generically not a clear separation of scales in the problem at hand. Indeed, the mass of the spectator field is not significantly larger than the Hubble rate for small $\nu$ values, meaning that horizon entry effects cannot be completely ignored. To surpass these limitations we will i) consider the inflation-to-kination transition to be instantaneous as compared to the temporal evolution of the spectator field, ii) focus on a large $\nu$ limit where the impact of horizon-entry effects is minimized by increasing the separation between the horizon scale and the dominant momentum scale of the spectator field fluctuations (cf. figure 3) and iii) assume the initial state of the system to be described by a vacuum Hermitian correlation matrix

$$\Sigma_\kappa(0,0) = \begin{pmatrix} \frac{\kappa}{2} & -\frac{i}{2} \\ \frac{i}{2} & \frac{1}{2\kappa} \end{pmatrix} \tag{5.10}$$

at $z_r = 0$. Under these assumptions, the solution of the equation of motion (5.9) with initial conditions $f_\kappa(0) = 1/\sqrt{2\kappa}$ and $f'_\kappa(0) = -i\sqrt{\kappa/2}$ takes the form

$$f_\kappa(z) = \sqrt{z+\nu}\left[A_\kappa\,\mathcal{J}_\nu(\kappa(z+\nu)) - B_\kappa\,\mathcal{Y}_\nu(\kappa(z+\nu))\right], \tag{5.11}$$

with $\mathcal{J}_\nu$ and $\mathcal{Y}_\nu$ the Bessel's functions of the first and second kind,

$$A_\kappa = \mathcal{Y}_\nu(\kappa\nu)\,\delta\,, \qquad B_\kappa = \mathcal{J}_\nu(\kappa\nu)\,\delta - \frac{f_\kappa(0)}{\sqrt{\nu}\mathcal{Y}_\nu(\kappa\nu)}\,, \tag{5.12}$$

and

$$\delta = \frac{\pi f_\kappa(0)}{4\sqrt{\nu}}\left[1 - 2\nu + 2\nu\left(\kappa\frac{\mathcal{Y}_{\nu-1}(\kappa\nu)}{\mathcal{Y}_\nu(\kappa\nu)} - \frac{f'_\kappa(0)}{f_\kappa(0)}\right)\right]. \tag{5.13}$$

Using this analytical solution we can evaluate the equal-time correlation matrix following from (5.8), namely

$$\Sigma_\kappa(z,z) = \begin{pmatrix} |g_\kappa(z)|^2 & F_\kappa(z) - \frac{i}{2} \\ F_\kappa(z) + \frac{i}{2} & |f_k(z)|^2 \end{pmatrix}, \tag{5.14}$$

with

$$F_\kappa(z) \equiv \frac{1}{2}\langle \Pi_{\vec{\kappa}}(z)\,Y^\dagger_{\vec{\kappa}}(z) + Y_{\vec{\kappa}}(z)\,\Pi^\dagger_{\vec{\kappa}}(z)\rangle = -\frac{i}{2}\left(g_\kappa f^*_\kappa - f_\kappa g^*_\kappa\right) = \mathrm{Im}\left(f^*_\kappa g_\kappa\right) \tag{5.15}$$

the so-called WKB phase. The form of the different entries in this correlation matrix is illustrated in figure 4. As clearly appreciated there, the behaviour of the functions at a given instant $z$ can be roughly divided into 3 distinct regions. First, there exists a highly-infrared amplification band at $\kappa_{\min}(z) < \kappa < \kappa_{\max}(z)$ where all modes are almost democratically amplified. Second, we can identify a strongly oscillating region $\kappa_{\max}(z) < \kappa < \kappa_{\max}(0)$ associated with modes that, although initially amplified by the tachyonic instability, are currently unaffected by it. Finally, there is a stable region $\kappa > \kappa_{\max}(0)$ where modes never gets amplified. To get some analytical understanding on this numerical result, let us note that among the two terms in eq. (5.11), the first one gives the largest contribution, allowing us to approximate the mode function as

$$f_\kappa(z) \simeq A_\kappa\,\sqrt{z+\nu}\mathcal{J}_\nu(\kappa(z+\nu)). \tag{5.16}$$
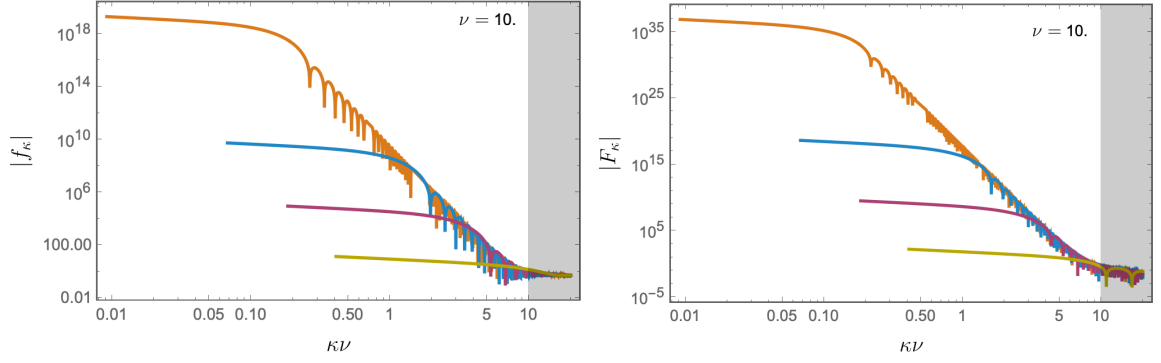
**Figure 4**. The functions $|f_\kappa|$ and $|F_\kappa|$ entering in the equal-time correlation matrix (5.14) and following from eq. (5.11) at $N = 0.1, 0.5, 1$ and 2 *e*-folds (from bottom-up). The shaded region corresponds to momenta $\kappa \geq \kappa_{\max}(0)$. The position of the knee in the spectrum matches the value of $\kappa_{\max}(z)$.

A simple description of this growing mode for the relevant *subhorizon* momenta we are interesting in can be obtained by expressing it in terms of the generalized hypergeometric series ${}_0F_1$ [41]

$$f_\kappa(z) = \frac{A_\kappa \sqrt{z+\nu}}{\Gamma(\nu+1)} \left(\frac{\kappa(z+\nu)}{2}\right)^\nu {}_0F_1\left(\nu+1, -\left(\frac{\kappa(z+\nu)}{2}\right)^2\right). \tag{5.17}$$

Further approximating[4]

$$|A_\kappa| \simeq \frac{|2\nu-1|}{8}\left(\frac{2}{\kappa\nu}\right)^{\nu+1/2}\Gamma(\nu), \qquad {}_0F_1\left(\nu+1, -\left(\frac{\kappa(z+\nu)}{2}\right)^2\right) \approx \exp\left(-\frac{1}{4}\frac{\kappa^2}{\kappa_*^2(z)}\right), \tag{5.19}$$

with $\Gamma(\nu)$ the Euler gamma function and

$$\kappa_*(z) \equiv 2\sqrt{\nu+1}\,\kappa_{\min}(z) \tag{5.20}$$

a typical momentum scale, we get

$$|f_\kappa(z)|^2 \approx \left(\frac{2\nu-1}{4\sqrt{2\nu}}\right)^2 \left(1+\frac{z}{\nu}\right)^{2\nu+1}\frac{1}{\kappa}\exp\left(-\frac{1}{2}\frac{\kappa^2}{\kappa_*^2(z)}\right), \tag{5.21}$$

$$|g_\kappa(z)|^2 \approx \frac{|f_\kappa(z)|^2}{2}\left(\frac{2\nu+1}{z+\nu} - \frac{\kappa^2(z+\nu)}{1+\nu}\right)^2, \tag{5.22}$$

$$|F_\kappa| \approx \frac{|f_\kappa(z)|^2}{2}\left|\frac{2\nu+1}{z+\nu} - \kappa^2\frac{z+\nu}{\nu+1}\right|. \tag{5.23}$$

Note that the growth factor of fluctuations for infrared modes $\kappa \ll \kappa_*(z)$ coincides with that of the zero mode in eq. (4.3). This rapid enhancement is intimately related to the emergence of

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[4]Note that this approximation becomes *exact* in the large $\nu$ limit,

$$\lim_{\nu\to\infty}\Gamma(\nu+1)\left(\frac{2}{\kappa(z+\nu)}\right)^\nu \mathcal{J}_\nu(\kappa(z+\nu)) = \exp\left(-\frac{(\kappa(z+\nu))^2}{4\nu}\right). \tag{5.18}$$
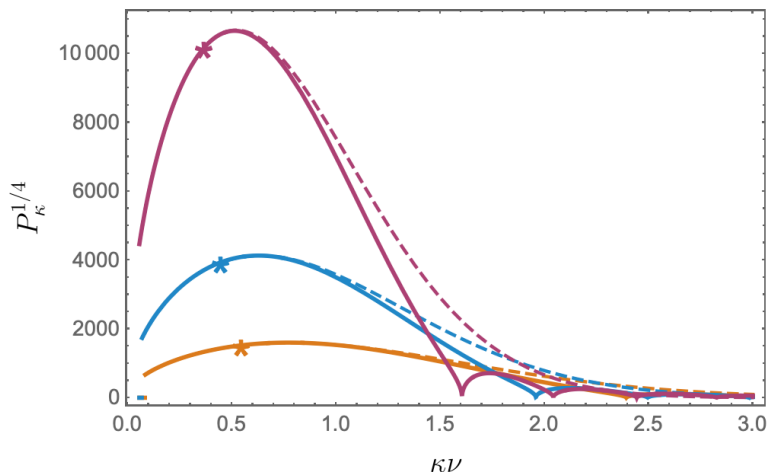
**Figure 5**. Comparison between the late-time behaviour of the power spectrum following from the exact solution (5.11) (solid line) and the Gaussian approximation (5.26) (dashed line) for $\nu = 10$ and $N = 0.9, 1$ and $1.1$ e-folds. The markers refers to the position of $\kappa_* \nu$ at the corresponding number of e-folds.

classicality in the corresponding range of momenta. In particular, the Heisenberg uncertainty principle in Fourier space can be written as [37–40]

$$\Delta Y_\kappa^2 \, \Delta \Pi_\kappa^2 = |F_\kappa(z)|^2 + \frac{1}{4} \geq \frac{1}{4} \left| \langle [Y_\kappa(z), \, \Pi_\kappa^\dagger(z)] \rangle \right|^2, \tag{5.24}$$

meaning that for $|F_\kappa(z)| \gg 1$ (or equivalently for $\langle \{Y_\kappa(z), \, \Pi_\kappa^\dagger(z)\} \rangle \gg \langle |[Y_\kappa(z), \Pi_\kappa^\dagger(z)]| \rangle = \hbar$), the ambiguity in the ordering of operators becomes quantitatively negligible. Thanks to this quantum-to-classical transition, the statistical properties of the system can be completely described in terms of a (dimensionless) power spectrum

$$P_\kappa(z) \equiv \kappa^3 |f_\kappa(z)|^2 \tag{5.25}$$

consistent with the conservation of probabilities along classical trajectories. Several interesting quantities characterising this classical Gaussian random field in position space can be computed by Fourier transforming expressions of the form $\kappa^p(\kappa) P(\kappa)$, with $p$ a positive even integer. These mode integrals are ultraviolet divergent ($|f_\kappa|^2 \sim 1/\kappa$, $P(\kappa) \sim \kappa^2$ at large $\kappa$) and must be regularized in order to perform any practical computation. A measurable quantity in the effective field theory sense can be defined, for instance, by integrating out momenta above a given cutoff scale. When doing that, quadratic and logarithmic divergences can be absorbed in the renormalized constants of the theory [37, 42–47]. A natural way of introducing this cutoff in our setting is to replace the mode function $|f_\kappa(z)|^2$ by its approximate form (5.21). This effective truncation smears out the problematic high-momentum modes leading behind an approximately Gaussian spectrum

$$P_\kappa(z) \approx \left( \frac{2\nu - 1}{4\sqrt{2\nu}} \right)^2 \left( 1 + \frac{z}{\nu} \right)^{2\nu + 1} \kappa^2 \exp\left( -\frac{1}{2} \frac{\kappa^2}{\kappa_*^2(z)} \right), \tag{5.26}$$

which can be analytically integrated over. The comparison between this simple expression and that obtained from the exact solution (5.11) is depicted in figure 5. As clearly appreciated

in this plot, soon after the beginning of the phase transition, the spectator field power spectrum develops a peak around the typical momentum scale (5.20) which redshifts and grows with time until eventually dominating the infrared part of the spectrum. Note that the Gaussian approximation (5.26) accurately captures this leading behaviour, failing only in the numerically subdominant oscillatory part at large $\kappa$ values, as otherwise expected.

The low-momentum structure of the power spectrum translates into the formation of localized domains in position space. The size of the correlated regions is determined by the spatial correlation function, defined as

$$\zeta(\vec{y}, z) \equiv \langle Y(\vec{y}, z) Y(0, z) \rangle = \int \frac{d^3\kappa}{(2\pi)^3} e^{i\vec{\kappa}\cdot\vec{y}} |f_\kappa(z)|^2 = \frac{1}{2\pi^2} \int e^{i\vec{\kappa}\cdot\vec{y}} P_\kappa(z) \, d\ln\kappa \,. \tag{5.27}$$

Taking into account eq. (5.26) and reducing the 3-dimensional integral in this expression to a 1-dimensional integral by switching to spherical coordinates, we get

$$\zeta(r, z) = \frac{1}{2\pi^2} \int_0^\infty P_\kappa(z) \, j_0(\kappa r) \, d\ln\kappa \simeq \zeta(0, z) \, G_1(\kappa_* r) \,, \tag{5.28}$$

with $j_0(\kappa r) = \sin(\kappa r)/(\kappa r)$ the spherical Bessel function.[5] Here

$$\zeta(0, z) \equiv Y_{\rm rms}^2(z) = \left(\frac{2\nu - 1}{8\pi\nu}\right)^2 \left(1 + \frac{z}{\nu}\right)^{2\nu+1} \kappa_*^2(z) \tag{5.29}$$

stands for the square of the time-dependent dispersion determining the root mean-square perturbation (rms) and

$$G_1(\kappa_* r) \equiv \frac{\sqrt{2}}{\kappa_* r} D\left(\frac{r\,\kappa_*}{\sqrt{2}}\right) = \sqrt{\frac{\pi}{2}} \frac{1}{\kappa_* r} \exp\left(-\frac{1}{2}\kappa_*^2 r^2\right) \mathrm{erfi}\left(\frac{\kappa_* r}{\sqrt{2}}\right) \tag{5.30}$$

is a shape function approaching unity at $r \to 0$, with $D$ the Dawson function and erfi the imaginary error function [41]. Note that, although the mean field value is zero when averaged over the entire ensemble, typical field configurations within the ensemble are non vanishing, but rather sample positive and negative values. At a given time $z$, the correlation length is dominated by wavelengths of the order of the inverse momentum $\kappa_*^{-1}(z)$ and therefore much smaller than the horizon radius for large $\nu$ values. This is also reflected in the profile of the domains, which, following similar steps to those above, can be written as [48]

$$\rho(r, z) = \frac{1}{\sqrt{2}\,\pi} \int_0^\infty P_\kappa^{1/2}(z) \, j_0(\kappa r) \, d\ln\kappa \simeq \rho(0, z) G_2(\kappa_* r) \,, \tag{5.31}$$

with

$$\rho(0, z) \equiv \pi^{1/2} \frac{|2\nu - 1|}{8\pi\nu} \left(1 + \frac{z}{\nu}\right)^{\nu+1/2} \kappa_*(z) \,, \qquad G_2(\kappa_* r) \equiv \frac{\sqrt{\pi}}{2} \frac{\mathrm{erf}(\kappa_* r)}{\kappa_* r} \,, \tag{5.32}$$

and erf the error function, cf. figure 6.

Although classical and sufficiently localized, the identification of the created topological defects in a Gaussian random field is a generically a complicated task. The number density

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[5]Note that we have omitted an IR cutoff at $\kappa = \kappa_{\min}(z)$. The error we are making is, however, small, namely of the order of percent for $\nu = 10$. In fact, $\delta\zeta/\zeta = 1 - \exp[-1/(8(\nu+1))]$ in the $r \to 0$ limit.
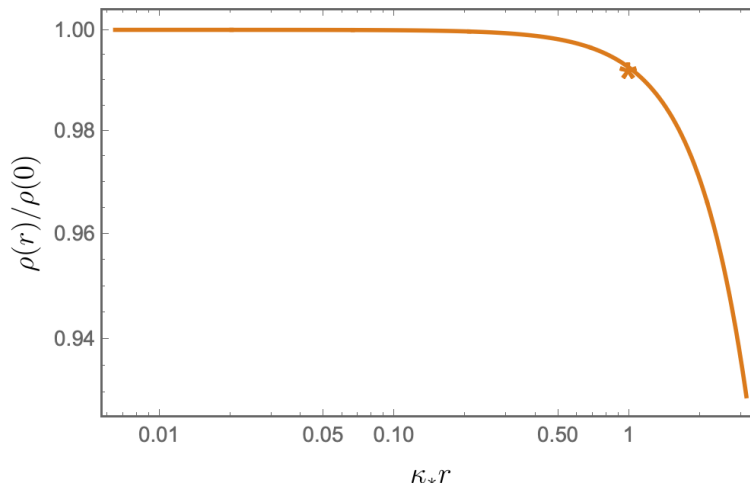
**Figure 6**. Density profile (5.32) normalized to the central density. The star signals $\kappa_* r = 1$.

of domain walls can be estimated by counting the number of zeros of the classical field, given by [49, 50]

$$n_{\text{walls}}(z) = \frac{1}{\pi^2} \left( -\frac{\zeta''(0,z)}{\zeta(0,z)} \right)^{3/2} , \tag{5.33}$$

with

$$-\frac{\zeta''(0,z)}{\zeta(0,z)} = \frac{\int d\ln\kappa \, \kappa^2 P_\kappa(z)}{\int d\ln\kappa \, P_\kappa(z)} . \tag{5.34}$$

We get

$$n_{\text{walls}}(z) = \left( \frac{2}{3} \right)^{3/2} \frac{\kappa_*^3(z)}{\pi^2} . \tag{5.35}$$

## 6   Symmetry restoration and domain wall annihilation

The applicability of the previous section techniques is restricted to the early times at which the self-interactions of the created particles can be safely neglected ($\lambda \to 0$). A useful formalism able to account for potential backreaction effects in a time-dependent setting is the two-particle-irreducible close-time path or in-in formalism, whose truncation at the two-loop order becomes equivalent to the time-dependent Hartree-Fock approximation. In this limit, the quartic non-linearity is replaced by $3/2 \, \lambda \langle Y^2 \rangle Y^2$, leading to the following effective frequency for the spectator field fluctuations

$$\omega_\kappa^2(z) \equiv \kappa^2 - M(z)^2 + 3\lambda Y_{\text{rms}}^2(z) . \tag{6.1}$$

The tachyonic instability at small momenta will stop when this modified frequency becomes positive, leading in practice to symmetry restoration. This symmetry restoration can be "*effective*" if the field is stabilized by its own interactions and starts oscillating with a sizable amplitude around the origin of the potential (cf. section 4) or "*exact*" if the Universe is heated through some additional sector before the onset of non-linearities.

As argued in ref. [14], the heating effects can be parametrized by a *heating efficiency* parameter

$$\Theta \equiv \frac{\rho_R}{\rho_\phi}\Big|_{z_{\rm kin}} = \left(\frac{a_{\rm kin}}{a_{\rm rad}}\right)^2 , \qquad (6.2)$$

with $\rho_R$ the energy density of the heating products and $a_{\rm kin}$ and $a_{\rm rad}$ the values of the scale factor at the onset of kinetic and radiation domination. The minimal value of the heating efficiency

$$\Theta \gtrsim 10^{-16} \left(\frac{H_{\rm kin}}{10^{11}\,{\rm GeV}}\right)^2 , \qquad (6.3)$$

is restricted by the (integrated) nucleosynthesis constraint on the gravitational waves density fraction [14]. Taking into account the scaling of the radiation and inflaton energy densities during kinetic domination,

$$\frac{\rho_R(a)}{\rho_\phi(a)} = \Theta \left(\frac{a}{a_{\rm kin}}\right)^2 , \qquad (6.4)$$

we can easily determine the number of *e*-folds after the end of inflation at which the $Z_2$ symmetry is "*exactly*" restored by heating effects ($a = a_{\rm rad}$, $\rho_{\rm R}(a_{\rm rad}) = \rho_\phi(a_{\rm rad})$, $R \simeq 0$),

$$N_{\rm exact} = \ln\left(\frac{a_{\rm kin}}{a_{\rm rad}}\right) = \frac{1}{2}\ln\Theta . \qquad (6.5)$$

This temporal scale should be compared with the "*effective*" restoration time at which the last two terms in eq. (3.6) become approximately equal $[\lambda Y_{\rm rms}^2/2 \simeq M^2]$, namely

$$N_{\rm eff} = \frac{1}{2}\ln\left(1 + \frac{z_{\rm eff}}{\nu}\right) \simeq \frac{1}{2\nu+1}\ln\left[\frac{8\pi\nu^2}{(2\nu-1)\sqrt{3\lambda(\nu+1)}}\left(1 - \frac{1}{4\nu^2}\right)^{1/2}\right] . \qquad (6.6)$$

Beyond the smallest of these quantities, the approach presented in section 5 becomes inaccurate and the theory cannot be longer described as a free theory where the gaussianity of the initial state is preserved by the evolution.

Immediately after symmetry restoration the Universe is divided into regions with a rather homogeneous field distribution on scales $L \lesssim \kappa_*^{-1}$, cf. figure 6. Any homogeneous or perturbative approximation ignoring this fact will be unable to provide a realistic description of the problem at hand, disregarding in the process interesting physical effects such as the potential generation of gravitational waves by the walls interpolating among domains [7]. The precise impact of the domain wall decay at symmetry restoration seems difficult to estimate without the use of lattice simulations. A naive extrapolation of the usual kink solution, strictly applicable only in the static regime, would suggest the gradient contribution to be commensurable with the potential energy density of the field at that time [51]. Being this the case, the effective restoration of the symmetry due to backreaction effects will not generically translate into a smooth oscillatory pattern as that depicted in figure 1, but rather into a non-perturbative stage where the created defects will percolate among themselves, releasing gradient energy, producing inhomogeneities and heating the system. We postpone the study of this scenario to a future publication.

## 7 Conclusions

The spontaneous breaking of discrete symmetries in the early decelerating Universe need not be a cosmological disaster. In this work, we have performed a detailed analysis of the early

symmetry breaking dynamics of a non-minimally coupled $Z_2$-symmetric spectator field in quintessential inflation scenarios. The presence of a kinetic dominated era in this type of models triggers a Hubble-induced symmetry breaking for the spectator field soon after the end of inflation that forces its evolution towards large expectation values. The transition to the new minima proceeds through a tachyonic or spinodal instability able to extract energy from the gravitational field itself. The associated production of fluctuations in the infrared part of the spectrum turns out to be very efficient, giving rise to a semiclassical but stochastic picture where sizable and rather homogeneous field configurations emerge. The homogeneity of these spatial domains is, however, a purely local concept since, as required by the model symmetries, the mean value of the scalar field remains zero at all times. From a global point of view, the Universe looks rather inhomogeneous, with regions of positive and negative field values separated by domain wall configurations. These topological defects are, however, short-lived and present only till the onset of radiation domination or the appearance of sizable backreaction effects, whatever happens first. Beyond that time, the Hubble-induced symmetry breaking ceases effectively to exist and the domain walls are doomed to disappear. The domain wall problem is consequently exorcised and replaced by a quite rich cosmological scenario. Among other interesting consequences, the temporal existence of topological defects may lead to the production of a detectable gravitational waves' background [7] even in those cases in which the amplification of the primordial spectrum of gravitational waves during kinetic domination does not lead to any observable consequence ($\omega \simeq 1$) [28, 52, 53]. Our basic results can be applied to alternative symmetry breaking patterns leading to other topological defects such as strings or textures by simply replacing $\chi$ by its modulus. In particular, it would be interesting to apply our results to the Standard Model Higgs, extending the analysis of ref. [28] to non-homogeneous field configurations. Some work in refining the present treatment and the parameter space scanning is however necessary before extracting quantitative conclusions. In particular, the results presented in this paper rely on simple analytical methods within the linear regime and should be consequently understood just as a preliminary step towards the proper characterization of the defects and their decay via lattice simulations. We believe, however, that the qualitative picture presented here is robust enough.

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