About the calculus of the relative frequency shifts for a beam with multiple cracks

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Abstract. We propose in this paper a mathematical relationship to calculate the natural frequency shifts of beams due to multiple cracks. The relationship is based on the application of the superposition principle. A crack alters the frequencies for the bending vibration modes in a particular manner, with respect to the crack depth and location. We have shown that the affected beam have the same natural frequencies as a healthy beam with a smaller thickness; this thickness has to be calculated separately for each vibration mode. As a consequence, we can consider the next crack affects healthy beams, one different beam for each considered mode. Subsequent cracks affect a bigger number of beams. For the last crack, we have to calculate the frequencies for a number of beams that is the number of the considered vibration modes to the power equal with the number of cracks. To automate the process of calculating the resulted natural frequencies, we have written a program in Visual Basic for Excel Applications (VBEA), which permits finding the natural frequencies of a beam with up to nine cracks. This application can calculate the natural frequencies for four beam types: with fixed-free, fixed-fixed, simply supported and free-free restraints. The results obtained by calculating the frequencies for several damage scenarios and boundary conditions fit those obtained from simulations by involving the finite element method.

1. Introduction

In the last decades, structural integrity evaluation based on the analysis of vibration becomes an important concern of researchers and practitioners. As a consequence, numerous techniques that permit estimating the size and position of a crack in the beam are available [1]. These vibration-based assessment methods consider either the natural frequencies shifts or the changes of the mode shapes, the modal curvatures respectively the modal damping [2-5].

The natural frequency is the most commonly used modal parameter because it can easily be measured and does not imply the use of complex or sensitive equipment. However, a high accuracy of the frequency estimate must be ensured by advanced signal processing methods [6] in order to observe the crack occurrence as early as possible [7], [8]. Mathematical relationships between the damage parameters (i.e. the location and severity) and the frequency decrease are available in the literature; see for instance references [9-12]. The methods can be employed as well for an instable environment, but corrections have to be applied for the frequencies to eliminate the environmental changes; see for instance references [13-15].

To find the crack location and severity, in most of the cases the frequency shift caused by cracks is compared with those achieved by using theoretical models. If more vibration modes are involved in the analysis, the comparison is made by using dissimilarity estimators that perform a mode-by-mode evaluation [16], [17]. Another option to assess damages from vibration measurement is the use of artificial intelligence and optimization techniques [18-20].

If the structure is affected by one crack, it is possible to separate the problem of finding the crack location and severity by applying two consecutive normalizations [21]. Initially, the location is determined involving the Damage Location Coefficients (DLC) deduced from the Relative Frequency Shifts (RFS) found after a procedure described in [22]. Afterward, the severity is found for the crack for which the location is identified [23].

From our prior research, we found that for a beam with more cracks directly employing the DLCs is not possible since the cracks can achieve different severities and normalization is hence improper. Different approaches to detect multiple cracks are largely described in the literature [24-26]. The key finding is that the superposition principle generally applies, except the case if the damages are closely located each to the other [27]. We propose here an algorithm to find the DLCs for a beam with up to three cracks that we have implemented in VBEA. It ensures an automated generation of damage scenarios which is useful to easily create a database that can be used in the assessment of multiple damages. The algorithm's efficiency is tested by comparing the results obtained from the VBEA with the frequencies obtained from the modal analysis performed using the finite element method (FEM) for a series of simulated damages.

2. The model of cracked beams

Models used to investigate real systems should describe their behavior or at least the aspect of which the study is focused, as accurate as possible. In this paper, we analyze prismatic beams with transverse cracks and the phenomenon refers to the transverse vibration, more precisely to the frequencies of these modes of vibration. The dimensions of the intact beam considered in this work are: the length $L = 1000\,\text{mm}$, the width $B = 50\,\text{mm}$ and the thickness $H = 5\,\text{mm}$. The dimensions of the beam qualify it to be considered as an Euler-Bernoulli beam and It is made of steel, and has the mass density $\rho = 7850\,\text{kg/m}^3$ and the longitudinal modulus of elasticity $E = 2 \cdot 10^{11}\,\text{N/m}^2$.

We perform the study on a cantilever beam, i.e. with fixed-free boundary conditions, but in the paper we permanently demonstrate that the developed theory and the resulting mathematical relationships are valid for any other boundary conditions that can be applied to the beam.

2.1. The model of the beam with one crack

The aim of this sub-section is to demonstrate the behavior in terms of frequencies of a prismatic beam with a crack of depth d and location c can be modeled employing a beam with a smaller but constant thickness or with a reduced density. We base this demonstration on the well-known fact that the ratio of the frequencies of two beams is proportional with the ratio of the energies stored in the two beams.

Let us design the original beam with constant stiffness the Intact Beam (IB), the beam with a crack the Damaged Beam (DB) and the model of the DB, which has reduced but constant thickness or density, the Equivalent healthy Beam (EHB). The DB and the EHB are presumed to achieve under dead mass the same deflection at the free end. This, according to Castigliano's second theorem indicates that these store the same amount of energy. Hence, based on the proportionality between frequencies and stored energies, the EHB will have the same frequency as the DB. Now, let's see how the thickness or the density of the EHB can be deduced.

The dimensions of the beam qualify it to be considered as an Euler-Bernoulli beam and in consequence, the frequency of the i^{th} transverse vibration mode of the IB can be calculated as:

$$f_i = \frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI}{\rho A L^4}} \tag{1}$$

In equation (1), λ_i is the wavenumber for transverse vibration mode $i = 1 \ldots n$, which depend on the boundary conditions [10]. In addition, we have in this equation the area *A* of the constant cross-section and the second moment of area *I*.

The maximum deflection of the IB under dead mass can be calculated with the well-known equation:

$$\delta_{IB} = \frac{\rho \cdot g \cdot A \cdot L^4}{\kappa \cdot E \cdot I} \tag{2}$$

where *g* is the gravitational acceleration and κ is a parameter who's value depends on the boundary conditions. For the cantilever beam, which is our concern in this paper, $\kappa = 8$. Other examples are: $\kappa = 384/5$ for the simply supported beam and $\kappa = 384/2.08$ for the fixed-hinged beam.

One can observe that the mathematical relationships that express the frequency and the deflection contain similar terms. Hence, by a simple substitution, for the cantilever beam we can write:

$$f_{i-IB} = \frac{\lambda_i^2}{2\pi} \sqrt{\frac{g}{8\delta_{IB}}} \tag{3}$$

The maximum deflection increases in the case a crack occurs in the beam, and becomes:

$$\delta_{DB}(c,d) = \delta_{EHB}(c,d) = \frac{\rho \cdot g \cdot A_{eq}(c,d) \cdot L^4}{\kappa \cdot E \cdot I_{eq}(c,d)} = \frac{\rho_{eq}(c,d) \cdot g \cdot A \cdot L^4}{\kappa \cdot E \cdot I} \tag{4}$$

where A_{eq}, I_{eq} respectively ρ_{eq} represent the values found for the EHB to fit the deflection of the DB that has a crack, and the bracket (*c,d*) indicates the crack parameters. One can observe from equation (4) that $\delta_{DB}(c,d) = \delta_{EHB}(c,d)$ can be obtained either by changing the beam thickness, or its density. Note that the shape of the DB is similar to that of the IB, except an additional rotation in the damaged slice.

From equation (1) and (4), we can obtain after performing a simple substitution:

$$f_{i-EHB}(c,d) = \frac{\lambda_i^2}{2\pi} \sqrt{\frac{g}{8\delta_{EHB}(c,d)}} \tag{5}$$

Hence, the mathematical relationship to express the natural frequency of the damaged beam results, from equations (3) and (5), as:

$$f_{i-EHB}(c,d) = f_{i-IB} \sqrt{\frac{\delta_{IB}}{\delta_{EHB}(c,d)}} \tag{6}$$

Now, we can estimate the RFS, denoted $\Delta\bar{f}_i(c,d)$, from the deflections of the IB and DB under static loads, given that f_{i-IB} are known. The contrived relation is:

$$\Delta\bar{f}_i(c,d) = \frac{f_{i-IB} - f_{i-EHB}(c,d)}{f_{i-IB}} = \frac{\sqrt{\delta_{EHB}(c,d)} - \sqrt{\delta_{IB}}}{\sqrt{\delta_{EHB}(c,d)}} \tag{7}$$

One can observe in equations (6) and (7) that the parameter κ belongs both to the nominator as well as to the denominator, so it can be reduced by simplifying it in the ratios. This permits concluding that the square root in equation (6) is the same for any boundary conditions the beam gets assigned, so that the deflection for the cantilever beam can be used to quantify the damage irrespective to the beam end supports.

It was shown in our prior research, see for instance [25], that the RFS achieves the biggest value if the crack is located on a slice where the bending moment or the modal curvature attains it highest value. This happens at the fixed end, which is $c = 0\,\text{mm}$ for the cantilever beam.

For all other positions of the crack, the relation between the deflections and the curvature is [5]:

$$\Delta\bar{f}_i(c,d) = \frac{\sqrt{\delta_{EHB}(c,d)} - \sqrt{\delta_{IB}}}{\sqrt{\delta_{EHB}(c,d)}} = \frac{\sqrt{\delta_{EHB}(0,d)} - \sqrt{\delta_{IB}}}{\sqrt{\delta_{EHB}(0,d)}}\left[\bar{\phi}_i''(c)\right]^2 = \Delta\bar{f}_i(0,d)\left[\bar{\phi}_i''(c)\right]^2 \tag{8}$$

where $\left[\bar{\phi}_i''(c)\right]^2$ is the square of the normalized modal curvature (SNMC). We can denote

$$\Delta\bar{f}_i(0,d) = \frac{\sqrt{\delta_{EHB}(0,d)} - \sqrt{\delta_{IB}}}{\sqrt{\delta_{EHB}(0,d)}} = \gamma(0,d) \tag{9}$$

where $\gamma(0,d)$ is the damage severity. If due to the boundary conditions the curvature achieves maxima at other locations as the beam end, that crack location must be taken into consideration when deriving the maximum beam deflection. In a similar way, we can designate $\Delta\bar{f}_i(c,d) = \gamma(c,d)$ the pseudo-severity. The damage severity is the same for a given crack, irrespective to the boundary conditions, while the pseudo-severity depends on the crack location and the boundary conditions.

From equations (9) we can find the frequency of the frequency shift:

$$\Delta f_i(0,d) = \gamma(0,d)\cdot f_{i-IB} \qquad \text{or} \qquad f_{i-IB} - f_{i-EHB}(0,d) = \gamma(0,d)\cdot f_{i-IB} \tag{10}$$

hence

$$\frac{f_{i-EHB}(0,d)}{f_{i-IB}} = \left[1-\gamma(0,d)\right] \tag{11}$$

But, after replacing A_{eq} and I_{eq} with their explicate expressions in equation (7), we obtain

$$\frac{f_{i-EHB}(c,d)}{f_{i-IB}} = \frac{\sqrt{\delta_{IB}}}{\sqrt{\delta_{EHB}(c,d)}} = \frac{\sqrt{H_{eq}^2(0,d)}}{\sqrt{H^2}} = \frac{H_{eq}(0,d)}{H} \tag{12}$$

After making the substitutions in equations (11) and (12), we obtain the value of the EHB thickness

$$H_{eq}(0,d) = \left[1-\gamma(0,d)\right]H \tag{13}$$

In a similar way we can find the density of the EHB as:

$$\rho_{eq}(0,d) = \left[1-\gamma(0,d)\right]^{-2}\rho \tag{14}$$

Obviously, for other crack locations as the fixed end, the pseudo-severity should be involved, that mean equation (13) becomes:

$$H_{eq}(c,d) = \left\{1-\gamma(0,d)\left[\bar{\phi}_i''(c)\right]^2\right\}H \tag{15}$$

and equation (14) becomes:

$$\rho_{eq}(c,d) = \left\{1-\gamma(0,d)\left[\bar{\phi}_i''(c)\right]^2\right\}^{-2}\rho \tag{16}$$

From equation (8), after multiplying both sides with f_{i-IB} and considering the equivalence in equation (9), we can deduce the frequency of the DB as:

$$f_{i-DB}(c,d) = f_{i-EHB}(c,d) = f_{i-IB}\left\{1-\gamma(0,d)\left[\bar{\phi}_i''(c)\right]^2\right\} \tag{17}$$

This relation permits finding the frequency for a beam with a crack in a simple and reliable way, by simply involving the severity and SNMC calculated for the considered crack.

2.2. The EHB – poof of the concept

We performed tests to show the EHB is a suitable model to predict frequencies for beams with one crack. To this aim, we performed simulations by means of the ANSYS software and derived the free-end deflections under dead mass for the IB and DB with a crack at the fixed end that has the depth $d = 1\text{mm}$. For these values we found the damage severity involving equation (9) and subsequently the equivalent thickness and density using relations (13) and (14). The severity is used to find the natural frequencies of the DB by calculus, with equation (17) and considering the SNMC = 1 because the crack is at the fixed end. The benchmark was the natural frequency of the IB found from simulation. The two other features, namely the $H_{eq}(0,d)$ and the $\rho_{eq}(0,d)$, are used to find the natural frequencies of the EHB by simulation. We also performed a modal analysis to find the natural frequencies for the DB with a simulated crack. Finally, we compared the frequencies obtained from calculus with those achieved by simulation to find if the developed concept of the EHB and the contrived mathematical relations are suitable to predict frequency changes due to damage. The test was also made for a crack located at $c = 160\,\text{mm}$ from the fixed end.

The deflections of the IB and the DB are shown in Table 1. Here, also the equivalent thickness and the equivalent density calculated with the equations (13) respectively (14) are specified.

Table 1. Equivalent healthy beam thickness and density calculated by employing the severity

Mode no.	Deflection at the free end		Severity $\gamma(0,1)$	Beam thickness		Beam density	
	δ_{i-IB} [mm]	$\delta_{i-DB}(0,1)$[mm]		H[mm]	$H_{eq}(0,1)$[mm]	$\rho[kg/m^3]$	$\rho_{eq}(0,1)[kg/m^3]$
1	22.948	23.092	0.00312	5	4.98435	7850	7899.259

For the severity deduced for the EHB, we calculated the frequencies for the DB with the crack at $c = 0\,\text{mm}$ and found the results presented in the last column in Table 2. In this table is also indicated the frequency obtained from FEM simulation for the DB, the EHB with reduced thickness and the EHB with increased density.

Table 2. Frequencies found from simulation and by calculus for the crack position $c = 0\,\text{mm}$

Mode no.	Frequency from FEM simulation [Hz]			Frequency from calculus [Hz]
	Original beam with the crack at $c = 0$	Reduced thickness $H_{eq}(0,1)$	Increased density $\rho_{eq}(0,1)$	Using the damage severity $\gamma(0,1)$
1	4.07720	4.07720	4.07723	4.07723
2	25.54700	25.54708	25.54725	25.54597
3	71.53300	71.53212	71.53261	71.52992
4	140.20000	140.19151	140.19246	140.18843
5	231.81000	231.79964	231.80122	231.79468

After this test, we conclude that irrespective to the method the frequencies of the DB are found, i.e. by employing the severity, the equivalent thickness or the equivalent density respectively calculus or FEM simulation, we obtain accurate results.

Our second approach was to test the method for a crack located at a certain distance from the fixed end. To this aim, we selected the crack position $c = 160\,\text{mm}$ and performed modal analysis for the DB. Afterward, we calculated the values of the SNMC for the considered crack position and calculated the natural frequencies for the first five transverse vibration modes by involving equation (17). To do this, we used the frequencies of the IB and the severity indicated in Table 1. All parameters used in the calculus are indicated in Table 3.

Table 3. Frequencies found from simulation and by calculus for the crack position $c = 160\,\text{mm}$

Mode no.	Frequency for the IB from simulation [Hz]	Squared Normalized Modal Curvature	Frequency for the EHB from calculus [Hz]	Frequency for the DB from simulation [Hz]
1	4.09000	0.60822	4.082231588	4.08227
2	25.62600	0.05950	25.62123818	25.62226
3	71.75400	0.03351	71.7464904	71.75043
4	140.62759	0.25657	140.5149174	140.52670
5	232.52081	0.43521	232.2047921	232.22581

The second experiment has shown that the relation can be applied for crack locations that are not at the fixed end, or at the beam slice subjected to the most important curvature. These findings validate the deduced equations and the concept of the EHB and guarantee that the model can be used to predict the natural frequencies of cracked beams.

2.3. *A predictive model for the behaviour of the beam with multiple cracks*

Knowing that the behavior of a beam with a crack can be described using an intact beam with changed mechanical or geometrical parameters, in fact the EHB, we can add to this beam a new crack with other location c_2 and depth d_2. The method described in sub-section 2.1 can be repeated, and natural frequencies are found for the beam with two cracks. Also, new equivalent thickness and equivalent density can be found to model the behavior, in terms of frequency response, of the beam with two cracks. Note, the procedure can be applied iteratively for any number of cracks, provided that the cracks are not close to each other.

In the following, we present some relation to be applied for the case that multiple cracks affect a beam. The natural frequency of a beam with several cracks can be calculated with the mathematical relation:

$$f_{i-DB}(c_1,d_1;c_2,d_2;...) = f_{i-IB}\left\{1-\gamma(0,d_1)\left[\bar{\phi}_i''(c_1)\right]^2\right\}\left\{1-\gamma(0,d_2)\left[\bar{\phi}_i''(c_2)\right]^2\right\}... \tag{18}$$

where for each new crack a new parenthesis is added to the left term of the equation.

The equivalent thickness of the EHB that model an IB with several cracks is calculated with the mathematical relation:

$$H_{eq}(c_1,d_1;c_2,d_2;...) = H\left\{1-\gamma(0,d_1)\left[\bar{\phi}_i''(c_1)\right]^2\right\}\left\{1-\gamma(0,d_2)\left[\bar{\phi}_i''(c_2)\right]^2\right\}... \tag{19}$$

The equivalent density of the EHB that model an IB with several cracks is calculated with the mathematical relation:

$$\rho_{eq}(c_1,d_1;c_2,d_2;...) = \rho\left\{1-\gamma(0,d_1)\left[\bar{\phi}_i''(c_1)\right]^2\right\}^{-2}\left\{1-\gamma(0,d_2)\left[\bar{\phi}_i''(c_2)\right]^2\right\}^{-2}... \tag{20}$$

The mathematical relation that is most often used in damage detection is equation (18), because it permits calculating the RFSs, which are directly used in vibration-based damage assessment as a benchmark. But, because of the huge number of possible scenarios, calculating all possible the RFSs is time and resource consuming. That is why it is necessary to automate the process of creating the RFSs.

3. Implementing the model in VBEA

The algorithm implemented in VBEA use the equation (20) to calculate the natural frequencies of the cracked beam for the first nine weak-axis transverse vibration modes. The utilizer can define up to nine crack locations and has the possibility to ensure too every crack the desired depth. The interface for introducing the data is presented in Fig. 1. The reference frequency values belonging to the IB must also be introduced as input data.

Vibration mode	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Intact beam	4.09	25.627	71.757	140.63	232.53	347.46	485.47	646.59	830.81
	Crack1	Crack2	Crack3	Crack4	Crack5	Crack6	Crack7	Crack8	Crack9
Crack depth - d	**1**	**1**							
Beam thickness - h	**5**								
Relative crack depth - d/h	0.2	0.2							
Severity	0.00312	0.00312							
Crack location - x	**160**	**700**							
Beam length - L	**1000**								
Relative crack location - x/L	0.16	0.7							
Calculated frequencies for the multi-cracked beam	**4.082**	**25.6**	**71.622**	**140.44**	**232.19**	**346.56**	**484.72**	**646.57**	**829.81**

Figure 1. The interface for introducing the input data

The IB is described in rows five (thickness) and nine (length) in Figure 1. The maximum nine cracks are described in absolute values at rows four (depth) and eight (location). The program automatically calculates the relative values, the damage severity and the SNMC. The relative values for the chosen cracks are displayed in the interface presented in Figure 1. Afterward, the program calculates the natural frequencies for the damaged beam if the frequencies of the IB are introduced (row two) and the support type is selected. This selection process can be made using a drop-down window, which permits choosing four beam types, as shown in Figure 2. The calculus is performed by pushing the button “Generate frequencies for the multi-cracked beam” also shown in Figure 2.

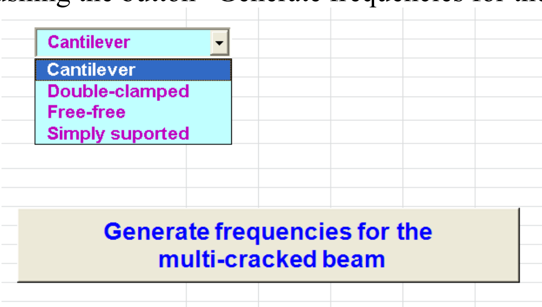

Figure 2. The drop-down window with the options to be selected for the boundary condition and the “generate frequencies” button

The program also calculates the RFSs, which are displayed as shown in Figure 3. These can be further used in the damage assessment process.

RFS Mod 1	RFS Mod 2	RFS Mod 3	RFS Mod 4	RFS Mod 5	RFS Mod 6	RFS Mod 7	RFS Mod 8	RFS Mod 9
0.002	0.0011	0.001883	0.0014	0.0015	0.0026	0.0016	3E-05	0.0012

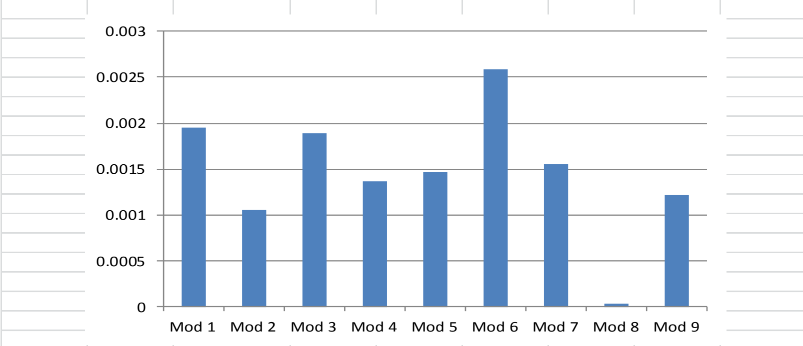

Figure 3. The RFSs for the first nine modes displayed numerical and in form of a diagram

Another facility of the program is the calculus of as many damage scenarios the utilized ask for. This is made by selecting some possible severities, as shown in Figure 4 in the first row. Note that the number of severities is not limited, but the higher the number the longer the generation process is. The number of scenarios is also defined by the step with which the cracks are iteratively repositioned on the beam. In the analysis presented in Figure 4, the step is 0.005, meaning that the crack attain 200 locations on the beam.

Severity	0	0.0025	0.003	0.0035	0.004				
Mode number	1	2	3	4	5	6	7	8	9
Frequencies for the IB	4.0899957	25.627281	71.75669	140.63163	232.52735	347.46291	485.47392	646.58675	830.8137
Frequencies for the DE	4.081992215	25.60029783	71.62157095	140.4399042	232.1866621	346.563875	484.7202725	646.5669397	829.8074619
Cantilever		Calculate table with defined scenarios							
x/L	1	2	3	4	5	6	7	8	9
0.0001	0.999724717861	0.999044072903	0.998430882894	0.997802027313	0.997174569089	0.996547233540	0.995920099520	0.995293162854	0.994666423281
0.005	0.986282315477	0.952763637840	0.923053571692	0.893064372520	0.863626904483	0.834680456031	0.806229215513	0.778273517874	0.750813932160
0.01	0.972659378169	0.906670400168	0.849189720281	0.792183425860	0.737270716824	0.684339676200	0.633402645769	0.584464728011	0.537531176942
0.015	0.959131210461	0.861721090089	0.778414215616	0.697377282511	0.620980990264	0.549076365747	0.481690180971	0.418835504038	0.360520514806
0.02	0.945697848926	0.817916948034	0.710735440654	0.608673005246	0.514818138560	0.428996139242	0.351242915087	0.281559968751	0.219920849198

Figure 4. The RFSs for the first nine modes displayed numerical and in form of a diagram

For all single-crack scenarios the bracket in equation (17) is calculated, see as an exemplification the last rows in Figure 4. The brackets are used in equation (18) and frequencies that cover all possible defined scenarios are created. For these frequencies the RFSs are calculated and compared by employing one of following two metrics: Minkowski or Gillich. The selection of the metric is a choice of the utilizer. The "best" fit between the RFS belonging to the cracked beam found by fem simulation or experiment and the calculated table indicate the cracks' location and severities. The result is displayed as shown in Figure 5.

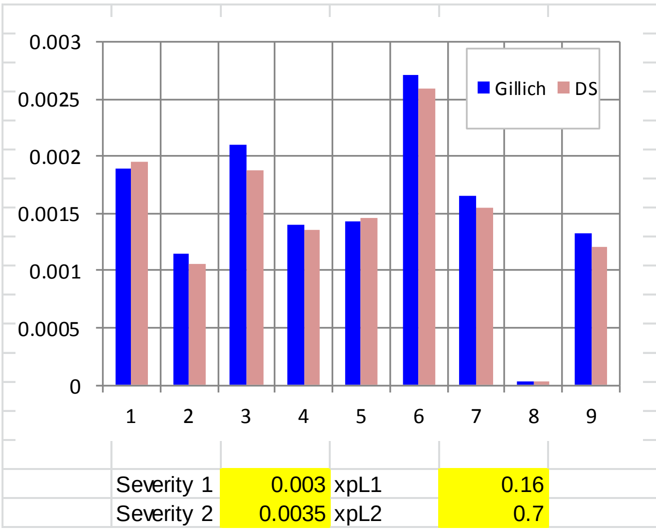

Figure 5. Crack assessment results indicating the estimated severities and locations and the RFSs found by calculus that fit the measurement results

We made tests using the facility of the developed program in VBEA to find if the generated frequencies fit the simulations and if the damage assessment is accurate. So, we performed FEM simulations considering several scenarios, from which we present the beam with cracks at $c_1 = 160\,\text{mm}$ and $c_2 = 700\,\text{mm}$, both cracks having the depth $d = 1\,\text{mm}$. The frequencies for the IB and the DB obtained from simulation are presented in Table 4. In the last row in this table we also present the values calculate utilizing the VBEA program based on equation (18). Observe the accuracy of predicting the natural frequencies of the damaged beam, the errors being framed in the range ±0.2%.

Table 4. Frequencies found from simulation and by calculus for the cracks with depth $d = 1$mm and the locations $c = 160$mm and $c = 160$mm

Mode no.	Frequency for the IB from simulation [Hz]	Frequency for the DB from simulation [Hz]	Frequency for the EHB from calculus [Hz]	Error [%]
1	4.089996	4.081992	4.081	0.0243
2	25.62728	25.6003	25.621	-0.0807
3	71.75669	71.62157	71.747	-0.1751
4	140.6316	140.4399	140.51336	-0.05232
5	232.5274	232.1867	232.19236	-0.0024

In addition, we exactly identified the crack locations and accurately found the severities, see for conformity Figure 5. Finer setting the targeted severities in the interface in Figure 4 lead to an improvement of the severity estimation and subsequently crack depth assessment.

4. Conclusion

The paper presents the concept of the EHB model and demonstrates it can be used to predict the natural frequencies of beams with any cracks. We have developed a program written in VBEA which calculates the frequencies of the cracked beam in real-time. By employing this program we found very small errors, less than 0.25, if comparing the frequencies obtained by calculus and these obtained from FEM simulation.

A second aspect approached in the paper is the assessment of multiple cracks, which is also performed by the program developed by the authors. It also bases on the EHB model and mathematical relations deduced by the authors. Again good results are obtained; exact crack locations and very accurate severities were found. Future research will focus on testing the program for experimental data.

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