Upon the efficiency of gear transmissions

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Abstract. The work proposes an experimental method and device for finding the efficiency of the spur gear mechanism. A simple device that can measure the work lost by friction only in teeth contacts was constructed. The test rig was designed to ensure measurements for two configurations with the same inertial characteristics but for different gear friction conditions. The principle is based on the difference of energy losses for the two mounting cases that provides the energy lost by friction between flanks. Recording the variation of the decreasing rotational speed for the case of constant actuating torque, the frictional losses can be deduced. The values of the efficiency obtained are smaller than in literature. The method is simple and rapid and the experimental results validate the theoretical model.

1. Introduction

Gear transmissions are widely used in engineering applications. The main advantage of these mechanisms is the fact that the transmission ratio ensured is rigorously constant. On the other hand, the main disadvantage is the presence of the higher pair between the teeth of the gears where the sliding friction is present and manifests as a great energy consumer and wear generator. Finding the work lost by friction is an important goal when estimating the quality of the gear transmission [1-6].

One of the main drawbacks encountered during theoretical modelling of the dynamic evolution of a gear mechanism is the fact that during gearing there are periods when two pairs of teeth are in the gearing field and the system is statically undetermined. Another difficulty resided in the stipulation of the dependencies of friction force and rolling friction torque upon the normal reaction [7-10].

The field of gear transmission - the region between the addendum circles of the two wheels [11] is presented in Figure 1. The contact points between the flanks move on the tangent lines (lines of action) $K'_1 K'_2$, the contact points C_1, C_2 and $K''_1 K''_2$, the contact points being now C_3 and C_4.

The next reactions occur in the points of contact between the flanks:
- the normal reactions, parallel to the line of action N ;
- the friction forces normal to the lines of action T ;
- the rolling friction torques, normal to the plane of the figure $\tilde{M}_r$.

A theoretical model of gear transmission with friction is difficult to develop, due to:
- the intricacy of establishing the number of contact points – there is possible the existence of one point of contact or of two points of contact, and to the difficulty of précising the sense of

the friction forces between the flanks - sense that depends on the relative motion between flanks, which, at its turn, is influenced by the friction forces;
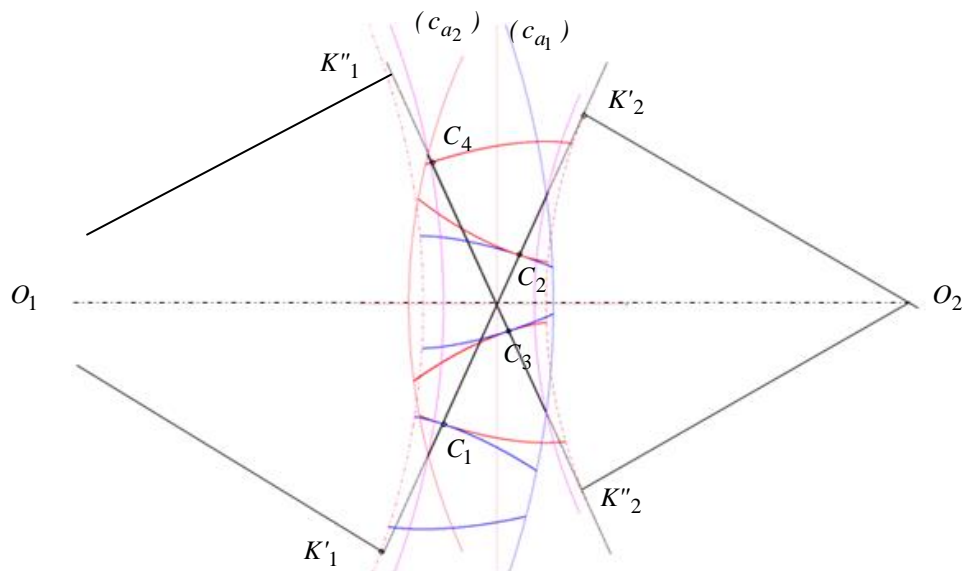
- the difficulty of stipulating the sense of the friction forces between the flanks - sense that depends on the relative motion between flanks, which, at its turn, is influenced by the friction forces.

Figure 1. The field of gear transmission multiple contacts

One of the main drawbacks encountered during theoretical modeling of the dynamic evolution of a gear mechanism is the fact that during gearing there are periods when two pairs of teeth are in the gearing field and the system is statically undetermined.

2. Proposed experimental method and device for establishing the work lost by friction in a gear mechanism

The present paper proposes an experimental method and the device for establishing the work lost by friction in a gear mechanism, Figure 2. The device consists in three cylindrical gear wheels 1, 2 3 which form parallel gear mechanisms. Two flywheels 4 and 5 are mounted on the shafts 6, 7 and 8. An inextensible wire 9 is winded on the main shaft, having a mass 10 at its end and thus ensuring a constant actuating torque.

The system is let to perform freely, while the variation of the decreasing rotational speed is registered. The manner the angular velocity of the shafts decreases offers information about the frictional losses from all the pairs of the assembly. All the ball bearings from the structure of the test rig were washed and the friction from joints is independent on the motion of the system.
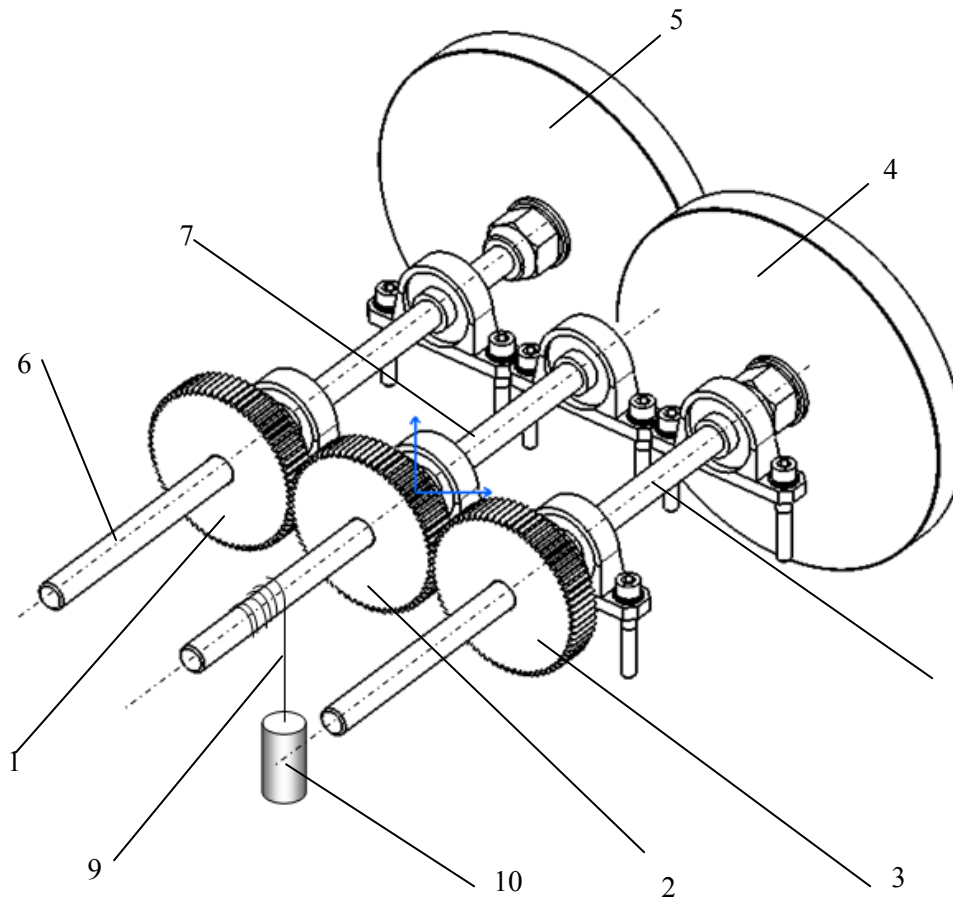
Figure 2. The proposed test rig

3. The dynamical equations

Aiming only the losses due to teeth friction, the joints between the wheels are uncoupled and all the flywheels are mounted on the same shaft. The above operation is repeated and now the work lost in the bearings of the mechanism is obtained. This should be eliminated when the wheels are coupled and the work is measured.

   a)   the case of the two gear wheels coupled, as in Figure 3

   The angular momentum theorem is applied for the central shaft in the case of the two gear wheels coupled:

$$\left(J_0 + mr^2\right)\varepsilon_1 = mgr - 2Nr_b - 2M_f - M_{0a} \tag{1}$$

and for each of the side shafts:

$$J\varepsilon_1 = Nr_b - M_f - M_a \tag{2}$$

In the equations (1) and (2), J_0 is the axial moment of inertia of the middle shaft together with the parts mounted on it (wheel, ball-bearing rings); J is the moment of inertia of the lateral shaft considered together with all the parts fixed to the shaft (wheel, flying wheel, ball-bearing rings, fixing elements); ε_1 is the angular acceleration of the shafts (the same for all the shafts as the wheels are identical); N is the magnitude of the normal force from the contact between the flanks; r_b is the radius of the base circle of the wheel; M_f is the resistant moment from the teeth contact due to both sliding friction between the flanks and to rolling friction; M_{0a} is the friction torque from the central

joint; M_a is the moment from the joint of the side shaft; m is the mass of the actuating part; r is the winding radius of the actuating wire.
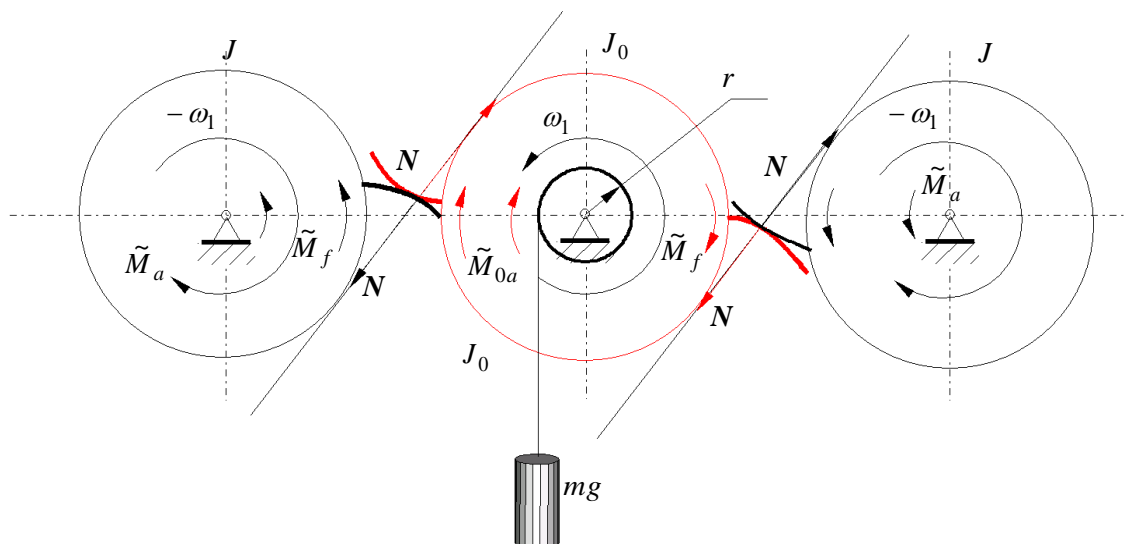

Figure 3. Running of the device with the two gear wheels coupled

The second equation is multiplied by 2 and added to the first equation and it results:

$$(J_0 + mr^2 + 2J)\varepsilon_1 = mgr - 4M_f - (M_{0a} + 2M_a) \tag{3}$$

b) the case when the side shafts are uncoupled

As shown in Figure 4, for the test rig in the new operating position the wheels and the flywheels from the side shafts are now fixed on the central shaft and therefore the moment of inertia of the shaft with all the elements is identical to the reduced moment of the dynamical system $(J_0 + 2J + mr^2)$:

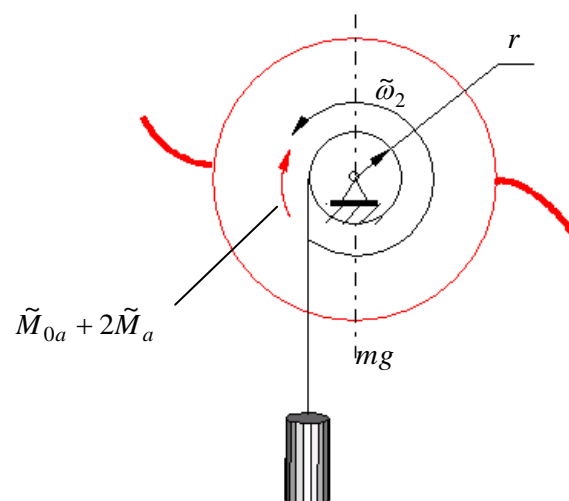$$(J_0 + 2J + mr^2)\varepsilon_2 = mgr - (M_{0a} + 2M_a) \tag{4}$$

Figure 4. Schematics of the running of the device with uncoupled wheels

In the equation (4) it was accepted that the magnitude of the reaction force from the joint of the shaft depends only on the weights of the parts mounted on the shaft. Since the ball bearings were washed, there is dry friction that generates friction forces proportional to the normal forces.
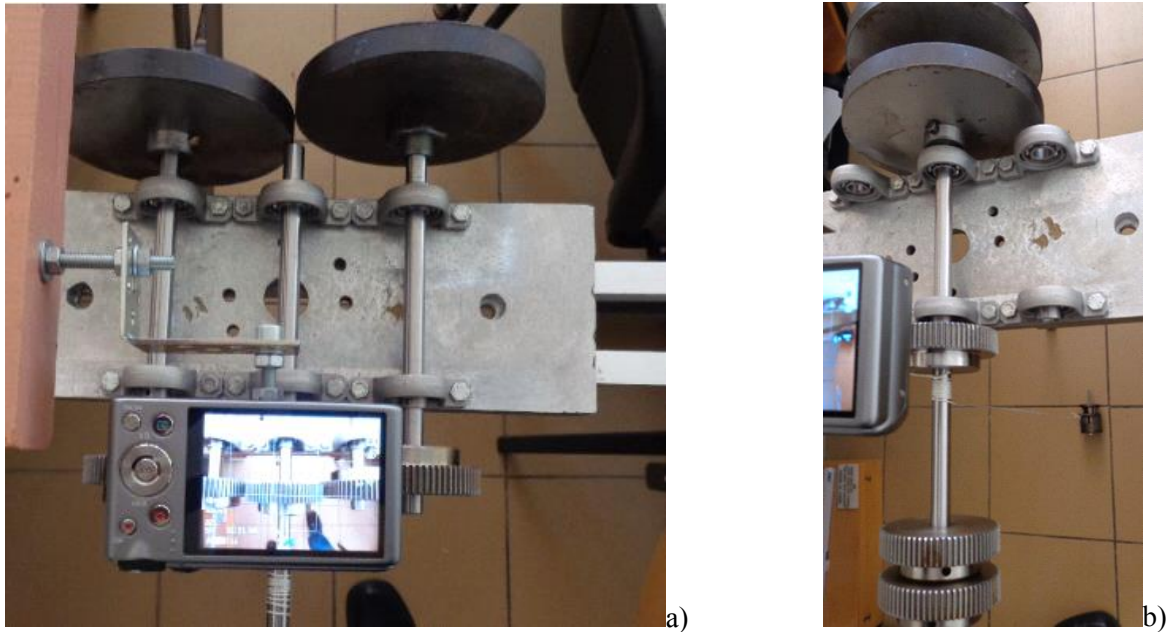

Figure 5. The experimental device in the two running cases: a) coupled side shafts; b) uncoupled side shafts

The experimental data and the interpolation curves are presented in Figure 6 where the ideal case when there is no friction in the system is traced with black line.
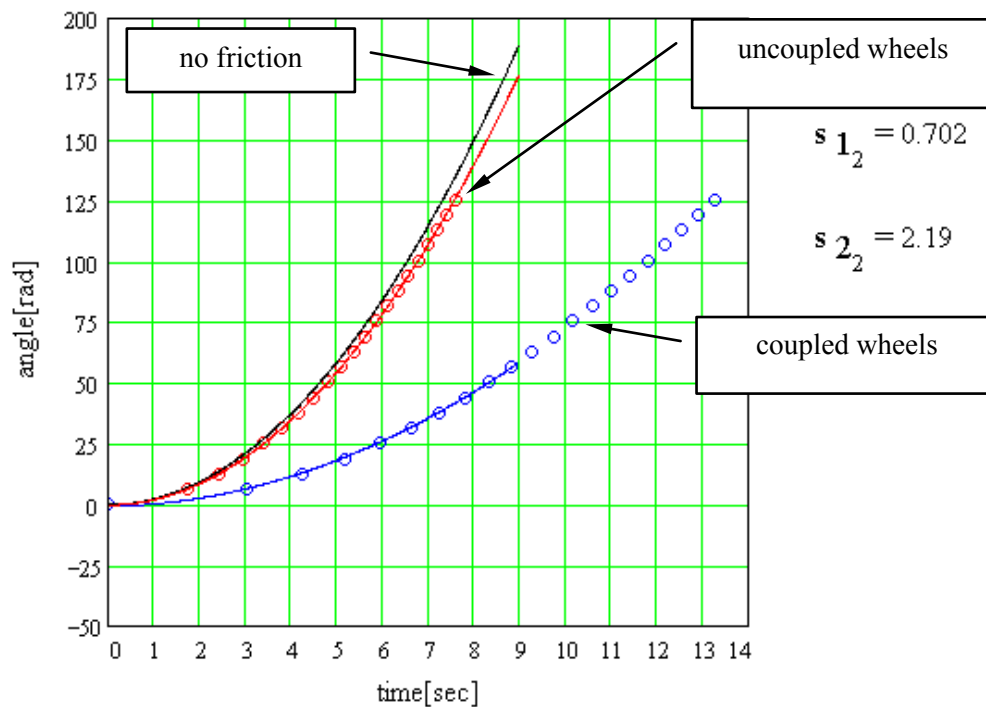

Figure 6. Experimental data and the interpolation curves

The accelerations of the main shaft are $\varepsilon_0 = 4.679$ in the absence of friction, $\varepsilon_1 = 2s_{2_2} = 4.381$ for the uncoupled gear wheels case and $\varepsilon = s_{1_2} = 1.404$ for the coupled gear wheels situation. For the calculus of the moment of inertia of the system it was thought-out that the moments of the shafts are negligible and only the moments of inertia of the wheels J_r and of the flywheels J_v are to be considered. With this approximation, it results:

$$J_0 + 2J + mr^2 = 3J_r + 3J_v = J_{tot} \tag{5}$$

For the laboratory device, the actual values are $J_v = 5 \cdot 10^{-3} kgm^2$, $J_r = 3 \cdot 10^{-4} kgm^2$, $r = 6 \cdot 10^{-3} m$ and the mass of the actuating part is $m = 0.85 kg$.

The friction moment between the flanks is obtained by subtracting the relation (3) from relation (4). The total friction torque from the joints of the wheels is denoted:

$$M_{at} = M_0 + 2M_a \tag{6}$$

and considering the system formed by the equations (3) and (4) of unknowns M_f and M_{at}, the solutions are immediately found:

$$M_f = J_{tot}(\varepsilon_2 - \varepsilon_1)/4 \; ; M_{at} = mgr - J_{tot}\varepsilon_2 \tag{7}$$

The consequence of accepting that the position angle of the central shaft has a parabolic variation is the fact that the moments M_{at} and M_f have constant values. The laboratory test rig can now be modeled as a system for which the reducing element is the central shaft. When the wheels are uncoupled, the torque acting upon the model is ($mgr - M_{at}$) and when the wheels are coupled, the acting torque is ($mgr - M_{at} - M_f$). In both cases, the driving torque is mgr. The efficiency of the transmission is the ratio between the useful work output and the total work input; for the uncoupled wheels has the expression:

$$\eta = \frac{dL}{dL_m} = \frac{M \cdot d\theta}{M_m \cdot d\theta} = \frac{mgr - M_{at}}{mgr} \tag{8}$$

resulting the value $\eta = 93.6\%$ while for the coupled wheels case takes the form:

$$\eta = \frac{dL}{dL_m} = \frac{M \cdot d\theta}{M_m \cdot d\theta} = \frac{mgr - M_{at} - M_f}{mgr} \tag{9}$$

and after the calculus is made, it is obtained the $\eta = 77.7\%$ efficiency. If the losses from the joints of the wheels are neglected, it results:

$$\eta = 1 - \frac{J_{tot}}{4} \frac{\varepsilon_2 - \varepsilon_1}{mgr} \tag{10}$$

and for the actual figures the calculus conducts to the value $\eta = 84.1\%$.

Two observations are arising:
- the friction from the joint has an insignificant effect upon the value of the efficiency of the transmission;
- the values of the efficiency are inferior to the numbers given in the technical literature [12].

The causes of this difference consist in:

a) the gear wheels are unfinished: measurements were made using a *Nanofocus laser-scan* and as noticed from Figure 7, the laser scanner analysis reveals a roughness $R_a = 4.068\,\mu m$ while the literature recommends values of $R_a = 1\,\mu m$ [13].

b) the gear wheels are not heat treated;

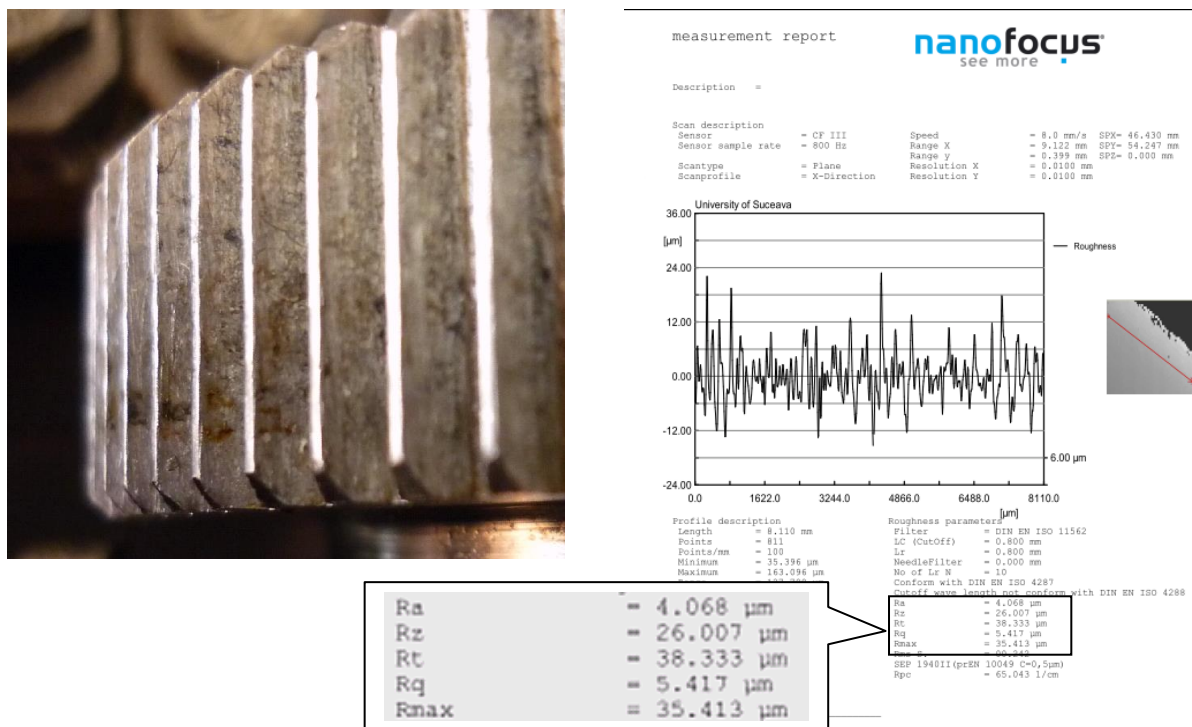c) the execution impreciseness and mounting impreciseness of the device.

Figure 7. Laser scan analysis report for the teeth flank of the gear

4. Conclusions

The paper presents an experimental method and device for finding the efficiency of the spur gear transmission. There are discussions due to disagreement between theoretical predicted values of efficiency and the actual values, found by users; the experimental values obtained for the mechanical efficiency are much reduced than the ones predicted in theoretical technical literature.

The causes of these results are:
- the use of dry contacts, chosen in order to eliminate the effect of viscosity;
- the finishing of the spur gears used, proved by the roughness, is coarse;
- the material of the gears is not heat treated;
- low manufacturing and assembling (parallelism of axes) accuracy.

Despite these aspects, the device can be improved as it presents the next advantages:
- the proposed method is validated by the concordance between theoretical model and the experimental results;
- it is a rapid and economically convenient method.

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