Nonuniform stability in mean-square for stochastic differential equations in Hilbert spaces

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Abstract. In this paper we study the relation between the nonuniform stability in mean square and admissibility of stochastic differential equation in Hilbert spaces. We consider an adapted norms and thus we obtain a variant for the stochastic case of nonuniform exponential stability in mean square due to in deterministic case. In the qualitative theory of evolution equations, nonuniform exponential stability is one the most important asymptotic properties and in last years it was treated from various perspectives The main objective is to give a more general concept of nonuniform exponential stability in mean square of stochastic differential equations in Hilbert spaces.

1. Introduction

During the last decades, considerable attention has been devoted to the problem of asymptotic behaviors of solutions of the stochastic differential equations in Banach spaces. In this paper we consider the more general concept of nonuniform exponential stability in mean square. In comparison with the notion of uniform exponential stability in mean square, this is much weaker requirement.

The notion of exponential stability for linear differential equations was studied in the paper of Perron in [1], which was concerned with the problem of conditional stability of a deterministic differential equation $x_0(t) = A(t)x(t)$ and its connection with the existence of bounded solutions of the perturbed equation $x_0(t) = A(t)x(t) + \alpha(t)$.

In deterministic case many authors obtain important results for nonuniform behaviors, as for example the papers [2], [3]. Nonuniform stability of stochastic differential equations has been considered in [4], [5].

A principal motivation for weakling the notion of uniform exponential behavior is that from the point of view of ergodic theory, almost all linear variational equations in a finite dimensional space, have a nonuniform exponential behavior.

Our main is to study the relation between the notions of nonuniform exponential stability in mean square and admissibility for stochastic differential equations in Hilbert spaces, and our approach is based on the extension of the techniques for ordinary differential equations which was developed in [6]. Some of the results for asymptotic behaviors in mean square of stochastic differential equations in Hilbert spaces was presented in the papers [7], [8].

2. Preliminaries

Let $(\Omega, F_t, \{Ft\}_{t\geq 0}, P)$ be a standard filtered probability space, U and H a real separable Hilbert spaces and L(U; H) be the set of all linear bounded operators from U into H, equipped with the usual operator norm $\|\bullet\|$ and let be the set $\Delta = \{(t,s) \in R_+^2 | t \geq s \geq 0\}$. Let Q be the self-adjoint nonnegative operator on space K and suppose that W(t), $t \geq 0$ is a U-valued, Q-Wiener process on a probability space with covariance operator Q, adapted to filtration $\{F_t\}_{t\geq 0}$. Let $U_0 = Q^{\frac{1}{2}}(U)$ be a Hilbert space and $L_2^0 = L_2(U_0; H)$ the Hilbert space of all Hilbert-Schmidt operators from U_0 into H (see [9]).

In this paper we consider the following infinite-dimensional stochastic differential equations

$$\begin{cases} dX(t) = [AX(t) + \alpha(t)]dt + BX(t)dW(t), & t > s \\ X(s) = \xi \end{cases} \tag{1}$$

where $A: D(A) \subset H \rightarrow H$ is the infinitesimal generator of a C_0-semigroup $S(t), t \geq 0$, $B: D(B) \subset H \rightarrow L_2^0$ a linear bounded operator and ξ is a H-valued random variable, F_s-measurable with $E\|\xi\| < \infty$ and α is H-valued stochastic predictable process with Bochner integrable trajectories.

The mild solution of equation (1) is a stochastic process X(t; s); $t \geq s$ of form

$$X(t,s) = S(t-s)\xi + \int_s^t S(t-\tau)\,\alpha(\tau)d\tau + \int_s^t S(t-\tau)\,BX(\tau,s)dW(\tau) \tag{2}$$

where $X(s,s) = \xi$ is the initial condition of equation (1), see [10].

In the next we assume that the linear homogeneous equation

$$\begin{cases} du(t) = Au(t)dt + Bu(t)dW(t), & t > s \\ u(s) = \xi \end{cases} \tag{3}$$

has a mild solution $u(t) = \Phi(t,s)\xi$, for all $(t,s) \in \Delta$ with the initial condition $u(t,s) = \Phi(s,s)\xi = \xi$, where Φ is called the stochastic evolution semigroup related to the equation (3), see [11]. In this hypothesis we have that

$$X(t,s) = \Phi(t,s)\xi + \int_s^t \Phi(t,\tau)\alpha(\tau)\,d\tau\,, \quad \forall (t,s) \in \Delta, \xi \in H \tag{4}$$

is a mild solution of equation (1). [12]

Definition 1. *The stochastic evolution semigroup $\Phi(t,s): \Delta \rightarrow L(H)$, is with exponential growth in mean square if there exist the constants $\gamma > 0$ and a measurable function $M: R_0^+ \rightarrow R_0^+$ with*

$$E\|\Phi(t,s)\xi\|^2 \leq M(s)e^{(t-s)}E\|\xi\|^2 \tag{5}$$

for all $((t,s) \in \Delta$ and for all H-valued random measurable ξ, F_s-measurable.

In the next we denote the new norm by:

$$\|\xi\|_t^2 = \sup\{E\|\Phi(\tau,t)\xi\|^2 e^{-\gamma(\tau-t)} : \tau \geq t\}, \quad \forall\, \xi \in H, \forall\, t \geq 0 \tag{6}$$

With respect to these norm we have the following properties for stochastic evolution semigroup.

Proposition 1. *If $\Phi(t,s): \Delta \rightarrow L(H)$ is the stochastic evolution semigroup related to the equation (3) then*

$$\|\Phi(t,s)\xi\|_t^2 \leq e^{\gamma(t-s)} \quad \|\xi\|_s^2\,, \quad \forall\, \xi \in H, \forall\, (t,s) \in \Delta \tag{7}$$

Proof. We have that

$$\begin{aligned} \|\Phi(t,s)\xi\|_t^2 &= \sup\{E\|\Phi(\tau,s)\xi\|^2 e^{-\gamma(\tau-t)} : \tau \geq t\} \\ &\leq e^{\gamma(t-s)}\sup\{E\|\Phi(\tau,s)\xi\|^2 e^{-\gamma(\tau-s)} : \tau \geq s\} = e^{\gamma(t-s)}\|\xi\|_s^2 \end{aligned} \tag{8}$$

■

We introduce the next Hilbert spaces:
$L^2 = \{\alpha - \text{stochastic process on } H, F_t - \text{measurable}: \|\alpha\|_2 < \infty\}$,
$L^\infty = \{\alpha - \text{stochastic process on } H, F_t - \text{measurable}: \|\alpha\|_\infty < \infty\}$
with the norms $\|\alpha\|_2 = \left(\int_0^\infty \|\alpha(\tau)\|_H^2\, d\tau\right)^{1/2}$, respectively $\|\alpha\|_\infty = \text{esssup}_{t\geq 0}\|\alpha(t)\|$.

For each Hilbert spaces we have the space

$$\mathrm{L}^2(\mathrm{H}) = \{\alpha - \text{stochastic process Bochner integrable on H}, : \mathrm{t} \to \|\alpha\|_{\mathrm{t}}^2 \in \mathrm{L}^2\}$$

with the norm $\|\alpha\|_2^2 = \|\Lambda\|_2$, where $\Lambda(\mathrm{t}) = \|\alpha(\mathrm{t})\|_{\mathrm{t}}^2$, and respectively for $\mathrm{L}^\infty(\mathrm{H})$ the norm $\|\alpha\|_\infty^2 = \mathrm{esssup}_{\mathrm{t}\geq 0}\|\alpha(\mathrm{t})\|_{\mathrm{t}}^2$.

3. Admissibility and nonuniform stability in mean square

In this section we give a characterization of nonuniform exponential stability in mean square in terms of admissibility of stochastic differential equation (1)

Definition 2. *We say that the pair $(L^2(H), L^\infty(H))$ is admissible for the stochastic differential equation (1) if for each $\alpha \in L^2(H)$ the stochastic process defined by*

$$x_\alpha(t) = \int_0^t \Phi(t,\tau)\alpha(\tau)\, d\tau \tag{9}$$

is in the space $L^\infty(H)$.

Definition 3. *We say that the stochastic differential equation (3) is nonuniformly exponentially stable in mean square if there exists a constant $\nu > 0$ and a measurable function $N: R_0^+ \to R_0^+$ such that*

$$E\|\Phi(t,s)\xi\|^2 \leq N(s)e^{-\nu(t-s)}E\|\xi\|^2 \tag{10}$$

for all $((t,s) \in \Delta$ and for all H-valued random measurable ξ, F_s-measurable.

Lemma 1. *If the pair $(L^2(H), L^\infty(H))$ is admissible for the stochastic differential equation (1) then there exists a constant $K > 0$ such that for every $\alpha(t) \in L^2(H)$ we have*

$$\|x_\alpha\|_\infty^2 \leq K\|\alpha\|_2^2\,. \tag{11}$$

Proof. Let $\mathrm{C}: \mathrm{L}^2(\mathrm{H}) \to \mathrm{L}^\infty(\mathrm{H})$ be the linear operator defined by $\mathrm{C}\alpha = \mathrm{x}_\alpha$ and to establish propriety (11) we use the closed graph theorem to show that C is a bounded operator.

Thus, let $\{\alpha_n\}_{n\in N}$ be a sequence of $\mathrm{L}^2(\mathrm{H})$, and so there exists a limit of this $\alpha \in \mathrm{L}^2(\mathrm{H})$, such that

$$\|\alpha_n(t) - \alpha(t)\|_2^2 \to 0\ ,\ n \to \infty, \forall t \geq 0.$$

Then there exists $y \in \mathrm{L}^\infty(\mathrm{H})$ such that $\left\|x_{\alpha_n}(t) - y(t)\right\|_\infty^2 \to 0, n \to \infty$. It follows that exists a subsequence $\{\alpha_{n_k}\}$ of $\{\alpha_n\}$ in $\mathrm{L}^2(\mathrm{H})$ such that $\left\|x_{\alpha_{n_k}}(t) - y(t)\right\|_\infty^2 \to 0, k \to \infty$. in the space $\mathrm{L}^\infty(\mathrm{H})$, for all $t \geq 0$. From

$$\|y(t){-}x_\alpha(t)\|_\infty^2 = \sup\{E\left\|\Phi(\tau,t)\big(y(t){-}x_\alpha(t)\big)\right\|^2 e^{-\gamma(\tau-t)} :\ \tau \geq t\}$$

$$\leq \sup\left\{2\left(E\left\|\Phi(\tau,t)\left(y(t){-}x_{\alpha_{n_k}}(t)\right)\right\|^2 + E\left\|\Phi(\tau,t)\left(x_{\alpha_{n_k}}(t){-}x_\alpha(t)\right)\right\|^2\right)e^{-\gamma(\tau-t)} :\ \tau \geq t\right\}$$

$$\leq 2\left\|y(t){-}\ x_{\alpha_{n_k}}(t)\right\|_t^2 + 2\sup\left\{E\left\|\int_0^t \Phi(\tau,s)\left(\alpha_{n_k}(s) - \alpha(s)\right)ds\right\|^2 e^{-\gamma(\tau-t)} :\ \tau \geq t\right\}$$

$$\leq 2\left\|y(t){-}\ x_{\alpha_{n_k}}(t)\right\|_t^2 + 2\sup\left\{\int_0^t E\left\|\Phi(\tau,s)\left(\alpha_{n_k}(s) - \alpha(s)\right)\right\|^2 e^{-\gamma(\tau-s)}\, e^{\gamma(t-s)}ds :\ \tau \geq t\right\}$$

$$\leq 2\left\|y(t){-}\ x_{\alpha_{n_k}}(t)\right\|_t^2 + 2\sup\left\{\int_0^t\left\|\alpha_{n_k}(s) - \alpha(s)\right\|_s^2 e^{\gamma(t-s)}ds :\ \tau \geq t\right\}$$

$$\leq 2\left\|y(t){-}\ x_{\alpha_{n_k}}(t)\right\|_t^2 + 2e^{\gamma t}\int_0^t\left\|\alpha_{n_k}(s) - \alpha(s)\right\|_s^2 ds$$

for $k \to \infty$ we obtain that $y(t) = x\alpha(t) = C\alpha(t)$ for all $t \geq 0$, and by the closed graph theorem we deduce that C is a bounded operator and so we obtain the relation (11). ■

The next theorem is our main results and presents a sufficient conditions, some "Perron type" conditions, for nonuniform exponential stability in mean square of stochastic differential equation (3) in Hilbert spaces.

Theorem 1. *Suppose that the stochastic evolution semigroup Φ related to the equation (3) has exponential growth in mean square. If the pair $(L^2(H), L^\infty(H))$ is admissible for the stochastic equation (1) then the equation (3) is nonuniformly exponentially stable in mean square.*

Proof. Let be $\xi \in H$ and $t_0 \geq 0$ and we define the stochastic process H-valued by:

$$\alpha(t) = \begin{cases} \Phi(t, t_0)\xi, & t \in [t_0, t_0 + 1] \\ 0, & \text{in rest} \end{cases} \tag{12}$$

Thus we have that

$$\|\alpha(t)\|_t^2 = \sup\{E\|\Phi(\tau, t)\alpha(t)\|^2 e^{-\gamma(\tau-t)} : \tau \geq t\}$$
$$\leq e^{\gamma(t-t_0)} \sup\{E\|\Phi(\tau, t_0)\xi\|^2 e^{-\gamma(\tau-t_0)} : \tau \geq t\} \aleph_{[t_0,t_0+1]} = e^{\gamma(t-t_0)} \|\xi\|_{t_0}^2 \aleph_{[t_0,t_0+1]}$$
$$\leq e^{\gamma} \|\xi\|_{t_0}^2 \aleph_{[t_0,t_0+1]}, \qquad \forall t \in [t_0, t_0 + 1], \forall\, \xi \in H.$$

That we observe that

$$\|\alpha\|_2^2 = \left(\int_0^\infty \|\alpha(\tau)\|_H^2\, d\tau\right)^{1/2} \leq e^{\frac{\gamma}{2}} \|\xi\|_{t_0}^2 \left\|\aleph_{[t_0,t_0+1]}\right\|_2 = e^{\frac{\gamma}{2}} \|\xi\|_{t_0}^2 \tag{13}$$

and thus result that $\alpha \in L^2(H)$. From definition of stochastic process x_α with α defined by relation (12), we obtain

$$x_\alpha(t) = \int_{t_0}^{t_0+1} \Phi(t, \tau)\Phi(\tau, t_0)\xi d\tau = \Phi(t, t_0)\xi, \quad \forall\, t \geq t_0 + 1, \text{and } \xi \in H.$$

From this we have that $\|\Phi(t, s)\xi\|_t^2 = \|x_\alpha(t)\|_t^2 \leq \|x_\alpha\|_\infty^2$. So, by Lemma 1 and relation (13) obtain

$$\|\Phi(t, t_0)\xi\|_t^2 \leq \|x_\alpha\|_\infty^2 \leq K\|\alpha\|_2^2 \leq Ke^{\frac{\gamma}{2}} \|\xi\|_{t_0}^2 \quad \forall\, t \geq t_0 + 1, \text{and } \xi \in H. \tag{14}$$

From Proposition 1 we have $\|\Phi(t, t_0)\xi\|_t^2 \leq e^\gamma \|\xi\|_{t_0}^2$, for all $t \in [t_0, t_0 + 1]$and $\xi \in H$, and if denote

$$|||\Phi(t, s)||| \coloneqq \sup_{\xi \neq 0} \sqrt{\frac{\|\Phi(t,s)\xi\|_t^2}{\|\xi\|_s^2}} \tag{15}$$

than from (14) obtain $|||\Phi(t, s)|||^2 \leq L$, for all $t \geq s$, $\xi \in H$, where $L = e^\gamma \max\{K, 1\}$. In the next we define the stochastic process of form

$$\beta(t) = \begin{cases} \Phi(t, t_0)\xi, & t \in [t_0, t_0 + \delta] \\ 0, & \text{in rest} \end{cases}, \delta > 0. \tag{16}$$

with the property $\|\beta(t)\|_t^2 \leq \|\Phi(t, t_0)\xi\|_t^2 \leq L\|\xi\|_{t_0}^2$, and thus result

$$\|\beta\|_2^2 \leq L\delta^{1/2} \|\xi\|_{t_0}^2 \quad \text{and} \quad \beta \in L^2(H). \tag{17}$$

For $t = t_0 + \delta$ we have

$$\boldsymbol{\frac{\delta^2}{2} \|\Phi(t, t_0)\xi\|_t^2 = \left\| \int_{t_0}^{t} (s - t_0)\Phi(t, t_0)\xi ds \right\|_t^2}$$
$$= \sup\left\{ E \left\| \Phi(\tau, t) \int_{t_0}^{t} (s - t_0)\Phi(t, t_0)\xi ds \right\|^2 e^{-\gamma(\tau-t)} : \tau \geq t \right\}$$
$$\leq \sup\left\{ \int_{t_0}^{t} (s - t_0)\, E\|\Phi(\tau, t_0)\xi\|^2 ds\, e^{-\gamma(\tau-t)} : \tau \geq t \right\}$$

$$= \int_{t_0}^{t} (s - t_0) \sup\{E\|\Phi(\tau,t)\Phi(t,t_0)\xi\|^2 \, e^{-\gamma(\tau-t)} : \tau \geq t\} ds$$
$$\leq \int_{t_0}^{t} (s - t_0) \|\Phi(t,t_0)\xi\|_t^2 ds = \int_{t_0}^{t} (s - t_0) \|\Phi(t,s)\Phi(s,t_0)\xi\|_t^2 ds.$$

Since, for $t \in [t_0, t_0 + \delta]$ and $\xi \in H$, the stochastic process β has the form $x_\beta = (t - t_0)\,\Phi(t,t_0)\xi$, and it results

$$\frac{\boldsymbol{\delta^2}}{\mathbf{2}} \|\boldsymbol{\Phi(t,t_0)\xi}\|_t^2 \leq \boldsymbol{L} \int_{t_0}^{t} (\boldsymbol{s - t_0}) \|(\boldsymbol{s,t_0})\boldsymbol{\xi}\|_t^2 \boldsymbol{ds} = \boldsymbol{L} \int_{t_0}^{t} \left\|\boldsymbol{x_\beta(s)}\right\|_t^2 \boldsymbol{ds}$$

Thus from Lemma 1 and from $|||\Phi(t,s)|||^2 \leq L$, we obtain

$$\frac{\boldsymbol{\delta^2}}{\mathbf{2}} \|\boldsymbol{\Phi(t,t_0)\xi}\|_t^2 \leq \boldsymbol{L\,\delta} \left\|\boldsymbol{x_\beta}\right\|_\infty^2 \leq \boldsymbol{KL\,\delta} \|\boldsymbol{\beta}\|_2^2 \leq \boldsymbol{KL^2\,\delta^{3/2}} \|\boldsymbol{\xi}\|_{t_0}^2$$

and so, it results

$$|||\boldsymbol{\Phi(t,t_0)}|||^2 \leq \boldsymbol{L}\ |||\boldsymbol{\Phi(t_0 + n\delta_0, t_0)}|||^2$$
$$\leq L\ |||\Phi(t_0 + n\delta_0, t_0 + (n-1)\delta_0)|||^2|||\Phi(t_0 + (n-1)\delta_0, t_0 + (n-2)\delta_0)|||^2 \ldots.$$
$$\ldots|||\Phi(t_0 + 2\delta_0, t_0 + \delta_0)|||^2|||\Phi(t_0 + \delta_0, t_0)|||^2 \leq L2^n K^n L^{2n} {\delta_0}^{-n/2}.$$

If we denote $D = 2KL^2{\delta_0}^{-1/2} < 1$ then

$$|||\Phi(t,t_0)|||^2 \leq m\, e^{-\mu(t-t_0)}\ , \qquad (18)$$

for positive constants $m = L/D$ and $\mu = -\frac{1}{\delta_0} lnD$. From the norm defined by (6) and by exponential growth in mean square of stochastic evolution semigroup Φ we have

$$\|\xi\|_{t_0}^2 \leq M(t_0)\, E\|\xi\|^2, \text{ for all } t_0 > 0, \xi \in H\ .$$

Since

$$E\|\Phi(t,t_0)\xi\|^2 \leq \|\Phi(t,t_0)\|_t^2$$

from the relation (18) we obtain

$$E\|\Phi(t,t_0)\|^2 = sup_{\xi\neq 0}\frac{E\|\Phi(t,t_0)\xi\|^2}{E\|\xi\|^2} \leq M(t_0) sup_{\xi\neq 0}\frac{\|\Phi(t,t_0)\xi\|_t^2}{\|\xi\|_t^2} \leq mM(t_0)\, e^{-\mu(t-t_0)}$$

for all $(t,t_0) \in \Delta, \xi \in H$, where $N(t_0) = mM(t_0)$ and thus the stochastic differential equation (3) is nonuniformly exponentially stable in mean square. ■

For the converse to Theorem 1, we consider the Hilbert space

$$L_N^2 = \{\alpha - stochastic\ process\ on\ H, F_t - measurable: \|\alpha\|_{2,N} < \infty\}$$

with the norm $\|\alpha\|_2 = \left(\int_0^\infty \|\alpha(\tau)\|_H^2 N^2(\tau)\, d\tau\right)^{1/2}$.

For each Hilbert spaces H, we set the space $L_N^2(H)$ with the norm $\|\alpha\|_{2,N}^2 = \|\Lambda\,\|_{2,N}$, where $\Lambda\,(t) = \|\alpha(t)\|_t^2$.

Theorem 2. *If the stochastic differential equation (3) is nonuniformly exponentially stable in mean square then the pair* $(L^2(H), L^\infty(H))$ *is admissible for the stochastic differential equation (1).*

Proof. Consider a stochastic process $\alpha \in L_N^2(H)$ and thus

$$\|x_\alpha(t)\|_t^2 = sup\left\{E\left\|\int_0^{t} \Phi(\tau,t)\Phi(t,s)\alpha(s)ds\right\|_t^{\mathbf{2}} e^{-\gamma(\tau-t)} :\ \tau \geq t\right\}$$
$$\leq sup\left\{\int_0^{\boldsymbol{t}} E\|\Phi(\tau,s)\|^2 E\|\alpha(s)\|^2 e^{-\gamma(\tau-t)} ds :\ \tau \geq t\right\}.$$

From the nonuniform exponential stability in mean square of stochastic differential equation (3) obtain

$$\|x_\alpha(t)\|_t^2 \le \sup\left\{\int_0^{\mathbf{t}} N(s)e^{-\mu(\tau-s)}E\|\alpha(s)\|^2 e^{-\gamma(\tau-t)}ds :\ \tau \ge t\right\}$$

$$\le \sup\left\{\int_0^{\mathbf{t}} N(s)e^{-\mu(t-s)}E\|\alpha(s)\|_{\mathbf{s}}^{\mathbf{2}}ds :\ \tau \ge t\right\}$$

$$\le \|\alpha(s)\|_{2,N}^{\mathbf{2}} \sup\left\{\left(\int_0^{\mathbf{t}} e^{-2\mu(t-s)}ds\right)^2 :\ \tau \ge t\right\} \le \left(\frac{1}{2\mu}\right)^{1/2} \|\alpha(s)\|_{2,N}^{\mathbf{2}}.$$

From this relation we obtain

$$\|x_\alpha(t)\|_\infty^2 = \sup_{t\ge 0}\|x_\alpha(t)\|_t^2 \le \sup_{t\ge 0}\left(\frac{1}{2\mu}\right)^{1/2} \|\alpha(t)\|_{2,N}^{\mathbf{2}} < \infty,$$

and so the pair $(L^2(H), L^\infty(H))$ is admissible for the stochastic differential equation (1). ■

4. Conclusion

Theorems 1and 2 present the relation between the notions of nonuniform exponential stability in mean square and admissibility for stochastic differential equations in Hilbert spaces. The demonstrations of these theorems are based on the extension of the techniques for ordinary differential equations.

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