

# Research of probability characteristics of current and voltage unbalance based on using graphs of load for the duration at the substation

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**Abstract.** The theoretical study and recommendations on the practical application of methods for calculating the probability of asymmetry in the four-wire networks of 0.38/0.22 kV were carried out through using the load curves in the duration at the substation. We cited a group of curves, which show the variation of coefficient of filling the graphs from the share of power consumers in the total load for the consumer substations with different installed capacity.

## 1. Introduction

Efficient using of electricity is determined by creating conditions of consumption, which provide the necessary quality of electricity, which influence additional loss in Network [1–4]. The relevance of the problem of improving quality and reducing additional losses is particularly increasing in rural power grids voltage of 0.38/0.22 kV. The asymmetry of currents and voltages in networks leads to increasing in units of load voltage deviation from acceptable value, reduction the life of agriculture sector's electric motor and other negative phenomena in networks and electricity receivers. Therefore, knowledge of asymmetry in the network value, allows you to specify the level of energy losses and take appropriate measures to reduce them [5–8].

Numerous studies on the analysis of modes of rural networks with voltage up to 0.38/0.22 kW [9–11] showed that the current asymmetry is caused by communal work load, the bulk of which are unevenly distributed in phases single-phase electro-receivers that have the random nature of inclusion [12–16]. There are two types of asymmetry: probabilistic and systematic or random. Systematic asymmetry conditioned by uneven load phase constant in time; probabilistic asymmetry corresponds to the load, which vary randomly over time [17], [18].

The aim of research is to estimate a probabilistic asymmetry in the distribution networks 0.38/0.22 kV on the basis of load charts for the duration at the substation.



## 2. Experiment

Usually in practice, curves of load probability distribution (graphs of density probability) are unknown. However, graphs of load expressed with capacity or currents (which is less common in the operation of transformer substations) can be known. There is a simple dependence between the graphs of density probability of load and graphs of load which are rebuilt in duration. If the equation of the load graph in duration is known, we can obtain the graph of density probability of load equation the and vice versa.

Let the load graph in the duration of are given by the equation

$$t = t(I). \quad (1)$$

The probability of the current  $I$  is:

$$P(I) = dt = -t'(I) dI. \quad (2)$$

On the other hand, considering the equation of the probability density graph

$$P = f(I) \quad (3)$$

we can see that

$$P(I) = f(I) dI. \quad (4)$$

Comparing (2) and (4), we obtain the ratio:

$$f(I) = -t'(I). \quad (5)$$

Expression (5) establishes a connection between the probability density graph and the load graph in duration.

Consider the method of solving the problem of probabilistic estimates of asymmetry in the three-phase four-wire networks. Let set distribution curves (graphs of density probability) of three independent random variables with their equations:

$$P_1 = f_1(\xi), P_2 = f_2(\eta), P_3 = f_3(\zeta). \quad (6)$$

Values  $\xi$ ,  $\eta$  and  $\zeta$  adopted with random variables change in intervals “n”:

$$a_1 \leq \xi \leq b_1, a_2 \leq \eta \leq b_2, a_3 \leq \zeta \leq b_3, \quad (7)$$

where  $f_1(\xi), f_2(\eta), f_3(\zeta)$  – continuous function.

Let us have some dependence (in the form of inequality) between values  $\xi$ ,  $\eta$  and  $\zeta$ :

$$F(\xi, \eta, \zeta) \leq M. \quad (8)$$

We have to find a probabilistic assessment of the implementation of this ratio:

$$P(F(\xi, \eta, \zeta) \leq M), \quad (9)$$

where  $M$  – positive or negative real constant.

According to [5] the expression (9) is written:

$$P(F(\xi, \eta, \zeta) \leq M) = \iiint_V f_1(\xi) f_2(\eta) f_3(\zeta) d\xi d\eta d\zeta \quad (10)$$

and the area of integration is the intersection of the cube, which is limited to planes

$$\xi = a_1, \xi = b_1, \eta = a_2, \eta = b_2, \zeta = a_3, \zeta = b_3, \quad (11)$$

with multiple points of space  $O\xi\eta\zeta$ , defined by inequality

$$F(\xi, \eta, \zeta) \leq M. \quad (12)$$

After entering the expression (10) instead of the probability density  $f_1(\xi), f_2(\eta), f_3(\zeta)$  of the function, describing the graphs load in duration  $t_1(I_1), t_2(I_2), t_3(I_3)$ , we obtain

$$P(F(I_1, I_2, I_3) \leq M) = \iiint t'_1(I_1) t'_2(I_2) t'_3(I_3) dI_1 dI_2 dI_3. \quad (13)$$

Thus, to solve this problem we need to know the equation graphs load in duration and function  $F(I_1, I_2, \dots)$ , that defines the area of integration. At the substation which are not serviced, load graphs can be obtained from special measurements but at the substation, which are serviced, they built regularly in indexes meters. As for the function  $F(I_1, I_2, \dots)$ , i.e. the ratio between the currents, it is determined by the conditions of the problem.

We move solve the problem of asymmetry probabilistic assessment provided by angular symmetry of currents and average values of phase equality of an average values of power factor.

Let's define by  $x, y, z$  ( $I_{\min} = x, I_{cp} = y, I_{\max} = z$ ) loads of three phases. Then, according to (2) write:

$$dt(x) = -t'(x) dx, \quad (14a)$$

$$dt(y) = -t'(y) dy, \quad (14b)$$

$$dt(z) = -t'(z) dz. \quad (14c)$$

If we consider these differentials as the probability of occurrence of the relevant loads, the duration of the simultaneous action of load  $x, y$  and  $z$  will be equal to:

$$dt(x) dt(y) dt(z) = -t'(x) t'(y) t'(z) dx dy dz. \quad (15)$$

Perform an assessment of unequal loads due to the relative difference of phase currents:

$$a = \frac{I_{cp} - I_{\min}}{I_{\max}} \quad b = \frac{I_{\max} - I_{cp}}{I_{\max}}, \quad (16)$$

where  $I_{\max}, I_{cp}, I_{\min}$  are respectively the valid values of currents of maximum, medium and minimum loaded phase.

The asymmetry, caused by phases of the load different on value, can be determined by the task of two values  $a$  and  $b$  by the formula:

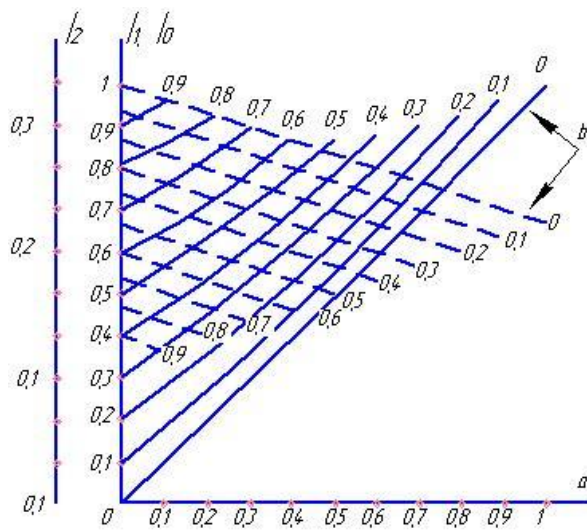
$$y = x + a, \quad (17a)$$

$$z = y + b. \quad (17b)$$

Figure 1 shows the graphs of dependence of currents symmetrical components in four-wire networks 0,38 / 0,22 kV of values  $a$  and  $b$ . Asymmetry duration which is above or equal to the value of  $a$  and  $b$ , is a function of these values; denote it  $S(a, b)$ ; it is a probability that the inequalities will be used.

$$y - x \geq a \text{ and } z - y \geq b, \quad (18)$$

$$S(a, b) = P\{y - x \geq a; z - y \geq b\}. \quad (19)$$



**Figure 1.** Graphs of currents symmetrical components dependence in four- network from the values of  $a$  and  $b$

According to the adding probability theorem [19], [20], we get this value by a triple integration (13) in region  $V$ , which is caused by inequality:

$$0 \leq x \leq 1, \quad 0 \leq y \leq 1, \quad 0 \leq z \leq 1, \quad y - x \geq a, \quad z - y \geq b, \quad (20)$$

$$S(a, b) = \iiint_V t'(x) t'(y) t'(z) dx dy dz. \quad (21)$$

Taking into account (20), the expression (21) solves the problem.

Apart the limits of integration, we get finally:

$$P\{y - x \geq a; z - y \geq b\} = \int_0^{1-a-b} t'(x) dx \int_{x+a}^{1-b} t'(y) dy \int_{y+b}^1 t'(z) dz. \quad (22)$$

When performing the calculations the condition must be observed:

$$a + b \leq 1. \quad (23)$$

Three wire branch can be considered as a special case when, for example,  $z=0$ . It is necessary to implement the conditions:

$$S(a) = P(y - x \geq a). \quad (24)$$

That is, to solve the problem we need to take a double integral over the region  $D$ , which caused in the  $xoy$  plane with the following conditions  $0 \leq x \leq 1, 0 \leq y \leq 1, x - y \geq a$ :

$$S(a) = \iint_D t'(x) t'(y) dx dy. \quad (25)$$

Apart limits of integration, we obtain:

$$S(a) = 2 \int_a^1 dx \int_0^{x-a} t'(x) t'(y) dy. \quad (26)$$

Equation (26) solves this problem.

### 3. Results and discussion

Comparative analysis of approximation methods proposed by various authors [20–23] showed that for solving this problem the most appropriate is an approximation function of the form:

$$t = (1 - i)^n, \quad (27)$$

where

$$n = \frac{1 - k_3}{k_3}, \quad (28)$$

$k_3$  – duty cycle of the graph

Calculations show that the error during charts approximation of community consumers using the expression (27, 28) does not exceed 5%.

In cases where  $n = 1, 2, 3 \dots, r, r + 1$ , the probability integral (20) is calculated exactly; if  $n$  is a fractional value the expression integrates approximately.

A fractionally-linear function can also be offered as an approximation. The formula is:

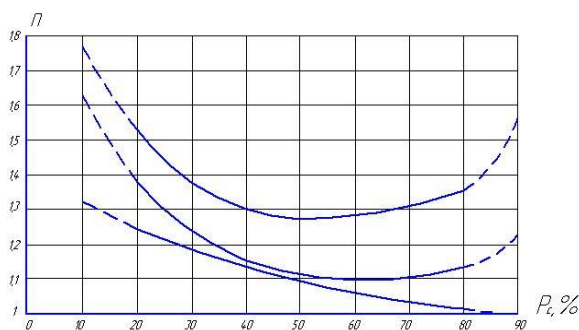
$$i = \frac{d + et}{q + t}. \quad (29)$$

With the proper selection of  $d$  and  $e$  coefficients, the function (26) satisfies the basic requirements:

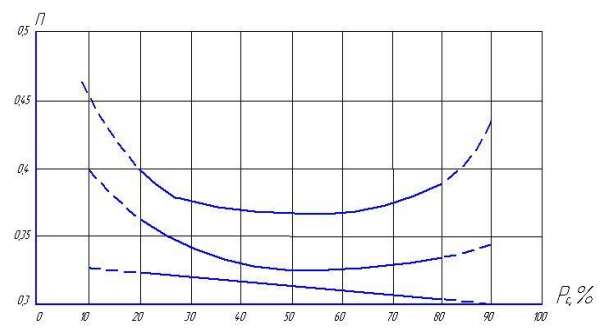
- 1) it is decreasing;
- 2) at  $t = 0, i = 1$ ;
- 3) at  $t = 1, i = i_{min}$ ;
- 4) the area of curvilinear trapezoid, defined by the equation  $i = f(t)$ , is equal to the amount of consumed electricity.

Figure 2 shows a family of curves [23] showing the change in graphs fill factor from the share of power consumers in the overall composition of the load  $k_3 = f(P_c\%)$  for consumer substations with installed capacity of 0 - 50 kW; 50 - 100 kW and 100 - 300 kW.

Basing on these curves, we calculated dependence of the degree of approximation function (25) for all types of load demand consumer substations 10 / 0.4 kV. The curves  $n = f(P_c\%)$  are shown in Figure 3.



**Figure 2.** The coefficient of filling graphs variation from the share of power consumers in the overall composition of the load  $k_3 = f(P_c\%)$



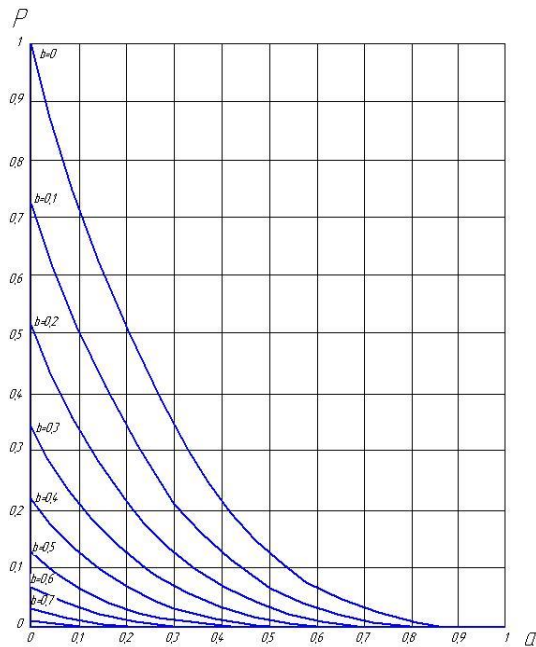
**Figure 3.** The dependence of the degree of approximation function for all kinds of load graphs of consumer substations 10/0.4 kV  $n = f(P_c\%)$

The analysis shows that virtually all real cases where the composition includes load power consumers, the exponent is within  $1 \leq n \leq 2$ .

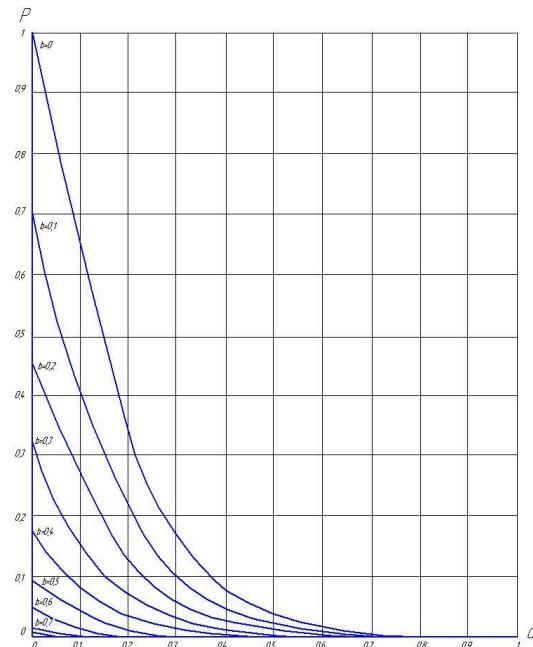
In cases where there is no power load ( $P_c = 0$ ), the exponent increases and for the typical graph of  $P_c = 0$  is close to 3.

In this way, we can assume that for the full range of the load  $0\% \leq P_c \leq 100\%$  will be  $3 \geq n \geq 1$ .

Based on the established boundaries change of  $n$ , the calculated values of the integral probability and constructed curves  $P\{y - x \geq a; z - y \geq b\} = S(a, b)$  for  $n = 1, 2, 3$ , shown in Figure 4 - Figure 6.



**Figure 4.** The value of integral probability for  $n = 1$



**Figure 5.** The value of integral probability for  $n = 2$

The graphs duty cycle of certain types of load are within 0.60 - 0.85. Such loads include, for example, farms with quadruple milking, poultry farms, incubators and others. In addition, the substations (feeders) feed singlephase domestic loads and coverages, distributed in phases.

Based on the above, it is obvious that the approximation of the graphs of such cumulative loads can be performed using out function  $n \leq 0.5$ .

Calculating the probability integral to four-wire networks with  $n = 0.5$  associated with known difficulties, and can be done only approximately.

Checking the probability integral conducted on the statistical analysis results of numerous direct measurements of phase currents on the findings of 0.38 kV transformer of consumer substations 10/0.4 kV.

Statistical curves were built based on processing of the measurement

$$P\{y - x \geq a; z - y \geq b\} = S(a, b), \quad (30)$$

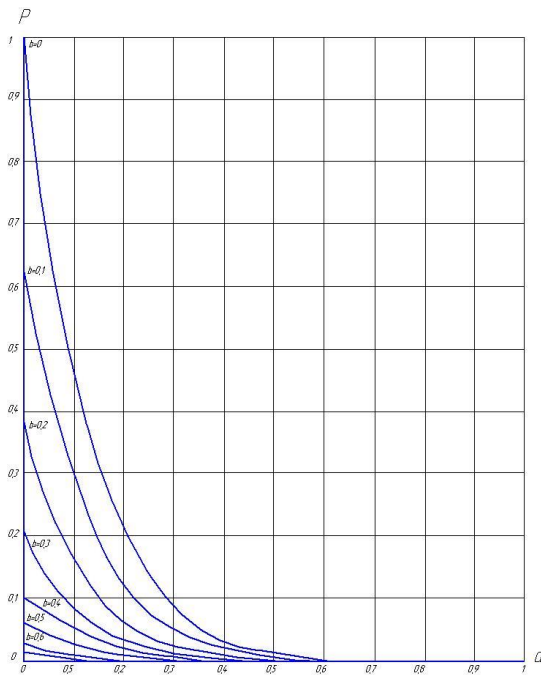
for  $b = 0$  and  $b = 0.2$  shown in Fig. 7. Probability curves (dotted line) for the same values for  $n = 1, 2$ , and 3 are also displayed.

As we can see, the character of statistical curves is quite close to the theoretical correlations.

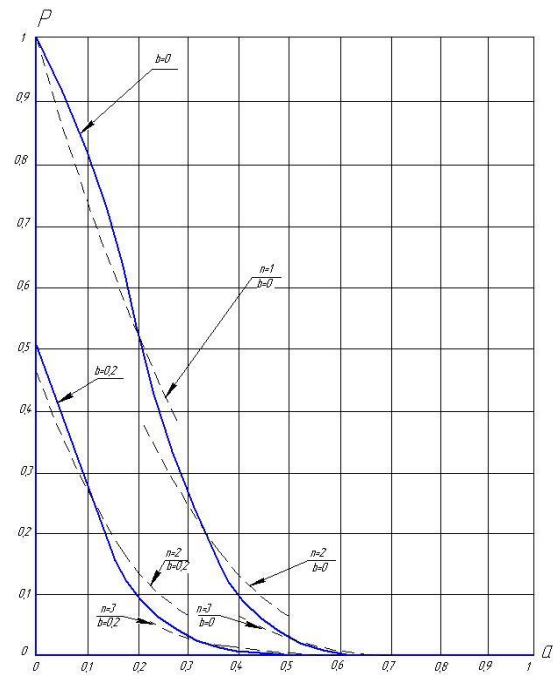
Statistical curves divided conditionally into parts, within which their approach to the probability can be considered as satisfactory.

Participation of power consumers in the total load substations, which conducted measurements, varied; that is why the duty cycles of the graph also were not the same.

Different components of load at tested substations has led to, for example, statistical curves for  $b = 0$  at the top approaching to theoretical for  $b = 0$  when  $n = 1$ , at the medium - to  $n = 2$ , and the bottom - to  $n = 3$ .



**Figure 6.** The value of integral probability for  $n = 3$



**Figure 7.** Statistical curves

Statistical analysis of mass measurements data in networks of 0.38/0.22 kV shows the distribution of currents in phases obeys the normal law. Accordingly, we can propose a practical calculation method of integrated probability of unbalance currents in full-phase elements and parts of the four-wire networks, as well as three wire networks (two-phase and zero) which are branches of the four-wires networks.

In general, the integral of probability can be written as:

$$P = \frac{1}{(2\pi)^{\frac{3}{2}} \prod_{i=1}^3 (\sigma_i)} \iiint_V e^{-\frac{1}{2} \left( \frac{I_a^2}{\sigma_1} + \frac{I_b^2}{\sigma_2} + \frac{I_c^2}{\sigma_3} \right)} di_a di_b di_c, \quad (31)$$

Solutions of the integral (28) present certain difficulties. Moving to spherical coordinates let to determine the limits of integration and change and get a decision after some assumptions.

Thus, considering

$$I_a = \rho \sin \Theta \cos \varphi, I_b = \rho \sin \Theta \sin \varphi, I_c = \rho \cos \Theta, \quad (32)$$

we get

$$P = \frac{1}{(2\pi)^{\frac{3}{2}} \prod_{i=1}^3 (\sigma_i)} \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} e^{-a\rho^2} \rho^2 \sin \Theta d\rho d\varphi d\Theta, \quad (33)$$

where

$$a = \frac{1}{2} \left( \frac{\cos^2 \varphi \sin^2 \Theta}{\sigma_1} + \frac{\sin^2 \varphi \sin^2 \Theta}{\sigma_2} + \frac{\cos^2 \Theta}{\sigma_3} \right), \quad (34a)$$

$$\delta = \sqrt{1 - \sin \Theta (\sin \Theta \sin \varphi \cos \varphi + \cos \Theta \sin \varphi + \cos \Theta \cos \varphi)}, \quad (34b)$$

$$M = I_a^2 + I_b^2 + I_c^2 - I_a I_b - I_a I_c - I_b I_c, \text{ that is } M = I_{\text{н}} = 3I_0. \quad (34c)$$

Accordingly

$$0 \leq I_{\text{н}} \leq 1. \quad (35)$$

Solutions of the integral can be obtained in the form of

$$P = \frac{M^3}{12\sqrt{2\pi} \prod_1^3 (\sigma_i)}. \quad (36)$$

The error in the result during the integration based on assumptions is

$$\Delta < \frac{a\rho^5}{5} = \frac{a}{5} \left( \frac{M}{\sigma} \right)^5. \quad (37)$$

Accordingly, for the three wire branches

$$P = \frac{M^2}{2\sqrt{3} \prod_1^2 (\sigma_i)}. \quad (38)$$

#### 4. Conclusions

We propose a calculations methodology of the currents and voltages at asymmetrical four-wire networks 0.38/0.22 kV; the information base of the methodology is using of load graphs; it let to determine the probability characteristics of a random network asymmetry and refine the calculated value of additional losses.

The problem of determination of unbalanced possible outcomes on currents corner symmetry condition and equality of phases of power factors average values is solved. The graphs show dependencies of currents symmetrical components in four-wire networks of 0.38/0.22 kV, which showed that the error under graphs approximation of household consumers does not exceed 5%.

The dependence of the exponents approximating function for all kinds of load profiles of consumer transformer substations of 10/0.4 kV are defined. The verification of the integral probability completed. It based on the statistical processing of the results of numerous direct measurements of the phase currents on the transformer bushings consumer substations of 10/0.4 kV. A statistical analysis of measurement data in the rural distribution network 0.38/0.22 kV completed. It showed that the distribution of the currents in the phases obeys the normal law.

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