Exact solution for the severity of transverse cracks in prismatic beams

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Abstract. We propose herein a mathematical relation to calculate the severity of transverse cracks which affect a prismatic beam. It is known that the damage severity depends on the depth of the crack located at the slice on which the biggest bending moment is achieved. The frequency drop due to this crack is proportional to the severity associated with it. For all other crack locations, the effect is diminished in relation to the curvature registered at the affected slice. It is essential to obtain a relationship to correctly express the severity with respect to the crack depth. To find it, we designed a model of a cantilever beam with cracks located near the fixed end. We observed that the deflection of the beam's free end increases the closer the crack to the fixed end is, but after a certain limit, the deflection decreases. We concluded that the deflection decrease occurs because the deformation around the crack is restricted by the fixing condition. To find the true severity, we derived the pseudo-severity for six crack positions and estimate the severity using the linear and second-order polynomial regression curves. Next, we used twelve points for interpolation and found a very similar severity. Finally, we performed modal analysis for the beam with cracks at different positions and found the mathematical relation developed to predict frequency changes that involves the severity provides accurate results.

1. Introduction

Due to geometrical discontinuities produced by cracks the structure's global stiffness is modified and the mass can change too. Due to these changes, the modal parameters as the natural frequencies, damping ratios, modal shapes, and modal curvatures are altered. The main effect on the modal parameters is produced by the stiffness loss because it diminishes the capacity of the structure to store energy. This effect directly depends on the crack position [1-3], its shape [4], [5] and dimensions [6-9]. It also depends on the number and type of fastening systems [10]. The literature contains many works that formalize the link between the characteristics of the crack and the modal parameter changes. In some approaches the damage severity is found by involving fracture mechanics theory, while others are based on energy methods. The theory was also developed for changing environment [11-13], variable beam cross-section [14] and multiple-cracked structures [15-17].

In previous research, we developed robust techniques for assessing cracks in beams [18] and plates [19], regardless of the structure's boundary conditions. All these techniques are based on the natural frequency shifts and evaluate the severity by energetic methods.

A problem we face when we evaluate the damage severity is that the frequency drop for the transverse crack located at a fixed end of a beam is smaller than the drop if the crack is located at a certain distance from this end. This is in disagreement with the relationship we developed for the frequency shift curves [20], which represent the frequency of the damaged beam versus the crack position. In addition, for cracks that have a longitudinal extent [5], it is impossible to estimate the frequency that should be obtained if the crack is located at the fixed end.

We investigate in this study, by involving simulation performed using the finite element method (FEM), how the frequency shift of beam-like structures can be estimated for the cracks located at the fixed end. These values should concord with those obtained from the theory.

2. A mathematical relation to calculate the damage severity from an energy method

A beam with a crack has usually a model consisting of two segments linked by a massless torsion spring [21], as shown in Figure 1. In this case, resolving the equation of motion for each segment, the solution in terms of displacement results in two equations that are:

$$\phi_1(x_1) = C_1 \sin(\alpha x_1) + C_2 \cos(\alpha x_1) + C_3 \sinh(\alpha x_1) + C_4 \cosh(\alpha x_1) \tag{1}$$

$$\phi_2(x_2) = C_5 \sin(\alpha x_2) + C_6 \cos(\alpha x_2) + C_7 \sinh(\alpha x_2) + C_8 \cosh(\alpha x_2) \tag{2}$$

where $C_1 \dots C_8$ are eight coefficients and α is the dimensionless wavenumber to be derived.

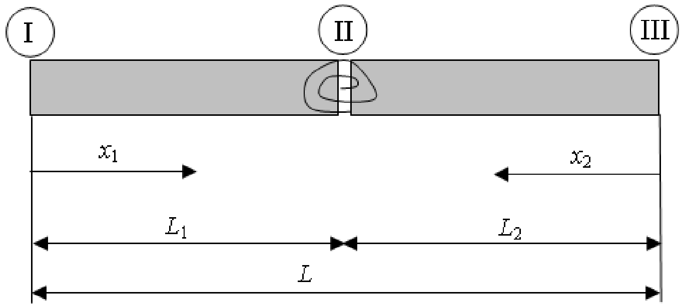

Figure 1. Model of a cracked beam

Imposing four end conditions (two for each of the beam ends I and III) in respect to the support types and four continuity conditions applied for the joint between the two segments (point II), one obtain a system with eight equation and nine unknowns. To properly set the continuity conditions, it is necessary to correctly estimate the spring stiffness. This is usually made by means of the fracture mechanics approach, from the numerous empirically deduced relations being available in the literature see for instance [7-9], [21]. By eliminating the coefficients $C_1 \dots C_8$ from the system results a transcendental equation in α, with infinite solutions that correspond with the vibration modes $i = 1 \dots \infty$. Denoting $\lambda_1 = \alpha L_1$ and $\lambda_2 = \alpha L_2$ we obtain the eigenvalues for the two segments used to derive the natural frequencies and mode shapes of the beam. The frequency f_{iD} of the i-th mode is:

$$f_{iD} = \frac{\lambda_{1i}^2}{2\pi L_{1i}^2} \sqrt{\frac{EI}{\rho A}} = \frac{\lambda_{2i}^2}{2\pi L_{2i}^2} \sqrt{\frac{EI}{\rho A}} \tag{3}$$

where E is the Young's modulus, ρ is the mass density, A is the cross-sectional area and I is the moment of inertia of the healthy beam, respectively.

This approach is well-known and largely applied to estimate the frequencies of a damaged structure. However, it requests extensive time and computational resources since numerous experiments are necessary to find the spring stiffness for different crack depths and afterward solving the transcendental equation for different crack positions. For three or more cracks the difficulty increases linearly with the number of cracks.

A simpler approach is proposed by the authors [22]. We found a relation that expresses the natural frequencies of the damaged beam f_{iD} according to the frequency of the intact beam f_{iU} and the position and severity of the crack, as:

$$f_{iD}(x,a)=f_{iU}\left\{1-\gamma(0,a)\left[\bar{\phi}_i''(x)\right]^2\right\} \tag{4}$$

where $\bar{\phi}_i''(x)$ is the normalized modal curvature at distance x from the fixed end that obviously take values between -1 and 1, and $\gamma(0,a)$ is the damage severity due to a crack of depth a.

It was shown in [4] that the damage severity is in direct relation with the energy loosed by the beam due to the crack. For a cantilever beam, the severity $\gamma(0,a)$ is calculated for the damage located at the fixed end $x=0$, where the crack produces the highest effect (i.e. frequency drop). Since the energy los can be found from the free end deflection [2], the severity becomes:

$$\gamma(0,a)=\frac{\sqrt{\delta_{D\max}(0,a)}-\sqrt{\delta_{U\max}}}{\sqrt{\delta_{D\max}(0,a)}} \tag{5}$$

where δ_U is the deflection at the free end of the intact beam and $\delta_D(0,a)$ is the deflection at the free end of the cantilever beam with a damage of depth a located at the fixed end. Similar to the torsion spring stiffness, the severity $\gamma(0,a)$ is the same for any boundary conditions imposed to the beam [22]. For other boundary conditions as the cantilever beam has, the location of the crack must be taken at the slice where the bending moment achieves the biggest value. For example, the simply supported beam requires the crack positioned at the middle of the beam.

Returning to the cantilever beam, we also noticed that a crack located elsewhere as the fixed end has a diminished effect and will produce a lower frequency drop. This fact is taken into account in relation (4) by using the term that is the square of the normalized modal curvature (SNMC). We can make the substitution:

$$\gamma(x,a)=\gamma(0,a)\left[\bar{\phi}_i''(x)\right]^2 \tag{6}$$

in which case we speak about the pseudo-severity. From prior research [23] we found that the pseudo-severity can be found from the deflections of the beam's free end in damaged and healthy state, as:

$$\gamma(x,a)=\frac{\sqrt{\delta_{D\max}(x,a)}-\sqrt{\delta_{U\max}}}{\sqrt{\delta_{D\max}(x,a)}}=\frac{\sqrt{\delta_{D\max}(0,a)}-\sqrt{\delta_{U\max}}}{\sqrt{\delta_{D\max}(0,a)}}\left[\bar{\phi}_i''(x)\right]^2 \tag{7}$$

if the crack is located at distance x from the fixed end. The relations for the severity and the pseudo-severity being obtained after formalizing physical phenomena and not using data achieved stochastically from experiments, conduct to precise results. Because of their simplicity and precise results ensured, the use of these relationships has a clear advantage over the approach that involves fracture mechanics. Therefore, the relative frequency shift (RFS) for a beam with known depth and position can be easily calculated with the mathematical relation:

$$\Delta\bar{f}_{iD}(x,a)=\frac{f_{iU}-f_{iD}(x,a)}{f_{iU}}=\frac{\sqrt{\delta_D(0,a)}-\sqrt{\delta_U}}{\sqrt{\delta_D(0,a)}}\left[\bar{\phi}_i''(x,a)\right]^2=\gamma(0,a)\left[\bar{\phi}_i''(x,a)\right]^2 \tag{8}$$

Since the damage severity is not dependent on the boundary conditions, relation (8) can be applied for all kind of the beam's end supports by simply choosing the right SNMC.

3. Deriving the exact damage severity from several pseudo-severities

Tests made during prior research have shown that the severity calculated for the crack at the fixed end of the cantilever beam under-evaluates a little bit the frequency drops. This was found to happen because the beam can deform just on one side of the crack, the other side of the beam being fixed. The behavior of the beam under dead mass is shown in the figure below, which illustrates the deflection in the vertical direction of the beam with the crack at the fixed end (Figure 2.a) and near the fixed end (Figure 2.b), respectively. One can observe that the deflection is smaller if the crack is located in the fixture, depicted in Figure 2.a.

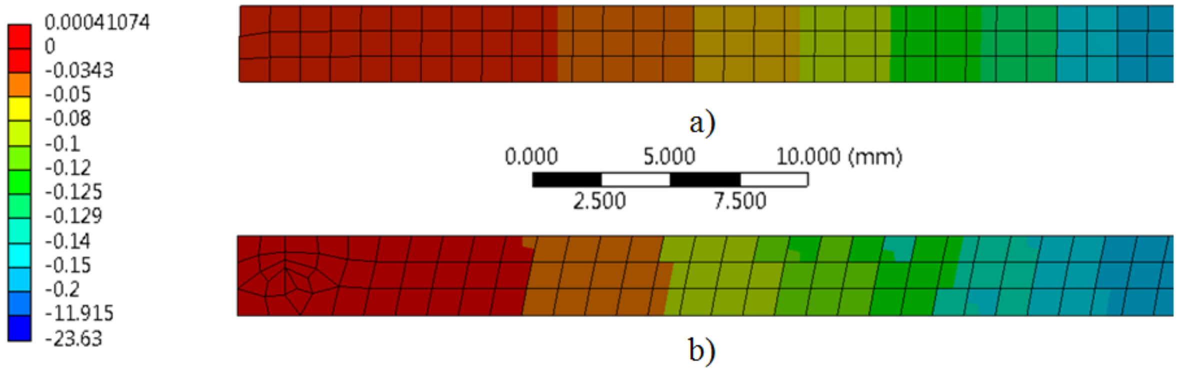

Figure 2. Deflection of the beam's segment near the fixed end, if the crack is located: a) at the fixed end; b) in the vicinity of the fixed end

When extracting the vertical displacement produced due to dead mass, we found that the beam deflects stronger if the crack is not located exactly at the fixed end, as shown in Table 1. The locations where pointers are positioned to indicate the deflections are at the upper beam face at distances 6, 18, 30 and 36 mm from the fixed end (fixture). The crack location, if not at the fixed end, is x=12 mm.

Table 1. Vertical displacements for the beam with two damage cases

Analysis case	Deflection (mm)			
	Point A (6 mm)	Point B (18 mm)	Point C (30 mm)	Point D (36 mm)
Healthy beam	0.0018755	0.015055	0.040329	0.057456
Crack at the fixed end	0.0019633	0.020344	0.049073	0.067942
Crack at 12 mm from the fixture	0.0018802	0.019074	0.052334	0.073454

The study presented in this paper to find the damage severity is carried out on a cantilever beam with length L=1000 mm, width w=50 mm and a thickness h=5 mm. Both the beam and crack geometries were modeled using the computer-aided design software SolidWorks. The analysis is carried out using the engineering simulation software ANSYS. In order to obtain accurate results, for this study, we have defined a hexahedral mesh with the maximum element edge of 2 mm. The physical-mechanical properties for the structural steel assigned for the cantilever are extracted from the ANSYS Workbench library and are: Yield strength 250 MPa; Ultimate strength 460 MPa; Mass density ρ 7850 kg/mm^3; Young modulus E $2{\bullet}10^{11}$ N/m^2 and Poisson ratio ν 0.3.

The damage considered here is a breathing crack, which has depth d=2 mm. A schematic of the beam, with a detailed view on the fixed end, and the crack positions is presented in Figure 2, the exact position of the cracks being specified in Table 2. Note that the step between two consecutive cracks is s=3 mm until the distance x_{12}=36 mm is achieved.

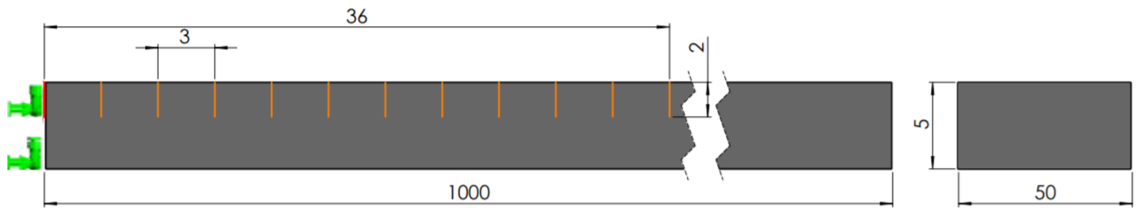

Figure 3. The position of the transverse cracks considered one-by-one in the FEM analysis

The aim of this study is to find the correct severity by extending the regression curve plotted for the points indicated in Table 2. To this aim, we performed simulations by employing the *static analysis module* and found the deflections for all crack locations.

Table 2. The location of the transverse cracks in the near-field of the fixed end and the resulted deflections

Parameter	Crack 1	Crack 2	Crack 3	Crack 4	Crack 5	Crack 6
Distance x_k (mm)	3	6	9	12	15	18
Deflection δ_k (mm)	23.63	23.62	23.615	23.608	23.606	23.593

Table 3. The location of the transverse cracks in the far-field of the fixed end and the resulted deflections

Parameter	Crack 7	Crack 8	Crack 9	Crack 10	Crack 11	Crack 12
Distance x_k (mm)	21	24	27	30	33	36
Deflection δ_k (mm)	23.587	23.581	23.581	23.565	23.571	23.557

With the results presented in Tables 2 and 3 we calculate the mathematical expression for the regression curve in two cases: linear and second order polynomial. The results are depicted in Figure 4 for the deflections due to the crack in the near-field of the fixed end, respectively in Figure 5 for an extended domain. In these figures we also present the R-squared value, which indicates the close the estimated values are to the real one.

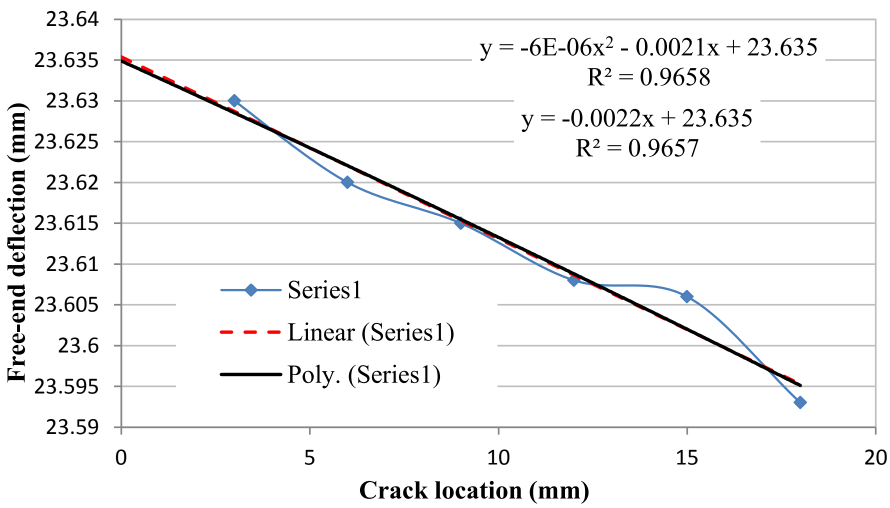

Figure 4. The extended regression curves plotted for the deflections of the damaged beam considering the first six cracks

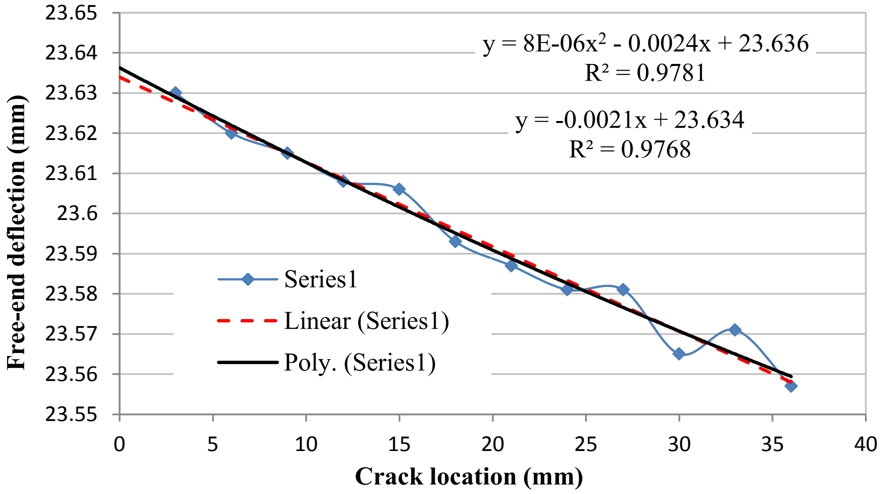

Figure 5. The extended regression curves plotted for the deflections of the damaged beam considering twelve cracks

One can observe from Figures 4 and 5 that the values for the free-end deflection estimated from the regression curves, regardless of the curve type (linear or second-order polynomial) and number of points used, are very close to 23.635 mm.

Similar evolution of the deflection with the crack position was achieved, in prior research [23], for a crack with depth 1 mm. We can conclude that, to accurately find the damage severity of a transverse crack, it is enough to involve 6 cracks equidistantly distributed near the fixed end, with the distance between them 1/30 times the beam length.

4. Numerical validation

We prove in this section that the severity calculated by using the procedure described above can be successfully used to estimate the frequencies of beams with cracks, irrespective to the boundary conditions. Hence, we perform modal analysis for several crack location on the beam with different supports and found the natural frequencies. The results are compared with those obtained by involving relation (4), where the severity is calculated with relation (5) first with the deflection obtained directly from simulation and afterward for the severity calculated from the regression curve.

To this aim, we first find from FEM simulation the free-end deflection for the healthy beam, respectively the deflections for the crack at the fixed end and at several locations it its vicinity. From the latter we estimate the deflection by involving the regression curve. All results, along with the calculated severities are given in Table 4.

Table 4. The severity calculated directly from simulation and involving the regression curve

Analysis case	Deflection (mm)	Severity (-)
Healthy beam	22.948	-
Values for the crack at the fixed end	23.243	0.006366
Values from the regression curve	23.635	0.014641

The values of the free-end deflections under dead mass for the healthy beam and the beam with a crack at the fixed end are found from FEM simulation in a similar way and beam configuration as all other values presented in the above tables. One can observe the big discrepancy between the severity γ^{FEM} found from simulation results directly and the severity γ^{REG} estimated using the regression curve.

4.1. *Cantilever beam*

The first tests are made for the cantilever beam, for which the severity was derived. The locations where randomly selected, and concern the distances 160, 250 and 570 mm from the left (fixed) end. The first 4 natural frequencies are calculated with relation (4) both for γ^{FEM} and γ^{REG} and the results compared with these directly obtained from simulation. The results are presented in Tables 5 to 7.

Table 5. Frequencies calculated for the beam with fixed-free end conditions considering the severities derived in two ways and directly from FEM for the crack located at 160 mm from the fixed end

Mode no.	SNMC (-)	Severity (-) γ^{FEM}	Severity (-) γ^{REG}	Frequency from calculus (Hz) Using γ^{FEM}	Frequency from calculus (Hz) Using γ^{REG}	Frequency from simulation (Hz)
1	0.6136034	0.006366	0.014641	4.0741	4.0535	4.0538
2	0.0624528	0.006366	0.014641	25.6165	25.6043	25.604
3	0.0331268	0.006366	0.014641	71.7367	71.7143	71.727
4	0.2660330	0.006366	0.014641	140.3793	140.0566	140.14

Table 6. Frequencies calculated for the beam with fixed-free end conditions considering the severities derived in two ways and directly from FEM for the crack located at 250 mm from the fixed end

Mode no.	SNMC (-)	Severity (-) γ^{FEM}	Severity (-) γ^{REG}	Frequency from calculus (Hz) Using γ^{FEM}	Frequency from calculus (Hz) Using γ^{REG}	Frequency from simulation (Hz)
1	0.4362515	0.006366	0.014641	4.0787	4.0641	4.0641
2	0.0187494	0.006366	0.014641	25.6229	25.6181	25.621
3	0.3545076	0.006366	0.014641	71.5921	71.3748	71.409
4	0.4130015	0.006366	0.014641	140.2578	139.792	139.85

Table 7. Frequencies calculated for the beam with fixed-free end conditions considering the severities derived in two ways and directly from FEM for the crack located at 570 mm from the fixed end

Mode no.	SNMC (-)	Severity (-) γ^{FEM}	Severity (-) γ^{REG}	Frequency from calculus (Hz) Using γ^{FEM}	Frequency from calculus (Hz) Using γ^{REG}	Frequency from simulation (Hz)
1	0.068848	0.006366	0.014641	4.0883	4.0860	4.0859
2	0.515644	0.006366	0.014641	25.5423	25.4336	25.442
3	0.1610203	0.006366	0.014641	71.6749	71.5722	71.601
4	0.2721607	0.006366	0.014641	140.3998	140.1037	140.11

As it can be observed from Tables 5 to 7, the frequencies calculated using the severity γ^{REG} lead to much better results as these calculated with the severity γ^{FEM} if the frequencies obtained by simulation are taken as a reference. This shows that the use of the severity found from the regression curves should be considered in calculus. Else, an under-estimated frequency shift results.

4.2. *Beam fixed at both ends*

Similar tests as in the previous sub-section are made for the beam fixed at both ends. Here, the severities are the same but the SNMC differs. The results are presented in the Tables 8 to 10.

Table 8. Frequencies calculated for the beam with fixed-fixed end conditions considering the severities derived in two ways and directly from FEM for the crack located at 160 mm from one end

Mode no.	SNMC (-)	Severity (-)		Frequency from calculus (Hz)		Frequency from simulation (Hz)
		γ^{FEM}	γ^{REG}	Using γ^{FEM}	Using γ^{REG}	
1	0.071864	0.006366	0.014641	26.087	26.0715	26.071
2	0.032262	0.006366	0.014641	71.912	71.893	71.896
3	0.248691	0.006366	0.014641	140.767	140.4767	140.5
4	0.424972	0.006366	0.014641	232.439	231.62	231.67

Table 9. Frequencies calculated for the beam with fixed-fixed end conditions considering the severities derived in two ways and directly from FEM for the crack located at 250 mm from one end

Mode no.	SNMC (-)	Severity (-)		Frequency from calculus (Hz)		Frequency from simulation (Hz)
		γ^{FEM}	γ^{REG}	Using γ^{FEM}	Using γ^{REG}	
1	0.009840	0.006366	0.014641	26.0974	26.0952	26.096
2	0.341930	0.006366	0.014641	71.7704	71.5669	71.575
3	0.385819	0.006366	0.014641	140.6437	140.1936	140.22
4	0.065547	0.006366	0.014641	232.9727	232.8463	232.84

Table 10. Frequencies calculated for the beam with fixed-fixed end conditions considering the severities derived in two ways and directly from FEM for the crack located at 570 mm from one end

Mode no.	SNMC (-)	Severity (-)		Frequency from calculus (Hz)		Frequency from simulation (Hz)
		γ^{FEM}	γ^{REG}	Using γ^{FEM}	Using γ^{REG}	
1	0.318771	0.006366	0.014641	26.0460	25.9771	25.981
2	0.128292	0.006366	0.014641	71.8683	71.7919	71.794
3	0.263325	0.006366	0.014641	140.7536	140.4465	140.46
4	0.350455	0.006366	0.014641	232.55	231.8741	231.92

One can observe that the severity estimated for the cantilever beam applies in this case too. Again, the severity γ^{REG} lead to much better results as the severity γ^{FEM}.

4.3. Beam free at the two ends

A last test is made for the beam with free ends. Again, the severities are these derived for the cantilever beam, but the SNMC is selected for the free-free ends case. The achieved results are presented in Tables 11 to 13.

Table 11. Frequencies calculated for the beam with free-free end conditions considering severities derived in two ways and directly from FEM for the crack located at 160 mm from one end

Mode no.	SNMC (-)	Severity (-)		Frequency from calculus (Hz)		Frequency from simulation (Hz)
		γ^{FEM}	γ^{REG}	Using γ^{FEM}	Using γ^{REG}	
1	0.073740	0.006366	0.014641	25.931821	25.915991	25.926
2	0.376470	0.006366	0.014641	71.359561	71.136740	71.305
3	0.787096	0.006366	0.014641	139.577076	138.663476	139.35
4	0.999608	0.006366	0.014641	230.513669	228.594861	230.09

Table 12. Frequencies calculated for the beam with free-free end conditions considering severities derived in two ways and directly from FEM for the crack located at 250 mm from one end

Mode no.	SNMC (-)	Severity (-)		Frequency from calculus (Hz)		Frequency from simulation (Hz)
		γ^{FEM}	γ^{REG}	Using γ^{FEM}	Using γ^{REG}	
1	0.295375	0.006366	0.014641	25.895214	25.831806	25.873
2	0.916528	0.006366	0.014641	71.113627	70.571161	70.989
3	0.821370	0.006366	0.014641	139.546468	138.593087	139.34
4	0.142221	0.006366	0.014641	231.779952	231.506949	231.72

Table 13. Frequencies calculated for the beam with free-free end conditions considering severities derived in two ways and directly from FEM for the crack located at 570 mm from one end

Mode no.	SNMC (-)	Severity (-)		Frequency from calculus (Hz)		Frequency from simulation (Hz)
		γ^{FEM}	γ^{REG}	Using γ^{FEM}	Using γ^{REG}	
1	0.918802	0.006366	0.014641	25.792245	25.595007	25.727
2	0.254877	0.006366	0.014641	71.414933	71.264078	71.382
3	0.441335	0.006366	0.014641	139.885862	139.373596	139.77
4	0.608796	0.006366	0.014641	231.090863	229.922243	230.84

In this case, the frequencies calculated with γ^{REG} and γ^{FEM} lead to equally good. This show on one hand that the severity for the cantilever beam, which is the easiest to be found, can be employed for any other boundary conditions. In addition, we conclude that the severity γ^{REG} calculated from several cracks located near the fixed end, we recommend six, is accurate and permits a precise estimation of the frequencies of beams with known location and depth.

5. Conclusion

The paper investigates the conditions in which the severity of a transverse crack can be estimated with accuracy from static measurements. We found that, when the energy method proposed by the authors is utilized, the result obtained by direct FEM simulation considering the crack located at the position where the beam achieves the biggest bending moment is not enough accurate. Important improvement is achieved if the severity is estimated using statistical dta processing after a method proposed by the authors in this paper.

As a second finding of the study is the possibility of using the severity derived for a given boundary condition for all other boundary conditions imposed to the beam.

In future research, we will tackle the problem of the severity estimation for complex-shaped cracks and beams with multiple supports.

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