PAPER

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Computed tomography, sinograms, and image reconstruction in the classroom

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Abstract

As part of an undergraduate introductory survey course on medical physics, x-ray computed tomography (CT) was used to illustrate fundamental principles and mathematics in imaging science. A qualitative description of sinograms was presented to students through a hands-on activity involving simple classroom materials, then the basics of tomographic image reconstruction were presented. Modern applications of CT imaging, including for the diagnosis and treatment of disease, were used to emphasize the utility of medical physics and medical imaging. A simple, qualitative description of convolution, including a very elementary presentation on Fourier transforms and inverse transforms, was included to offer a basic introduction to some of the mathematical tools used in medical imaging physics. Electronic media and materials for the lesson plan are available upon request.

1. Introduction

Many students are familiar with x-ray computed tomography, or CT (formerly 'computed axial tomography', or CAT). At the high school or introductory undergraduate level, however, these students may not yet have been introduced to mathematical concepts involved with medical imaging science, such as Fourier analysis or the central slice theorem [1]. CT imaging may be used as a way to illustrate simple concepts involving the mathematics of imaging science, and as an example of techniques from physics and mathematics applied to problems in medicine. This learning unit was designed for an undergraduate student population with a range of educational backgrounds and majors, with prerequisites of at least the equivalent of one semester of undergraduate calculus and one semester of introductory undergraduate physics.

Previously published works [2–6] detail comprehensive laboratory exercises examining various aspects of computed tomography. For a survey course with a single 50 min class period devoted to CT imaging and reconstruction, there was insufficient time for an extensive project-based learning unit or full laboratory activity. Instead, a brief and straightforward hands-on activity was developed to supplement a classroom discussion introducing these topics.

2. Computed tomography (CT)

In a previous class, students learned that photons used in radiography and other medical imaging modalities passing through media are attenuated exponentially, based on properties of the media and on the energy of the radiation. Because some materials in the body attenuate more than others (for example, bone attenuates keV x-rays to a greater degree than soft tissue), x-rays travelling through a body may be used to acquire a planar image of that body detailing different anatomical features. X-ray computed tomography involves acquiring many two-dimensional projections, or planar images at known angles, of a three-dimensional object to construct a three-dimensional image. These projections are acquired using an x-ray tube (a source of x-rays) that is rotated around the object, and attenuated x-rays are detected by a series of x-ray detectors positioned opposite the tube [7].

The prefix ‘tomo’ derives from the Greek word for ‘slice’ or ‘section’ [8]. Computed tomography is the process of building a three-dimensional image out of individual slices, and these slices are themselves built from data acquired at many different angles. The result is an incredibly important tool in modern medicine that is used for the diagnosis, monitoring, and treatment of disease. For this class, only a subset of possible CT geometries and algorithms was discussed for simplicity. Only parallel-beam, single-slice, and axial scanning geometries (as opposed to fan-beam, multi-slice, or helical geometries) were discussed, and only filtered backprojection (as opposed to iterative reconstruction) was illustrated as a reconstruction method.

3. Sinograms

For each image (in the case of a three-dimensional object to be imaged, each cross-sectional plane or slice), the projection data may be compiled and displayed in an array called a sinogram. This acquisition and organization of raw data is the first step to reconstructing an image. Raw imaging data is arranged based on the angle of projection that it was acquired as well as its spatial position on the imaging plane. A simple example was presented in class and is included in figure 1.

An in-class, hands-on activity was designed to guide students in building a sinogram. Students were divided into seven groups (Group A to Group G). A white paper circle with two three-dimensional black building blocks was displayed at the front of the classroom as the object to be imaged. It was overlaid on black paper indicating seven projection angles (corresponding to the seven groups); see figure 2. Each group was given a strip of paper the same length as the length of the sides of the black paper indicating projection angle. Students were asked to draw what they saw from their assigned observation angle. Following the exercise, the projection drawings were arranged in order of projection angle to reveal the approximate sinogram shape. Students were asked to hypothesize how the sinogram would change if more precise methods of observation were used and many additional projection angles were included.

While simple objects, such as those included in figures 1 and 2, are illustrative for explaining the concept of sinograms, in general, it is not straightforward to recover an imaged object from raw data just by observing the sinogram. This is true for anatomical data as well as any image data, such as a photograph. To illustrate this point, a photograph of a CT scanner was displayed in class along with its corresponding sinogram. The images are included in figure 3. How is a sinogram—a set of data for which it is not straightforward to view as an image—reconstructed into an image? This question leads to a discussion of backprojection and filtered backprojection.

4. Backprojection and filtered backprojection

A sinogram acquired using projection imaging (forward projection) may be used to reconstruct an image of the original object through a process called backprojection. Information acquired at each projection angle is spread out along the path of that projection. Over many projection angles, this procedure results in an image. An illustration of simple backprojection was presented in class and is included in figure 4.

With simple backprojection, the resulting image will always experience a blurring effect (related to $1/r$) [9]. Often this blurry image is inadequate for medical purposes, but it is possible to ‘deblur’ image data. Many students are familiar with deblurring options available on smart phone photography applications, and this was discussed in class. A similar process, called filtered backprojection, may be accomplished with backprojected data. Figure 5 illustrates the same image of a CT scanner used in figure 3. The original data were first blurred using a Gaussian blurring filter, then the blurred data were sharpened using a deblurring filter. Filtered backprojection

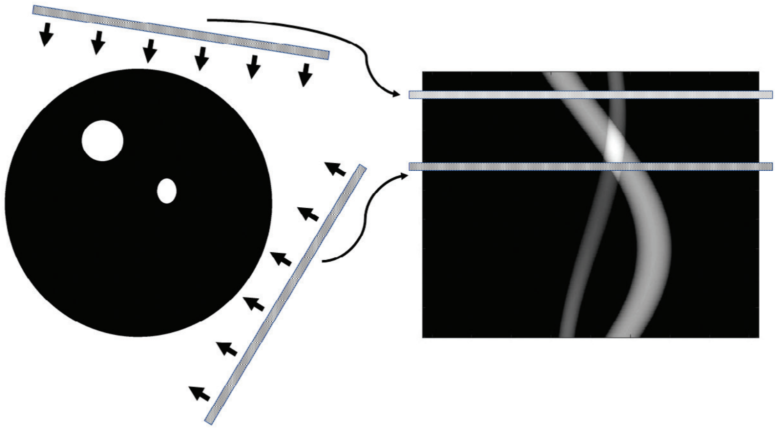
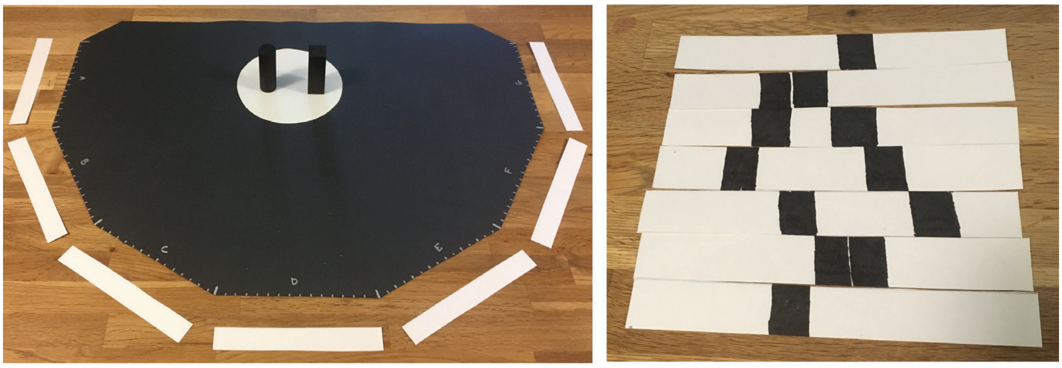


Figure 1. (Left) The object that is to be imaged consists of a dark circle with two light-colored, round embedded elements. (Right) The resulting sinogram is illustrated. The projection angles are displayed along the vertical axis of the sinogram, and the spatial information along each projection is displayed along the horizontal axis of the sinogram. Two planes at different projection angles are highlighted in relation to the object and in their approximate corresponding position on the sinogram.



(*a*) (*b*)

Figure 2. Materials for the hands-on activity illustrating sinograms. (a) The white paper circle and black building blocks represent the 'slice' of the object to be imaged. The dark paper divided into seven segments indicates the projection angle from which student groups will observe the object. Each student group was given a strip of paper to draw the image from their perspective. Following student participation, strips of paper may be arranged according to their projection angle, as shown in (b), to reveal the approximate sinogram shape as acquired from a small number of projection angles.

uses a similar process. The basic mathematics of these processes will be introduced in the following section.

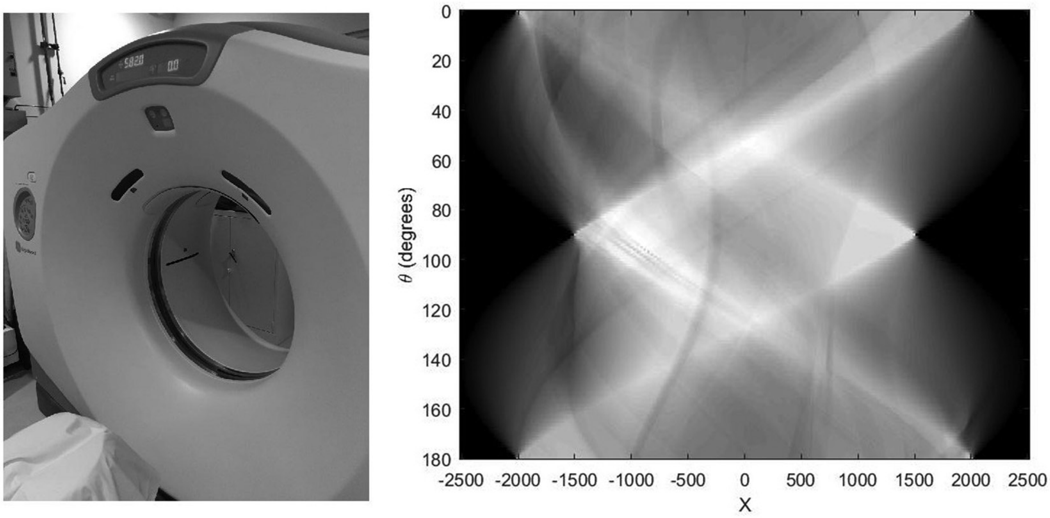
5. Convolution and Fourier transforms

The filter function discussed in the previous section is often applied using the mathematical operation of convolution, denoted $*$ or $\circledast$. Convolution is the expression of how two signals or functions may be combined to derive a resulting signal or function [9]. In the one-dimensional case, convolution may be expressed:

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(\tau) \cdot g(x-\tau)\mathrm{d}\tau \qquad (1)$$

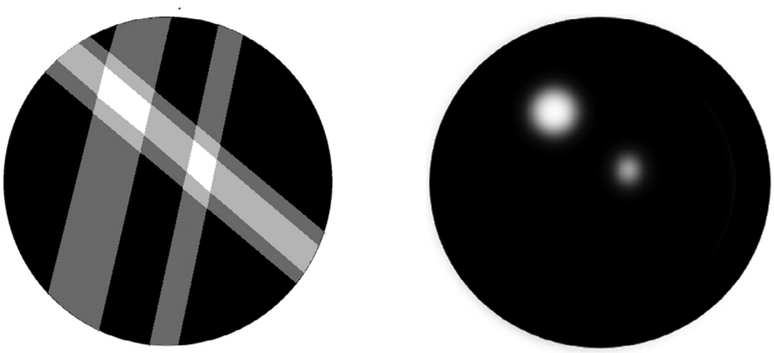
where f and g are functions of a continuous spatial variable x.

The photograph of the CT scanner in figure 5(a) was blurred in figure 5(b) by convolving it with a Gaussian function, and then the blurred image was deblurred in figure 5(c) by



(*a*) (*b*)

Figure 3. (a) A photograph of a CT scanner is shown as an example of a two-dimensional object that may be imaged. (b) The resulting sinogram is shown, where the projection angle is displayed on the vertical axis and the pixel number is displayed on the horizontal axis.

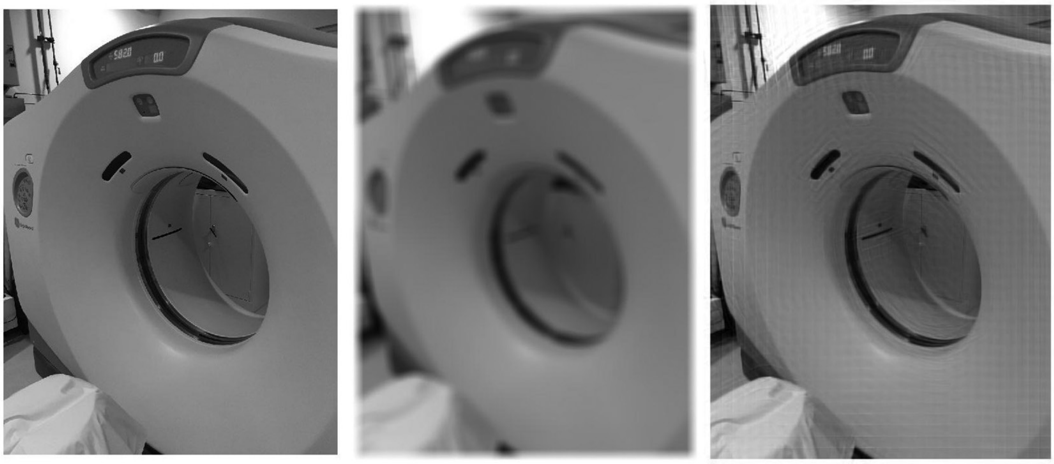


(*a*) (*b*)

Figure 4. The sinogram data from figure 1 is reconstructed to form an image. (a) The two angles highlighted in figure 1 are used for backprojection using only two projection angles. An approximation of the backprojected data results in the bright spots spread out along the directions of the two projection angles. (b) An approximation of the data from many projection angles results in a blurry image.

convolving it with a deblurring filter. How this is often accomplished in practice is by completing a Fourier transform—a new mathematical concept to many students at this level. Equation (1) gives the one-dimensional expression for convolution. In a CT scan, however, large amounts of two-dimensional data must be convolved, which is a computationally expensive process. To complete this task efficiently, convolution may be completed first by transforming to frequency space using a Fourier transform. While in the spatial domain, convolution is a slow process, in Fourier space, this process becomes multiplication (a much faster process). The multiplied data in Fourier space may then be transformed back into the spatial domain.

The Fourier transform process expresses a composite signal in the spatial domain (an image) in its component signals in the frequency domain (Fourier space) [11]. Often, operations in



(*a*) (*b*) (*c*)

Figure 5. (a) The same image of a CT scanner as was used in figure 3. (b) The image is blurred using a Gaussian blurring filter. (c) The blurred image is deblurred using a Weiner filter algorithm [10].

one domain have corresponding operations in the other domain that are significantly faster to perform, such as convolution in the spatial domain performed as multiplication in the frequency domain. Tomographic reconstruction of CT data may be used as an example of using Fourier analysis as a means for practical implementation of convolution of a large amount of data.

6. Discussion

This lesson was designed to introduce an undergraduate student population to basic qualitative concepts in computed tomography and tomographic image reconstruction in a short time period. Many students are likely familiar with medical applications of radiography and computed tomography; however, the basics of medical imaging physics are likely new to many students at this level.

While some more advanced students may be aware that all functions may be expressed as a sum of sines and cosines of varying frequencies and amplitudes from studying Fourier series in previous undergraduate math or physics courses [12–15], they may not be cognizant of the range of applications for Fourier analysis. A thorough discussion of Fourier transforms and inverse transforms were beyond the scope of the short course period. However, CT image reconstruction may be used to introduce students to one practical application for Fourier analysis, and to encourage interested students to pursue the line of study further. To support this learning unit, following the class period, students were assigned to read a nonmathematical summary of tomographic reconstruction by Currie *et al* [16]. The work is aimed at medical radiation technologists and therapists, but offers students a more in-depth (while still qualitative) overview of topics covered during the classroom instruction period.

Principles of tomographic image reconstruction are used in other imaging modalities, including positron emission tomography (PET) and single photon emission computed tomography (SPECT). Though computed tomography was used to showcase the basics of this learning unit, many students may also be interested in other facets of medical physics, such as nuclear medicine with its associated imaging modalities. This lesson plan may serve as a way to generate enthusiasm for a new topic for students who are interested in medical physics.

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Declaration of interest

There are no relevant conflicts of interest to disclose.

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