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Estimating the yield of the *Trinity* test with a simple kinetic energy analysis

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Abstract
The order of magnitude of the explosive yield of the 1945 *Trinity* nuclear weapon test is estimated through a high-school-level kinetic energy argument.

Nuclear weapons are always sure to catch the attention of students, and physics instructors are ideally positioned to nurture this curiosity. While nuclear explosions are extremely complicated phenomena, some of their aspects can be treated with approximate analyses that are accessible to high-school or lower-college level students [1]. The purpose of this paper is to present such an analysis of how one can roughly estimate the explosive yield of the July 1945 *Trinity* test of the Manhattan Project's plutonium implosion bomb by applying a conservation of kinetic energy argument to publicly available information on the growth of the radius of the resulting fireball as a function of time. The *Trinity* fireball at 25 milliseconds after the explosion is shown in figure 1.

A brief history lesson is relevant here. The characteristics of the shock wave which would be formed by the sudden release of a great amount of energy in a small volume were studied theoretically by British physicist Geoffrey Taylor in a secret report prepared in 1941. Taylor's analysis was dauntingly complex, involving advanced differential equations to treat the thermodynamics of highly ionized and compressed air. The net result, however, was a prediction that in its initial stages, the fireball radius r should grow as the two-fifths power of elapsed time: $r \propto t^{2/5}$. The two-fifths power dependence can also be argued on dimensional grounds[1].

Taylor's analysis was published in 1950 along with a companion paper which analyzed the growth of the *Trinity* fireball as deduced from declassified films of the explosion [2, 3]. His data comprised measurements of the fireball radius over the time span 0.10 to 62 milliseconds following the explosion, and he found, somewhat to his own surprise in view of approximations invoked in his original analysis, that the two-fifths-power law held remarkably closely over this span. That the predicted power law held so well is even more surprising in that the fireball hit the ground within a millisecond of the explosion; this must have absorbed some of the available energy.

Empirically, Taylor found that for r in meters and t in seconds, the fireball radius could be expressed as

$$\log(r) = 2.766 + \frac{2}{5}\log(t), \tag{1}$$

or

$$r = 583.5t^{2/5}. \tag{2}$$

As is described in what follows, the two-fifths power dependence can be 'derived' via a

[1] A dimensional analysis of the *Trinity* test can be found at www.atmosp.physics.utoronto.ca/people/codoban/PHY138/Mechanics/dimensional.pdf

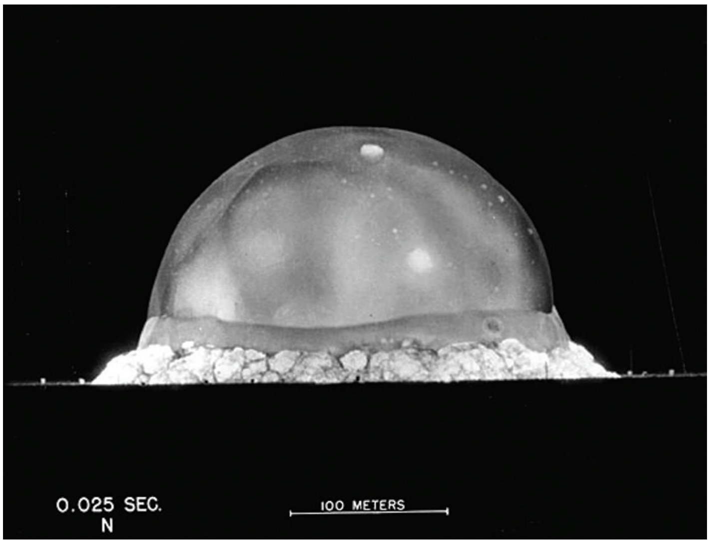

Figure 1. The *Trinity* fireball 25 milliseconds after detonation source: http://commons.wikimedia.org/wiki/File:Trinity_Test_Fireball_25ms.jpg. Does the radius of the fireball accord with equation (2)?

straightforward conservation of energy argument. The empirical factor of 583.5 can then be used to estimate the energy of the explosion.

Conservation of energy

When a nuclear weapon is detonated, an enormous amount of energy is released. According to Glasstone and Dolan (1977; [4]), results gleaned from years of nuclear tests indicate that ~50% of the energy yield goes into a shock wave of compressed air that spreads outward from the explosion, with the remainder distributed between thermal radiation (~35%), prompt ionizing radiation (~5%), and longer-term residual fallout radiation (10%); these authors also give a detailed description of fireball formation and evolution. Since the nuclear explosion itself takes only about one microsecond, all of the energy released in the *Trinity* explosion was emitted well before the 0.1 ms initial time of Taylor's analysis; we can assume that no further energy is generated during the span of his data. For the present purpose, I will assume that an amount of energy E goes into the kinetic energy of a bubble of air which expands outward, accumulating more air as it does so. After securing an estimate of E, the overall yield of the bomb can be estimated by multiplying by a factor of two to account for the shock-wave share of the total energy as indicated above.

If the mass of the bubble at any time is m, then $E = mv^2/2$, where the expansion speed v is $(\mathrm{d}r/\mathrm{d}t)$. If the density of air is ρ, then when the bubble has radius r we must have $m = 4\pi r^3\rho/3$. Hence we have, at any moment,

$$E = \frac{1}{2}\left(\frac{4}{3}\pi r^3\rho\right)\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2. \tag{3}$$

Take the square root of this expression and separate variables to give

$$\sqrt{\frac{3E}{2\pi\rho}}\mathrm{d}t = r^{3/2}\mathrm{d}r. \tag{4}$$

Integrating from $r = 0$ at $t = 0$ to some general later time t gives

$$r = \left(\frac{5}{2}\sqrt{\frac{3E}{2\pi\rho}}\right)^{2/5} t^{2/5}. \tag{5}$$

While the 2/5 power might be argued as to be expected on the basis that energy must be conserved no matter how complex a phenomena, it seems surprising that we can recover Taylor's fundamental radius-time behavior with such a simple argument on considering that in reality the fireball will be a complex soup of bomb debris, fission products, photons, and ionized air.

What do the numbers give? Taking $\rho \sim 1.3\,\mathrm{kg\ m^{-3}}$ and the factor of 583.5 in equation (2), students should be able to verify that $E \sim 2.95 \times 10^{13}$ J. An explosion of one kiloton (kt) of TNT liberates 4.2×10^{12} J, so we have $E \sim 7.0$ kt. Accounting for the factor of two described above, our estimate of the total yield comes in at ~14 kt.

The yield of the *Trinity* test is officially estimated as 21 kt, so our estimate is low by a factor of about one-third. Considering the approximations involved here, however, this is not at all a bad result.

With the energy of the fireball in hand, a good student exercise would be to compute the time-evolution of pressure and temperature within the fireball, treating the enclosed air as an ideal gas; this would help drive home the phenomenal orders of magnitude of physical quantities involved in such explosions.

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