

Multiple traffic states and Braess' paradox in dynamical networks with limited buffer size

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Abstract – Traffic dynamics has always been a research hotspot of complex networks. In this letter, dynamical networks in which the nodes are moving with limited buffer size are studied. We propose an adaptive routing strategy where Euclidean distance and node load are combined by a tunable parameter. The packet loss and traffic congestion can be observed in our model due to limited buffer size. Traffic congestion will occur unless the tunable parameter is in a critical interval. We mainly focus on the impact of the buffer size on traffic congestion and obtain four different traffic states: partial-, short-, no- and long-congestion state. Moreover, a phenomenon similar to the Braess' paradox can be observed in our model. We also find that the higher the node speed, the worse the traffic capacity.

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Introduction. – The dynamic processes on complex networks [1–3] have been extensively studied in the past few years, including phase transition phenomena [4–6], scaling of traffic fluctuations [7–9] and routing strategies [10–12]. The transmission of data on the Internet, the flying of aircrafts between airports, the movement of vehicles in urban networks, and the migration of carbon in bio-systems are all realistic prototypes of network traffic. The topological properties of the network (*e.g.*, degree distribution, average path length and clustering) have an important impact on the dynamic processes. Research on dynamic processes of complex networks is developing rapidly, but there are still some challenging problems including traffic congestion [13–15], cascade failures [16–18], epidemic spreading [19–21].

Traffic congestion on complex networks is one of the hot issues in current research. When the traffic load of the entire network increases over time, the network will get to be congested. Generally speaking, traffic congestion does not occur unless the packet generation rate exceeds a threshold. The root cause of traffic congestion is that the node cannot deliver the packet to the neighbor node in

time due to the limited processing capability of the node. From previous studies, it can be found that scale-free networks are more prone to cause traffic congestion than homogeneous networks [10,22], and nodes with larger degree are more likely to be blocked than those with smaller degree. In our real life, traffic congestion not only increases travel time, but also seriously restricts economic development. In order to solve the congestion problem in these network systems, researchers put forward many improvement measures. The simplest way is to augment the traffic resources [23,24], such as increasing the line bandwidth, widening the road, etc. But these methods are costly. Therefore, it is a necessary issue to explore new routing strategies and optimize network traffic resources [25–27] to solve traffic congestion. In path choice process, more information brings better traffic performance of the network, but leads to higher computational cost. The most famous shortest path protocol is a simple and widely used technology in static routing algorithm. The shortcomings of the shortest path protocol on scale-free networks are also exposed. For example, previous studies [10] have found that nodes with high degree (*e.g.*, hub nodes) are very easy to be blocked when using the shortest path protocol, and the congestion of hub nodes will gradually cause

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a global congestion of the entire network. Therefore, the traffic capacity is very limited by the shortest path routing strategy on the scale-free network.

In recent years, the dynamical network [14,21,28,29] has attracted extensive attention as an important part of complex networks. Compared to static networks, nodes in dynamical networks are moving all the time, so the network structure is constantly changing. A typical example of dynamical network in real world is the wireless *ad hoc* network (WANET) [12], specifically vehicular *ad hoc* networks (VANETs), smart phone *ad hoc* networks (SPANs), wireless sensor networks, robots, and the like. Generally, the bandwidth of the wireless *ad hoc* network is much smaller than that of the cellular network, and the carrier of the mobile node can be either a person, a vehicle or even a UAV, so the moving speed of the node may be very fast. Since the structure of dynamical networks is much more complex than that of static networks, the characteristics of traffic congestion in dynamical networks is different from that of static networks. Yang *et al.* [28] proposed a random routing to study the transportation dynamics on mobile networks. A theory is provided to explain the dependence of the throughput on the speed of agent movement and the communication range. Then, Yang *et al.* [29] proposed an adaptive routing strategy that incorporates geographical distance with local traffic information through a traffic-awareness parameter to improve traffic capacity of dynamical networks.

In most previous studies, the capacity of storing packets at nodes is assumed to be infinite. Recently, multilayer networks with finite storage capacity have been studied and some interesting phenomena, such as the slower is faster effect and Braess' paradox, were observed [15]. In this letter, we introduce a dynamical network model, in which each node has a limited and identical buffer size. We also propose an adaptive routing strategy to enhance network performance. Then, we further look into the impact of the buffer size and obtain four different traffic states. A phenomenon similar to the Braess' paradox [30] can be observed in our model. Finally, we study how the node speed affects the traffic capacity of the network. In Manfredi's model [15], times spent on links and on nodes are always comparable, and the message generation rate is constant at a small value. But in our model, travel times can be heavily dominated by waiting times on the nodes, and the message generation rate is varied and later tuned to a value close to system overload.

This letter is organized as follows. The network model and the traffic model are described in detail, respectively. Then the simulation results are presented and discussed. In the last section, the work is concluded.

The network model. – We study the traffic behavior on a dynamical network. Take a wireless *ad hoc* network as an example, in which mobile nodes communicate with each other via wireless links and a node can transmit data packets to the other node if their distance is shorter than

a critical value. We let N nodes move on a $L \times L$ square area with periodic boundary conditions. At time $t = 0$, all nodes are randomly distributed on the square area. At each time step, the moving direction θ of each node changes randomly, but the moving speed v is the same for all nodes in the whole simulation process. Since the nodes are moving, their positions change with time t . The evolution of each node i 's position and moving direction is the following:

$$\begin{aligned} x_i(t+1) &= x_i(t) + v \cos \theta_i(t), \\ y_i(t+1) &= y_i(t) + v \sin \theta_i(t), \\ \theta_i(t+1) &= \theta_i(t) + \psi_i(t), \end{aligned} \quad (1)$$

where $x_i(t)$ and $y_i(t)$ are the horizontal and vertical coordinates of node i at time t , respectively. $\psi_i(t)$ represents the change of moving direction of node i between time $t+1$ and time t , which is a random number in the interval $[-\pi, \pi]$. We use the Euclidean distance to describe the position relationship between nodes. The Euclidean distance between node i and j at time t is defined as

$$D_{ij}(t) = \sqrt{[x_i(t) - x_j(t)]^2 + [y_i(t) - y_j(t)]^2}. \quad (2)$$

Each node has the same communication radius r . Two nodes can communicate with each other if the Euclidean distance between them is less than r in a time step, and those two nodes are regarded as temporary neighbors. Therefore, all temporary neighbors of a node are defined as all nodes within the current communication area of that node.

The traffic model. – In our traffic model, each node can create, buffer, deliver, and receive packets. At each time step, for each node, a packet will be generated with a probability ρ to be delivered to another node of the dynamical network. Thus, there are on average $N\rho$ packets generated on the network at each time step, and the destination node of each packet is selected randomly from the network. Every node has a limited buffer size to store packets. For simplicity, the buffer size of each node is homogeneous and denoted as B . This implies that the queue length in all nodes cannot exceed B . The first-in-first-out (FIFO) rule is applied to all queues. One node can deliver at most C packets to its current neighbors at each time step. To deliver a packet to its destination, a node performs a local search within its neighbors. If the packet's destination is a neighbor of the node where the packet is currently located, the packet will be delivered directly to its destination node at the next time step and then be immediately removed from the network. Otherwise, the packet is forwarded to the appropriate neighbor node selected based on the adaptive routing strategy we proposed below. Let us assume that at time t the packet is in node s , and its destination node j is not a neighbor of node s . For each neighbor node i of node s , there is an effective distance from node j , denoted by

$$d_{eff}^{ij}(t) = h \frac{D_{ij}(t)}{L} + (1-h) \frac{q_i(t)}{B}, \quad (3)$$

where $q_i(t)$ is node i 's queue length at time t and h is a tunable parameter ($0 \leq h \leq 1$). In order to make the first item and the second item in eq. (3) in the same order of magnitude, we set $L' = L/2$. Then, at time $t + 1$ node s will send the packet to the neighbor node i with the smallest value of $d_{eff}^{ij}(t)$. Note that when $h = 1$, the packet will be delivered to a neighbor node whose Euclidean distance away from the destination node is shortest. If a node currently has no neighbors, it will keep the packet and deliver it later.

It is important to note that packet loss may occur in the model due to the limited buffer size. The queuing process of packets in nodes is described as follows [15]. We denote the resulting net flow in node i at time t as $R_i(t)$:

$$R_i(t) = I_i(t) + \delta I_i(t) - O_i(t) - \delta O_i(t), \quad (4)$$

where $I_i(t)$ and $O_i(t)$ are the queue incoming and outgoing rate from and to other nodes, respectively, while $\delta I_i(t)$ and $\delta O_i(t)$ are the number of packets generated or removed in node i at time t . Particularly, the outgoing rate $O_i(t)$ is related to the capacity of node i to deliver packets towards other nodes. As mentioned above, one node can deliver at most C packets at each time step, so $O_i(t) \leq C$. Once all the values of $R_i(t)$ are calculated, we update the queue length according to the following rule:

$$q_i(t+1) = \begin{cases} 0 & q_i(t) + R_i(t) \leq 0, \\ q_i(t) + R_i(t) & 0 < q_i(t) + R_i(t) < B, \\ B & B \leq q_i(t) + R_i(t) \end{cases} \quad (5)$$

for each node i , $i = 1, \dots, N$. Notice that when $B \leq q_i(t) + R_i(t)$, $q_i(t+1) = B$, which means that $q_i(t) + R_i(t) - B$ packets are lost in node i at time $t+1$. Packets may be lost in the process of generation and transportation, and not all packets can be actually generated and reach their respective destinations. Therefore, we define the traffic flow as the total number of packets that reach their respective destinations during the system process.

Simulation results. – In the following simulations, we set the total number of nodes $N = 1500$, the size of the square region $L = 10$, the communication radius $r = 1$, and the node delivery capacity $C = 1$. The moving speed v is fixed to 0.1 if there is no special mention. Each simulation runs T_m time steps, and $T_m = 1 \times 10^5$.

Figure 1(a) shows the relationship between the traffic flow and the packet generation rate ρ with different values of the tunable parameter h . One can see that for a fixed value of h , as the packet generation rate ρ enhances, the traffic flow first increases linearly until a suddenly drop after reaching the peak, and then decreases slowly. The maximum value of the traffic flow depends on h and the optimal value of ρ is denoted as ρ_c ($\rho_c \approx 0.18$ for $h = 0.2$ or 0.9, while $\rho_c \approx 0.2$ for $h = 0.5$). Figure 1(b) shows the network density Φ as a function of ρ with different values of h . The network density Φ is defined as the ratio of the total number of packets in the network to the sum of the

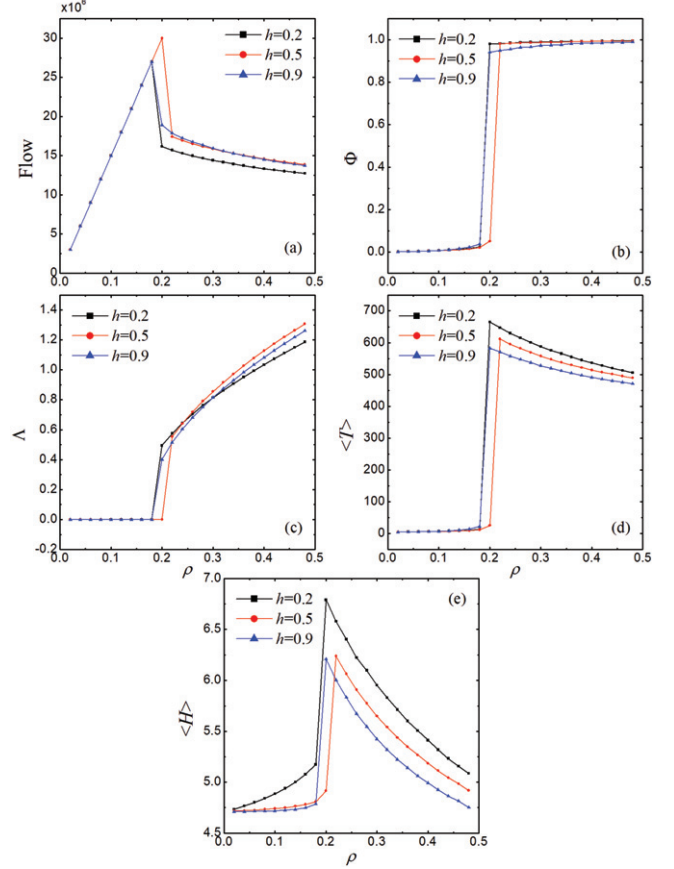


Fig. 1: (a) The traffic flow, (b) network density Φ , (c) packet loss Λ , (d) average travel time $\langle T \rangle$ and (e) average number of hops $\langle H \rangle$ as a function of the packet generation rate ρ with three different values of the tunable parameter $h = 0.2, 0.5, 0.9$. The simulation parameters are $N = 1500$, $L = 10$, $C = 1$, $r = 1$, $v = 0.1$, and $B = 100$.

buffer sizes of all nodes at the end of the simulation, which can be expressed as

$$\Phi = \Phi(T_m) = \frac{\sum_{i=1}^N q_i(T_m)}{NB}. \quad (6)$$

For a fixed value of h , Φ mutates from about 0 to about 1 at $\rho \approx \rho_c$, indicating that there is a balance between the number of generated and removed packets when $\rho \leq \rho_c$, but congestion occurs and packets accumulate in the system until the buffers of all nodes are full when $\rho > \rho_c$. Figure 1(c) shows the packet loss Λ as a function of ρ for different values of h . The packet loss Λ is defined as the percentage of packets that are unable to reach their destination with respect to the total number of actually generated packets. One can observe that for a fixed value of h , no packets are lost when ρ is small and Λ increases linearly as ρ surpasses ρ_c . Obviously, if ρ exceeds this critical value, the faster the packet is generated, the faster the network reaches congestion. Then, we define the travel time as the time steps that a packet spends traveling from its source to its destination. In our simulation, the travel time of a packet is only counted when the traffic condition

is stable. Figure 1(d) shows the average travel time $\langle T \rangle$ as a function of ρ with different values of h . For a fixed value of h , with the increase of ρ , $\langle T \rangle$ is very small and increases slowly when $\rho \leq \rho_c$. When ρ surpasses ρ_c , $\langle T \rangle$ increases suddenly to a very large value, which is also the maximum value, and then decreases slightly. The rate of decline of $\langle T \rangle$ depends on h . From the first four subgraphs in fig. 1, one can find that all curves in the same subgraph show the same trend regardless of the values of h and have a sudden transition when ρ is around ρ_c . It is indicated that congestion begins to occur when $\rho \approx \rho_c$.

When the moving speed is not very large (here $v = 0.1$), it is easy to understand why the critical packet generation rate ρ_c is around 0.2: The network has a physical size of $L = 10$, so the distance of packet source and sink is about 5 on average. A packet hops a distance of almost one per hop on average. Therefore, once in about five time steps a packet reaches its destination and is removed, resulting in a removal rate around 0.2. Therefore, for $\rho \leq 0.2$ the nodes are almost empty, for $\rho > 0.2$ they are almost completely filled. Figure 1(e) shows the average number of hops $\langle H \rangle$ as a function of ρ with different values of h . When $\rho > \rho_c$, a packet needs slightly more than five hops, and on every node the packet waits almost 100 time steps (because $B = 100$), so the travel time is totally dominated by waiting on the nodes. After ρ surpasses ρ_c , as ρ increases, $\langle H \rangle$ decreases instead. This can explain why the average travel time $\langle T \rangle$ decreases slightly with the increase of ρ when $\rho > \rho_c$. The faster the packet is generated, the more congested the node is, and the easier it is for the packet to be deleted during the delivery process. This will cause that only nodes with few hops successfully reach their destination. In a congestion state, the fewer hops means that the waiting time is greatly reduced, so the average travel time will be shorter. In addition, the smaller h is, the more the packet tends to be delivered to the node with a relatively small load, leading to an increase in the number of hops. However, the network at $h = 0.5$ is less prone to congestion, compared with that at $h = 0.2$ and $h = 0.9$. So there may be a better value of h to optimize the traffic capacity of the network.

Figure 1 shows that routing strategies that tend to be distance-based ($h \rightarrow 1$) or load-based ($h \rightarrow 0$) are prone to congestion, and that the effective combination of the two ($h \approx 0.5$) can improve the traffic capacity of the network. If ρ takes a very small or very large value, then for the routing strategy we propose, no matter what value h takes, the network will be completely unblocked or congested. In order to reflect the advantages of the proposed routing strategy and some noteworthy characteristics, the packet generation rate ρ needs to be controlled within a certain interval. When $h = 0, 0.5, 1$, let $\rho_c = \rho_{c0}, \rho_{c0.5}, \rho_{c1}$, respectively. It is acceptable for ρ to take any value in the interval $(\max(\rho_{c0}, \rho_{c1}), \rho_{c0.5}]$. When ρ is close to $\rho_{c0.5}$, congestion is more likely to occur and the simulation time can be shortened. Therefore, in the following simulations, without loss of generality, we fix $\rho = 0.2$.

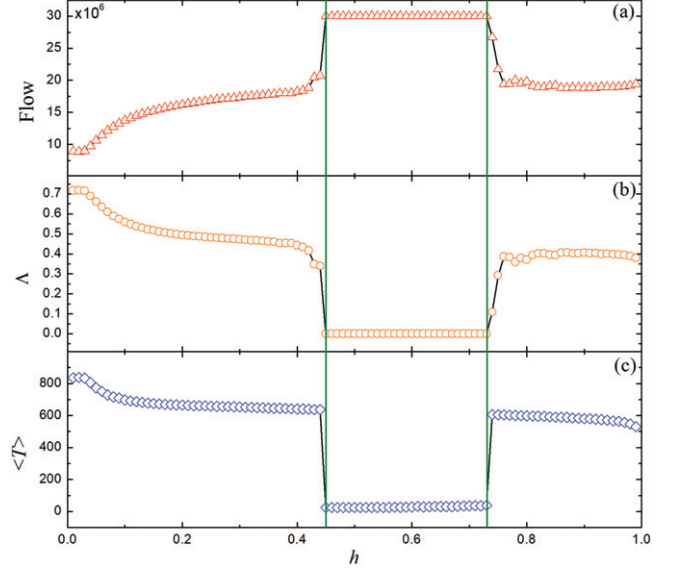


Fig. 2: (a) The traffic flow, (b) packet loss Λ , and (c) average travel time $\langle T \rangle$ as functions of the tunable parameter h . The simulation parameters are $N = 1500$, $L = 10$, $C = 1$, $r = 1$, $v = 0.1$, $B = 100$, and $\rho = 0.2$.

Then, we study the effect of h on the traffic capacity of our model. We fix $\rho = 0.2$ and $B = 100$ based on the result of fig. 1. From fig. 2, one can see that the traffic state changes significantly with the change of h . With the increase of h , all three functions have significant transitions at $h = 0.45$ and $h = 0.73$. In fig. 2(a), when $0.45 \leq h \leq 0.73$, the traffic flow fluctuates slightly around a certain value and is much larger than that when $h < 0.45$ or $h > 0.73$. However, in figs. 2(b) and (c), the trends of the packet loss Λ and the average travel time $\langle T \rangle$ are nearly the same and completely opposite to the traffic flow as h increases. For fix values of ρ and B , if the value of h is too large or too small, congestion will occur. Only if h is within a certain interval can congestion be alleviated. Therefore, the tunable parameter h considering both distance and node load can effectively enhance the traffic capacity of the dynamical network.

Next, we mainly focus on the impact of the buffer size B on traffic congestion. We set $\rho = 0.2$ and $h = 0.5$ according to above results. Figure 3(a) shows the traffic flow as a function of the buffer size B . It can be observed that with the increase of B , when $0 < B \leq B_1$ ($B_1 = 25$), the traffic flow increases; when $B_1 \leq B \leq B_2$ ($B_2 = 85$), the traffic flow first decreases, then increases sharply; when $B_2 \leq B \leq B_3$ ($B_3 = 280$), the traffic flow fluctuates slightly around its maximum; when $B \geq B_3$, the traffic flow displays a decreasing trend with fluctuation. Figure 3(b) shows the network density Φ as a function of B . When $0 < B \leq B_2$, with the increase of B , Φ first increases greatly, and gradually converges to $\Phi = 1$, but then suddenly drops to the minimum. When $B_2 \leq B \leq B_3$, Φ fluctuates around its minimum. When $B \geq B_3$, Φ first increases rapidly, then fluctuates around its maximum. In

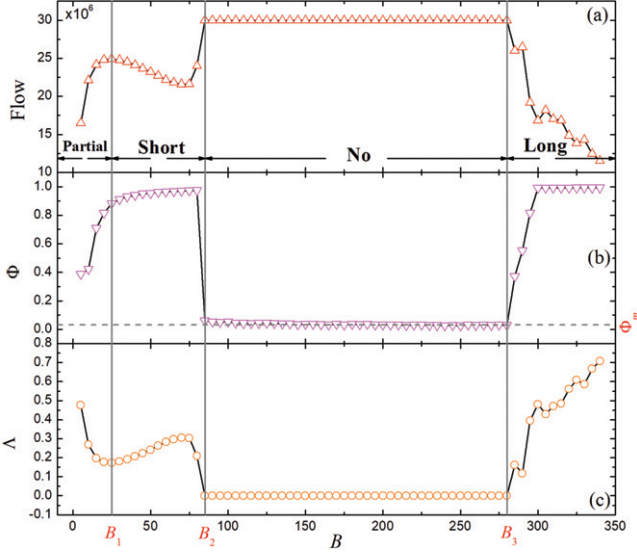


Fig. 3: (a) The traffic flow, (b) network density Φ , and (c) packet loss Λ as a function of the node buffer size B . B_1 , B_2 and B_3 are the critical values for partial-, short-, no-, and long-congestion states, respectively. The simulation parameters are $N = 1500$, $L = 10$, $C = 1$, $r = 1$, $v = 0.1$, $\rho = 0.2$, and $h = 0.5$.

fig. 3(c), the trend of the packet loss Λ is completely opposite to the traffic flow in fig. 3(a) as B increases.

The reasons for the different traffic states in fig. 3 are explained as follows. We first observe the traffic state when the buffer size $B \in [B_2, B_3]$. Obviously, the traffic flow is the largest and no packets are lost in this interval. We call this traffic state *no-congestion state*. In this state, no matter what the value of B is, the network density Φ is very small and almost the same (the average value Φ_m is shown in fig. 3(b)). It can be seen from eq. (6) that $\Phi(t)$ is the average value of ratio $q_i(t)/B$ of all nodes at time t , which can measure the weight of the second term in eq. (3) at this time. From the analysis of fig. 2, we know that only when the two terms in eq. (3) are combined with a certain weight, can the traffic capacity in the system be optimal. Therefore, a necessary condition to avoid traffic congestion is that the network density $\Phi(t)$ is always approximately equal to Φ_m when $t \rightarrow \infty$.

When the buffer size B is very small ($0 < B \leq B_1$), a small difference in queue length q of each node will lead to a large difference in ratio q/B , so the effective distance expressed in eq. (3) is mainly determined by the second item. Thus, packets are mainly delivered to nodes with smaller queue length, causing congestion in these nodes. Because only a small number of nodes are congested, the network density Φ and the packet loss Λ are not very large. With the increase of B , this difference effect decreases, and the effect of the first term in eq. (3) increases gradually, so that the traffic flow increases and Λ decreases. We call this traffic state *partial-congestion state*.

When the buffer size B is not very small ($B_1 < B < B_2$), traffic congestion also apparently occurs. At the beginning

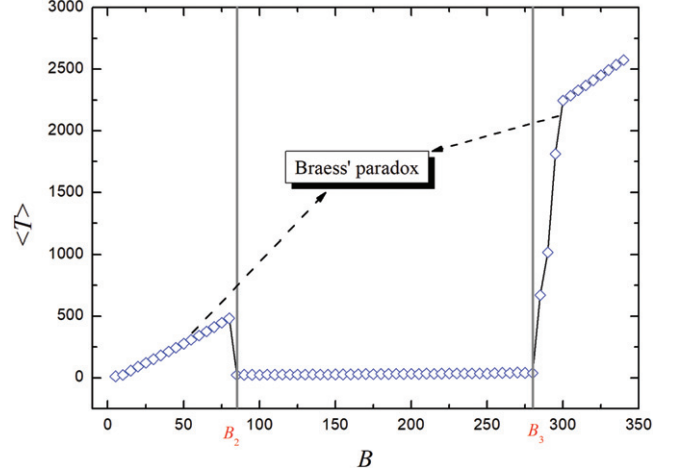


Fig. 4: The average travel time $\langle T \rangle$ as a function of the node buffer size B . B_2 and B_3 are the critical values for short-, no-, and long-congestion states, respectively. The simulation parameters are as in fig. 3. The arrows denote the area where the Braess' paradox occurs.

of the simulation, the network density $\Phi(t)$ increases gradually as packets are injected into the network at each time step. At some time steps, $\Phi(t)$ is bound to reach Φ_m , but why does congestion eventually occur? In the no-congestion state, there is a balance between the number of generated packets and removed packets without packet loss. It is required that the total number of packets in the network should not be too small. When $\Phi(t)$ reaches Φ_m , the total number of packets in the network is too small to reach that balance according to eq. (6). At this time, the queue length of each node is also very short. Once $\Phi(t)$ continues to increase, traffic congestion is inevitable. Eventually all nodes will be congested. The larger B , the longer the average waiting time of the packet in the node. Note that the first term in eq. (3) has timeliness, *i.e.*, the longer the time after calculation, the worse the effect of the equation. As a result, the traffic flow decreases and the packet loss Λ increases with the increase of B . We call this traffic state *short-congestion state*. Obviously, as B approaches B_2 , the traffic state changes to no-congestion state.

When the buffer size B is too large ($B > B_3$), traffic congestion occurs because of the reason opposite to the short-congestion state. When the network density $\Phi(t)$ reaches Φ_m , the total number of packets in the network is too large, *i.e.*, the average queue length in nodes is very long. Due to the timeliness of the first item in eq. (3), the adaptive routing strategy has no effect, and thus congestion will also inevitably occur. Over time, all nodes become congested. When the total number of packets in the network is small, the second term in eq. (3) has little difference. The larger B is, the less difference it has, and the faster congestion will occur. As a result, the traffic flow decreases and the packet loss Λ increases with the increase of B . However, due to the randomness of the dynamical

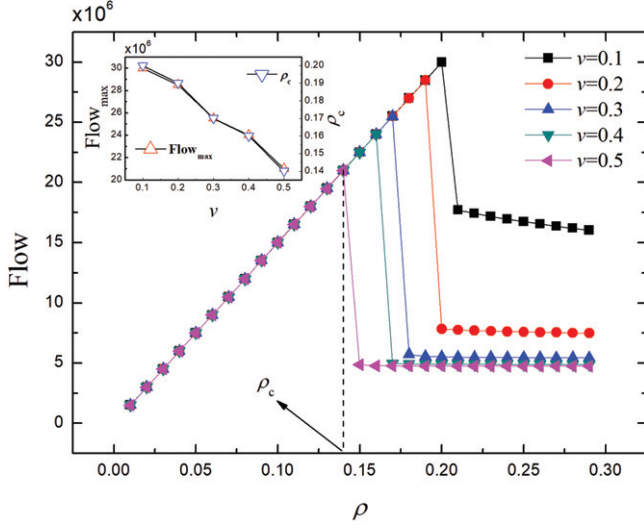


Fig. 5: The traffic flow as a function of the packet generation rate ρ for five different values of node speed $v = 0.1\text{--}0.5$. The arrow denotes the position of the critical value ρ_c . The inset shows the maximum traffic flow $Flow_{max}$ and ρ_c as functions of v . The simulation parameters are $N = 1500$, $L = 10$, $C = 1$, $r = 1$, $B = 100$, and $h = 0.5$.

network, the results show some fluctuations. We call this traffic state *long-congestion state*.

Moreover, a phenomenon similar to the Braess' paradox, in which adding extra resources to a network will reduce network performance in some cases, can be observed in our model. Figure 4 shows the average travel time $\langle T \rangle$ as a function of the buffer size B . One can see that when $B < B_2$ or $B > B_3$, $\langle T \rangle$ increases with the increase of B . According to the analysis of fig. 3, traffic congestion occurs when $B < B_2$ or $B > B_3$, resulting in almost full buffer for many nodes. In these two regimes, the average waiting time for all packets is positively correlated with the buffer size B . Therefore, the buffer size enhancement (*i.e.*, the addition of physical space to the nodes) during congestion will not improve the system, but will even cost more travel time. Note that when B approaches B_2 , the congestion may be eliminated and the average travel time $\langle T \rangle$ plummets. We can also see that when $B_2 \leq B \leq B_3$, the average travel time $\langle T \rangle$ is very small and remains almost unchanged due to the unblocked network.

Figure 5 shows the dependence of traffic flow on the packet generation rate ρ for different values of v . For a fixed value of v , the traffic flow first increases linearly as ρ increases. When ρ surpasses a critical value denoted as ρ_c , the traffic flow drops suddenly, and then remains almost unchanged, *i.e.*, congestion occurs. The traffic flow at ρ_c is denoted as $Flow_{max}$, which is also the maximum traffic flow at this speed. From the insets of fig. 5, one can observe that when v increases from 0.1 to 0.5, both ρ_c and $Flow_{max}$ decrease continuously. Therefore, increasing v within a certain range will lead to slower packet processing rate and thus worse traffic capacity of the network.

Conclusions. – The queue length of nodes is usually regarded as infinite in most papers, but in fact the buffer size is a finite resource. In this letter, we study the dynamical network with limited buffer size and propose an adaptive routing strategy to deliver packets. The routing strategy combines Euclidean distance and node load with a tunable parameter. The packet loss and traffic congestion can be observed in our model due to limited buffer size. We conclude that traffic congestion will occur unless the tunable parameter is in a critical interval, which means that the adaptive routing strategy can play a positive role. We mainly discuss the impact of buffer size on traffic congestion. Based on the buffer size, four different traffic states are obtained: partial-, short-, no-, and long-congestion state. In the no-congestion state, there is no packet loss and the traffic capacity of the network is the best. In the other three states, traffic congestion occurs for various reasons related to the routing strategy. In the partial-congestion state, some nodes are blocked, while in the short- or long-congestion state, all nodes are blocked. Moreover, a phenomenon similar to the Braess' paradox can be observed in our model. When traffic congestion occurs, the average travel time increases with the buffer size due to the increasing waiting time. In addition, increasing v within a certain range will lead to slower packet processing rate and thus worse traffic capacity of the network. It seems that the assumption that all nodes know the status of other nodes is also a complex issue. However, it has been solved in reality. Taking the wireless *ad hoc* network as an example, each node can use GPS for positioning, and transmit the location information to other nodes through the wireless *ad hoc* network system. Because location information is very important, it can be sent by other routes as a priority. In reality, the location information is relatively small, and it only takes up very little bandwidth and buffer. This letter studies a simplified physical model, so this issue is ignored. Existing research models [14,28,29] also use simplified processing similar to the model in this letter. We believe that our work can provide some inspiration for alleviating real-world traffic congestion.

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