IOP Publishing Physica Scripta

Phys. Scr. **95** (2020) 055212 (3pp) https://doi.org/10.1088/1402-4896/ab7654

Kinetic friction force in a three-layer model with commensurate substrates

Li-Ping Jia[1,3] and Jasmina Tekić[2]

[1] Department of Physics, Longdong University, Qingyang, People's Republic of China
[2] Vinča Institute of Nuclear Sciences, Laboratory for Theoretical and Condensed Matter Physics, University of Belgrade, Belgrade, Serbia

E-mail: pp2008634@163.com

Received 19 November 2019, revised 28 January 2020
Accepted for publication 13 February 2020
Published 2 March 2020

Abstract

A chain of harmonically interacting particles confined between two sinusoidal substrate potentials has been analysed. When the top substrate is driven through an attached spring with a constant velocity, the influence of the system parameters on the kinetic friction force was examined in the system with commensurate substrates. It was shown that the friction reached its maximum value when the equilibrium distance of the intermediate particles was an integer multiple of the space period of the upper and lower substrate potentials, which led to the maximum energy loss. This loss can be reduced by the increase of interparticle interaction of the middle layer.

Keywords: the kinetic friction force, the energy loss, the lattice constant, the interaction strength

(Some figures may appear in colour only in the online journal)

1. Introduction

When two objects are sliding relative to each other, there is a force between the contacts that blocks the sliding of the objects. This force is called kinetic friction, and it is a leading cause of the energy losses in the system. Understanding friction in the microcosm and how to reduce energy losses are the problems that have been attracting the interest of researchers for years [1–3]. In the theoretical descriptions, the Frenkel-Kontorova model best captures the essence of friction phenomena [4–15].

The standard FK model, a chain of harmonically interacting particles subjected to an external periodic substrate potential, provides a good description of a 'dry friction' [3, 16, 17]. However, in the studies of friction phenomena, particularly interesting are the three layer systems, which contain a layer, the so called 'third body or bodies', between the substrate potential and the chain. It acts like a lubricant film, and it has been described as a series of atomic chains confined between two substrate potentials by O M Braun *et al* [3]. It was shown that when the upper substrate potential was driven by a constant moving spring, the Golden mean structure was more favourable to sliding than the Spiral mean structure. The golden mean and the spiral mean structures: the upper substrate potential period is c, the interatomic equilibrium is b, the bottom substrate potential period is a; for golden structure, $a/b = 233/144$, $c/a = 144/89$; for spiral mean structure, $a/b = 351/265$, $c/a = 265/200$. Kinetic friction was studied intensively in incommensurate systems [3], while on the other hand, three-layer models with commensurate substrates were seldom studied.

In the present paper, we modelled a one-dimensional system of two rigid sinusoidal substrates and a chain of interacting particles embedded between them. We will examine the influence of the lattice constant and the interaction strength of the middle layer on the kinetic friction force when the top substrate of mass M is driven by a constant moving spring. In the examination of the lattice constant dependence, the kinetic friction force decreases as the interaction strength increases while it oscillates with the lattice constant. Dependence of the kinetic friction force on the lattice constant and the interaction strength will be also investigated. We will show that when the lattice constant is taken near the positive integer, the kinetic friction force reaches the maximum value.

[3] Author to whom any correspondence should be addressed.

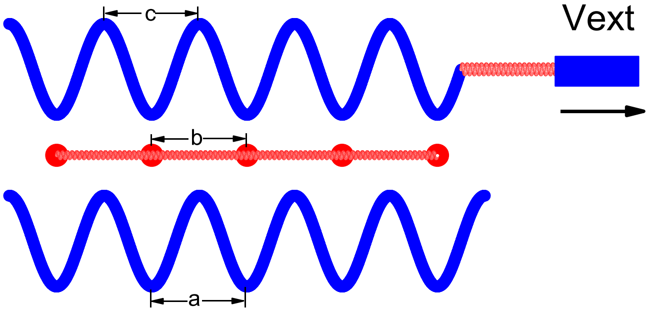

Figure 1. Schematic of the model. The model parameters are $a = c = 1.0$, $Vext = 0.3$, $Kext = -0.03$.

This paper is organized as follows. The model is introduced in section 2. The numerical results are presented and analyzed in section 3. The conclusion is given in section 4.

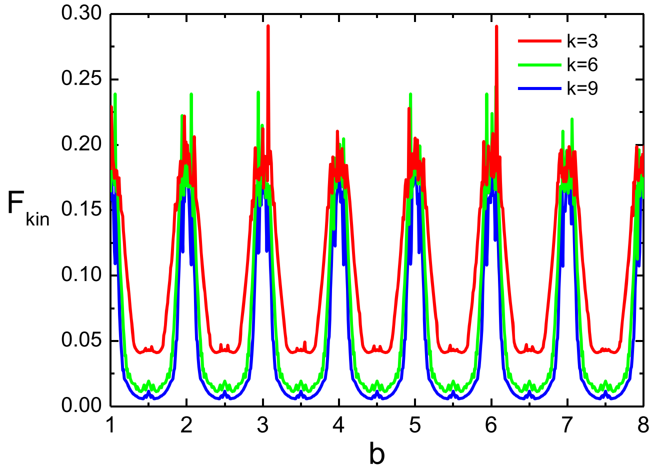

Figure 2. Dependence of the kinetic friction force F_{kin} on the lattice constant b for different values of the interaction strength k.

2. Model

In our study the friction phenomena, we will consider the three-layer model, as shown in figure 1.

The middle layer consists of a chain of particles interacting harmonically, while both the upper and lower layer are rigid substrates. The upper layer is pulled by a spring with the elastic constant *Kext*. The spring is connected to a stage that moves with a velocity *Vext*. The dynamic is described by the following system of equations:

$$m\ddot{x}_i = -\gamma\dot{x}_i - \gamma(\dot{x}_i - \dot{X}_{top}) + \frac{d}{dx_i}\sum_{i\neq j} V(|x_i - x_j|) + \frac{1}{2}\left[\sin\frac{2\pi x_i}{a} + \sin\frac{2\pi(x_i - X_{top})}{c}\right], \tag{1}$$

$$M\ddot{X}_{top} = -\sum_{i=1}^{N}\gamma(\dot{X}_{top} - \dot{x}_i) + \sum_{i=1}^{N}\frac{1}{2}\left[\sin\frac{2\pi(X_{top} - x_i)}{c}\right] + K\,\mathrm{ext}(Vext^{*}t - X_{top}), \tag{2}$$

here, $x_i (i = 1, 2, 3, ..., N)$ represents the coordinate of i-th particles of the middle layer, while X_{top} is the coordinate of the particle in the upper substrate potential. The mass of the particles of the rigid substrates and the middle layer are M and m, respectively. The period of the substrate potentials are equal, i.e., $a = c = 1$, where we use dimensionless units. In the equations (1) and (2), the damping force is described by the terms with damping coefficient γ(in this system, we choose $\gamma = 0.2$). The interaction potential between particles in the middle layer has the following form:

$$V(X) = \frac{k}{2}[(X - b)^2], \tag{3}$$

where k is the interaction strength between particles and b is the lattice constant. X is the difference of the coordinates between the nearest neighbours. The sine terms of equations (1) and (2) represent the interaction between particles and the substrate potential. The equations have been integrated by using the fourth-order Runge–Kutta method. In order to enforce a fixed density condition for the chain, we introduce periodic boundary conditions [16–18]:

$$x_{N+1} = x_1 + Nb, \tag{4}$$

$$x_0 = x_N - Nb, \tag{5}$$

At the beginning, the middle particles are placed at rest at uniform separation b. The upper substrate potential is rigid, so at the initial time we set its coordinates $X_{top} = 0$. After relaxing the starting configuration, spring starts to move at speed *Vext*. Given enough time for the system to reach a stable state, we start to measure the relevant data.

3. Results and discussion

In tribology, one of the main problems is the energy loss caused by kinetic friction force F_{kin}, which in our model could be easily determined. If in some time t, the substrate potential of the upper layer moves the distance ΔX_{top} [3], then the energy loss of the system can be calculated as

$$E_{loss} = F_{kin}\Delta X_{top} = F_{kin}\dot{X}_{top}\Delta t, \tag{6}$$

In our system, all of energy loss [3], comes from the $\gamma-$damping terms:

$$E_{loss} = \sum_{i=1}^{N}\int_{t_1}^{t_2}\gamma[x_i^2 + (\dot{x}_i - \dot{X}_{top})^2]dt, \tag{7}$$

where $\Delta t = t_2 - t_1$. According to equation (6) we obtain that the kinetic friction force of the system [3] is

$$F_{kin} = \sum_{i=1}^{N}\lim_{\Delta t\to\infty}\int_{t_1}^{t_2}\gamma[x_i^2 + (\dot{x}_i - \dot{X}_{top})^2]/\dot{X}_{top}dt, \tag{8}$$

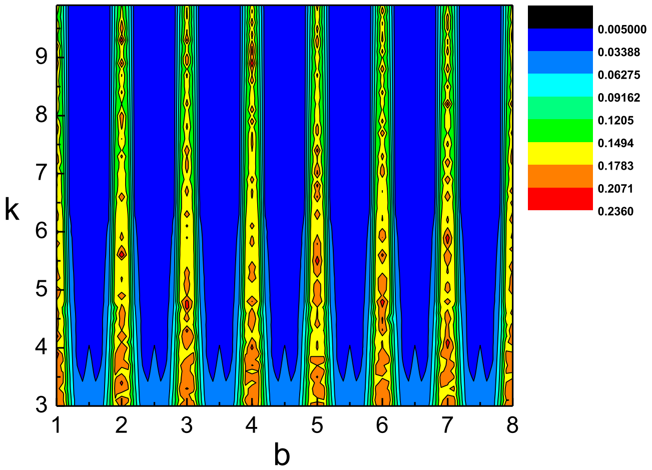

Figure 3. Contour plot of the kinetic friction force F_{kin} as a function of the lattice constant b and the interaction strength k.

Dependence of the kinetic friction force F_{kin} on the lattice constant b for different values of the interaction strength k is presented in figure 2. As we can see the kinetic friction force F_{kin} decreases as k increases while it oscillates with the lattice constant b.

In order to further examine the influence of the lattice constant and the interaction strength on the kinetic friction force, in figure 3 the kinetic friction force is presented by different colors in $k - b$ plane. The dependence of the kinetic friction force on the parameter b is quasi-periodic, where the period is approximately equal to 1. Therefore, when the lattice constant is taken near the positive integer, the kinetic friction force reaches the maximum value, and the energy loss of the system is also maximum. We can also see that the value of the kinetic friction force F_{kin} decreases with the increase of k.

4. Conclusions

In conclusion, the kinetic friction of a three-layer system was modelled, where the influence of the lattice constant and the interaction strength of the middle layer on the friction force was examined. The results have shown that the system exhibited maximum energy loss when the lattice constant was taken near the positive integer, and the kinetic friction force reached the maximum value. The increase of the interaction strength led to the decrease of the kinetic friction force, and therefore to the reduction of the energy loss of the system.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant No. 11847163) and the Doctor Science Foundation of LongDong University (Grant No. XYBY1701). This work was supported by the Serbian Ministry of Education and Science under Contracts No. III-45010.

ORCID iDs

Li-Ping Jia https://orcid.org/0000-0002-1134-3071

References

[1] Persson N J B 1999 *Surf. Sci. Rep.* **33** 83
[2] Braun O M and Kivshar Y S 1998 *Phys. Rep.* **306** 1
[3] Braun O M, Vanossi A and Tosatti E 2005 *Phys. Rev. Lett.* **95** 026102
[4] Jia L P, Tekic J and Duan W S 2015 *Chin. Phys. Lett.* **4** 040501
[5] Wang C L, Duan W S, Chen J M and Hong X R 2008 *Appl. Phys. Lett.* **93** 153116
[6] Jia R J, Wang C L, Yang Y, Gou X Q, Chen J M and Duan W S 2013 *Acta Phys. Sin.* **62** 068104
[7] Lin M M, Duan W S and Chen J M 2010 *Chin. Phys. B* **19** 026201
[8] Ma M, Benassi A, Vanossi A and Urbakh M 2015 *Phys. Rev. Lett.* **114** 055501
[9] Yang Y, Tekic J, Hong X R, Jia L P, Wei P and Duan W S 2019 *Waves in Random and Complex Media* **29** 236–46
[10] Wei P, Tekic J, Yang Y, Jiang X, Duan W S and Yang L 2016 *Waves in Random and Complex Media* **26** 592–98
[11] Yang Y, Wang C L, Duan W S, Chen J M and Yang L 2015 *Commun Nonlinear Sci Numer Simul.* **20** 1
[12] Hasnain J, Jungblut S, Troster A and Dellago C 2014 *Nanoscale* **6** 10161
[13] Vanossi A, Manini N and Tosatti E 2012 *Proc Nat Acad Sci.* **109** 16429
[14] Vanossi A and Tosatti E 2012 *Nature Mater* **11** 97
[15] Jia L P, Li X F, Yin H M and Cao P F 2019 *J. Phys.: Conf. Ser.* **1325** 012174
[16] Vanossi A, Manini N, Divitini G, Santoro G E and Tosatti E 2006 *Phys. Rev. Lett.* **97** 056101
[17] Vanossi A, Manini N, Caruso F, Santoro G E and Tosatti E 2007 *Phys. Rev. Lett.* **99** 206101
[18] Braun O M and Kivshar Yu S 2004 *The Frenkel-Kontorova Model: Concepts, Methods, and Applications* (Berlin: Springer)