IOP Publishing Physica Scripta
Phys. Scr. 95 (2020) 055214 (9pp) https://doi.org/10.1088/1402-4896/ab7851

A class of unexcited hyperjerk systems with megastability and its analog and microcontroller-based embedded system design

Rajagopal Karthikeyan[1], Serdar Çiçek[2], Viet-Thanh Pham[3], Akif Akgul[4] and Prakash Duraisamy[1]

[1] Center for Nonlinear Dynamics, Institute of Research and Development, Defence University, Ethiopia
[2] Department of Electronic and Automation, Vocational School of Hacıbektaş, Nevşehir Hacı Bektaş Veli University, Hacıbektaş 50800, Nevşehir, Turkey
[3] Faculty of Electrical and Electronic Engineering, Phenikaa University, Yen Nghia, Ha Dong district, Hanoi 100000, Vietnam
[4] Department of Electrical and Electronic Engineering, Faculty of Technology, Sakarya University of Applied Sciences, Sakarya, Turkey

E-mail: aakgul@sakarya.edu.tr and aakgul@subu.edu.tr

Received 15 December 2019, revised 8 February 2020
Accepted for publication 20 February 2020
Published 2 March 2020

Abstract

Discovering chaotic systems with special features is of interest in the recent years. In this paper we introduce a new class of simple hyperjerk systems with infinitely coexisting chaotic attractors usually termed as megastability. The novelty of the proposed systems is that the systems shows megastability without external excitation which was not the case in most of the existing megastable attractors discussed in the literatures. Various dynamical properties of one of the proposed systems like the stability of equilibrium points, bifurcation and Lyapunov spectrum are discussed. Also, a circuit realization using off-the-shelf components is shown to prove the implementation feasibility of the systems. In addition, microcontroller based embedded design with graphic LCD of obtained new simple hyperjerk megastable oscillators was realized. Thus, the obtained new oscillators can be used both in a variety of real digital applications such as random number generators, encryption, communication and for educational purposes.

Keywords: chaos, megastable, hyperjerk, bifurcation, microcontroller-based embedded design, electronic circuit implementation

(Some figures may appear in colour only in the online journal)

1. Introduction

If a physical variable at time't' is denoted with $x(t)$ then the third derivative $\frac{d^3x}{dt^3}$ represents the jerk and the fourth derivative $\frac{d^4x}{dt^4}$ can be represented as hyperjerk as defined mainly by Sprott *et al* [1, 2]. Jerk oscillator which holds chaotic behavior finds its attraction in secure communication because of its simple configuration. Many encryption algorithms were developed using such oscillators [1, 3]. Compare to jerk oscillators, hyperjerk oscillators show intricate characteristics with similar simple algebraic equation [4].

Hyperjerk systems are found in various real time systems such as secure communication, neuron modeling, etc, In application point of view many hyperjerk oscillators found with chaotic attractors, which are very useful to model neurons [5]. In order to mimic neuron synapse, researchers seek complex dynamical systems which shows different special

 © 2020 IOP Publishing Ltd Printed in the UK

Table 1. Various cases of SHMO.

System	$f(y, z, w)$	Parameters
SHMO-1	$-by - dw + cz - ez^5$	$d = 1,\ b = 2, c = 9,\ a = 2,\ e = 0.7,\ n = 1$
SHMO-2	$-by - cz - dw$	$a = -13, b = 5, c = 5, d = 5, n = 3$
SHMO-3	$-by - z - w$	$a = 1, b = 0.7, n = 1$
SHMO-4	$-y - z - cw - b\sin(z)$	$a = 1, b = 0.1, c = 5, n = 1$
SHMO-5	$-z - w - b\sin(y)$	$a = 1, b = 1, n = 1$

properties. Hyperjerk systems found more suitable for such applications [4]. Recently some literatures display hyperjerk oscillators possess special properties like multi scroll [6], multi wing [7] chaotic attractors, coexistence of multiple attractors, symmetricity [8] etc.

Multistability represents the existence of more than one attractor in a nonlinear dynamical system and transition from one state to other by changing the initial condition [9]. It increases the flexibility of a system to deal among different coexisting states without changing parameters. The attracting state of a multistable system highly sensitive to the initial conditions compared with monostable systems. Multistability can be categorized into two; Extreme multistability which holds infinite uncountable number of coexisting attractors [10] where as Megastability refers infinite countable number of coexisting attractors [11].

Megastability is identified and categorized by Sprott *et al* [12], while investigating a periodically forced oscillator with a spatially periodic damping term. It has been portrayed with coexisting attractors including limit cycles, tori and chaotic attractors which form a layered cabbage-like structure. Wang *et al* [13] developed a novel oscillator with infinite coexisting asymmetric attractors. Tang *et al* [14, 15] designed a chaotic system with megastable type which is similar to a carpet-like structure. Li *et al* [16–18] construct 2D and 3D in a Programmable Chaotic Circuit and studied the behavior of Infinitely Many Attractors. Vo *et al* proposed two-dimensional nonlinear oscillator which holds feature of having layer–layer self excited and hidden coexisting attractors and proved that the complexity of the system behavior increases with such special properties [19].

In circuit designs with analog circuit devices, tolerance values of analog devices vary depending on temperature. Furthermore, it is difficult to update analog circuit designs. In embedded system circuit designs, the temperature-dependent tolerance value is very small and the circuit design is much easier to update. In addition, today's embedded system designs consume much less energy. For these reasons, nowadays, embedded system devices have started to be used in the design of chaotic systems [20–25].

Motivated from the above discussion in this paper we are proposing a family of simple hyperjerk chaotic system with megastable property. The novelty of the proposed system is there is no external excitation.

Table 2. Functions used in the Jacobian matrix (2).

System	Functions of (2)		
	f_y	f_z	f_w
SHMO-1	$-b$	$c - 5ez$^4	$-d$
SHMO-2	$-b$	$-c$	$-d$
SHMO-3	$-b$	-1	-1
SHMO-4	-1	$b\cos(z) - 1$	$-c$
SHMO-5	$-b\cos(y)$	-1	-1

2. Simple hyperjerk megastable oscillators (SHMO)

Discovering chaotic attractors with infinite number of coexisting attractors [16–18] is of interest in the recent years. Of these some of them are named as 'Megastable' after [12]. But most of these megastable oscillators are 3D nonautonomous systems and excited by periodic or quasiperiodic forcing [26–29]. Hence we are interested in proposing a few new megastable oscillators which are 4D hyper jerk systems showing megastability without external excitation. The dimensionless model of the SHMO is,

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= w \\ \dot{w} &= f(y,\ z,\ w) + a\cos(nx) \end{aligned} \tag{1}$$

and depending on the various selections of function $f(y,\ z,\ w)$, we can derive different cases of SHMO as shown in table 1.

The Jacobian matrix of the SHMO system can be derived as,

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -an\sin(nx) & f_y & f_z & f_w \end{bmatrix} \tag{2}$$

where the functions f_y, f_z, f_w are given in table 2.

It can be verified that the equilibrium points of the SHMO systems are $\left(\frac{(2k+1)\pi}{2n}, 0, 0, 0\right)$ where k is an arbitrary integer number. Using the equilibrium points with the Jacobian matrix (2), the characteristic polynomial is

$$\lambda^4 - f_w\lambda^3 - f_z\lambda^2 - f_y\lambda + an\sin(nx) \tag{3}$$

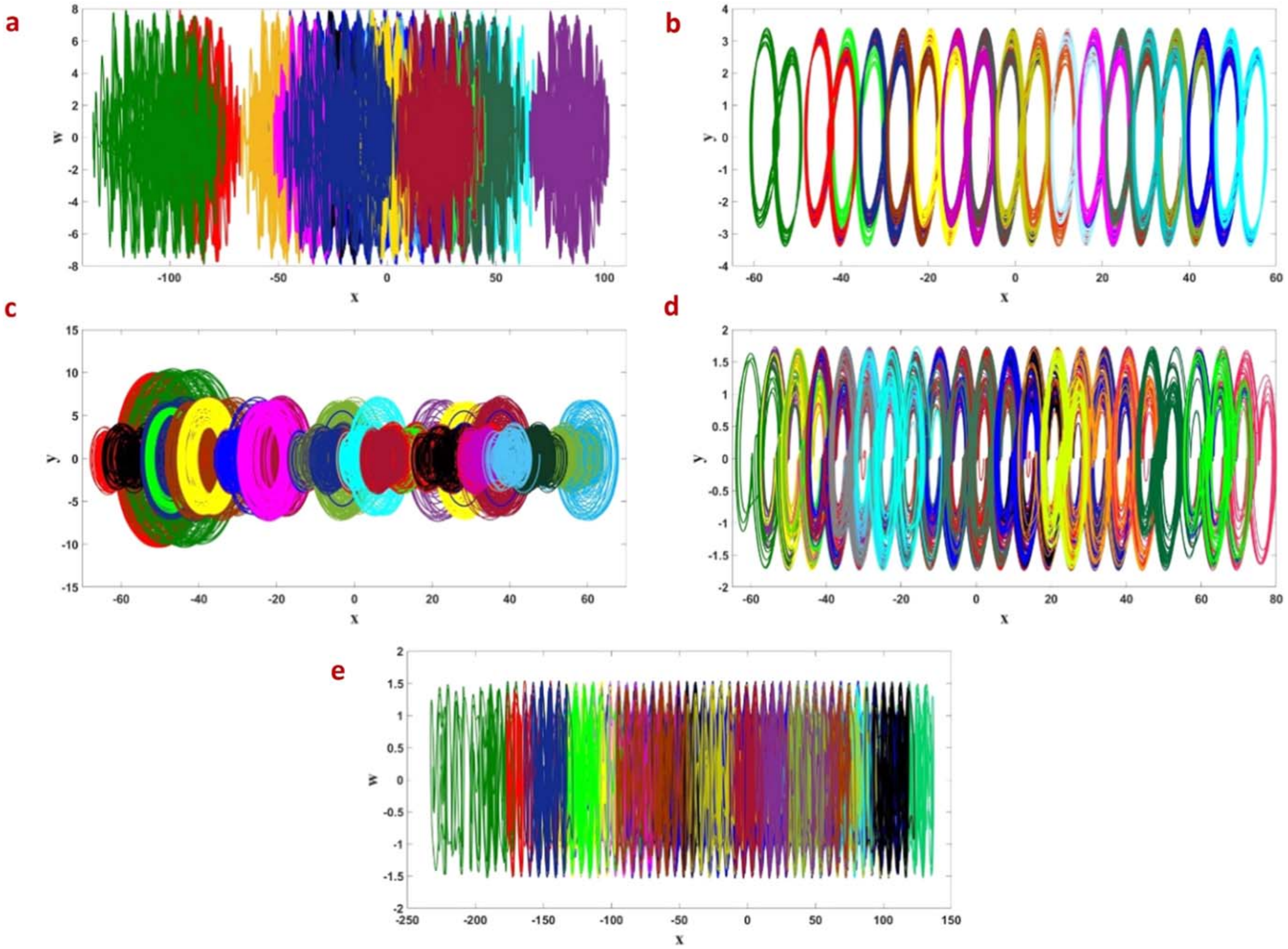

Figure 1. Phase portrait of SHMO systems ((a)–(e) for SHMO-1 to SHMO-5 respectively) for 26 initial conditions located on the x-axis (from $x = -50$ to $x = +50$ with steps equal to 4) while the other states initial conditions are kept to 0.

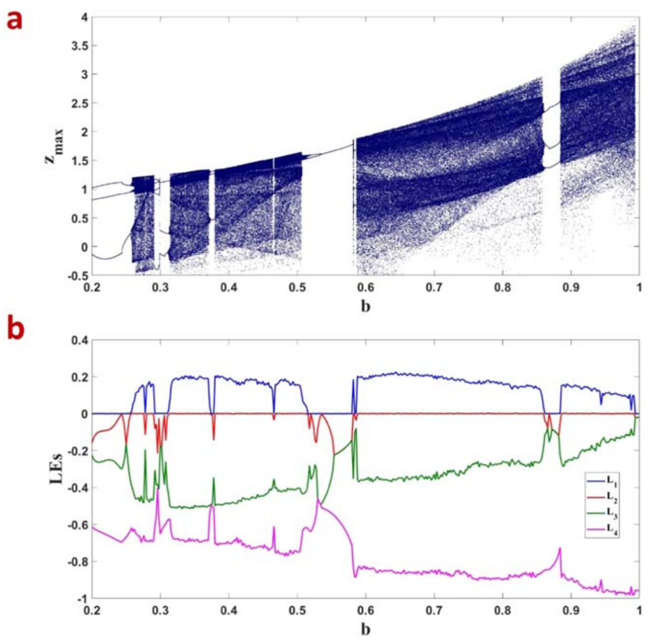

Figure 2. (a) The bifurcation of the SHMO-3 system with parameter b; (b) The corresponding Les.

As per the Routh-Hurwitz criterion the principal minors are

$$\Delta_1 = -f_w;\ \Delta_2 = \begin{bmatrix} -f_w & 1 \\ -f_y & -f_z \end{bmatrix};$$
$$\Delta_3 = \begin{bmatrix} -f_w & 1 & 0 \\ -f_y & -f_z & -f_w \\ 0 & an\sin(nx) & -f_y \end{bmatrix}$$
$$\Delta_4 = \begin{bmatrix} -f_w & 1 & 0 & 0 \\ -f_y & -f_z & -f_w & 1 \\ 0 & an\sin(nx) & -f_y & -f_z \\ 0 & 0 & 0 & an\sin(nx) \end{bmatrix} \tag{4}$$

and the principal minors have to be positive for the system to be stable. The condition for principal minors for positive the conditions are,

$$\Delta_1 = -f_w;\ \Delta_2 = f_y + f_w f_z;\ \Delta_3 = -an\sin((\pi(2k+1))/2)f_w^2 - f_z f_y f_w - f_y^2$$
$$\Delta_4 = -an\sin((\pi(2k+1))/2)(an\sin((\pi(2k+1))/2)f_w^2 + f_z f_y f_w + f_y^2) \tag{5}$$

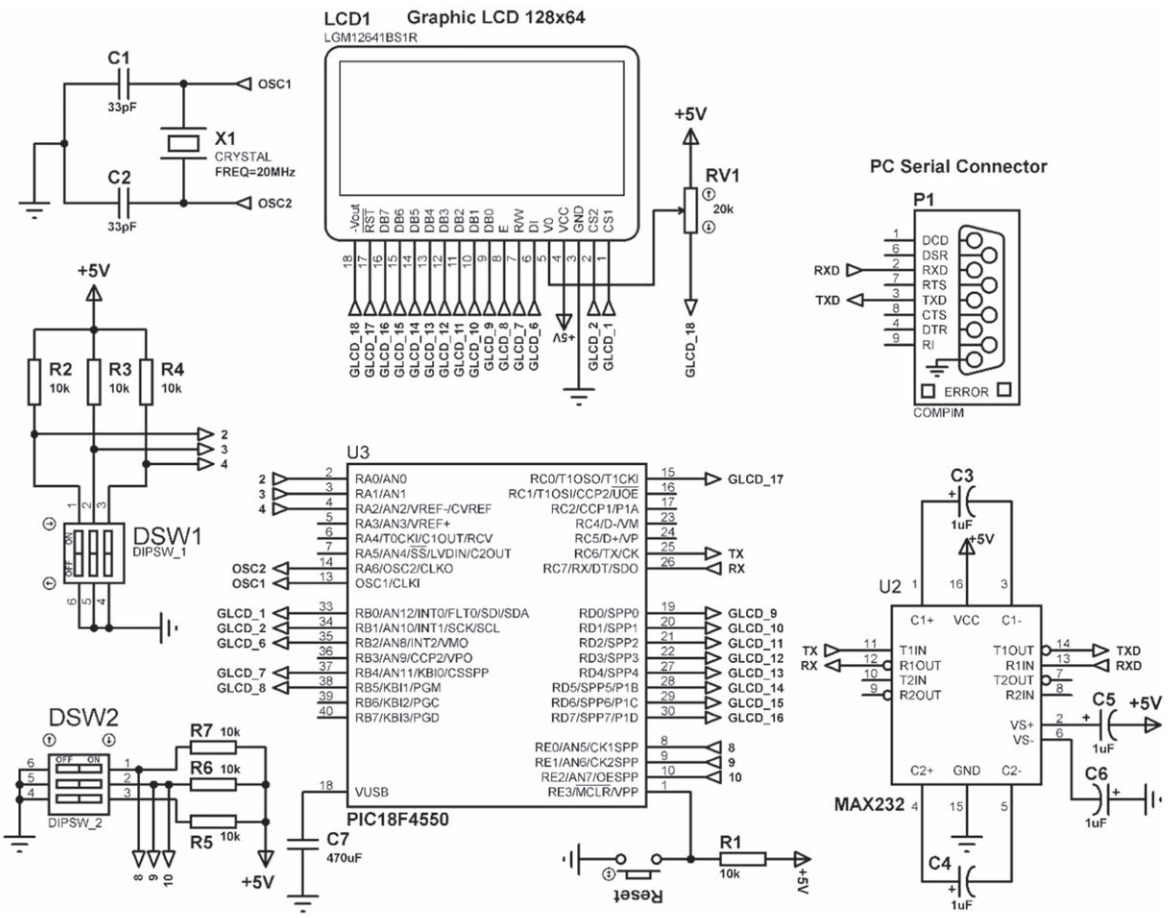

Figure 3. The designed microcontroller based embedded system circuit diagram for SHMO.

Table 3. Values of DSW1 dip-switch (1-On, 0-Off).

000	SHMO-1 system
001	SHMO-2 system
010	SHMO-3 system
011	SHMO-4 system
100	SHMO-5 system

Table 4. Values of DSW2 dip-switch (1-On, 0-Off).

000	*x-y* phase portrait
001	*x-z* phase portrait
010	*x-w* phase portrait
011	*y-z* phase portrait
100	*y-w* phase portrait
101	*z-w* phase portrait

We can easily verify that the principal minors are not positive for the parameter values in table 1 and confirm that the SHMO systems are unstable. The phase portraits of the SHMO systems for different initial conditions selected from $-50 \leqslant x \leqslant 50$ with a step of 4 are shown in figure 1. It should be noted that all the attractors shown in figure 1 (different colors) are chaotic and never disintegrates to a torus as like in many other cases of megastable oscillators [26–29]. From figure 1 we could see that when the initial conditions for the state variable x is shifted, the attractor also changes to the environment around the equilibrium and thus shows infinitely coexisting attractors.

To analyze the dynamical behavior of the SHMO systems, we considered the SHMO-3 system. The bifurcation plot of the SHMO-3 system is derived for the parameter b and the local maxima of the state variable z is plotted. We could see that the system shows period doubling and inverse period doublings in the bifurcation plot shown in figure 2(a). To be more specific, the SHMO-3 system shows chaotic oscillations for the values of $b \in [0.257, 0.289]$, $[0.312, 0.367]$, $[0.38, 0.505]$, $[0.5868, 0.857]$, $[0.885, 0.993]$. It can be noted that every time the SHMO-3 enters chaotic region by period doubling route and takes an inverse period doubling exit from chaos. The corresponding finite time Lyapunov exponents (LEs) using the Wolfs algorithms [30] for a finite time of 20000 s are shown in figure 2(b).

3. Microcontroller based embedded design of the SHMO systems

In this section, microcontroller based embedded design with graphic LCD was designed in order to use the obtained

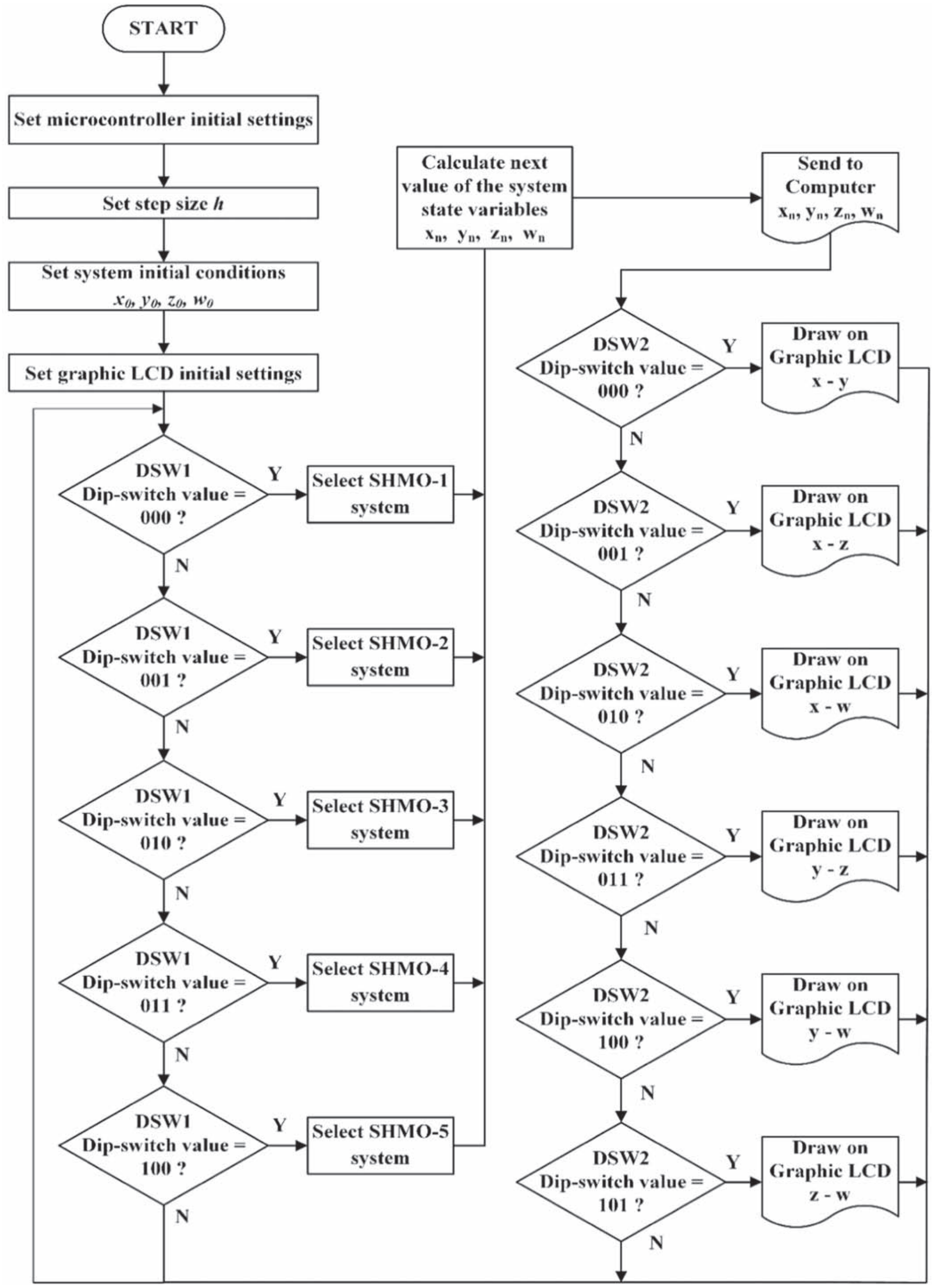

Figure 4. The flow diagram of the microcontroller program.

SHMO systems in various digital applications and for educational purposes. Microchip PIC18F4550 product was used as microcontroller in the design. The PIC18F4550 microcontroller features 25KB program memory, 2KB data memory, 48Mhz operating speed, built-in USB module. The designed circuit diagram of microcontroller based embedded system is given in figure 3. The design has a 128 × 64 graphic LCD and two dip-switches to monitor the phase portraits of SHMO systems for educational purposes. DSW1 dip-switch used in the design can select which SHMO system to operate. By using DSW2 dip-switch used in the design, it is possible to select which phase portrait of the SHMO system will be drawn on the graphic LCD. Tables 3 and 4 as presented below show the values of the DSW-1 and DSW-2 dip-switches, respectively.

State variables (x, y, z, w) calculated by microcontroller can be transferred to computer by asynchronous serial communication. In this way, the results of the selected SHMO system can be used in various digital applications. The microcontroller program is written according to the CCS C compiler, which is based on the standard C programming language. Euler method was used to calculate the systems

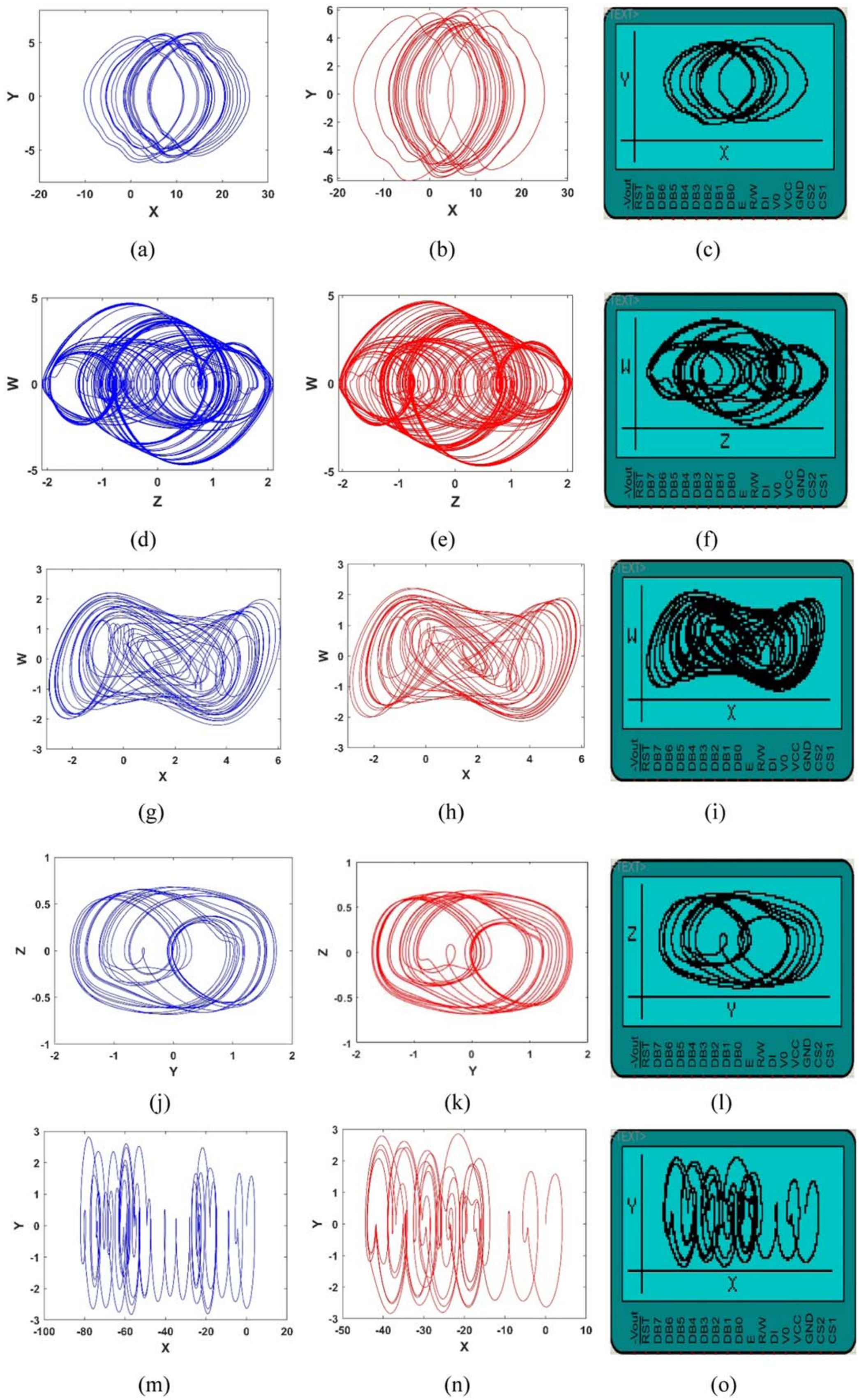

Figure 5. Phase portraits of Matlab and microcontroller results and graphic LCD views, of the SHMO systems, respectively (a)–(c) SHMO-1 (d)–(f) SHMO-2 (g)–(i) SHMO-3 (j)–(l) SHMO-4 (m)–(o) SHMO-5.

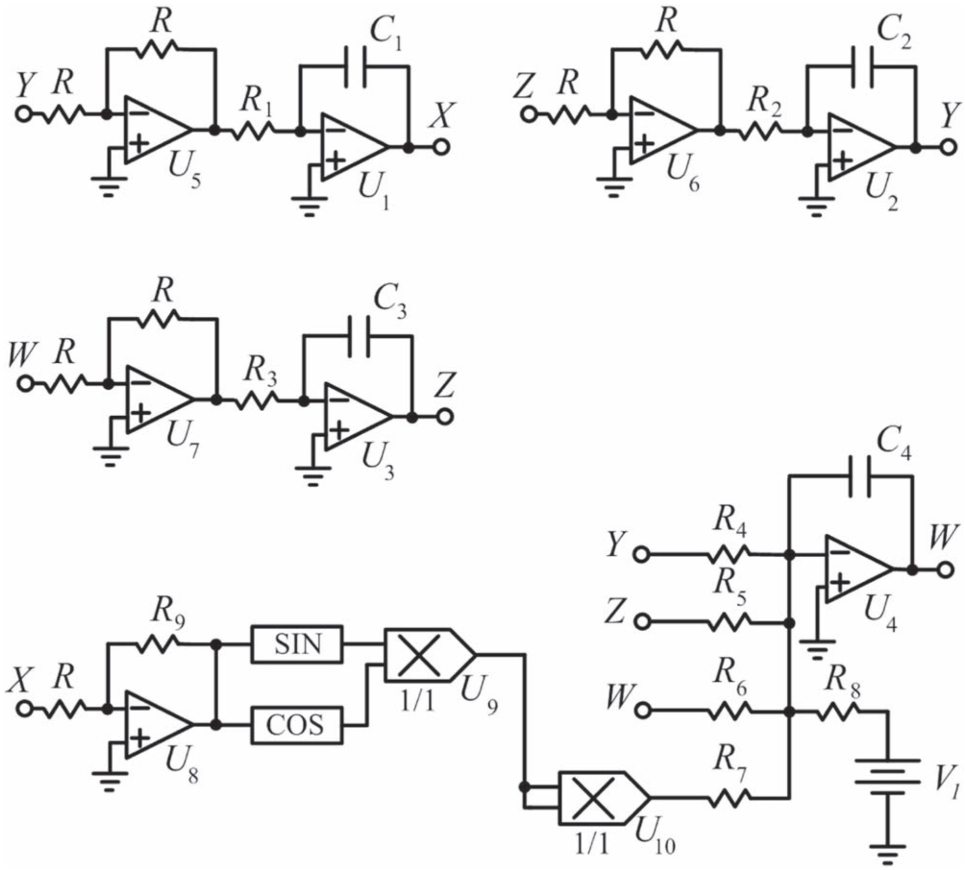

Figure 6. A possible circuital design for emulating System SHMO-3.

state variables. The flow diagram of the program loaded on the microcontroller is given in figure 4.

For testing purposes, 3,000,000 data were obtained from microcontroller and Matlab numerical simulation for each the SHMO systems. Initial conditions of the SHMO systems were taken as (x_0, y_0, z_0, w_0) = (0, 0, 0, 0). In the microcontroller calculation, step size h = 0.0001 is taken. There is difference between the numerical computational and trigonometric (sine, cosine) calculation algorithm sensitivity of the Matlab program and the computational sensitivity of the microcontroller. This difference occurred due to the very sensitive dependence of chaotic systems on values. It was seen that only x state variable had higher difference between Matlab and microcontroller results than other state variables. The phase portraits of the systems were drawn from these data and the numerical simulation results obtained from the Matlab program. In figure 5, Matlab numerical simulation results, microcontroller results and graphical LCD views of some phase portraits of the SHMO systems are given. When the figure 5 is examined, the numerical analysis results of the Matlab program, the results obtained from the microcontroller output and the results displayed on the graphic LCD confirm each other. Due to the difference between Matlab program and microcontroller calculation accuracy, very small calculation differences were observed. With DSW-1 and DSW-2 dip-switch devices on the circuit, the user can select the desired SHMO system and see the path of the desired phase portraits of the selected system on the graphic LCD and take the system state variables values from the computer.

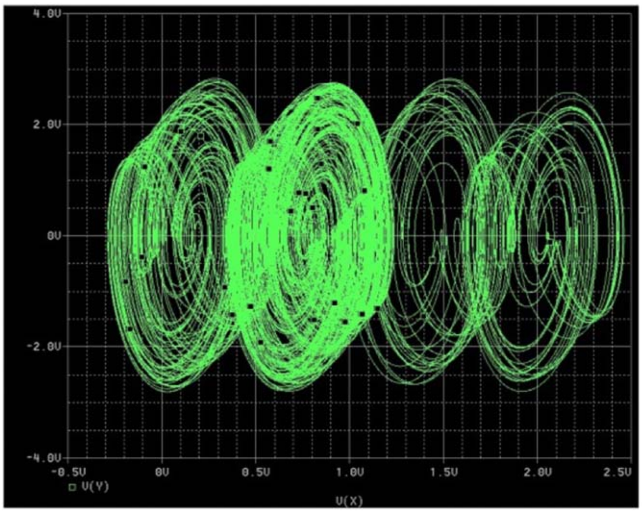

Figure 7. Attractor observed in the circuit for the duration of time 300 ms.

4. Circuit implementation

It is possible to design an electronic circuit for System SHMO-3 (see figure 6). The circuit is based on four operational amplifiers ($U_1 - U_4$), which are connected as integrators [31–34]. As can be seen in figure 6, the voltages of such four integrators are X, Y, Z, and W. The components in circuit are $R_1 = 100\ \mathrm{k}\,\Omega$,

$R_2 = R_3 = R_5 = R_6 = R_8 = R = 10$ k Ω, $R_4 = 14.28$ k Ω, $R_7 = 1.25$ k Ω, $R_9 = 25$ k Ω, $V_1 = -1\ V_{DC}$, and $C_1 = C_2 = C_3 = C_4 = 10$ nF. By implementing the circuit in PSpice, the circuit's attractor is reported in figure 7. As can be seen in figure 7, the designed circuit displays multi-scroll attractor. It is noted that the observed attractor in figure 7 is chaotic. There is an agreement between the theoretical results and circuit-based ones.

5. Conclusion

A new class of hyperjerk chaotic systems showing megastability is discussed. The novelty of the proposed work is that these systems exhibit megastability without external excitation which was not the case in the megastable oscillators discussed in the literatures so far. Secondly, most of the megastable systems discussed in the literatures have the character of disintegrating in to a non-chaotic attractor (tori) as the initial conditions are changes whereas the proposed systems are always chaotic irrespective of the initial conditions. Also it is worthy to mention that this is the first type of hyperjerk system showing megastability as all the megastable systems in the literatures are jerk type systems. Various dynamical behavior of one of the proposed system is discussed using Lyapunov spectrum and bifurcation plots. A PSpice simulation is conducted for one of the proposed systems whose circuit realization is done using off-the-shelf components. In addition, microcontroller based embedded design with graphic LCD was designed in order to use the obtained SHMO systems. Thus obtained SHMO systems can be used in various digital engineering applications such as encryption, communication, random number generator and also for educational purposes.

ORCID iDs

Akif Akgul https://orcid.org/0000-0001-9151-3052

References

[1] Chlouverakis K E and Sprott J C 2006 Chaotic hyperjerk systems *Chaos Solitons Fractals* **28** 739–46
[2] Sprott J C 1997 Some simple chaotic jerk functions *Am. J. Phys.* **65** 537–43
[3] Rajagopal K, Kingni S T, Kuiate G F, Tamba V K and Pham V-T 2018 Autonomous jerk oscillator with cosine hyperbolic nonlinearity: analysis, FPGA implementation, and synchronization *Advances in Mathematical Physics, Hindawi* **2018** 7273531
[4] Dalkiran F Y and Sprott J C 2016 Simple chaotic hyperjerk system *Int J Bifurc Chaos* **26** 1650189
[5] Li X and Rakkiyappan R 2013 Impulsive controller design for exponential synchronization of chaotic neural networks with mixed delays *Commun. Nonlinear Sci. Numer. Simul.* **18** 1515–23
[6] Cafagna D and Grassi G 2003 New 3D-scroll attractors in hyperchaotic Chua's circuits forming a ring *Int. J. Bifurc. Chaos.* **13** 2889–903
[7] He S, Sun K and Zhu C-X 2013 Complexity analyses of multi-wing chaotic systems *Chin. Phys.* B **22** 050506
[8] Li C, Sprott J C and Xing H 2017 Constructing chaotic systems with conditional symmetry *Nonlinear Dyn.* **87** 1351–8
[9] Li C and Sprott J C 2014 Multistability in the Lorenz system: a broken butterfly *Int J Bifurc Chaos* **24** 1450131
[10] Chen M, Sun M, Bao B, Wu H, Xu Q and Wang J 2018 Controlling extreme multi-stability of memristor emulator-based dynamical circuit in flux–charge domain *Nonlinear Dyn.* **91** 1395–412
[11] Wei Z *et al* 2018 A modified multistable chaotic oscillator *Int J Bifurc Chaos* **28** 1850085
[12] Sprott J C, Jafari S, Khalaf A J M and Kapitaniak T 2017 Megastability: coexistence of a countable infinity of nested attractors in a periodically-forced oscillator with spatially-periodic damping *The European Physical Journal Special Topics* **226** 1979–85
[13] Wang Z *et al* 2018 A new oscillator with infinite coexisting asymmetric attractors *Chaos Solitons Fractals* **110** 252–8
[14] Tang Y-X, Khalaf A J M, Rajagopal K, Pham V-T, Jafari S and Tian Y 2018 A new nonlinear oscillator with infinite number of coexisting hidden and self-excited attractors *Chin. Phys.* B **27** 040502
[15] Tang Y *et al* 2018 Carpet oscillator: a new megastable nonlinear oscillator with infinite islands of self-excited and hidden attractors *Pramana* **91** 11
[16] Li C, Sprott J C, Hu W and Xu Y 2017 Infinite multistability in a self-reproducing chaotic system *Int. J. Bifurcation Chaos* **27** 1750160
[17] Li C, Sprott J C and Mei Y 2017 An infinite 2D lattice of strange attractors *Nonlinear Dyn.* **89** 2629–39
[18] Li C and Sprott J C 2018 An infinite 3D quasiperiodic lattice of chaotic attractors *Phys. Lett.* A **382** 581–7
[19] Vo T P, Shaverdi Y, Khalaf A J M, Alsaadi F E, Hayat T and Pham V-T 2019 A giga-stable oscillator with hidden and self-excited attractors: a megastable oscillator forced by his twin *Entropy* **21** 535
[20] Chiu R, Mora-Gonzalez M and Lopez-Mancilla D 2013 Implementation of a chaotic oscillator into a simple microcontroller *IERI Procedia* **4** 247–52
[21] Kaçar S 2016 Analog circuit and microcontroller based RNG application of a new easy realizable 4D chaotic system *Optik* **127** 9551–61
[22] Akgül A, Arslan C and Arıcıoğlu B 2019 Design of an interface for random number generators based on integer and fractional order chaotic systems *Chaos Theory and Applications* **1** 1–18 (https://dergipark.org.tr/tr/pub/chaos/issue/50396/653247)
[23] Yener S C, Barbaros C, Mutlu R and Karakulak E 2017 Implementation of microcontroller-based memristive chaotic circuit *Acta Phys. Polon.* A **132** 1058–60
[24] Mèndez-Ramìrez R, Arellano-Delgado A, Cruz-Hernàndez C and Martìnez-Clark R 2017 A new simple chaotic Lorenz-type system and its digital realization using a TFT touch-screen display embedded system *Complexity* **2017** 6820492
[25] Giakoumis A E, Volos C K, Stouboulos I N, Polatoglou H M and Kyprianidis I M 2018 Chaos generator device based on a 32 bit microcontroller embedded system *7th Int. Conf. on Modern Circuits and Systems Technologies (MOCAST), 7–9 May, Thessaloniki, Greece*
[26] Rajagopal K, Singh J P, Roy B K and Karthikeyan A 2019 Dissipative and conservative chaotic nature of a new quasi-periodically forced oscillator with megastability *Chin. J. Phys.* **58** 263–72
[27] Pankaj Prakash K, Rajagopal J P, Singh and Roy B K 2018 Megastability, multistability in a periodically forced

conservative and dissipative system with signum nonlinearity *Int. J. Bifurcation Chaos* **28** 1830030
[28] Prakash P, Rajagopal K, Singh J P and Roy B K 2018 Megastability in a quasi-periodically forced system exhibiting multistability, quasi-periodic behaviour, and its analogue circuit simulation *AEU—International Journal of Electronics and Communications* **92** 111–5
[29] Jahanshahi H, Rajagopal K, Akgul A, Sari N N, Namazi H and Jafari S 2018 Complete analysis and engineering applications of a megastable nonlinear oscillator *Int. J. Non Linear Mech.* **107** 126–36
[30] Wolf A, Swift J B, Swinney H L and Vastano J A 1985 Determining lyapunov exponents from a time series *Physica D* **16** 285–317
[31] Li C, Sprott J C, Akgul A, Iu H H and Zhao Y 2017 A new chaotic oscillator with free control. Chaos: an interdisciplinary *Journal of Nonlinear Science* **27** 083101
[32] Pehlivan İ, Kurt E, Lai Q, Basaran A and Kutlu M C 2019 A Multiscroll chaotic attractor and its electronic circuit implementation *Chaos Theory and Applications* **1** 29–37 (https://dergipark.org.tr/tr/pub/chaos/issue/50396/653298)
[33] Ma J, Wu X, Chu R and Zhang L 2014 Selection of multi-scroll attractors in jerk circuits and their verification using Pspice *Nonlinear Dyn.* **76** 1951–62
[34] Lü J H and Chen G R 2006 Generating multiscroll chaotic attractors: theories, methods and applications *Int. J. Bifurcation Chaos* **16** 775–858