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Quantum reversibility in cooperative interaction of the atom system with bi-modal cavity field in Raman conversion

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Abstract

The restoration of the initial state in the Raman interaction of the two atoms with the quantified pump and Stokes (anti-Stokes) cavity modes is analytically obtained. For this the *su* (2) symmetry representation of the bimodal field and a system of two atoms in scattering interaction is proposed for the simplification of the problem. Using the proprieties of the generators *su* (2) algebra the exact non-stationary solution of the system of two atoms in interaction with the bimodal field is found, taking into consideration the initial disentangled state between the atomic and field superposition. It is demonstrated that after this collapse and revivals of the inversion of flying atoms in the processes of absorption and emission of the photons from Stokes and pump modes the system atoms and field becomes entangled. The conditions of the restoration of the initial state of the atom-field system were established as a function of the flying time through the resonator possible. A set of discrete flying time intervals for the restoration of the initial disentangled state was found.

Keywords: quantum entanglement and disentanglement, quantum reversibility, Raman cooperative nutation, nonlinear cooperative cavity effects

(Some figures may appear in colour only in the online journal)

1. Introduction

The interaction of multi-level radiators with a two-mode cavity field was in the focus of the attention of many experimental and theoretical [1–7]. This investigation is connected with the application of the multi-level system as q-bits in quantum computing and quantum processing of information [8–10]. In many cases, the distinguished ensembles of q-bits are used for the realization of quantum registers [11–15]. In order to treat the interaction of a multi-level atom with a cavity field of arbitrary detuning, a suitable quantum mechanical theory has been proposed in [16] and developed for special situations in [17, 28]. Since the total Hamiltonian connects the Fock states in the small region of the Hilbert space, it can be solved in each subspace independently. An attractive approach of the interaction of the atomic ensemble with the single-mode cavity field with an arbitrary distribution of the photons on the Fock states was proposed in [8–10].

The Bose–Einstein condensation of atomic ensembles provides an opportunity to study ensembles of two-level atoms as an indistinguishable ensemble of radiators [20–23]. According to the principle of indistinguishably between the particles of quantum mechanics, the 2^N states of N two-level atoms can be reduced to $N+1$ states in the processes of coherent excitation of the system situated in the volume with the dimensions than radiation wavelength [8, 24–26]. This happens because the superposition of two atomic states is considered as a single collective state: atom, A, is in its excited state, B, in the ground state, and vice-versa : $(|0_A, 1_B\rangle + |1_A, 0_B\rangle)/\sqrt{2} = |1_A\rangle$. The similar conclusions maybe given for two excitation in the ensemble of three atoms, A, B and C, where the superposition of the three states $\{|1_A, 1_B, 0_C\rangle + |1_A, 0_B, 1_C\rangle + |0_A, 1_B, 1_C\rangle\}/\sqrt{3}$ may be considered as a single collective excitation state. It happens when the system of atoms are situated in the volume less than radiation wavelength. This cooperative effect can be exported for the ensemble of atoms in scattering resonance with the two-mode cavity field.

The interaction of the pump field with nonlinear induced Raman was the subject of theoretical and application studies [27, 29–31]. For example in [27] the analytic and numerical solutions were obtained for the equations describing the propagation of several pulses of different frequencies through a Raman medium. The transient case is treated, and the analysis includes Stokes, pump, and anti-Stokes frequency components. The effect of phase mismatch Δk between these waves is investigated. An analytic solution is found that describes the phenomenon of transient Stokes-anti-Stokes gains suppression in the limit of $\Delta k = 0$. In [29] the authors propose the classical aspects of the theory of superradiance under Raman light scattering conditions allowing for pump depletion in the single-mode approximation. The author demonstrates that pump depletion imposes an additional constraint on the observation of scattering super-radiance.

The quantum aspect of Raman conversion of the photons between the above components and two-photon masers remains in the center of attention in many investigations of many investigations [33, 28]. For example in the open micro-resonators with a lifetime of atoms less than the Rabi frequency, the quantum lasing effect was studied in [28, 32–34]. This approach proposed to declare the two-mode states of the cavity as a possible laser conversion of the total number of photons, n = $2j$, between the pump and anti-Stokes modes. From a physical point of view, it is important to study the opposite situation when the flying time $\tau_0 = l/v$ of the atoms through the cavity is larger or comparable with the inverse value of mean quantum Rabi frequency [26] defined in the section 2. Here the l and v are the cavity length and velocity of atom during the flying time through it. In this situation, we were limited to the 'good cavity limit' in which the photon losses from the cavity $\kappa \sim c(1-r)/l$ is less than the quantum Rabi frequency and atomic 'losses' from cavity $1/\tau_0$, where r is the reflection coefficients of the mirrors. In other words, the lifetime of the photons in the cavity is larger than the flying time, τ_0, through the resonator. In this paper, we have studied the behavior of the ensemble of N-atoms in cooperative induced Raman interaction with the photons from a two-cavity field named pump and anti-Stokes. In comparison with the [33, 28] we have studied the nutation process of this ensemble in the process of the photon conversion from one mode to another. For n photons distributed between the pump field we name this state by the ket vector $|n\rangle_p|0\rangle_a$, where $|n\rangle_p$ and $|0\rangle_a$ are the pump and anti-Stokes states respectively. Acting with conversion operator $\hat{L}^+ = \hat{a}^\dagger\hat{b}$ on these states we can create with operator $\hat{a}^\dagger$ the photon in the anti-Stocks mode simultaneously annihilating the photon from the pump mode with operator $\hat{b}$: $\hat{L}^+|n\rangle_p|0\rangle_a = n_p|n-1\rangle_p|1\rangle_a$. Acting with this operator k times we convert k photons from the pump to anti-Stokes modes ($\hat{L}^k|n\rangle_p|0\rangle_a = n(n-1)\ldots(n-k+1)1\times 2\times 3\times\ldots\times k|n-k\rangle_p|k\rangle_a$).

This procedure can't be continued till infinity because the number of photons in pump mode is considered finite so that $(\hat{L}^+)^{n+1}|n\rangle_p|0\rangle_a = 0$. As follows from this representation the conversion procedure can be easily be described by the angular momentum state $|m, j\rangle$, where m is the difference between the number of photons in the anti-Stokes and Pump modes $m = N_a + N_P$.

Following the single-mode approach proposed in the [8] we export this conception to the induced scattering conversion between the two egan states of cavity modes (see figures 1, 2). The exact solution of the ensemble formed from one -five radiators can be analytically obtained. in the process of quantum exchange of energy with a single-mode cavity field [8]. The reversibility problem with the increasing number of atoms in the cavity becomes a complicated problem from analytically points of view. This effect consists of in the restoration of the initial disentangled state of atoms and filed after the flying time of the atomic ensemble through the cavity. In section 2 we develop such a method for undistinguished atoms in the scattering resonance with two-cavity modes. Following this approach, we proposed the new type of correlations between the photons of the pump and anti-Stokes modes in the process of cooperative conversion in the induced Raman process The dynamical reversible conditions for two undistinguished atoms flying through the bimodal cavity in the Raman are obtained. The exact solution of the Schrödinger equation for two-level radiators in the Raman induced interaction with the bimodal quantified electromagnetic field has been obtained.

2. Non-equilibrium approach for the description of cooperative interaction of *N* two-level atoms with a cavity field

Let us consider N two-level atoms in the scattering interaction with two-cavity modes of a resonator with the frequencies ω_a and ω_b (see figure 2). The Hamiltonian of the system which describes this interaction can be represented in the following form

$$H = \hbar\omega_0\hat{S}_z + \hbar\omega L_z + \lambda\hbar\,(\hat{S}^+\hat{L}^- + \hat{L}^+\hat{S}^-). \tag{1}$$

Here $\omega = \omega_b - \omega_a$ is difference between the frequencies of the modes b and a; and $\hbar\omega_0 = E_2 - E_1$ is the energy difference between the second excited and ground states of atoms. The scattering transition takes place through the virtual state $|3\rangle$ of three- level systems represented in figure 1. In the Hamiltonian (1) is introduced the following notations $\hat{S}_z = \sum_j^N(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)/2$ is the collective inversion operator for two level system in interaction with cavity modes; $\hat{S}^+ = \sum_j^N|e_j\rangle\langle g_j|$ and $S^- = \sum_j^N|g_j\rangle\langle e_j|$ are the collective excitation and lowering operators of two atoms which satisfy the following commutation relations: $[\hat{S}^+, \hat{S}^-] = 2\hat{S}_z$ and $[\hat{S}_z, \hat{S}^\pm] = \pm\hat{S}^\pm$. The similar operators may be introduced for two modes of electromagnetic field in the cavity. The expressions $\hat{L}_z = (\hat{b}^\dagger\hat{b} - \hat{a}^\dagger\hat{a})/2$ and $\hat{L}^+ = \hat{a}^\dagger\hat{b}$, $\hat{L}^- = \hat{a}\hat{b}^\dagger$ are the bi-boson operators of two mode fields which represent the photon difference between the

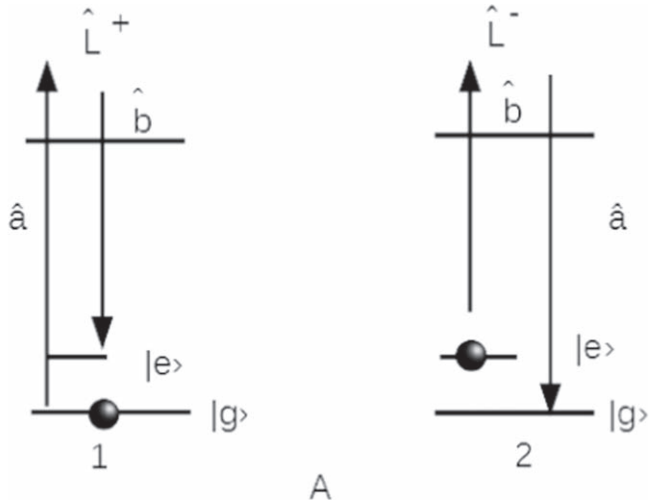

Figure 1. The ensemble of two atoms in the scattering resonance with two cavity modes a and b. The excitation process takes place with the absorption of a photon from the mode a and emission of another quantum in the mode b.

numbers of quanta in each mode and the conversions of photons from the a to b modes of cavity respectively. The creation $\hat{b}^{\dagger}$, $\hat{a}^{\dagger}$ and annihilation, $\hat{b}$, $\hat{a}$, operators describe the processes of simultaneous annihilation of photons in one of the mode and creation of such a photon in another mode. The new field bi-operators satisfy a similar commutation as atomic operators: $[\hat{L}^+, \hat{L}^-] = 2\hat{L}_z$, $[\hat{L}_z, \hat{L}^{\pm}] = \pm\hat{L}^{\pm}$.

We consider the situation in which the flying time through the resonator τ_0 is shorter than the photon lifetime of photons in it, $\tau_{ph} = l/[c(1-r)]$. In this approximation, we consider that the total number of atomic and field excitation in such resonator is conserved during the flying time. Let us consider that in the time moment, t_0, the system of atoms, concentrated in the volume less than the wavelength, achieved the resonator. As the interaction of the atom with the cavity field is larger than its interaction with the external field, we neglect the last type of interaction [26]. So the system of atoms begin to interact with cavity field during the flying time $\tau \leqslant \tau_0$.

$$i\hbar \frac{\partial|\tilde{\Psi}(t_0+\tau)\rangle}{\partial(t_0+\tau)} = \hat{H}|\tilde{\Psi}(t_0+\tau)\rangle \quad (2)$$

The exact solution for a large ensemble of atoms becomes complicated when the number of atoms in the system increases [8]. In this paper, we are interested in the restoration of the initial states of the atomic ensemble after the flying time through the cavity. Due to the increase of the degree of freedom in the interaction In the atomic system, we limit our model to the system of two identical atoms in interaction with the cavity bimodal field. For this, we introduce the new collective states like in the Dicke super-radiance [24] described in the introduction, $|G\rangle = |g_1, g_2\rangle$; $|I\rangle = [|g_1, e_2\rangle + |e_1, g_2\rangle]/\sqrt{2}$ and $|E\rangle = |e_1, e_2\rangle$, which corresponds to three-level system represented in the figure 2. The new operators are represented through the collective states $\hat{S}^+ = \sqrt{2}(|E\rangle\langle I| + |I\rangle\langle G|)$; $S^- = \sqrt{2}(|G\rangle\langle I| + |I\rangle\langle E|)$ and $S_z = |E\rangle\langle E| - |G\rangle\langle G|$. In the resonance case $\omega = \omega_0$, the Hamiltonian part $H_0 = \hbar\omega_0(S_z + L_z)$ commutes with interaction Hamiltonian part $H_I = \lambda\hbar(S^+ L^- + L^- S^+)$ of the total Hamiltonian (1). In this case, we can regard the Schrodinger equation (2) to similar equation in interaction picture with the time-independent interaction part of Hamiltonian

$$i\hbar \frac{\partial|\Psi(t_0+\tau)\rangle}{\partial\tau} = H_I|\Psi(t_0+\tau)\rangle \quad (3)$$

Let us consider that at time moment, t_0, the system of atoms achieved the cavity. In this case we considered that both atoms are prepared in the superposition of ground and excited states $|\Psi_i(t_0)\rangle = c_2|e_i\rangle c_1|g_i\rangle$, $i = 1, 2$. This initial superposition easily can be extended to the collective atomic system states: $|\Psi_A(t_0)\rangle = \alpha|E\rangle\beta|I\rangle\gamma|G\rangle$. Taking into consideration that the cavity field satisfies the similar commutation relations, we consider that at initial stage of the interaction of atoms with the two modes of the cavity field is prepared in the similar superposition of angular momentum states, $|\Psi_F(t_0)\rangle = \sum_{m=-j}^{m=j} C_m|m, j\rangle$, as the two atom ensemble (see figure 3). Here the collective photon states, $|m, j\rangle$ of the two-cavity modes are similar to the angular momentum states described in quantum mechanics [28]. If we consider that the total number of photons in the system is n, we observe that the quantum number $j = n/2$. The equations of L operators of the bimodal cavity field on the collective states $|m, j\rangle$ is well described in the quantum mechanics.

For analytical representation of the Schrodinger equation, $|\Psi(t_0+\tau)\rangle = e^{\frac{-i\hat{H}_I\tau}{\hbar}}|\Psi(t_0)\rangle$, we introduce the fallowing three operator-vectors $\hat{x}(\tau) = e^{-i\hat{H}_I\tau}|E\rangle$, $\hat{y}(\tau) = e^{-i\hat{H}_I\tau}|I\rangle$ and $\hat{z}(\tau) = e^{-i\hat{H}_I\tau}|G\rangle$, which depends on the field operators of the system. Following the method developed in the [25, 8, 10], this operator-vectors satisfy the closed system of differential equation

$$\begin{aligned}
\frac{d\hat{x}(\tau)}{d\tau} &= -ig\hat{y}(\tau)\hat{L}^+;\\
\frac{d\hat{y}(\tau)}{d\tau} &= -ig\hat{x}(\tau)\hat{L}^- - ig\hat{z}(\tau)\hat{L}^+;\\
\frac{d\hat{z}(\tau)}{d\tau} &= -ig\hat{y}(\tau)\hat{L}^-.
\end{aligned} \quad (4)$$

After the time derivation of the second equation of the system of operator equation (4) and introducing the first and the third equation in it we obtain the simple second order equation with constant operator coefficients

$$\frac{d\hat{y}(\tau)}{d\tau} = \hat{y}(\tau)(\hat{L}^-\hat{L}^+ + \hat{L}^+\hat{L}^-).$$

Solving this equation and taking into consideration the initial conditions for these vectors, $\hat{x}(0) = |E\rangle$, $\hat{y}(0) = |I\rangle$ and $\hat{z}(0) = |G\rangle$, we obtain the following solutions for the system

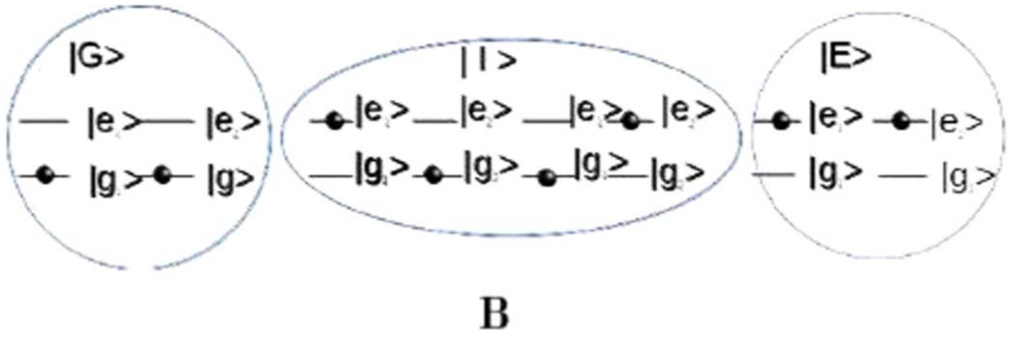

Figure 2. Energy level diagram showing the states involved in Raman excitation consists of three collective stats $|E\rangle$, $|I\rangle$ and $|G\rangle$.

of operator equations (4)

$$\hat{x}(\tau) = -ig|I\rangle \frac{\sin(\hat{\Omega}\tau)}{\hat{\Omega}}\hat{L}^{+} + g^2\{|E\rangle\hat{L}^{-} + |G\rangle\hat{L}^{+}\}\frac{\cos(\hat{\Omega}\tau) - 1}{\hat{\Omega}^2}\hat{L}^{+} + |E\rangle$$
$$\hat{y}(\tau) = -ig|I\rangle\cos(\hat{\Omega}\tau) - ig\{|E\rangle\hat{L}^{-} + |G\rangle\hat{L}^{+}\}\hat{\Omega}^{-2} \times \sin(\hat{\Omega}\tau);$$
$$\hat{z}(\tau) = -ig|I\rangle\frac{\sin(\hat{\Omega}\tau)}{\hat{\Omega}}\hat{L}^{-} + g^2\{|E\rangle\hat{L}^{-} + |G\rangle\hat{L}^{+}\}\frac{(\cos\hat{\Omega}(\tau))}{\hat{\Omega}^2}\hat{L}^{-} + |E\rangle - g^2\{|E\rangle\hat{L}^{-} + |G\rangle\hat{L}^{+}\}\frac{1}{\hat{\Omega}^2}\hat{L}^{-} + |G\rangle. \quad (5)$$

In this equation the expression, $\hat{\Omega} = g\sqrt{\hat{L}^{+}\hat{L}^{-} + \hat{L}^{-}\hat{L}^{+}}$, is the operator of the Rabi nutation frequency.

Acting on the ket-vectors the $su(2)$, $\hat{L}^{+}|m, j\rangle = \sqrt{(j+m+1)(j-m)}|m+1, j\rangle$ and $\hat{L}^{-}|m, j\rangle = \sqrt{(j+m)(j-m+1)}|m-1, j\rangle$, the the wave function is simply represented to the vector operators $|\Psi(t_0+\tau)\rangle = [\alpha\hat{x}(\tau) + \beta\hat{y}(\tau) + \gamma\hat{z}(\tau)]\sum_{m=-j}^{m=j} C_m|m, j\rangle$. Introducing the solutions (5) of the system of equations we obtain the following analytic representation of wave function

$$\begin{aligned}|\Psi(t_0+\tau)\rangle &= \textstyle\sum_m |E\rangle|m, j\rangle \\ &\times\{\alpha C_m\{g^2(j-m)(j+m+1) \\ &\times\Phi(m+1, \tau) + 1\} \\ &- i\beta C_{m+1}\sqrt{(j+m+1)(j-m)}F(m+1, \tau)g \\ &+ \gamma C_{m+2}g^2 \\ &\times\Phi(m+1, \tau)\} \\ &+ \textstyle\sum_m |I\rangle|m\rangle \\ &\times\{-\alpha ig C_{m+1}\sqrt{(j+m)(j-m-1)}F(m, \tau) \\ &+ \beta C_m\cos(\Omega(m)\tau) + ig\gamma F(m, \tau)C_{m-1}\} \\ &+ \textstyle\sum_m |G\rangle|m\rangle\{\alpha C_{m+2}g^2\Phi(m-1, \tau) \\ &\times\sqrt{(j+m+3)(j+m+4)(j-m+2)(j-m+1)} \\ &- i\beta g C_{m-1}\sqrt{(j+m)(j-m+1)}F(m-1, \tau) \\ &+ \gamma C_m\{g^2(j-m)(j-m+1)\Phi(m-1, \tau) + 1\}.\end{aligned} \quad (6)$$

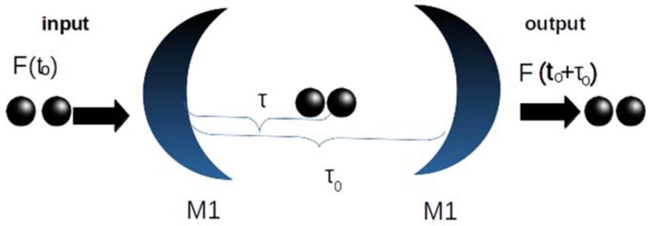

Figure 3. The flaying of the system of two-atoms in the scattering resonance with two cavity modes. It is required that in the input time $t \leqslant t_o$ the atomic ensemble is disentangled relative to cavity modes. The conditions for the restoration of the initial disentangled state of atomic and field systems are found as a function of flying time, τ_0, and superposition preparation of the atomic and cavity field.

Here $\Omega(m) = g\sqrt{2[j(j+1) - m^2]}$, $m = -j, \quad \ldots, j$ are the eigenvalue of operator of the Rabi nutation frequency $\hat{\Omega}$ on the vector state $|m, j\rangle$. In the solution (6) it is introduced the following notations : $\Phi(m, \tau) = (\cos[\Omega(m)\tau] - 1)/\Omega^2(m)$, $F(m, \tau) = \sin[\Omega(m)\tau]/\Omega(m)$.

We are interested in the restoration of the initial state of the system after the flying time $\tau = \tau_0$. The restoration conditions consist in the conservation of uncoupled density of the states for the cavity field and population of the atomic states. According to this condition we request that after the discrete interaction time interval τ_n the wave function of the pairs of atoms is restored so that only the phases of the coefficients α, β, γ, :$|\Psi_A(t_0)\rangle \Rightarrow |\Psi_A{}'(t_{nr})\rangle$ and $|\Psi_F(t_0)\rangle \Rightarrow |\Psi_F(t_{nr})\rangle$, where $|\Psi_A{}'(t_{nr})\rangle = \alpha\exp(i\varphi_\alpha)|E\rangle + \beta\exp(i\varphi_\beta)|I\rangle + \gamma\exp(i\varphi_\gamma)|G\rangle$. The phases φ_α, φ_β and φ_γ is determined from the restoration conditions.

Let us chose the phase for excites state $|E\rangle$ in the Esp. (6). Indeed, considering that phase φ_α is equal to $2\pi n$ (n integer number) we obtain the following expression for dynamical trapping condition $\exp[-iH_I\tau_0|E\rangle = \alpha|E\rangle$

$$\begin{aligned}&-\alpha g C_m\sqrt{(j-m)(j+m+1)}\frac{\sin[(\Omega(m+1)\tau_0/2]}{\Omega(m+1)} \\ &+ i\beta C_{m+1}\cos[\Omega(m+1)\tau_0/2] \\ &- \gamma C_{m+2}g\sqrt{(j+m+2)(j-m-1)} \\ &\times\frac{\sin[\Omega(m+1)\tau_0/2]}{\Omega(m+1)} = 0\end{aligned} \quad (7)$$

We must obtain a similar expression for the intermediary state, $|I\rangle$. This is possible if we chose the phase, $\varphi_\beta = \pi(1+2n)$, where n is integer number. According to this expression we replace the coefficient in the solution (6) $\exp(-iH_I\tau_0)|I\rangle = -\alpha|I\rangle$

$$-\alpha gC_{m-1}\sqrt{(j-m+1)(j+m)}\frac{\sin[\Omega(m)\tau_0/2]}{\Omega(m)} + i\beta C_m\cos[\Omega(m)\tau_0/2] - g\gamma C_{m+1}\frac{\sin[\Omega(m)\tau_0/2]}{\Omega(m)}\sqrt{(j+m+1)(j-m)} = 0 \quad (8)$$

The condition for ground state $|G\rangle$ is obtained in a similar way as the excited state $\exp(-iH_I t+\tau)|G\rangle = \alpha|G\rangle$

$$-\alpha C_{m-2}g\sqrt{(j-m+2)(j+m-1)}\frac{\sin[\Omega(m-1)\tau_0/2]}{\Omega(m-1)} + i\beta C_{m-1}\cos[\Omega(m-1)\tau_0/2] - \gamma gC_m\sqrt{(j+m)(j-m+1)}\frac{\sin\Omega(m-1)\tau_0}{\Omega(m-1)} = 0$$

As follows from the Exps. (7)–(9) during the flying time τ_0 calculated from one of these equations the quantum system formed from the bimodal cavity field and ensemble of two atoms restore their initial disentangled state. All recurrence expressions (7)–(9) are same. And the coefficients can be easily find from the recursion relation

$$C_m = -\frac{ig}{\beta\Omega(m)}\{\alpha\sqrt{(j-m+1)(j+m)}C_{m-1} + \gamma\sqrt{(j+m+1)(j-m)}C_{m+1}\}\tan[\Omega(m)\tau_0/2].$$

This time interval becomes periodical. All three Exps.(8, 9) are similar to the relation (7) if we make the substitution $m+1$ and $m-1$ through m in them respectively. We may find the stationary trapping condition [34, 35] from the Exps. (7)–(9). Indeed, dividing the Exp. (7) on the $g\sin[\Omega(m)\tau_0/2]/\Omega(m)$ and considering that intermediary state is unpopulated $\beta = 0$, we obtain the stationary trapping condition

$$-\alpha C_{m-1}\sqrt{(j-m+1)(j+m)} - \gamma C_{m+1}\sqrt{(j+m+1)(j-m)} = 0. \quad (10)$$

This Exp. (10) describes the stationary trapping for the three-level system in interaction with the bimodal cavity field. It describes the preparation of the cavity field according to the preparation of the inversion of excited state relative to the ground one.

3. Conclusions

The new solutions of two undistinguished two-level atoms with a bimodal electromagnetic field in the Raman interaction are proposed. This solution was simplified due to the fact that the bimodal field was reduced to a ‘single’ mode system of particles belonging to $su(2)$ algebra. The exact solution of the Schrodinger equations (3) described by matrix, representation was analytically found.

The dynamical trapping condition and stationary one are obtained. These effects may be applied for conditional processing of information as the function of trapping time [18, 19, 36] obtained from one of the Exps (7)–(9), and preparation of the atomic inversion of two undistinguished atoms flying through the bimodal resonator. Such a three-level system can be used to study the basic principles of interaction of single atom [36] and an ensemble of atoms with radiation in Raman induced processes [37]. Another application of trapping condition and cavity-enhanced Raman transitions involving localized excitation could potentially be used for gaining quantum control over the nanomechanical motion of atoms or molecules and open a route for molecular cavity optomechanics [37, 38].

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