

# PLANCKS 2019

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## Abstract

The sixth edition of the Physics League Across Numerous Countries for Kick-ass Students (PLANCKS) competition was held in the city of Odense, Denmark. The purpose of PLANCKS is to find the most prominent theoretical physics student team in the world. The competition was part of a three-day programme, featuring scientific as well as social events. The scientific programme featured three symposia all containing talks from prominent physicists, most notably the 2001 Nobel Prize laureate Wolfgang Ketterle of MIT. Both the competition and the symposia will enlighten the students about fields of physics which may not be present at their home university. The competition contained a total of 34 teams from 18 countries including the winning team ‘The Four Vectors’ from Germany, and the runners-ups ‘CV5 Irreducibles’ and ‘Komfur’ from Serbia and Denmark, respectively. This year’s problems featured 10 exciting exercises. Here we present two problems from PLANCKS 2019 and the line of reasoning behind making a PLANCKS problem, which is an opportunity for a scientist to highlight an interesting part of their field to young students.

Supplementary material for this article is available [online](#)

Keywords: competition, international student competition, IAPS, PLANCKS

(Some figures may appear in colour only in the online journal)

## 1. Introduction: what is IAPS and PLANCKS

### 1.1. What is PLANCKS

The Physics League Across Numerous Countries for Kick-ass Students (PLANCKS) is a theoretical competition for physics students, largely adapted from a similar competition originally from a Dutch student organisation, Studenten Physica in Nederland (SPIN) [1]. The first edition of PLANCKS took place in May 2014 in Utrecht, and was organised with the help of the International Association of Physics Students (IAPS). Since 2014 PLANCKS has been held annually in May.

PLANCKS is a theoretical physics competition for bachelor's and master's level students and is typically a three-day event with various activities such as an opening ceremony, lab excursions, and the competition itself. Besides these activities, the three days are stacked with numerous excursions and social activities, in which the competitors are encouraged to experience the research environment and also the culture of the host country. Through these activities PLANCKS aims to connect students from all over the world with each other such that new friendships can be formed in a unique environment where ideas and experiences can be exchanged; see figure 1 for the distribution of participants.

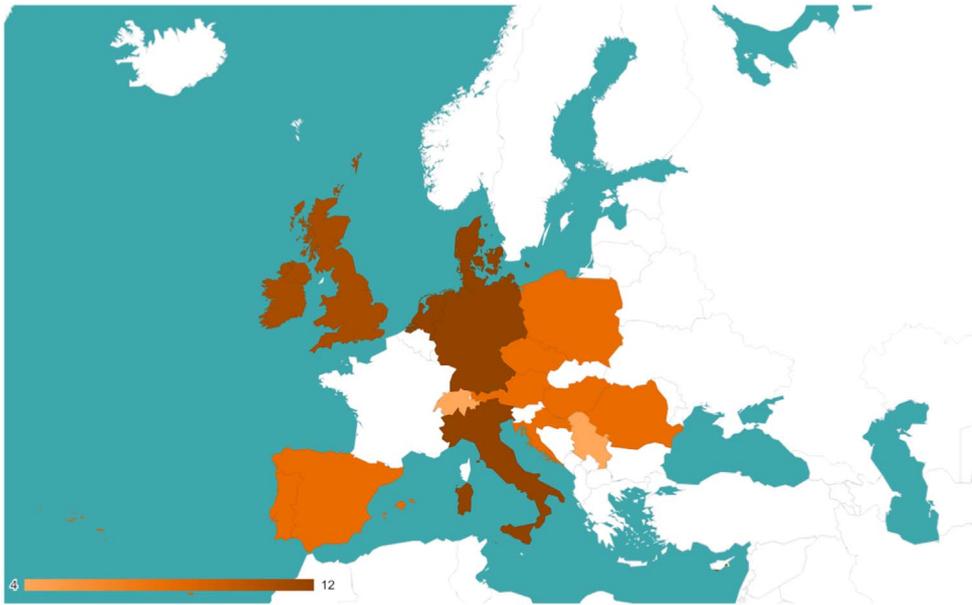
Participants of PLANCKS compete in self-formed teams consisting of three to four bachelor- or master-level students. These teams try to solve as much of 10 exercises for 4 h and will, preferably, be comprised of students from the same country although not necessarily of the same university. Each nation is allowed to enter a limited amount of teams in the PLANCKS finals, which has led numerous countries to organise national preliminary competitions, where the two or three winning teams qualify for the final competition, which was held in Odense, Denmark in 2019. Already at the national level these competitions are attracting a lot of students, e.g. in Germany where more than 20 teams compete in the DOPPLERS competition [2], which serves as the PLANCKS qualification, similar competitions are held in other countries where the interest attracts more than two teams.

The international finals of PLANCKS is hosted by a different country every year, and is organised by a committee of students at the host university. The selection of the hosting country is through competitive application at the annual general assembly of IAPS. The hosting country is determined at the general assembly two years before the competition date, and as soon as an organising committee have been elected, the detailed planning of this unique event begins.

### 1.2. What is IAPS?

The main organisation behind many international events for physics students, including PLANCKS 2019, is the International Association of Physics Students (IAPS). This organisation is run by students, and aims to promote international collaboration between physics students around the world. This is mainly done through educational events which allow students to travel and experience physics all over the world.

IAPS was founded in 1987 by physics students who wanted better relations between those passionate about physics, regardless of borders. IAPS have been very successful on this matter, with PLANCKS 2019 having participants from 18 countries, 17 from Europe and one from Asia, although the goal is to get participants from all continents.



**Figure 1.** A map showing the number of participants at PLANCKS 2019 from each country. There were also participants from Singapore, but this is not displayed in order to keep the map comprehensible.

## 2. Structure of PLANCKS 2019

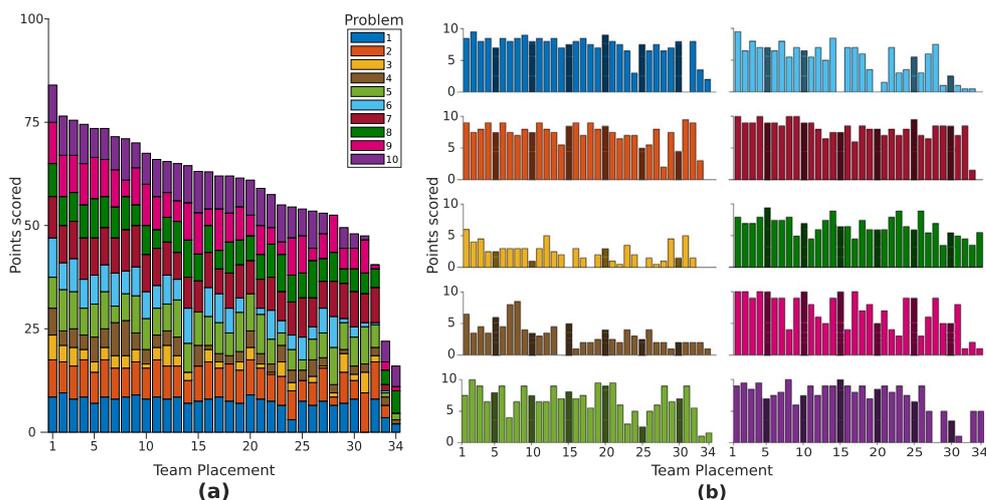
PLANCKS 2019 featured a wide variety of events, from social events with a focus on networking, to talks that expanded the students' academic horizons by peaking their interests in new fields of physics. And, of course, the competition itself, which challenged the students with exciting problems meant to develop the students' problem-solving skills. The relations built at student conferences and competitions form a solid foundation for an academic network upon which to build students' careers. The talks familiarised the students with topics such as Bose–Einstein condensates with Nobel Laureate Wolfgang Ketterle (Massachusetts Institute of Technology), anomalies in the standard model with Holger Bech Nielsen (University of Copenhagen) and many more<sup>3</sup>. For many students, especially the younger ones, these are the first international talks they attend. They play an important role in motivating the students and giving them a sense of the different fields' communities and current research, while giving the speakers and problem makers a unique opportunity to enlighten young students about their field of research.

## 3. The competition

### 3.1. Requirements for PLANCKS problems

For the competition itself, held on the second day, the 34 teams were distributed in separate rooms and given four printed copies of the 10 problems. The contestants had 4 h to solve the

<sup>3</sup> Recorded live streams of all plenary talks from the event are available online at <https://facebook.com/PLANCKS2019/>.



**Figure 2.** PLANCKS 2019 scores. (a) Total scores for each of the 34 teams; segments show scores on each problem. (b) Team scores on each problem consistent with color coding of (a). Every 5th team is marked with a darker color.

problems and produce written answers to said problems using only dictionaries, non-scientific calculators, pens and paper. The PLANCKS problems are designed to challenge the contestants, enlighten them and peak their interest. To meet this end, the organising committee seeks out competent faculty members to create the problems of the competition. The organising committee is in close contact with the professors to ensure the problems match the nature of the PLANCKS competition. A PLANCKS problem should

- Be difficult: the winning team should receive between 60 and 70 points out of 100.
- Be diverse: the problems should cover many different fields of physics.
- Focus on problem solving: the problems should test the contestants ingenuity, not their ability to remember formulas.
- Not require any books: only dictionaries and non-scientific calculators are available to the contestants.

By aiming for a level, which gives the winning team between 60 and 70 points out of 100, it is assured that the competition is challenging and the prestige of doing well on the PLANCKS problems is upheld. The distributions of the PLANCKS 2019 results are visualised in figure 2, where it can be noted that the winning team earned 84 points, which could indicate that some of the problems were too easy this year.

PLANCKS problems cover a wide array of topics. It is often a goal for PLANCKS problems to introduce the contestants to new topics, and should not be categorisable as ‘bachelor’s’ or ‘master’s’ level problems. This year, the problem covered topics like adsorption, spin dynamics, the Higgs mechanism, black holes and much more (see table 1). The contestants should go through a problem designed to teach them about new physics, while testing their skills and critical thinking. The focus in particular on problem-solving, instead of by-heart learnt knowledge, should be a welcome diversion from the typical focus of standard university lectures. Professors should expect students to return with increased motivation, carrying with them new approaches to familiar topics and inspiration to delve into new fields of physics.

**Table 1.** The problems of the PLANCKS 2019 problem set.

Problem	Title	Field
1	Mono- and multilayer adsorption	Membrane physics
2	Quantum optics	Quantum optics
3	Radical pair spin chemistry	Quantum mechanics
4	Relativistic orbit	Analytical mechanics
5	Polymers and rubber	Polymer physics
6	Topological phase transition in the 2D XY-model	Phase transitions
7	Higgs mechanism	Particle physics
8	Black hole picture	Astronomy
9	Solid state physics	Solid state physics
10	Freezing front	Thermodynamics

### 3.2. Correcting the problems

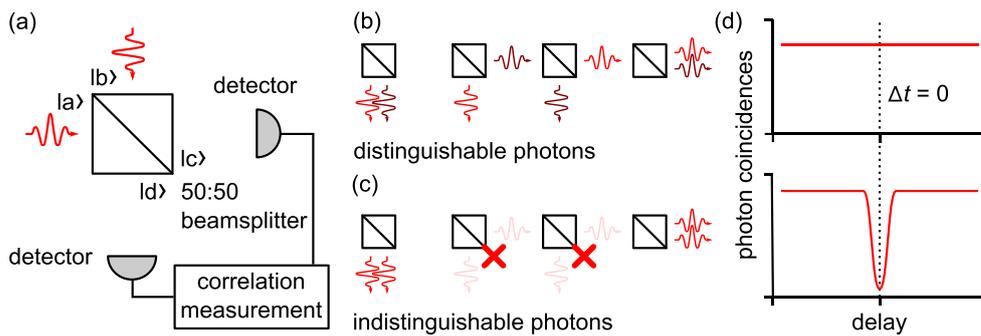
To ensure quality and fairness in the correction process, the jury members are expected to be of at least PhD-level. The authors of the problems create a correction sheet for reference when correcting. The authors themselves can be involved in the correction process, which helps handling any unorthodox solutions to the problems. In PLANCKS 2019 a double-checking system was employed, where two independent correctors would mark the problems and then discuss any disagreements. The results were announced at the closing ceremony the following morning.

## 4. Problems and solutions of PLANCKS 2019

In this following section we have invited the problem makers of two differently scoring problems to present their line of reasoning behind their respective problem. The presented problems are Problems 2 and 3, with Problem 2 devised and presented by Professor S Hofferberth (University of Southern Denmark), PhD student N Stiesdal and post-doc H Busche. This problem is about the Hong–Ou–Mandel effect, which is used in quantum optics. Any further questions regarding the problem can be directed towards Hofferberth@sdu.dk. The second problem, Problem 3 in the problem set, was made and presented by Professor I Solov'yov (Carl von Ossietzky Universität, Oldenburg, former University of Southern Denmark) and is about radical pair spin dynamics and its quantum-mechanical interpretation. Any further questions can be sent to ilia.solovyov@uni-oldenburg.de.

### 4.1. Quantum optics: Hong–Ou–Mandel effect

In this problem, the contestants investigated Hong–Ou–Mandel (HOM) interference [3], one of the most important effects in quantum optics. With respect to the criteria outlined in 3.1, the HOM effect is a well-suited competition problem since it can be used to test the abilities of the contestants beyond mathematical problem solving, i.e. to correctly interpret physical phenomena and experimental observations in conjunction with their own calculations. The problem is diverse and touches on a range of topics from classical electromagnetism and quantum optics, fundamental quantum mechanics, quantum statistics, and AMO physics. Despite the HOM effect's fundamental importance in quantum optics, only basic concepts from quantum mechanics, statistics, and electromagnetism, but no previous knowledge or formulae unique to quantum optics are required for the solution. The difficulty criterion is



**Figure 3.** Hong–Ou–Mandel interference. (a) Schematic of setup: two photons are simultaneously incident on two different inputs of a 50:50 beamsplitter corresponding to spatial modes described by quantum states  $|a\rangle$  and  $|b\rangle$ . Subsequently the photons are detected in the two output modes (quantum states  $|c\rangle$  and  $|d\rangle$ ) and correlations between detection events are analysed. (b) Distinguishable photons independently exit into any output mode. (c) Indistinguishable photons do not exit into different modes due to quantum interference. (d) In correlation measurements, coincidences are suppressed for zero time delay ( $\Delta t = 0$ ) between detection events for indistinguishable photons.

mostly addressed by the interpretation part, as confirmed by the results. Below, we briefly introduce the physics background, the exercises that formed the problem, and outline the concepts and approaches required to solve it. The original problem sheet with further explanations can be found in the supplementary information available online at [stacks.iop.org/EJP/41/034002/mmedia](https://stacks.iop.org/EJP/41/034002/mmedia). A model solution is available upon request.

**4.1.1. Physics background.** HOM interference occurs when a pair of indistinguishable photons, or more generally any other bosons, are simultaneously incident on both inputs of a beamsplitter (figure 3) [3]. As a result of interference and their indistinguishability, both photons will exit the beamsplitter through the same output, creating spatial entanglement between the states describing their spatial modes. This suppresses simultaneous detection events in both modes, which leads to a characteristic dip in the temporal correlation function. Incoming photons that are distinguishable or sufficiently separated in time, however, do not interfere and show no correlation.

The HOM effect is of high practical importance and exploited in many photon-based applications of quantum mechanics. This includes tests of the indistinguishability and spectral characteristics of single photon sources, e.g. for quantum networks [4], transfer of entanglement between remote qubits [5], or the implementation of quantum operations between photonic qubits in linear optics quantum information processing [6, 7]. HOM interference can also be observed between other bosons such as individual atoms [8], Bose–Einstein condensates [9], microwave photons in superconducting waveguides [10], or phonons [11].

**4.1.2. Structure of the problem set.** The problem itself (see [stacks.iop.org/EJP/41/034002/mmedia](https://stacks.iop.org/EJP/41/034002/mmedia) supplementary material) is divided into three sub-exercises. The first centres on the beamsplitter in classical optics and the mathematical description of its effect. The second focuses on the beamsplitter in quantum mechanics and the contestants are asked to calculate its output for different input states, i.e. the observation of HOM interference if the photons are indistinguishable. This tests the students’ understanding of and ability to

apply fundamental quantum mechanics. The final part focused on the interpretation of their results in comparison to experimental data and the ability to provide explanations for physical phenomena.

*Exercise 1.* Initially, the contestants lay the foundation for subsequent calculations on the effect of a beamsplitter, showing that a  $\pi/2$ -phase shift occurs for reflected light as a result of fundamental prerequisites such as energy conservation. During the following calculations, the contestants can draw parallels to this classical case.

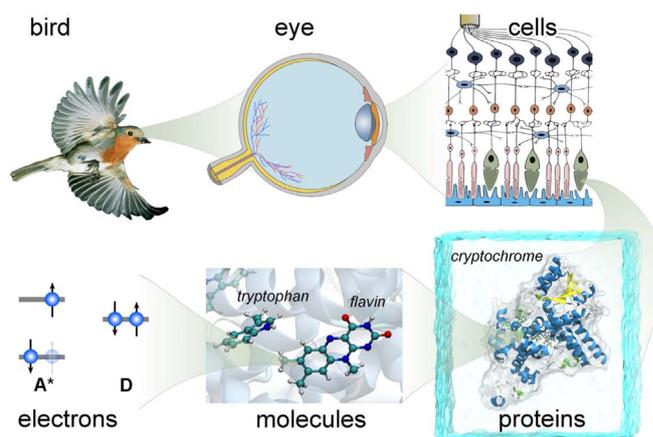
*Exercise 2.* In the second exercise, the contestants investigate the evolution of different quantum states on the beamsplitter and find that the output state depends on whether the photons are distinguishable or not. If two input photons (photon 1 in mode a and 2 in mode b) are distinguishable,  $|\psi\rangle_{\text{in}} = |a\rangle|b\rangle$ , the four cases shown in figure 3(b) are equally likely. For indistinguishable photons,  $|\psi\rangle_{\text{in}} = (|a\rangle|b\rangle + |b\rangle|a\rangle)/\sqrt{2}$ , both exit together,  $|\psi\rangle_{\text{out}} = (|c\rangle|c\rangle + |d\rangle|d\rangle)/\sqrt{2}$  (figure 3(c)).

The contestants also investigate the artificial case of ‘fermionic photons’, where the sign in the input state is flipped giving an antisymmetric fermionic wavefunction  $|\psi\rangle_{\text{in}} = (|a\rangle|b\rangle - |b\rangle|a\rangle)/\sqrt{2}$ . As expected due to the exclusion principle, they find that the photons never exit together,  $|\psi\rangle_{\text{out}} = i(|c\rangle|d\rangle - |d\rangle|c\rangle)/\sqrt{2}$ . This result can be observed for HOM interference between electrons [12].

*Exercise 3.* In the final part, the contestants must apply the results of their calculation to analyze the observations made by the original experiment in 1987 [3]. Based on the observation of a dip in the correlation function, the contestants should conclude that photons are indeed indistinguishable bosons which bunch together following interference on the beamsplitter. They were also asked to consider what influences the width and depth of the dip. Its width is determined by the spectral overlap of the photons, while the depth is determined by how distinguishable the photons are. The depth can also change if the beamsplitter has unequal transmission and reflection. Finally, they were asked how the observed anti-correlation can be in agreement with the fact that photons are noninteracting: the HOM effect is the result of interference, not interactions.

*4.1.3. Results and conclusion.* The problem was attempted by all but one of the groups and was amongst the better scoring ones with an average of 7.1 out of 10 points. While most groups scored near this, no team solved the problem fully with few achieving more than 8 points. Interestingly, scores were not correlated with overall performance. For example, the highest score of 9.5 points was achieved by a team finishing 31st overall, while several top teams only achieved average scores.

Most teams solved the calculation-heavy first parts correctly, while results were much more varied for the interpretation part. Many concluded correctly that the experiment shows the bosonic nature of photons, yet few related the shape and the depth of the HOM dip to the photons’ spectral properties and indistinguishability. In hindsight, a larger fraction of the total points should have been awarded for part 3 to better distinguish performance. Results indicate that all contestants have a very solid understanding of the basics of quantum mechanics, but their interpretations skills may vary due to different levels of experience and different foci in physics programmes at different universities.



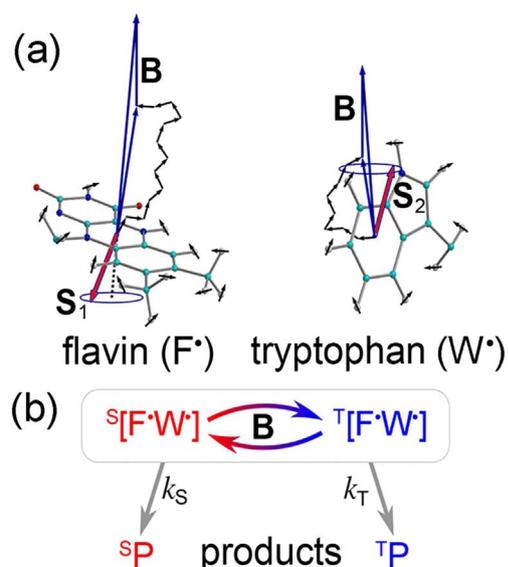
**Figure 4.** Avian magnetoreception is a multiscale problem.

#### 4.2. Radical pair spin chemistry

In this problem, the contestants were supposed to study a simplified model of the so-called radical pair spin system. The problem involves sub-fields of chemistry and physics and can be positioned at the intersection of chemical kinetics, photochemistry, magnetic resonance, and free radical chemistry. More specifically, the problem is dealing with magnetic and spin effects in chemical reactions. Spin chemistry is an interdisciplinary area of research with many applications. Important to mention here are the so-called magnetic isotope effects in chemical reactions, chemically induced dynamic nuclear polarization (CIDNP), chemically induced dynamic electron polarization (CIDEP), radiation-induced health effects, and avian magnetoreception. The latter is particularly relevant to the proposed problem as radical pair reaction kinetics may become dependent on the direction of external magnetic fields [13]. The problem included some basics of radical pair spin chemistry. It was tailored to probe the knowledge of the contestants in fundamental quantum mechanics, atomic interactions, and spin physics.

**4.2.1. Physics background.** The exact mechanism by which migratory passerines sense the geomagnetic field appears to rely on the quantum effects that involve spin dynamics of radical pairs induced by light in photoreceptor proteins cryptochromes found in the retina [14–17]; see figure 4.

Figure 5 illustrates the overall concept of a radical pair reaction. Here, two radicals [ $F^*W^*$ ] are formed through an ultrafast electron transfer reaction triggered by light absorption. The radicals, within the radical pair, can react and induce formation of distinct chemical products. Here the decisive factor is the overall spin state of the radical-pair, which can be either singlet ( $S$ ) or triplet ( $T$ ). The resulting products in figure 5 are labeled as  $^S P$  and  $^T P$ , being formed at a rate  $k_S$  and  $k_T$ , respectively. In the magnetic field, comprised of the internal (hyperfine) magnetic fields and the external magnetic field undergoes a coherent motion which is driven by the fields mentioned above. Constraining the radical pair in space will thus create a directional response of the radical pair products on the external magnetic field. Note, however, that this is only possible in the presence of the hyperfine interactions, which often appear anisotropic and can be written in a tensorial form. The radical pair could thus be seen



**Figure 5.** (a) Unpaired electron spins ( $S_1$  and  $S_2$ ) of the flavin and tryptophan radicals in cryptochrome precess about a local magnetic field due to an external magnetic field  $B$  and nuclear spins. The spin precession continuously alters the relative orientation of  $S_1$  and  $S_2$  causing a singlet  $^S[F^*W^*]$  to triplet  $^T[F^*W^*]$  interconversion, which underlies the magnetic field effect. (b) A generic radical pair reaction scheme.

as the primary block of the so-called chemical compass, that has inspired multiple research efforts in the area of magnetoreception [13, 14, 18–22].

**4.2.2. Structure of the problem set.** The problem (see [stacks.iop.org/EJP/41/034002/mmedia](https://stacks.iop.org/EJP/41/034002/mmedia) supplementary material) is divided into five sub-exercises. These sub-problems were designed to guide the students through the individual questions that need to be answered in order to predict a possible interconversion of a radical pair from one entangled quantum state (singlet) into the other quantum state (triplet). The first and second exercises focus on fundamental characterization of the overall spin system, the third exercise challenges the understanding of the fundamental laws of energy conservation in the studied spin system, while the fourth problem is just a follow-up numeric exercise. The final sub-problem offers the students to estimate the characteristic singlet-triplet transition times.

**Exercise 1.** The studied radical pair system includes three spin-1/2 particles and here the contestants need to elaborate what values can the total spin take. It requires some basic knowledge about the spin and how individual spins could be added.

**Exercise 2.** Here a hint is given that actually the studied radical pair system can be found in eight states and it is required to construct the wavefunctions of these states from the basis states. The successful completion of the task involves a bit of abstract thinking as the contestants had to choose these basis functions themselves. Since the studied system involves three spin-1/2 particles, these basis functions are known and are usually discussed in introductory quantum mechanics classes at universities.

*Exercise 3.* This sub-question requires understanding of energy conservation principles and some basic knowledge about selection rules. A transition from one quantum-mechanical energy level to another is only possible if the corresponding transition matrix element is non-zero. Now with the established wavefunctions from the previous sub-problem, and the Hamiltonian provided in the problem description it is possible to evaluate the different transition matrix elements and show that some of those will be non-zero. For the corresponding transition, it is however, also important to check that the transition matrix element will be greater than the energy difference between the two eigenstates of the Hamiltonian, as otherwise a transition would be impossible.

*Exercise 4.* This problem was introduced to give the contestants some feeling of the actual numbers that are expected in a typical radical pair system. By plugging in the provided numbers it is possible to estimate the characteristic energies for the different states in the system and the energy gaps for different transitions. This problem is also used to demonstrate that for a generic radical pair system, conditions in sub-problem 3 could be easily satisfied.

*Exercise 5.* In the final part of the exercise the contestants needed to compute the transition probability for the radical pair transitioning from a singlet to a triplet state. Here one essentially expects to compute the transition matrix element that will yield a time-dependent probability that will oscillate sinusoidally. The frequency of this oscillation would then readily deliver the characteristic transition time.

*4.2.3. Results and conclusion.* This problem was the problem which gave most 0 points of all, with six teams not even attempting the problem. With an average of 2.3 points, this is the lowest scoring problem of the PLANCKS 2019 competition, although it really separated the winners as the winners were the highest scorers of this problem and all teams in top 10 but the 10th place scored higher than the average on the problem (see figure 2). As explained above, the motivation for this PLANCKS problem is rather high as radical pair dynamics could be key to explain, for example, how migratory birds navigate. In the spirit of the contest this problem was designed as a rather challenging task and required from the contestants diverse knowledge across various areas of quantum mechanics. The resulting low scores for the problem could be seen twofold: (i) the problem was considered difficult by most of the contestants, and (ii) many of the sub-problems were interconnected which also resulted in some contestants failing to attempt specific sub-problem if another one was not solved. It is, however, important to stress that such hardcore problems are important for a competition like PLANCKS as such problems often make a difference and reveal the true winners.

## 5. Outlook

The sixth edition of PLANCKS took place from the 17th of May to the 20th of May in Odense, Denmark and found a winning team, ‘The Four Vectors’ from Germany, meaning Germany will be the defending country when PLANCKS visits London, England next year, for the seventh edition of the PLANCKS competition.

PLANCKS 2019 once again proved to be a unique experience for its 136 contestants, adding to these 136 contestants were 12 observers from, among others, the 2020 PLANCKS Organising Committee. Among the many unique experiences were in particular the Saturday evening containing a public talk by Nobel Prize Laureate Wolfgang Ketterle followed by a closed reception with food and beverages.

The problems this year had a great variety, as seen in the prior section, albeit the problems yielded more points than earlier PLANCKS competitions. Hopefully the PLANCKS 2020 competition will keep the variety while making the problems more difficult.

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## Conflict of interests

The authors declare no conflict of interest.

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