Unification of theories requires a postulate basis in common

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Abstract. A primary challenge of natural sciences in the new millennium is to cure the gap between metaphysics and empiricism – and puzzle out the obstacles to a unified theory and an understandable picture of reality. Antique science flourished via its strong philosophical impact but faded away due to the lack of supporting empirical science. The fast development of mathematical physics has led to the other opposite; theories are diversified, they are more like mathematical descriptions of observations; they provide precise predictions but lack a solid metaphysical basis and an understandable picture of reality. Anyway, modern science has increased our understanding of physics from elementary particles to cosmological structures and produced information that allows re-evaluation of the basis. By switching from an observer-oriented perspective to a system-oriented perspective, any local object is related to the rest of space and relativity appears as a direct consequence of the conservation of total energy in the system – without scarifying the absolute time and distance essential for human comprehension. Such a holistic approach has led to the Dynamic Universe theory (DU). After maturing for the last twenty-five years, DU produces precise, well-tested predictions for local and cosmological observables and an uncontradictory linkage to quantum mechanics.

1. Introduction

Physics is characterized as an empirical science. The accuracy of predictions for observations has become the main criteria of success, and theories producing accurate predictions our guides to the laws of nature and picture of physical reality. Fast progress in empirical science relies on successful mathematical descriptions of observations but limited metaphysical basis or linkage to primary laws of nature. Introduction of dark energy as a mathematical correction solved the mismatch between the predictions and observations of the magnitude-redshift relationship in standard cosmology but created a problem of the physics behind the dark energy. When observations on the velocity of light and the buildup of the momentum of accelerated electrons in the late 19th century did not fit in the linear Newtonian reality, the reality was corrected with dilated time and contracted length. "The corrected reality" turned out successful and obtained the form of spacetime that presently characterizes the physical environment we live in.

The Dynamic Universe theory (DU) [1] is an endeavor for identifying the basis for a unified formulation of the theories in physics and cosmology and answering unanswered questions in current formulations. DU relies on the zero-energy principle, first applied to space as a spherically closed whole, and further, to all local interactions in space. DU means a holistic perspective to the observable reality. Local phenomena are linked to the rest of space; motion in space is linked to the motion of space. Such an approach opens relativity as a direct consequence of the conservation of the overall energy balance in the system, e.g., the buildup of local kinetic energy in space reduces the rest energy of the object in

motion. In effect, the characteristic frequency of an atomic oscillator in motion is reduced. The corresponding effect occurs on the locally observed rest energy near mass centers due to local bending of space relative to the fourth dimension. There is no need for distorted time and distance needed in the kinematic solution of relativity theory. In the holistic perspective, relativity means relativity between the local and the whole rather than relativity between an object and the observer. Everything in space is interconnected.

Both the zero-energy principle and spherically closed space are well-known ideas. Combining the two is problematic without the fourth dimension of metric nature. The zero-energy universe has been proposed by Dennis Sciama [2]. As a zero-energy solution consistent with the space-time concept, Edward Tryon [3] proposed quantum fluctuation, an instant release of gravitational potential as negative quantum energy for the instant appearance of the energy of matter and radiation in the Big Bang.

In his Lectures on gravitation in the early 1960s, Richard Feynman [4] p. 164 introduced spherically closed space as an "intriguing suggestion" allowing an equal view to the surrounding expanding space at any location in space. Feynman [4] p. 10 also pondered the equality of the total gravitational energy and the rest energy in space as a great mystery but did not link the idea to spherically closed space.

DU is a dynamical solution linking Feynman's great mystery to his intriguing suggestion of spherically closed space. Following the cosmological principle of big bang cosmology and interpreting the fourth dimension of space-time a metric dimension with line element $dr_4 = cdt$, all locations in space are at about 14 billion lightyears distance from a "starting point in common" in the fourth dimension; such an interpretation means space as the 3D surface of a 4D sphere expanding at velocity c. Metric fourth dimension allows scalar, universal time that applies equally in dynamics in the three space dimensions and the fourth dimension.

The dynamics of spherically closed space is like that of a spherical pendulum in the fourth dimension. The energy of motion is gained against the release of gravitational energy in a contraction phase and converted back to gravitational energy in the ongoing expansion phase. Maintaining the zero-energy balance, the buildup of local structures in space converts part of the momentum in the fourth dimension into momentum in space via local bending of space. Such a process occurs in several steps creating a system of nested energy frames linking all local states of motion and gravitation to the state at rest in hypothetical homogeneous space that serves as a universal frame of reference.

2. Merits of the Dynamic Universe

2.1. Understandable picture of reality

Instead of an instant big bang, the buildup and energization of the observable universe are described as a contraction-expansion cycle from infinity in the past to infinity in the future or in repeated cycles passing the "essential infinity". In the contraction, mass in space gets its energy of motion from its own gravitation and releases it back to gravity in the ongoing expansion phase. Following the zero-energy principle, the energy structure of space is described as a system of nested energy frames that links any local energy state to the state of rest in hypothetical homogeneous space that serves as a universal reference. Time and distance are used as universal coordinate quantities essential for human comprehension. Phenomena that the relativity theory explains in terms of modified space-time metrics, are explained as consequences of their different energy states. Atomic clocks in motion or near mass centers run slower because part of their energy is bound to local motion or gravitation – not due to the different flow of time like in the framework of relativity theory. There is nothing discrete in DU space; any local object is linked to the rest of space.

2.2. Clear postulates and illustrative notations

DU relies on the zero-energy principle, which means double-entry energy-bookkeeping; for gaining energy of motion, potential energy is released, and vice versa. The energy-buildup of space reveals the rest energy of matter as the energy of motion due to the motion of space in the fourth dimension, the direction of the radius of the 4D sphere closing space. Maintaining the zero-energy balance, the buildup of local structures in space converts part of the momentum in the fourth dimension into momentum in space via local bending of space. Such a process occurs in several steps linking the local states of motion and gravitation to the state of rest in hypothetical homogeneous space. DU supports complex function notations comprising the real part that expresses the effects in space directions, and the imaginary part expressing the effect of whole space in the fourth dimension.

2.3. Phenomena that are explained better than by current theories

The overall energy balance in space requires that all gravitationally bound local systems expand in direct proportion to the expansion of space. Early planets have been closer to the Sun, which gives a natural explanation to the faint young Sun paradox, liquid water on Mars, and the higher ocean temperatures required by the early geological development of the Earth. The development of the number of days in a year can be followed from coral fossils originating back to 1 billion years. A precise match with data is obtained by combining the effect of tidal interactions with the effects of local expansion on the length of a day and the length of a year. (See Sipilä's article in this volume.)

Following the energy-bookkeeping, in free fall, kinetic energy is obtained against the release of the rest energy of a falling object which cancels the buildup of "relativistic mass" as suggested by general relativity due to the equivalence principle. Celestial mechanics in DU predicts perihelion advance equal to that in general relativity but cancels the instability of orbits near the critical radius of black holes as predicted by GR. Near the critical radius, DU predicts slow stable orbits that maintain the mass of the black hole. The DU prediction gives an excellent match to the periods observed around Sagittarius A* at the center of the Milky Way [5].

The cosmological appearance of space in DU is clear-cut; distance definitions are given in a closed, parameter-free form. The observed Euclidean appearance of galaxy space is confirmed, and the prediction for the magnitude-redshift relation of Ia Supernovae match observations accurately without dark energy or other experimental parameters. The expansion of space is not accelerating but decelerating due to the work expansion does against the gravitation of the structure.

2.4. Planck's constant and the nature of quantum and matter wave

Without any assumptions tied to DU, Planck's equation can be formally solved from Maxwell's equations by solving the energy that a single electron transition in a one-wavelength dipole emits into a cycle of electromagnetic radiation. A point source can be regarded as a one-wavelength dipole in the fourth dimension, where space moves the distance $cdt=\lambda$ in one cycle. The solution links Planck's constant to primary electrical constants and the velocity of light and discloses the nature of the fine structure constant as a pure numerical or geometrical factor. Removing the velocity of light from Planck's constant, $h_0=h/c$ [kg·m], reformulates Planck's equation into the form $E=h_0/\lambda \cdot c^2= m_\lambda \cdot c^2$ formally equal to the rest energy of mass $m_\lambda =h_0/\lambda$, which is the mass equivalence of a quantum of radiation, the counterpart of the Compton wavelength $\lambda_m =h_0/m$, the wavelength equivalence of mass m.

The reformulation of Planck's equation does not change physics but allows an illustrative picture of the nature of mass, quantum, and the expressions of energy. Following the new formulation, e.g. quantum states, like solutions of Schrödinger's equation in closed systems, appear as energy minima of mass wave states fulfilling relevant resonance conditions. The de Broglie wave can be derived from Compton wavelength as a mass wave carrying the momentum of a moving mass object – much in the way de Broglie was looking for.

2.5. Ontological considerations

The zero-energy approach of DU balances the rest energy of any mass object in space with the gravitational energy of the rest of space. Unification of physics is obtained via unified expressions of energy. The number of postulates needed in DU is radically smaller than that in contemporary physics. In DU, the zero-energy principle applies in all branches of physics; there are no conflicting postulates between different branches.

Following chapters summarize the basic principles and outcomes of the DU theory. A detailed mathematical derivation is documented in the book *The Dynamic Universe – Toward a unified picture of physical reality* [1]. The historical path guiding to DU is tracked in the book *The Short History of Science – or the long path to the union of metaphysics and empiricism* [6].

3. The zero-energy balance of motion and gravitation

3.1. Primary energy buildup in space

The primary energy buildup is described as a contraction-expansion process of spherically closed space. The rest energy appears as the energy of motion obtained against the release of gravitational energy in the contraction of spherical space toward singularity; in the ongoing expansion phase, the energy of motion is paid back to gravitational energy. Such an interpretation assumes a metric fourth dimension, representing the direction of the 4-radius of space and time as a universal scalar allowing the study of velocity and momentum equally in the three space directions and in the fourth dimension.
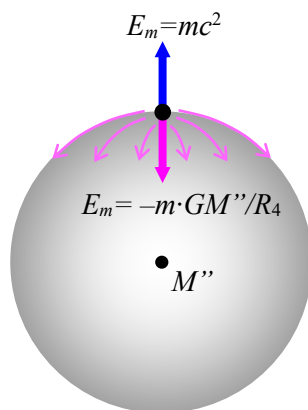
The gravitational energy of mass m in spherically closed space is expressed in terms of the mass equivalence $M'' = 0.776 \cdot M_\Sigma$ at the center of the 4D sphere closing space (Fig. 1). Mass M'' is obtained by integrating the gravitational energy in homogeneous space,

$$E_{g(m)} = -\frac{2}{\pi} \frac{GmM_\Sigma}{R_4} \int_0^\pi \frac{\sin^2 \theta}{\theta} \, d\theta = -\frac{0.776 \cdot GmM_\Sigma}{R_4} = -\frac{GmM''}{R_4} \tag{1}$$

where $G = 6.67 \cdot 10^{-11}$ [Nm^2/kg^2] is the gravitational constant, R_4 the 4-radius of space, and $M_\Sigma = \Sigma m$ the total mass in space. Applying the zero-energy principle, the sum of the total gravitational energy and the total energy of motion in the direction of the 4-radius, $E_m = c|\mathbf{p}_4| = M_\Sigma c^2$, is zero

$$E_m + E_g = M_\Sigma c^2 - GM_\Sigma M'' / R_4 = 0 \tag{2}$$

which means that the energy of motion in the contraction is obtained against release of gravitational energy and released back to the energy of gravitation in the ongoing expansion phase (Fig. 2).

$E_m = mc^2$

$E_m = -m \cdot GM''/R_4$

M''

Figure 1. The dynamics of spherically closed space is determined by the balance between the energies of gravitation and motion. The rest energy of a local object is counterbalanced by the gravitational energy arising from the rest of space. The gravitational energy of mass m due to the rest of space is expressed as the effect of the mass equivalence M'' representing the total mass M_Σ at the 4-center of space.
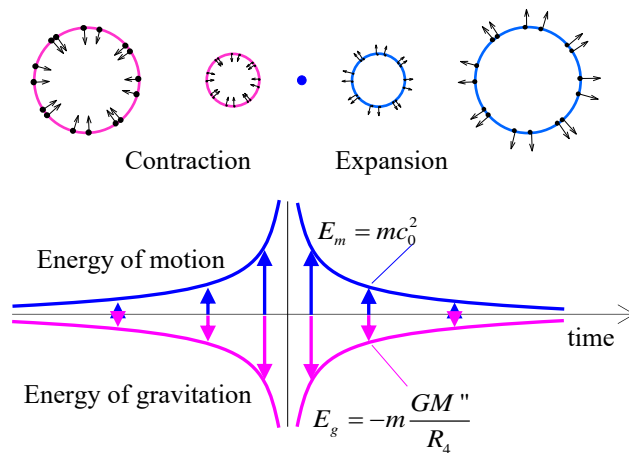
Figure 2. The buildup and release of the rest energy of matter as the energy of motion via contraction and expansion of spherically closed space.

The 4D velocity of space in the contraction and expansion is

$$c = \pm\sqrt{GM''/R_4} \approx 300\,000 \qquad \text{[km/s]} \tag{3}$$

The numerical value is obtained by applying the average mass density $\rho = 5 \cdot 10^{-27}$ [kg/m^3] which is the Friedmann critical mass equivalence in the DU framework, the gravitational constant $G = 6.67 \cdot 10^{-11}$ [Nm^2/kg^2], and R_4 = Hubble radius $\approx 13.7 \cdot 10^9$ [l.y.]. It is convenient to use the complex quantity notation with the real part expressing quantities in space directions and imaginary part expressing the related quantity in the fourth dimension.

Combining the rest momentum in the imaginary dimension with the momentum in a space direction, the total energy of motion can be expressed as

$$E_m = c\left|\mathbf{p}^{\alpha}\right| = c\left|\mathbf{p} + \mathbf{i}\,mc\right| = c\sqrt{p^2 + (mc)^2} \tag{4}$$

which is formally equal to the expression of total energy in special relativity but without any assumptions related to the theory of relativity. A consequence of the conservation of the total energy is that the maximum velocity in space and the velocity of light is equal to the velocity of space in the fourth dimension, $c=c_4$. The buildup of mass centers in space is associated with local bending of space in the fourth dimension which results in a reduction of the local velocity of light, observed as gravitational lensing and a reduction of the rest momentum and a corresponding reduction of the characteristic frequencies of atomic oscillators near mass centers in space (Fig. 3). Reduction of the 4-velocity and the associated reduction of the velocity of light is the DU replacement of the dilated time in the tilted space-time of general relativity.
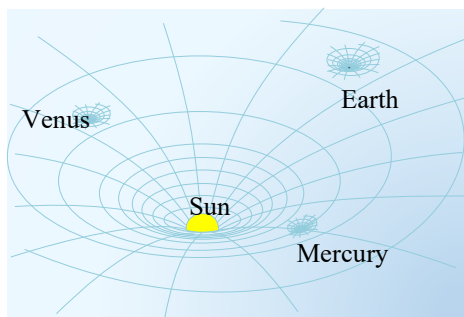
Figure 3 (a). The overall energy balance in space is conserved via tilting of space in local mass center buildup creating the kinetic energy of free fall and the local gravitational energy.
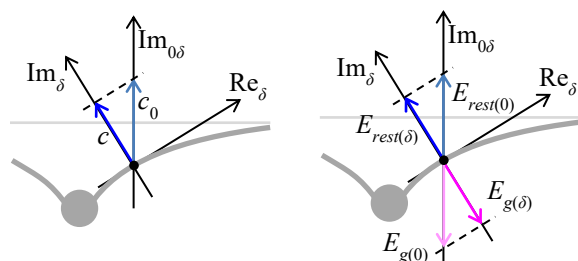
Figure 3 (b). Due to the tilting, the velocity of space in the local fourth dimension is reduced compared to the 4-velocity of the surrounding non-tilted space.

3.2. From instant Big Bang to continuous buildup and release of energy
The buildup of the rest energy in the pre-singularity contraction phase cancels the assumed instant Big Bang event of the standard model of cosmology. The singularity in DU is a state of extreme excitation of the energies of gravitation and motion, followed by turn to expansion at extreme velocity which has gradually slowed down to the present expansion velocity determining the current velocity of light. The deceleration rate of the present expansion of light is $dc_4/c_4 \approx -3.6 \cdot 10^{-11}$ /year. Such a change is observable only indirectly, because the frequency of atomic clocks and the rate of physical processes in general are directly proportional to the velocity of light. Following the zero-energy principle, the local velocity of light is a function of the local gravitational potential. Accordingly, also the ticking frequency of atomic clocks is a function of the local gravitational potential.

4. Linkage to GR space

4.1. Stress-energy tensor
In DU, the rest energy of a mass object in space is counterbalanced by the gravitational energy arising from the rest of space, $E=mc^2=m \cdot GM''/R_4$. Due to the spherical geometry of space, the balance of the complementary energies appears in the fourth dimension. For making sense with velocity, momentum and the corresponding energy of motion in the fourth dimension, the fourth dimension shall be studied as a metric dimension.

When interpreted in the light of Gauss's divergence theory or simply as the physical linkage of pressure and energy content, the stress-energy tensor in general relativity depicts similar symmetry and energy balance in the fourth dimension. On the cosmological scale, in homogeneous space, the stress-energy tensor can be expressed in the form

$$(T^{\mu\nu})_{\mu,\nu=0,1,2,3} = \begin{pmatrix} mc^2/dV & 0 & 0 & 0 \\ 0 & F_{11}/dA & 0 & 0 \\ 0 & 0 & F_{22}/dA & 0 \\ 0 & 0 & 0 & F_{33}/dA \end{pmatrix} \tag{5}$$

where, the energy density mc^2/dV is the rest energy of mass m in volume dV and the local net force densities F_{11}/dA, F_{22}/dA, and F_{33}/dA in the three space directions are equal to zero. The energy content of volume dV is equal to the pressure uniformly from all space directions, which can be interpreted as the integrated gravitational force from whole space. Once the global gravitation on element mc^2/dV appears in the element related to the fourth dimension in the stress tensor, the center of gravity must be in the fourth dimension at equal distance from any space location. Such a situation means spherically closed space.

Einstein [8] drew a similar conclusion in his Berlin Writings in 1914–1917 [7] p. 371: *"If we are to have in the universe an average density of matter which differs from zero, however small may be that difference, then the universe cannot be quasi-Euclidean. On the contrary, the results of calculation indicate that if matter be distributed uniformly, the universe would necessarily be spherical (or elliptical)."*

The concept of spacetime with time as the fourth dimension is confusing; the line element in the fourth dimension is $ds=c \cdot dt$, where c is the velocity of light [m/s] and dt the time differential [s]. It means that the extension in the fourth dimension is not measured in the units of time, seconds [s], but in the units of distance, meters [m]. Accordingly, when measured in the fourth dimension, the "age" of the universe is not 14 billion years, but the distance from the starting point to any location in space today is

$R = \int_0^{today} c \cdot dt = cT \approx 14$ billion lightyears, which means that space is the three-dimensional "surface"

of a four-dimensional sphere with radius $R \approx 14$ billion lightyears.

For conserving the balance of the energies in the local mass center buildup, the total gravitational energy is divided, via the tilting of local space, into orthogonal components with the local gravitational energy in a space direction and the reduced global gravitational energy in the fourth dimension. This means a reduction of the local rest energy of objects and consequently, e.g., reduction of the characteristic frequencies of atomic oscillators in tilted space.

4.2. Critical mass density

Based on measurements of microwave background radiation by the Wilkinson Microwave Anisotropy Probe (WMAP), the mass density in space is essentially equal to Friedmann's critical mass density

$$\rho_c = \frac{3H_0^2}{8\pi G} \approx 9.2 \cdot 10^{-27} \qquad \left[\text{kg/m}^3\right] \qquad (6)$$

where G ($\approx 6.67 \cdot 10^{-11}$ [Nm²/kg²]) is the gravitational constant and H_0 the Hubble constant [$\approx$70 (km/s)/Mpc]. In the framework of Standard Cosmology, such a condition means "flat space" expanding with the energies of motion and gravitation in balance. Assuming the volume of space as the volume of a 3D sphere with radius $R_H = c/H_0$, the total mass in space and the velocity of light can be expressed as

$$M = \rho_c \frac{4\pi R_H^3}{3} = \frac{3c^2}{R_H^2} \frac{4\pi R_H^3}{3 \cdot 8\pi G} = \frac{c^2 R_H}{2G} \qquad \Rightarrow c^2 = \frac{2GM}{R_H}, \qquad (7)$$

respectively.

Solved from the Friedmann's critical mass density, the rest energy of mass m and the total mass $M = \Sigma m$ in Standard Cosmology space are

$$mc^2 = \frac{2GMm}{R_H} \qquad ; \qquad \tfrac{1}{2}Mc^2 = \frac{GM^2}{R_H} \qquad \Rightarrow \quad c = \sqrt{\frac{2GM}{R_H}} \qquad (8)$$

Formally, the last form of equation (8) describes c as the Newtonian escape velocity at distance R_H from mass M at the barycenter representing the total mass in space. This means that the rest energy, as the Newtonian kinetic energy of mass m, is counterbalanced with the global gravitational energy arising from hypothetical mass M at distance R_H from mass m anywhere in space. Such a solution is possible only in 3D space which is the surface of a 4D sphere with radius R_H. The factor ½ in the rest energy Mc^2 in equation (8) comes from the numerical factors used in Einstein's field equations for making them consistent with Newtonian gravitation at a low gravitational field in 3D space.

5. Cosmological consequences

5.1. Development of the expansion of space

DU gives a precise prediction for the development of the expansion rate of space

$$c_0 = \frac{dR_4}{dt} = \left(\frac{2}{3}GM''\right)^{1/3} t^{-1/3} = \frac{2}{3}\frac{R_4}{t} \qquad (9)$$

where t is the time from the singularity. Today, the 4-radius R_4 is about 14 billion light years. Due to faster expansion rate in the past, the age of the expanding space is about 9.3 billion present years.

All gravitationally bound local systems, as well as the wavelength of electromagnetic radiation propagating in space, expand in direct proportion to the expansion of space as a whole (Fig. 4). Atoms and material objects do not expand. 2.8 cm of the measured 3.8 cm annual increase of the Earth to Moon distance comes from the expansion of space and only 1 cm from tidal interactions. Earth and Mars have been closer to the Sun at their infancy which offers an obvious solution to the early faint Sun paradox.
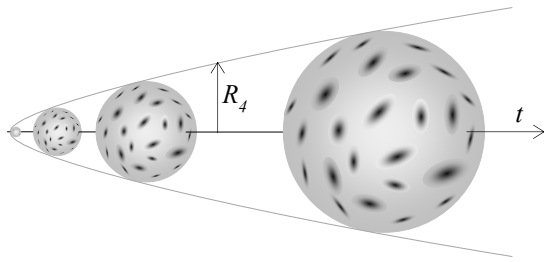
Figure 4. In DU, all gravitationally bound local systems like galaxies and planetary systems expand in direct proportion to the expansion of space.
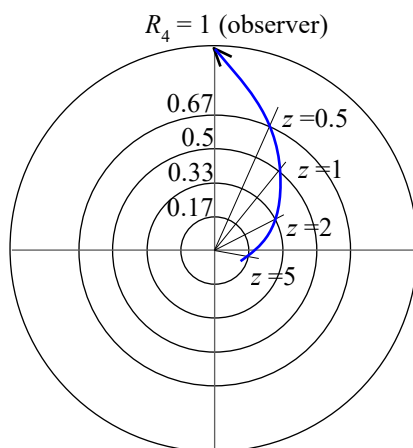
5.2. Optical distance

In DU space, everything is interconnected. The rest energy of any mass object in space is balanced with the gravitational energy arising from the rest of space. All gravitationally bound systems in space expand in direct proportion to the expansion of the 4-radius of space. The linkage of the velocity of light in space to the expansion velocity of space in the fourth dimension means, e.g., that the optical distance in space is equal to the increase of the 4-radius during the time light propagates from the object. Such a situation allows a simple, closed form expression for the optical distance versus redshift

$$D = R_0 \frac{z}{1+z} \tag{10}$$

where R_0 is the 4-radius of space at the time of the observation (Fig. 5).

The optical distance applies to angular size distance and, when corrected with Doppler dilution, to the luminosity distance. In DU, luminosity distance applies directly to the observed bolometric magnitudes (without reduction to the emitter's rest frame by the K-correction like in GR cosmology) and produces precise predictions, e.g., to Ia supernovae magnitudes without hypothetical dark energy. In DU, there is no basis for the reciprocity [8] of Standard Cosmology.


Figure 5. Light path in expanding space. The optical distance is the integrated tangential component of the of light path. The radial direction in the picture, is the fourth dimension showing the development of the expansion.

5.3. Euclidean appearance of galaxy space

The spherical geometry, the linkage of the velocity of light to the expansion velocity, and the linkage of the size of quasars and galaxies to the expansion of space result in Euclidean appearance of galactic space, fully supported by observations [9] (Fig. 6).
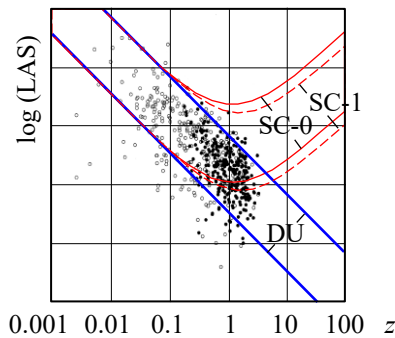
Figure 6. The data points fall well between the Euclidean DU prediction (blue lines). The red SC-0 and SC-1 curves are the Standard Cosmology predictions without (solid line) and with dark energy (red lines), respectively.

5.4. Magnitude of standard candle

DU produces a precise prediction for the bolometric magnitude of standard candles without dark energy or any other adjustable parameters. For applying the DU prediction to *K*-corrected magnitudes used in standard cosmology, the DU prediction obtains the form

$$m_{DU} = M + 5\log\left(\frac{R_4}{10\text{pc}}\right) + 5\log(z) + 2.5\log(1+z) \tag{11}$$

Figure 7 illustrates the match of (11) to the *K*-corrected observations of Ia supernovae [10].
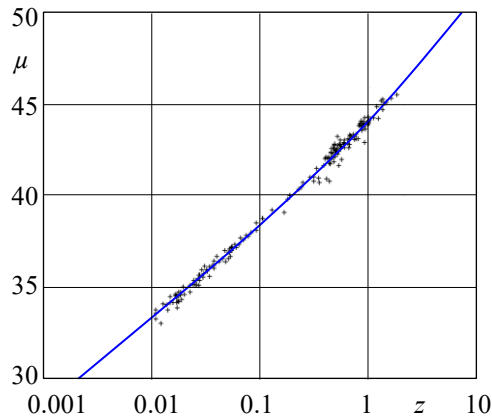
Figure 7. Distance modulus $\mu = m - M$, vs. redshift for Riess et al. "high-confidence" dataset and the data from the HST [10]. The DU prediction (solid curve) is based on equation (11).

5.5. Days in a year

Perhaps the most convincing cosmological support for the linkage between the size of planetary systems and the expansion of space comes from the prediction for the development of the number of days.

A unique possibility for studying the long-term development of the Earth's rotation comes from paleo-anthropological data available almost 1000 million years in the past. Fossil layers preserve both the daily and annual variations, thus giving the development of the number of days in a year [11, 12]. The lengthening of a day for the past 2700 years is also available from ancient Babylonian and Chinese eclipse observations [13,14]. The average lengthening of a day obtained from the eclipse observations is 1.8 ms/100y, which is about 0.7 ms/100y less than the estimated effect of tidal friction, 2.5 ms/100y. The length of a day has been measured with atomic clocks since 1955. An announced result for the lengthening of day by NASA is 1.5 ms/100y.

According to GR and Standard Cosmology, planetary systems do not expand with the expansion of space, and atomic clocks conserve their frequencies. It means that the length of a year is assumed unchanged, and the length of a day is affected only by tidal interactions with the Moon and Sun.
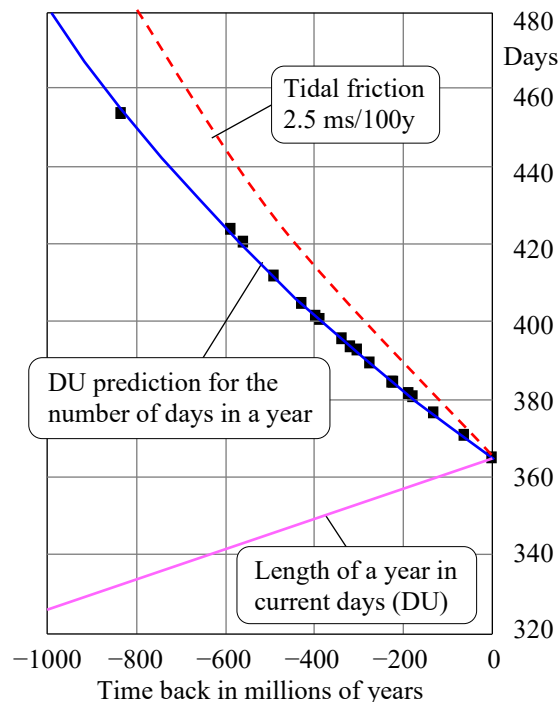
Figure 8. Development of the length of a year in the number of days. Black squares are data points from [12,13].

In DU framework, planetary systems expand in direct proportion to the expansion of space and the frequency of atomic clocks slows down in direct proportion to the decrease of the velocity of light. As a consequence, the length of a year, the length of a day, and the frequency of atomic clocks change with the expansion of space. Combining the change in the length of a year, 0.6 ms/100y, with the effect of tidal friction on the length of a day, 2.5 ms/100y we obtain 1.9 ms/100y which precisely matches the value obtained from the coral fossil data and is essentially the same as the result calculated from ancient solar eclipses (1.8 ms/100y) (Fig. 8). Correcting the atomic clock measurement by NASA with the DU correction due to the change of the frequency of atomic clocks, we get to 1.9 ms/100y.

5.6. The faint young Sun paradox and the lunar distance
At the time of the early development of the planets about 4 billion years ago, solar insolation is estimated to be about 25% fainter than it is today [15]. Based on geological observations, the temperature of oceans on the Earth has been about 30-40 °C. Also, there is evidence of liquid water on Mars at that time. According to DU, Earth and Mars have been about 30% closer to the Sun than they are today. Combining that with the fainter luminosity of the Sun, 30-40 °C ocean temperature on the Earth and liquid water on Mars are well in line with the DU prediction.

The distance of the Moon has been monitored in the Lunar Laser Ranging program since 1970s [16]. In the DU framework, 2.8 cm of the measured 3.8 cm annual increase of the Earth to Moon distance comes from the expansion of space and only 1 cm from the tidal interactions.

6. Motion and gravitation in local space

6.1. Momentum as complex function

6.1.1. Constant gravitational potential. Any motion in space is associated with the motion of space in the fourth dimension. It is convenient to express momentum and energy as complex quantities with the momentum in the fourth dimension as the imaginary part and the momentum in a space direction as the real part. In the complex function presentation, the total energy of motion is
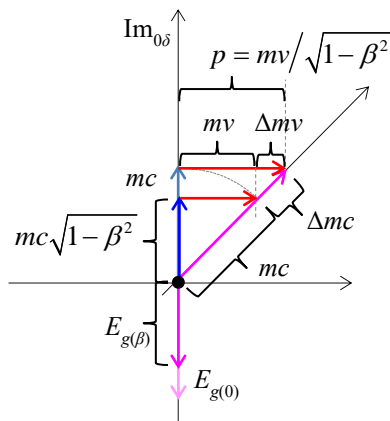
Figure 9. In DU space, buildup of velocity v at constant gravitational potential requires insertion of energy $c_0 \cdot \Delta mc$ which results in the momentum in the direction of real axis and total energy $E_{tot} = c_0 \cdot (m+\Delta m)c$, and the total momentum $\mathbf{p} = (m+\Delta m)\mathbf{v}$ in the direction of the real axis (space direction). Energy $c_0 \Delta mc$ is the energy insertion from the accelerating system resulting in the kinetic energy.

$$E_m^{\pi} = c_0 \cdot \left| \mathbf{p} + \mathbf{i}mc \right| = c_0 \left| \mathbf{p}^{\pi} \right| \tag{12}$$

where c_0 is the velocity of light in hypothetical homogeneous space, and c is the local velocity of light (in locally bent space). $\mathbf{p}$ is the momentum in space and $\mathbf{p}^{\pi}$ the complex total momentum. For mass m at rest in a local frame $\mathbf{p}=0$ and equation (12) gives the rest energy. For electromagnetic radiation $\mathbf{i}mc=0$, and the energy is $E=c_0|\mathbf{p}|$. For a moving mass object with momentum $\mathbf{p}$ in space the total energy of motion is

$$E = \text{Mod}\left\{ E_m^{\pi} \right\} = c_0 \cdot \sqrt{p^2 + (mc)^2} = c_0 (m + \Delta m) c \tag{13}$$

In the Earth gravitational frame, c is estimated as $c \approx 0.999\ 999 \cdot c_0$. Equation (13) conveys the total energy expression of special relativity without any assumptions behind the relativity theory. A detailed analysis of momentum $\mathbf{p} = (m+\Delta m)\cdot\mathbf{v}$ shows that the part $m\cdot\mathbf{v}$ of the momentum is the real part of the rotated rest momentum mc, and $\Delta m\cdot\mathbf{v}$ the real part of Δmc, the addition to the rotated rest momentum completing the total momentum (Fig. 9).

In spherically closed space, any motion is central motion relative to the 4-center of the structure. The work done by the central force against the gravitational force due to the rest of space in the fourth dimension is observed as the reduction of the rest energy of the moving object. This is the quantitative expression of Mach's principle.

The rest momentum and the corresponding rest energy of a moving object is reduced as

$$E_{rest(v)} = E_{rest(0)} \sqrt{1 - \beta^2} \tag{14}$$

where $\beta=v/c$. Equation (14) means, e.g., that atomic clocks in motion run slower – exactly in the way predicted by special relativity, however, not because of dilated time but as the consequence of reduced rest energy of the oscillating electrons in the clock. Also, equation (14) means that the frame of reference where velocity v is observed is the energy frame where the kinetic energy was created.

6.1.2. Momentum in free fall. The gravitational energy balancing the rest energy of a test mass m arises from all mass in space that is represented by mass equivalence $M''=0.776\cdot M_{tot}$ at the center of the 4D sphere, the barycenter of spherically closed space. Buildup of local mass centers means removal of mass from the symmetry to build up a mass center in a specific space direction. At distance R from the local mass center M in space the global gravitational energy arising from M" is reduced as,

$$E_{g(\delta)}^{''} = \frac{GM''m}{R''}\left(1 - \frac{GM}{Rc_0^2}\right) = E_{g(0)}^{''}(1-\delta) \tag{15}$$
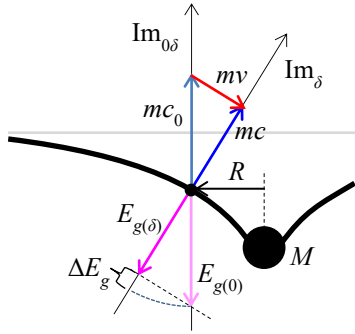
FIG. 10. Buildup of velocity v in free fall in a gravitational field is obtained against reduction of the local 4-velocity of space; there is no mass insertion, and the momentum is **p**=mv. The kinetic energy of free fall is equal to the gravitational energy released

$$E_{kin} = \Delta E_g^{''} = E_{g(0)}^{''} - E_{g(\delta)}^{''} = c_0 \cdot m\Delta c$$

which balances the local rest energy at distance R from the mass center

$$E_{rest(\delta)} = E_{g(\delta)}^{''} = E_{rest(0\delta)}\left(1-\delta\right) \tag{16}$$

where $E''_{rest(0\delta)}$ is the rest energy of mass m at rest far from the local mass center M (Fig. 10). As required by the conservation of the total energy, the kinetic energy obtained in free fall from homogeneous space to distance R from mass center M is equal to the reduction of the local rest energy, $\Delta E_{rest} = c_0\Delta mc$.

Combining the effects of motion and gravitation, the rest energy in a local gravitational frame is expressed

$$E_{rest(\beta,\delta)} = E_{rest(0\delta,0)}\left(1-\delta\right)\sqrt{1-\beta^2} \; . \tag{17}$$

As illustrated in figure 10 the reduction of the rest energy by local gravitation is associated with a reduced velocity of light. The frequency of atomic oscillators is directly proportional to the rest momentum of the oscillating electrons. Equation (17) conveys the combined effect of motion and gravitation on the frequency. Equation (17) is the DU replacement of Schwarzschildian time dilation of general relativity
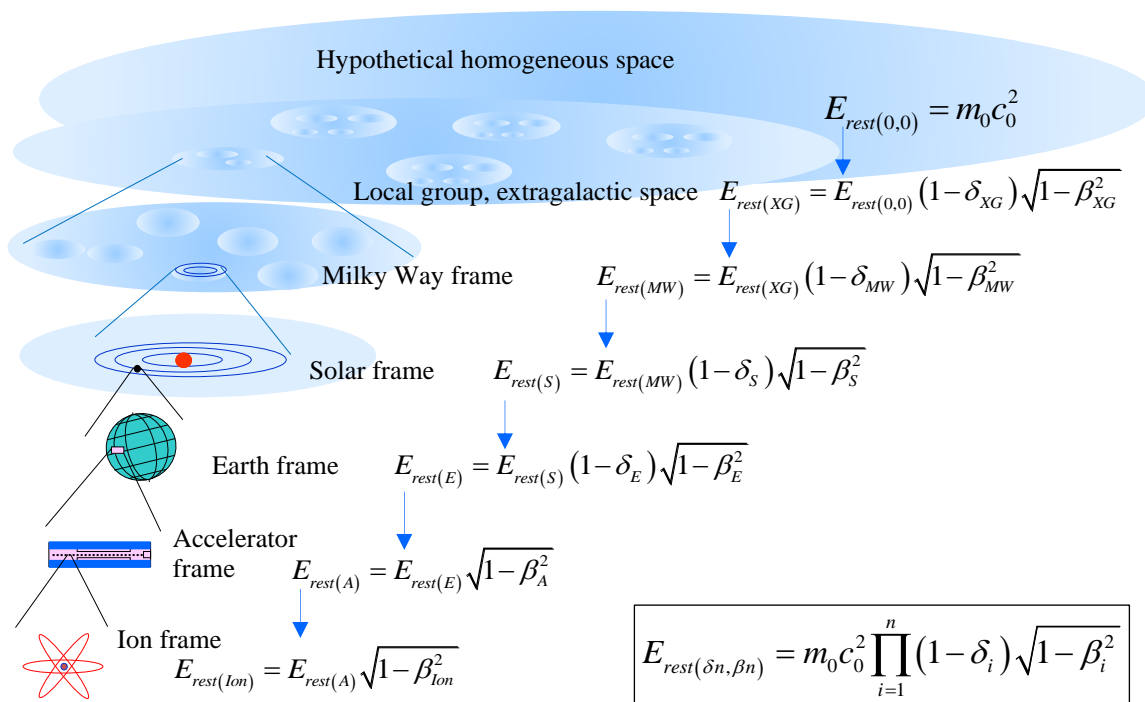
$$dt = dt_0\sqrt{1-2\delta-\beta^2} \tag{18}$$

In the Earth gravitational frame, the difference between equations (17) and (18) appears only in the 18th to 20th decimal. In DU space, the local velocity of light is locked to the local 4D velocity of space. A mass center like the Earth, orbiting the Sun, draws a dent in space with the orbital motion and conserves the local velocity of light at a fixed distance from the center.

6.1.3. The system of nested energy frames. The buildup of mass centers in space occurs in several steps. Following the conservation of the overall balance of the energies of motion and gravitation, the rest energy of mass m in any local frame can be related to the rest energy of mass m at rest in hypothetical homogeneous space

$$E_{rest(\beta,\delta)} = E_{rest(0,0)}\prod_{i=0}^{n}\left(1-\delta_i\right)\sqrt{1-\beta_i^2} \; . \tag{19}$$

Figure 11 illustrates the system of nested energy frames relating the rest energy of an accelerated ion in an accelerator on the Earth to the rest energy the electron would have at rest in hypothetical homogeneous space. The system of nested energy frames means full replacement of the observer-centered frames of reference applied in the framework of the theory of relativity.

Hypothetical homogeneous space

$$E_{rest(0,0)} = m_0 c_0^2$$

Local group, extragalactic space $\quad E_{rest(XG)} = E_{rest(0,0)}\left(1-\delta_{XG}\right)\sqrt{1-\beta_{XG}^2}$

Milky Way frame $\quad E_{rest(MW)} = E_{rest(XG)}\left(1-\delta_{MW}\right)\sqrt{1-\beta_{MW}^2}$

Solar frame $\quad E_{rest(S)} = E_{rest(MW)}\left(1-\delta_{S}\right)\sqrt{1-\beta_{S}^2}$

Earth frame $\quad E_{rest(E)} = E_{rest(S)}\left(1-\delta_{E}\right)\sqrt{1-\beta_{E}^2}$

Accelerator frame $\quad E_{rest(A)} = E_{rest(E)}\sqrt{1-\beta_{A}^2}$

Ion frame $\quad E_{rest(Ion)} = E_{rest(A)}\sqrt{1-\beta_{Ion}^2}$

$$E_{rest(\delta n, \beta n)} = m_0 c_0^2 \prod_{i=1}^{n}\left(1-\delta_i\right)\sqrt{1-\beta_i^2}$$

Figure 11. The system of nested energy frames. The rest energy in the *n*:th (local) frame is subject to reductions due to the motions and gravitational states of the local frame in all its parent frames – and is finally related to the rest energy the object would have at rest in hypothetical homogeneous space.

7. Celestial mechanics in DU space

### 7.1. Orbital precession and black holes

The DU prediction for the precession (in addition to interaction with other planets) of the orbit of the planet Mercury is the same 43 arcseconds/100 years as that given by the Schwarzschild's solution of general relativity. Schwarzschild solution generates a small cumulative term that increases the orbital radius. In textbooks, the perihelion is generally solved for a single revolution which allows omitting the cumulative term as a secondary secular [17], [18]. When calculated for about one million cycles, the cumulative term, however, grows large enough to cast Mercury out of the solar system (Fig. 12(a)). For orbits close to black holes, the cumulative term is large enough to throw the orbiting object out of the system in one revolution, which excludes orbits with the radius shorter than three times the critical radius in Schwarzschild space. The DU solution of the orbit around a mass center does not have cumulative terms, which means that all orbits, including orbits around black holes are stable (Fig. 12(b)).

In DU space, the orbital period has its minimum at the radii 2 times the DU critical radius $r_0 = GM/c^2$, which is half of the Schwarzschild critical radius. Orbital velocities in orbits with radius approaching the DU critical radius approach zero, which allows the mass at the slow orbits maintain the mass of the black hole.

Figure 13 illustrates the orbital period close to the critical radius of Sagittarius A* at the center of the Milky Way. The calculated minimum period in DU space is 14.8 min, which is short enough to explain the observed 16.8 min orbits [5].

(a) Schwarzschild space   (b) DU space
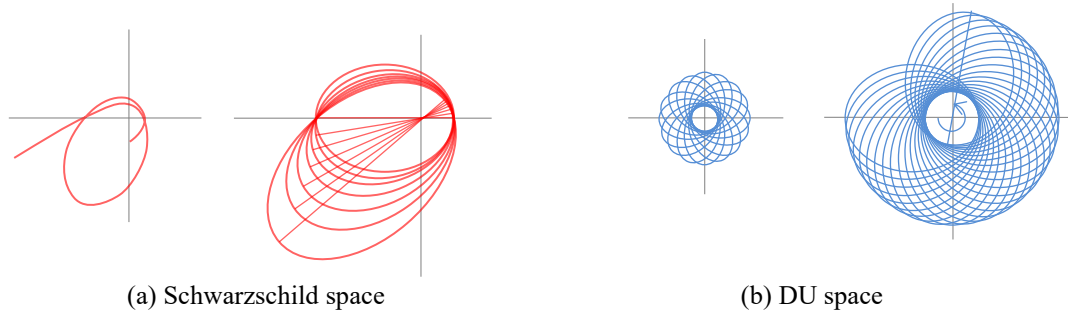
Figure 12. Development of the orbital precession for a near black hole orbit characterized by $r/r_0 =$ 20 and e=0.5, and for Mercury's orbit with $r/r_0 = 4 \cdot 10^7$ and $e = 0.2$. ($r_0 = GM/c^2$). (a) In Schwarzschild space the orbiting object escapes when the cumulated precession exceeds 45°. The calculation is based on the Schwarzschildian solution given in [17]. (b) In DU space orbits are stable. Mass at slow orbits near the critical radius maintain the mass on the black hole.
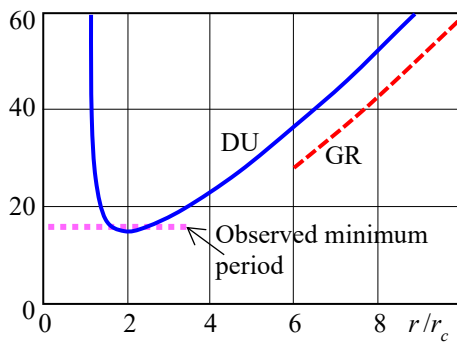
Figure 13. Predictions for the period of circular orbits around Sagittarius A* in the center of Milky Way. The shortest observed period is 16.8 ± 2 min [5] which is close to the minimum period of 14.8 minutes predicted by the DU. In GR space, the suggested explanation for the observed "too fast" periods is a rotating black hole referred to as the Kerr black hole.
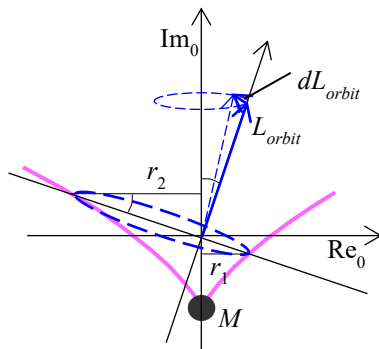
7.2. Orbital decay

In DU framework, the decay of the period of an elliptic orbit is a consequence of the periastron rotation and the related rotation of the orbital angular momentum in the fourth dimension (Fig.14).

Interestingly, the prediction (20) derived from the rotation of the 4D orbital angular momentum [1] gives essentially the same prediction as the GR prediction (21) based on the change of the quadrupole moment [19], [20]. The only difference is, that DU predicts orbital decay for eccentric orbits only, GR predicts decay for circular orbits, too (Fig. 15). The possible energy radiation (gravitational radiation) by the rotating 4D angular momentum in the DU has not been analyzed.

$$\text{DU:} \quad \frac{dP}{dt}_{(DU)} \approx 120 \cdot \frac{G^{5/3}}{c^5} \left(\frac{P}{2\pi} \right)^{-5/3} \left(2 \cdot \frac{\left[\sqrt{1+e_{0\delta}} - \sqrt{1-e_{0\delta}} \right]}{\left(1-e^2\right)^2} \right) \cdot \frac{m_p m_c}{\left(m_p + m_c\right)^2} \left(m_p + m_c\right)^{5/3} \quad (20)$$

$$\text{GR:} \quad \frac{dP}{dt}_{(GR)} \approx 123 \cdot \frac{G^{5/3}}{c^5} \left(\frac{P}{2\pi} \right)^{-5/3} \left(\frac{1+(73/24)e^2+(37/96)e^4}{\left(1-e^2\right)^{7/2}} \right) \cdot \frac{m_p m_c}{\left(m_p + m_c\right)^2} \left(m_p + m_c\right)^{5/3} \quad (21)$$

Figure 14. In DU framework the orbital decay of binary stars is calculated from the energy related to the rotation of orbital angular momentum due to the periastron advance.
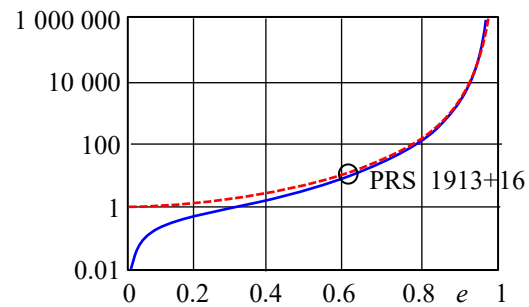
Figure 15. The eccentricity factor of the decay of binary star orbit period. At the eccentricity $e = 0.616$ of the PSR 1913+16 orbit the eccentricity factor of the GR and DU for the orbit decay are essentially the same. According to DU prediction the eccentricity factor goes to zero at zero eccentricity.

8. Mass and electromagnetic radiation

8.1. From Maxwell's equations to Planck's equation

Unlike generally understood, formally, Planck's equation can be derived from Maxwell's equations. Applying the standard solution of the Hertzian dipole, the energy emitted by a single oscillation of N electrons in one-wavelength dipole into a cycle of electromagnetic radiation is

$$E_\lambda = N^2 \cdot A \cdot 2\pi^3 e^2 \mu_0 c \cdot f \tag{22}$$

where A is the geometrical factor of the dipole which for a Hertzian dipole is $A=2/3$, e is the electron charge, μ_0 vacuum permeability, c the velocity of light, and $f=c/\lambda$ the frequency of the radiation emitted. (Note that equation (22) applies the vacuum permeability $\mu_0 = c^2/\varepsilon_0$ instead of vacuum permittivity ε_0 most commonly used.)

In DU framework, a point emitter, like an atom, moves the distance of one wavelength in the fourth dimension in a cycle (equal to the 4D line element cdt in the GR framework). A point emitter can be considered as a one-wavelength dipole in the fourth dimension with isotropic emission pattern suggesting A close to 1. For a single electron transition ($N=1$), with $A = 1.1049$ equation (22) becomes

$$E_\lambda = 1.1049 \cdot 2\pi^3 e^2 \mu_0 c \cdot f = h \cdot f \tag{23}$$

where the factor $1.1049 \cdot 2\pi^3 e^2 \mu_0 \cdot c = 6.62607 \cdot 10^{-34}$ [Js] is equal to Planck's constant h. An important message of equation (23) is that the Planck constant h has the velocity of light as a "hidden" internal factor. Defining the *intrinsic Planck constant* $h_0 = h/c$, Planck's equation obtains the form

$$E_\lambda = h \cdot f = h_0 c \cdot f = \frac{h_0}{\lambda} cc = m_\lambda c^2 \tag{24}$$

where the quantity $h_0/\lambda = m_\lambda$ is referred to as the *mass equivalence of electromagnetic radiation*. In DU framework, including the conversion factor c_0/c (estimated of the order of ppm) into factor A, equation (24) is written in form

$$E_\lambda = c_0 \frac{h_0}{\lambda} c = c_0 m_\lambda c = c_0 |\mathbf{p}| \ . \tag{25}$$

Applying the intrinsic Planck constant, Compton wavelength, as the *wavelength equivalence of mass* appears as the counterpart of the *mass equivalence of electromagnetic radiation*

$$\lambda_{Compton} \equiv \frac{h}{mc} = \frac{h_0}{m} = \lambda_m \tag{26}$$

allowing the wave expression of the rest energy in the form

$$E_{rest} = c_0 mc = c_0 \frac{h_0}{\lambda_m} c \tag{27}$$

The breakdown of Planck's constant into primary electrical constants discloses the physical nature of the fine structure constant α as a pure numerical or geometrical constant without connections to other natural constants

$$\alpha \equiv \frac{e^2 \mu_0 c}{2 h_0 c} = \frac{e^2 \mu_0}{2 \cdot 1.1049 \cdot 2\pi^3 e^2 \mu_0} = \frac{1}{4 \cdot 1.1049 \cdot \pi^3} \simeq \frac{1}{137.0360} \tag{28}$$

In DU, mass obtains the role of the wavelike substance for the expression of energy. Mass expresses energy via motion, gravitation or Coulomb energy, which for unit charges e can be expressed

$$E_C = \frac{e^2}{4\pi\varepsilon_0 r} = \frac{e^2 \mu_0}{4\pi r} c^2 = c_0 \alpha \frac{h_0}{r} c = c_0 m_C c \tag{29}$$

where m_C is the mass equivalence of unit Coulomb energy.

8.2. The frequency of atomic oscillators

The quantum mechanical solution of the emission/absorption frequency of atomic oscillators can be expressed in terms of the rest energy of the oscillating electrons, the Planck constant, and the quantum numbers characterizing the energy states related to the oscillation

$$f_{(n1,n2)} = \frac{\Delta E_{(n1,n2)}}{h} = \frac{E_{(e)rest}}{h} \Delta F \left(\alpha, n, l, m_l, m_s \right) \tag{30}$$

where $\Delta E_{(n1,n2)}$ is the difference of the rest energy of an electron in the two energy states relevant to the emission/absorption process, h is the Planck constant, m_e the rest mass of the electron of the atom in the local energy frame, and c the local velocity of light. The function $\Delta F(\alpha, n, l, m_l, m_s)$ is determined by the fine structure constant α and the quantum numbers characterizing the energy states in question. Applying the intrinsic Planck constant $h_0 = h/c$ equation (30) reduces to the form

$$f_{(n1,n2)} = \frac{m_e c}{h_0} \Delta F \left(\alpha, n, l, m_l, m_s \right) \tag{31}$$

which means that the characteristic frequency of an atomic oscillator is directly proportional to the rest mass of the oscillating electrons and the local velocity of light, which also guarantees that the velocity of light appears as constant when measured with an atomic clock. In DU framework, the rest mass is affected by motion as

$$m_{rest(n)} = m_{(0,0)} \prod_{i=0}^{n} \sqrt{1 - \beta_i^2} \tag{32}$$

and the local velocity of light by the local gravitational state as

$$c = c_{(n)} = c_{(0,0)} \prod_{i=0}^{n} (1 - \delta_i) \tag{33}$$

which give the general expression to the characteristic frequency

$$f_{(\beta_n, \delta_n)} = f_{(00,00)} \prod_{i=0}^{n} (1 - \delta_i) \sqrt{1 - \beta_i^2} \tag{34}$$

where $f_{(00,00)}$ is the frequency of the oscillator at rest in hypothetical homogeneous space. In a local energy frame, the frequency is expressed as

$$f_{(\delta, \beta)} = f_{(0\delta, 0)} (1 - \delta) \sqrt{1 - \beta^2} \tag{35}$$

where $f_{(0\delta,0)}$ is the frequency of the oscillator at rest (β=0) out of the gravitational interaction (δ=0) of the local frame like, e.g., the rest clock in the Earth gravitational frame (Earth Centered Inertial, ECI frame). Equation (35) applies to all clocks moving on the Earth and in near space. Equation (35) is the DU replacement of the dilated time in Schwarzschild space used to explain the changing clock frequencies in the Earth gravitational frame in GR framework

$$dt = dt_0 \sqrt{1 - 2\delta - \beta^2} \tag{36}$$

In the Earth gravitational frame, equations (35) and (36) give equal predictions up to the 18th to 20th decimal.

Closed systems like accelerators or centrifuges are subframes in the Earth gravitational frame. The clock frequency in such frames is

$$f_{(\delta, \beta)} = f_{(0\delta, 0)} \left(1 - \delta_{sf/Earth}\right) \sqrt{1 - \beta_{sf/Earth}^2} \sqrt{1 - \beta_{sf}^2} \tag{37}$$

where $\delta_{sf/Earth}$ and $\beta_{sf/Earth}$ are the gravitational factor and velocity of the subframe in the Earth gravitational frame, respectively, and β_{sf} the velocity of the clock in the subframe. When related to the frequency $f_{(sf=0)}$ of a reference clock at rest relative to and at the same gravitational state as the subsystem (the laboratory frame), equation (37) can be expressed as

$$f_{(\beta, \delta)} = f_{(sf=0)} \sqrt{1 - \beta_{sf}^2} \tag{38}$$

which corresponds to the time dilation equation in the framework of special relativity,

$$dt = dt_0 \sqrt{1 - \beta^2} \tag{39}$$

relating the "flow of time" in the clock's frame of reference to the flow of time in the observer's frame of reference.

8.3. The velocity of light

In DU framework, the velocity of light is linked to the local 4D velocity of space, which is a function of the local gravitational state. Bending of the light path passing a mass center as well as the Shapiro delay are direct consequences of the slower speed of light and the increased distance due to the dent around a mass center. Motion of a mass center in its parent frame, like the Earth in the Solar System gravitational frame draws the local dent with the motion, which conserves the velocity of light at a fixed gravitational state in the Earth gravitational frame.

The frequency of an atomic clock is directly proportional to the local velocity of light which means that the velocity of light is observed unchanged when measured with atomic clocks. The signal transmission time, e.g., from a satellite to a receiver on the rotating Earth can be calculated from the actual distance from the satellite at the time the signal is sent to the location of the receiver at the time

the signal is received. Such a calculation includes the Sagnac correction needed in the GR/SR framework as a correction for the motion of the receiver during the signal transmission time.

9. Ontological considerations

9.1. A. The nature of quantum

The wavelike nature of mass enabling the expression of the energy of electromagnetic radiation via mass equivalence $m_\lambda = h_0/\lambda$ or $m_\lambda = \hbar_0 k$ conveys many features obtained with the concept of wave function in the standard formalism of quantum mechanics. Energy eigenstates of electrons in atoms are considered discrete energy states. Resonant mass wave states show the same energy states, not as discrete energy states but as the energy minima of states fulfilling a resonance condition. Identification of Planck's equation as the energy conversion equation at the emitter and absorber instead of an intrinsic property of radiation, has important consequences in cosmology, especially on the interpretation of the effect of Planck's equation on the dilution of redshifted radiation [21].

The solution of Planck's equation from Maxwell's equations as the energy emitted to a cycle of electromagnetic radiation by a unit charge transition in the emitter re-establishes Planck's interpretation of the equation as the energy conversion equation at the emitter and absorber. In principle, an emitter may be isotropic or directional; in the first case the radiation emitted from a point source is spread uniformly to all space directions, in the latter case it is observed as a localized photon-like energy object like the emission from a laser.

Absorption of quantum is symmetric with the emission; the energy carried by a cycle of radiation is absorbed if the energy within the capturing area of the receiving "antenna" is enough to result in an electron transition corresponding to the energy characteristic of the wavelength of the radiation. It means, e.g., that we do not need localized photons for explaining the photoelectric effect.

As given by Maxwell's equations, the energy emitted into a cycle of radiation by an emitter with N oscillating electrons is

$$E_\lambda = N^2 \frac{h_0}{\lambda} c_0 c \qquad \left(= N^2 hf\right) \tag{40}$$

where N^2 is the intensity factor. A "quantum receiver" is not energy selective but wavelength selective like any radio antenna.

9.2. From Compton wavelength to de Broglie wavelength

In DU framework, localized mass objects can be described as "3D Compton wave resonators". At the state of rest, the momentum of the resonator, as the sum of opposite 3D waves, appears in the fourth dimension as the rest momentum. When the resonator moves at velocity βc in space, the rest momentum of the resonator is reduced as
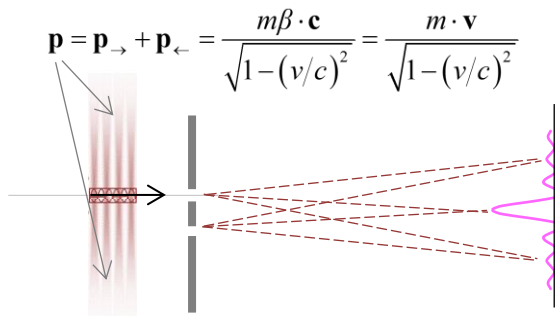
$$\mathbf{p}_{rest(\beta)} = \mathbf{i}\, mc\sqrt{1-\beta^2} \tag{41}$$

In the direction of the motion, the momentums of the Doppler shifted front and back waves in the rest frame are

$$\mathbf{p}_{rest(\beta)\rightarrow} = \tfrac{1}{2} \cdot \frac{mc\sqrt{1-\beta^2}}{1-\beta}\hat{\mathbf{r}} \qquad \text{and} \qquad \mathbf{p}_{rest(\beta)\leftarrow} = \tfrac{1}{2} \cdot \frac{mc\sqrt{1-\beta^2}}{1+\beta}\hat{\mathbf{r}}, \tag{42}$$

respectively, resulting in a net wave with momentum

$$\mathbf{p}_{\hat{\mathbf{r}}} = \mathbf{p}_{rest(\beta)\rightarrow} + \mathbf{p}_{rest(\beta)\leftarrow} = \frac{m\beta}{\sqrt{1-\beta^2}}c\,\hat{\mathbf{r}} = \frac{m}{\sqrt{1-\beta^2}}\beta c\,\hat{\mathbf{r}} = m_{eff}\mathbf{v} \tag{43}$$

Figure 16. The momentum of an object moving at velocity βc is the external momentum as the sum of the Doppler shifted front and back waves, which can be described as the momentum of a wave front propagating in the local frame in parallel with the propagating mass object.

which is the momentum de Broglie wave carries in space. The momentum wave can be interpreted as a wave with mass $m\beta / \sqrt{1-\beta^2} = \beta m_{eff}$ propagating at velocity c, or mass $m / \sqrt{1-\beta^2} = m_{eff}$ propagating at velocity $\beta c = v$. The momentum wave is observed propagating "beside" the moving object in the local frame, giving a natural explanation to the double slit experiment (Fig. 16).

9.3. Quantum states as energy minima of resonant mass wave structures
Applying the concept of a mass wave, the principal energy states of an electron in hydrogen-like atoms can be solved by assuming a resonance condition of the de Broglie wave in a Coulomb equipotential orbit around the nucleus. The Coulomb energy of Z electrons at distance r from the nucleus is

$$E_{Coulomb} = -Z\alpha \frac{h_0}{2\pi r} c_0 c = -Z\alpha \frac{\hbar_0}{r} c_0 c \tag{44}$$

For a resonance condition the de Broglie wavelength $n\lambda_{dB} = 2\pi r$, which is equal to the wave number boundary condition $k_{dB} = n/r$. The energy of an electron as the sum of kinetic energy and Coulomb energy in a Coulomb equipotential orbit with radius r is $E_n = E_{kin} + E_{Coulomb}$ and can be written in the form

$$E_n = \hbar_0 k_m c_0 c \left[\sqrt{1 + \left(\frac{n}{k_{dB}r}\right)^2} - 1 - \frac{Z\alpha}{k_{dB}r} \right] \tag{45}$$

The solution of equation (45) is illustrated in figure 17; for each value of n, the total energy E_n is a continuous function of r. The "quantized" energy states are energy minima of E_n for each value of n. The minima are obtained by derivation of equation (45)

$$E_{Z,n} = -mc_0 c \left[1 - \sqrt{1 - \left(\frac{Z\alpha}{n}\right)^2} \right] \approx -\left(\frac{Z}{n}\right)^2 \frac{\alpha^2}{2} mc^2 \tag{46}$$

showing the "relativistic" minima with an approximation equal to the non-relativistic solution obtained from Schrödinger's equation.

10. Philosophical considerations

10.1. The essence of mass
Breaking down Planck's constant into its constituents opens up the essence of mass as wavelike "substance" for the expression of energy. Mass is not a form of energy, but it expresses energy related to motion and potentiality. In DU framework, mass is conserved also in annihilation; the mass equivalence of emitted photons is equal to the rest mass of annihilated particles. The total mass in space
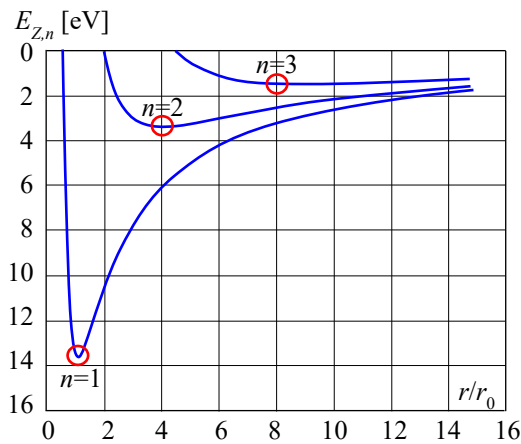
Figure 17. Total energy of electron in hydrogen-like atoms for principal quantum number $n = 1$, $n = 2$, $n = 3$ according to equation (45). Orbital radii of the energy minima are $r/r_{Bohr}=1$, $r/r_{Bohr}=4$, and $r/r_{Bohr}=8$, respectively.

is a primary conservable. The contraction of space builds up the excitation of complementary energies of motion and gravitation. The anti-energy for the rest energy of a localized mass particle is negative gravitational energy arising from all other mass in space.

10.2. Inertia and Mach's principle

In DU framework, inertial work is the work done against the global gravitational energy as the interaction in the fourth dimension, which means a quantitative explanation of Mach's principle. Inertia is not a property of mass; in DU framework, the "relativistic mass increase" Δm introduced in SR framework is the mass contribution by the accelerating system for the buildup of kinetic energy. In the complex quantity presentation, the real part of kinetic energy increases the momentum observed in space, and the imaginary part of kinetic energy reduces the global gravitational energy and the rest energy of the moving object, which is observed as the reduced ticking frequency of atomic clocks in motion.

Any motion in space is central motion relative to the barycenter of space in the center of the 4D sphere defining space. Inertial work can be understood as the work that the central force created by motion in space does against the global gravitational force in the fourth dimension. Energy objects like photons or electromagnetic radiation propagating at the velocity of light in space move like at a satellite orbit around the barycenter of space. Electromagnetic radiation has its mass equivalence; radiation is not massless but weightless.

10.3. Occam's razor

DU omits all central postulates of the relativity theory, the relativity principle, equivalence principle, the constancy of the velocity of light, dark energy, the space-time linkage. The primary DU postulate is the zero-energy balance in spherically closed space. DU gives at least as precise predictions as SR/GR but uses far fewer postulates and more straightforward mathematics [22]. Most importantly, DU uses time and distance as universal coordinate quantities essential for human comprehension and offers a framework for a unified theory comprising physics from cosmology to quantum phenomena.

10.4. Aristotle's entelecheia and the linkage of local to whole

In the spirit of Aristotle's entelecheia, the primary energy buildup is described as "actualization of potentiality", the conversion of gravitational energy into the energy of motion. In the spirit of entelecheia, DU follows the zero-energy principle or double-entry energy bookkeeping; for obtaining energy in one form the same amount of energy in another form is released. Any state of motion in space has its history that links it, through the system of nested energy frames, to the state of rest in hypothetical homogenous space. Velocity in space can be related to an observer in a kinematic sense; however, in

DU framework a state of motion is related to the state where the energy building up the kinetic energy was released. There are no independent objects in space, any local object is linked to the rest of space; the rest energy of any energy object is balanced by the global gravitational energy arising from all mass in space.

Acknowledgements

Appendix: Commentary

Reviewer A. Comment 1.
Dr. Suntola's theory of Dynamic Universe (DU) is a very interesting and thought-provoking theory in that it voices a candid criticism to the establishment of the modern theoretical physics. The most important element in Suntola's DU is the restoration of the classical universal time, which is courageous. I endorse Dr. Suntola's universal time completely.

It is well known that Einstein's theory of relativity is based on the fundamental idea that time and space are mutually dependent, and also dependent on velocity and the frame of reference. Without such relative time, the whole edifice of relativity (SR&GR) would not stand. Universal time, or absolute time, is a fundamental challenge to the theory of relativity.

DU also challenges another fundamental principle of relativity: The constancy of speed of light. With the relativistic time and constancy of speed of light removed, SR/GR could not stand. The two challenges distinguish DU from SR/GR.

Reply.
The theory of relativity is an observer-oriented theory. It relies on kinematics and metrics and postulates the principle of relativity, equivalence principle and the constancy of the velocity of light. In SR/GR, the rest energy of mass objects is independent of the state of gravitation and motion. The relativity of observations is conveyed by locally distorted time and distance.

Dynamic Universe is a system-oriented theory based on the zero-energy principle in spherically closed space, and the conservation of total energy in interactions in space. In DU, time and distance are universal coordinate quantities. The relativity of observations is conveyed by the energy state determined by the local state of gravitation and motion.

Reviewer A. Comment 2.
DU postulates an extra dimension in which the 3-D universe is a hyper spherical subspace embedded and oscillating in the 4-D space, an idea first proposed by Einstein in his effort to build a world model which was not successful. The past experience of Einstein, Dicke and Peebles may offer some reference for the DU model, but the more important issue with the hyper spherical idea is that the physical significance of the extra dimension could not be established and no empirical evidence supporting its existence.

Reply.
We cannot escape the need of a fourth dimension. In SR/GR framework, the fourth dimension is the direction of time, however, it is measured in the units of distance as *ds=c·dt*. In DU framework, time is a universal scalar and the fourth dimension is the metric dimension closing the 3-dimensional space.

An original trigger of the DU model was the interpretation of quantity *mc* in the well-established expression of total energy $E = c\sqrt{(mc)^2 + p^2}$ as the momentum in the fourth dimension perpendicular to momentum *p* in any space direction. Search for the physical message behind momentum in the fourth dimension led to spherically close space expanding in the direction of the 4-radius – and the zero-energy

balance of motion and gravitation in the structure. In spherically closed space, the barycenter of space is at the center of the 4D sphere. In a complex function presentation, the imaginary (4D) component of gravitational potential conveys the effect of whole space to local gravitational potential in space. The imaginary component of velocity and momentum link the velocity and momentum *of* space to the velocity and momentum *in* space. Such linkages convey the relativity of observations a direct consequence of the conservation of total energy in space.

Reviewer A. Comment 3.

DU is also obliged to answer a question: What is the force for the oscillation of the universe? No system would oscillate without a restoring force. An oscillating universe also needs a driving force responsible for its expansion and contraction. What force is it? Is such force a 3-dimensional or 4-dimensional? What is the physical meaning of a 4-dimensional force?

First, energy cannot be defined without the force. Energy is the ability or potential to do work, and work is defined as the scaler product of force and distance. Without force, we don't even know how to define work and energy. Could we define force without energy? Definitely. Force is defined as the mass times the acceleration. Energy is defined and derived from force. Both kinetic energy and potential energy is defined according to the ability or potential to do work.

Reply.

In Newtonian tradition, force is the postulated quantity and energy a derived quantity as integrated force. In DU, energy is the postulated quantity and force a derived quantity as the gradient of potential energy or time derivative of momentum. Force means the natural trend towards actualization of potentiality following Aristotle's entelechy as a primary law of nature.

Reviewer B. Comment 1.

Tuomo Suntola's contribution is on the essential aspects and observational support for his theory of the Dynamic Universe that obeys the zero-energy principle. This is a paradigm for cosmology and physics that needs scrutiny on details because it is based on a hypothesis based on a fact that is known for a long time, namely, that the gravitational energy of every particle in this universe, $-GMm/R$ is approximately equal to its mass-energy, mc^2. Here M and R are the notional total mass and radial extent of the universe. This implies that the total energy is also zero. This is attractive and reasonable as a principle or a constraint, indicating that the universe came into material being from "nothing".

Modern cosmology confirms this and interprets this as the universe being spatially flat. In such a universe ($k=0$), according to general relativity, parallel rays of light remain parallel as they propagate, as in familiar flat space. Despite conforming to this general fact, Suntola's theory of the dynamic universe (DU) is very different from standard general relativistic cosmology. DU does not use the theories of relativity as its basis. Many of its features are in fact contrary to the standard cosmology paradigm. Yet, the author confidently highlights its unique solutions that may explain certain observed features for which the standard paradigm offers no answer, or at best weak answers. It is in this comparison with observed features in phenomena on the cosmological scales of time and space DU deserves scrutiny and evaluation.

The zero-energy relation $E_m+E_g =mc^2-GM'm/R_4$ has a very specific meaning in DU theory. The quantity M' is an equivalent mass of the universe at a notional center of 4-dimensional space, in which our universe is the 3D spherical closed surface with radius R_4. The quantity c is interpreted as the 4-velocity of space itself. In this scheme, the rest energy is the energy of motion an object due to the motion of space in the fourth dimension.

Suntola has worked out various quantitative predictions from the DU theory for several physical effects that can be directly compared to the predictions of general relativity (GR) and standard cosmology. This is the most remarkable point about Suntola's theory – that it is presented as a totally testable and falsifiable premise with several direct predictions. Though I have not studied all these predictions and their details, even a cursory survey suggests that they deserve serious attention. One

aspect that caught my attention specifically was the prediction for the orbital decay of a binary star system (section 8.2). There is a significant difference in the rate of change of the orbital period, in DU and in GR because the quantity defined as the multiplicative "eccentricity factor" goes to zero in DU whereas it saturates to 1 in GR. Suntola states explicitly that there is no orbital decay for circular orbits. If the orbital decay stops or significantly reduces when the eccentricity approaches zero, I see an immediate problem: that would mean that there is no significant emission of gravitational waves and orbital decay to merger once the orbits become nearly circular, which seem in direct conflict with the gravitational wave events detected. This single comparison with data can falsify totally, or strengthen considerably, the DU theory. More importantly, if this is true GR is severely falsified. *My guess is that the gravitational wave data already go against stalling of orbital decay when orbits become circular, and hence against DU theory.* This serious issue should certainly be examined and clarified with priority. I leave it to the author to examine this issue urgently and comment on it.

Another noticeable feature is the link of the physical velocity of light to the quantity $c = \pm\sqrt{GM/R_4}$.

Since the universe is dynamic with R_4 changing, the speed of light changes as $dc/c = -dR_4/2R_4$ in the presently expanding universe. Several consequences are discussed. Most accessible in terms of tests are the Shapiro delay, variation in the number of days in a year, changes in the earth-moon separation etc., since all details are provided with complete calculations.

Reply.
It was interesting no notice that the DU-prediction for the orbital decay of binary systems is essentially the same as the corresponding GR-prediction although the derivation is completely different. The only difference in the predictions appears for binaries with essentially circular orbit – I am not aware of observations of the orbital decay of zero-eccentricity binary systems.

In DU, all local systems expand in direct proportion to the expansion of whole space, and all velocities in space are related to the velocity of space in the fourth dimension. Such a linkage conveys the relativity of observations as relativity between local and the whole rather than relativity between an observer and the object like in the theory of relativity. What SR/GR describes in terms of distorted time and distance, DU describes in terms of the local state of gravitation and motion.

Reviewer B. Comment 2.
One gap I notice is the lack of discussion on nucleosynthesis and comparison with observations. In fact, a discussion on the hot phase of the universe till the decoupling of the radiation is missing in the DU theory. Since CMBR observations are the primary source of several of the precision statements in cosmology, a comparison with what is expected in DU for thermal physics and nucleosynthesis in DU is essential.

The picture of the evolution of the universe in DU starts with a universe with near infinite "radius" that starts contracting and after crossing a singular moment, expends back again etc. If this is the case, the universe needs to go through the hot phase at least twice before the present expending phase. I do not see how this history takes care of the accurately observed features of the thermal history of the universe. This should be clarified, at least broadly indicated.

Reply.
Singularity, or the turning point means primarily an extreme excitation of the energy and momentum in the fourth dimension. "Hot" means thermal energy expressed by motion or vibrations in space directions. The features of the thermal history of the early big bang universe are based on calculations based on interpretations of CMBR observations in GR framework. We may assume that the analysis would be different in DU framework.

Reviewer B. Comment 3.

Another question to which the readers will definitely want an answer is about the age of structures and observable objects in our universe. DU concludes that the age since a singularity is about 9 billion years, much like what early bid bang theory had. But we know that there are older objects in this universe; stars that are nearly 13 billion years old. Therefore, the discussion on age of the universe becomes crucial. While the formation of elements etc. may not a problem if there are repeated contraction and expansion cycles, a thermal history needs to allow enough time for structures to form, and there are galaxies or globular clusters that looks older than 10 billion years.

Reply.

Assuming Hubble constant H_0=70 (km/s)/Mpc, the distance from the center of the 4D sphere closing space (the singularity), is about 14 billion light years, which corresponds the age of the universe in GR framework. In DU, the expansion velocity of space is determined by the zero-energy balance of motion and gravitation, which means that the expansion velocity, which determines the velocity of light and the rest energy of matter in space, slows down with time. It means that all physical processes including the radioactive decay have happened faster in the past, see chapter 6.4.2 in [1]. In DU framework, the age of stars is shorter than the values obtained with radiometric dating based on a constant decay rate. In no circumstances the age of stars exceeds the age of the expanding DU universe.

Reviewer B. Comment 4.

An easily understood, but likely to be most controversial, conclusion of DU is that gravitationally bound systems also expend with the universe, in direct proportion. I think it is a logically attractive and physically plausible possibility. Conventionally, gravitation theorists deny the possibility that a gravitationally bound system can expend with the universe, but the exact reason why this should be so is never cleared.

Reply.

The assumption of non-expanding local systems in FLRW cosmology apparently comes from the early paper by de Sitter [23], which assumes conservation of energy in local systems. In DU framework, the expansion of local systems with the expansion of whole space is a direct consequence of the conservation of the zero-energy balance; the gravitational energy of local systems develops in direct proportion to the gravitational energy of whole space.

Reviewer B. Comment 5.

There are very interesting observational evidence that are strongly presented in favor of DU theory by Suntola and this need to be thoroughly studied due the importance of the conclusion. There are several physical effects of DU in play here – the increase in the orbital distance of the earth resulting in the increase in the duration of the year and the number of days in a year, the decrease in the orbital velocity resulting in the increase in the duration of the year, the decrease in the rotational velocity of the earth leading to the increase in the length of the day and the decrease in the number of days in a year etc. The final prediction is compared to remarkable and impressive data of development related markings on ancient coral fossils. It gives number of days in a year as well as the number of days in a lunar month. What is not clear from the description in the paper is whether only an overall fit to the (decreasing) number of days in year has been obtained or whether both the lengthening of the duration of the year from expansion of space and the lengthening of the day due to the change in the rotation of the earth have been deduced from the yearly and monthly data. In any case, this is an interesting "local test" of the link between expansion of the universe and its effect or otherwise in the solar system, even independent of the theory of DU. This study has its own importance in cosmology and physics. I have not commented on those results that agree by and large with standard GR results.

Reply.

Strong support for the expansion of local systems is also obtained from the prediction for the angular size of quasars and galaxies, which show Euclidean appearance in the DU framework in a complete agreement with observations, see figure 6 in chapter 5.3.

In DU, the prediction for the magnitude-redshift relation of standard candela gives at least equal match with observations as the dark energy corrected prediction of GR. However, there is a crucial difference between the two predictions. The GR prediction relies on the 1930s interpretations of the Doppler effect, special relativity, general relativity, Planck's equation, and the reciprocity theorem. The GR prediction is applied to observations *"moved to emitter's rest frame"* with a *K*-correction that adds an extra $(z+1)^2$ dimming to bolometric observations. The DU prediction is derived for direct bolometric magnitudes by applying the DU optical distance of the objects and the effect of the expansion of space on the wavelength. Such a derivation allows filter by filter predictions for modern multi-bandpass detection, Chapter 6.3.3 – 6.3.4 in [1]. The DU prediction agrees with observations at least as accurately as the GR based prediction. The highest redshifts of SN Ia supernovas that are currently available, are about $z \approx 2$. DU and GR predictions for the magnitude deviate considerable from each other for objects with redshift higher than two, $z > 2$. We may have a chance to make the judgement between the theories in the near future.

Reviewer B. Comment 6.

The corrections in GPS etc. being different from what special relativity dictates is true, but the simplest explanation for that is the existence of a privileged frame (analogous to the aether) that determine relativistic physics, like the gravitational universe. Just accepting that explains the data, without an entirely new theory of the evolution universe.

Answer.

In DU, the privilege frame is the hypothetical homogeneous space as the ultimate parent frame in the system of nested energy frames. In GR framework, GPS clocks, like all Earth satellite clocks are studied in the Earth Centered Inertial frame, ECI-frame, with a hypothetical reference clock at rest relative to the rotating Earth, outside the gravitational interaction of the Earth. ECI frame ignores the effects of the orbital velocity of the Earth and the changing gravitational potential of the Sun experienced on Earth and in near space due to the eccentricity of the orbit. In DU framework, all relativity tests on clocks are expressed in the system of nested energy frames that gives a logical explanation on clock rates as functions of the gravitational state and velocity of the clock in the local frame and the gravitational state and velocity of the local frame in the parents frames. Importantly, the DU prediction is based on effects of motion and gravitation on the rest energy of the oscillating electrons in the clock which links the prediction to the quantum mechanical solution of the characteristic frequency of atomic oscillators. In DU, time is a universal scalar, relativistic effects are direct consequences of the conservation of total energy.

Reviewer B. Comment 7.

Mach's principle is briefly mentioned in the article, in relation to the philosophical dimension of the DU theory. Since DU works with zero-energy principle, mutual interaction all matter in the universe is its basis. However, the specific physics problem of the inertial forces is not addressed explicitly in the article. I suppose the conclusions will be very similar to those in Dennis Sciama's classical paper.

Reply.

The detailed derivation of Mach's principle is given in [1], chapters 1.2.2, 4.1.2, and 4.1.3. In chapters 1.2.2 and 4.1.2, Mach's principle is derived as the work done against the global gravitation when building up kinetic energy in space. Any motion in spherically closed space is central motion relative to the barycenter of the structure. Chapter 4.1.3 of [1] completes the analysis by the effect of the centrifugal force due to motion in space on the effective gravitational force in the direction of the 4-radius. The

result is the same: Mach's principle means the work done against global gravitation for obtaining a state of motion in space; in the complex function notation of DU the inertial work is the imaginary part of the complex kinetic energy – thus giving a quantitative expression for Mach's principle.

Reviewer B. Comment 8.
As to the mention of Occam's razor, one needs to be cautious because the scope of DU theory is still to be shown as comparable to the existing relativity physics and cosmology, before it can claim advantage on the criteria of Occam's razor. In particular, the principle of relativity, the equivalence principle etc. that are assumed in current theories are empirically true to high precision, independent of these theories. Therefore, any alternate theory should derive these as consequences of lesser assumptions.

Reply.
Relativity principle is needed in observer-oriented theories like the theory of relativity. Relativity principle is not relevant in system-based theories like thermodynamics, celestial mechanics or Dynamic Universe. A reference frame in DU is the frame where the energy conversion studied occurs. In most laboratory experiments, e.g., experiments with accelerators, the observer is at the state of rest relative to the accelerator – which makes the system frame look like an observer-oriented frame. Satellite clock experiments are studied in the Earth Centered Inertial frame, far space experiments the Solar system frame, and distant cosmological observations in the CMB frame, possibly representing the hypothetical homogeneous space frame.

Equivalence principle is needed in celestial mechanics based on Newton's mechanics and in the extension of special relativity to general relativity. Equivalence principle can be verified in many experimental setups. However, the analysis of free fall in DU-space shows that the equivalence principle infringes the conservation of total energy: buildup of kinetic energy at constant gravitational potential is obtained by feeding extra energy expressed as mass increase Δm, $E_{kin}=c \cdot \Delta mc$, which means *an increase of the total energy of the object* put in motion. Same is true at constant gravitational potential in DU framework, however, kinetic energy built up in free fall in gravitational field is obtained against reduction of the local rest energy due to the reduction of the local velocity of light at lower gravitational potential $E_{kin} = c_0 \cdot m \Delta c$,e which means that, unlike in GR framework, *the total energy of the falling object is conserved*.

The Schwarzschildean prediction for the perihelion advance of elliptic orbits contains a cumulative term that gradually increases the orbital radius – finally throwing away the orbiting object. At extreme conditions at distance, at 3x Schwarzschild's critical radius, that happens in one cycle. At low gravitational field, like in the case of Mercury it needs about one million cycles. In astronomy books, the cumulative term is eliminated as a secondary secular for preventing the perturbation of Mercury's orbit, however, it is just the same term creating the instability of orbits at extreme conditions close to black holes. In DU, thanks to the conservation of energy, orbits are stable down to the critical radius which in DU space is half of the Schwarzschild critical radius. Close to the critical radius, there are slow orbits that maintain the mass of a black hole.

The reason for the perturbation of the orbit in Schwarzschild space can be traced to the equivalence principle that requires buildup of relativistic mass in free fall. A thorough analysis shows that the Schwarzschildean orbital velocity exceeds the escape velocity when the orbital radius is 3xSchwarzschild's critical radius, see chapters 1.2.6 and 4.2.8 in [1].

This is a fundamental problem following from the principle of equivalence; the problem is severe enough to falsify the equivalence principle as the basis of general relativity.

Reviewer B. Comment 9.
In summary, T. Suntola's contribution compels us to pay attention to several aspects of observational cosmology and do some amount of rethinking and scrutiny of data and evidence. Particularly important is the data on the history of solar system, length of the day etc. that may support some of the assertions made in the paper. At the same time recent wealth of data available from gravitational wave detectors

can certainly offer decisive tests of the DU theory in my view. Specifically, the available data seem capable of testing the drastic prediction in DU theory, of slowing down and stopping of orbital decay of binary systems when the orbital eccentricity decreases to zero. An important topic that is missing in the DU paradigm is the thermal history of the universe and nucleosynthesis. The work on DU theory is tremendous amount of original work touching on many important aspects. What should be appreciated is the amount of calculations completed and presented, which enables direct comparisons and tests against observational data, guaranteeing clear falsifiability.

Reply.

An important difference between DU cosmology and GR based cosmology comes from the buildup of the energy in space. In GR based cosmology universe appeared as a stochastic ad hoc jump triggering the flow of time and the laws of nature. In DU, the laws of nature are eternal, the energy in space was built up gradually in a contraction phase before the ongoing expansion phase – following the same laws of nature, we observe in any interaction in space today. The singularity turning the contraction into expansion is an extreme excitation of the energy of motion and gravitation in the fourth dimension offering circumstances for the modelling nucleosynthesis and structure buildup in 3D space. Unlike GR, DU gives a precise prediction to the development of the expansion of space. DU relies essentially on the zero-energy principle, it does not need relativity principle, equivalence principle, reciprocity principle, dark energy, mass density parameters or postulates like the constancy of the velocity.

The original driver of the DU development was the main objective of science: to make nature understandable. A successful theory does not only give precise predictions but also relies on intelligible postulates, or in Aristotelian terms, on first causes – and gives an understandable picture of the observable reality.

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