 Communications in Theoretical Physics

Commun. Theor. Phys. 72 (2020) 025003 (6pp) https://doi.org/10.1088/1572-9494/ab617e

Optical solitons and modulation instability analysis of the (1 + 1)-dimensional coupled nonlinear Schrödinger equation

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Received 11 September 2019, revised 12 November 2019
Accepted for publication 15 November 2019
Published 3 February 2020

Abstract

This study successfully reveals the dark, singular solitons, periodic wave and singular periodic wave solutions of the (1 + 1)-dimensional coupled nonlinear Schrödinger equation by using the extended rational sine-cosine and rational sinh-cosh methods. The modulation instability analysis of the governing model is presented. By using the suitable values of the parameters involved, the 2-, 3-dimensional and the contour graphs of some of the reported solutions are plotted.

Keywords: the coupled nonlinear Schrödinger's equation, optical soliton, modulation instability

(Some figures may appear in colour only in the online journal)

1. Introduction

Various complex nonlinear physical aspects may be represented in the form of nonlinear partial differential equations (NLPDEs). Nonlinear Schrödinger type equations (NLSEs) are particular types of NLPDEs which are complex in nature. These types of equations can be used to express several nonlinear physical processes, such as plasma physics, fluid mechanics, photonics, ocean engineering, electromagnetism and so on [1–5]. The theory of optical solitons is one of the fascinating topics for the investigation of soliton propagation through nonlinear optical fibers. The propagation of ultrashort pulses of electromagnetic radiation is a multidimensional process in a nonlinear medium. The communication between different physical aspects, such as dispersion, material dispersion, diffraction and nonlinear response, affects the pulse dynamics [6]. Over the years, this field has captured the assiduity of many scientists. Various approaches have been utilized in obtaining the solutions of different kinds of nonlinear evolution equations, such as the modified $\exp(-\Psi(\eta))$-expansion function method [7–9], the first integral method [10, 11], the improved Bernoulli sub-equation function method [12, 13], the trial solution method [14, 15], the new auxiliary equation method [16] and several others [17–38].

In the present study, the extended rational sine-cosine and rational sinh-cosh techniques [39, 40] will be utilized to construct some optical soliton solutions of the (1 + 1)-dimensional coupled NLSE [41]. The modulation instability analysis of the studied nonlinear model is also going to be discussed.

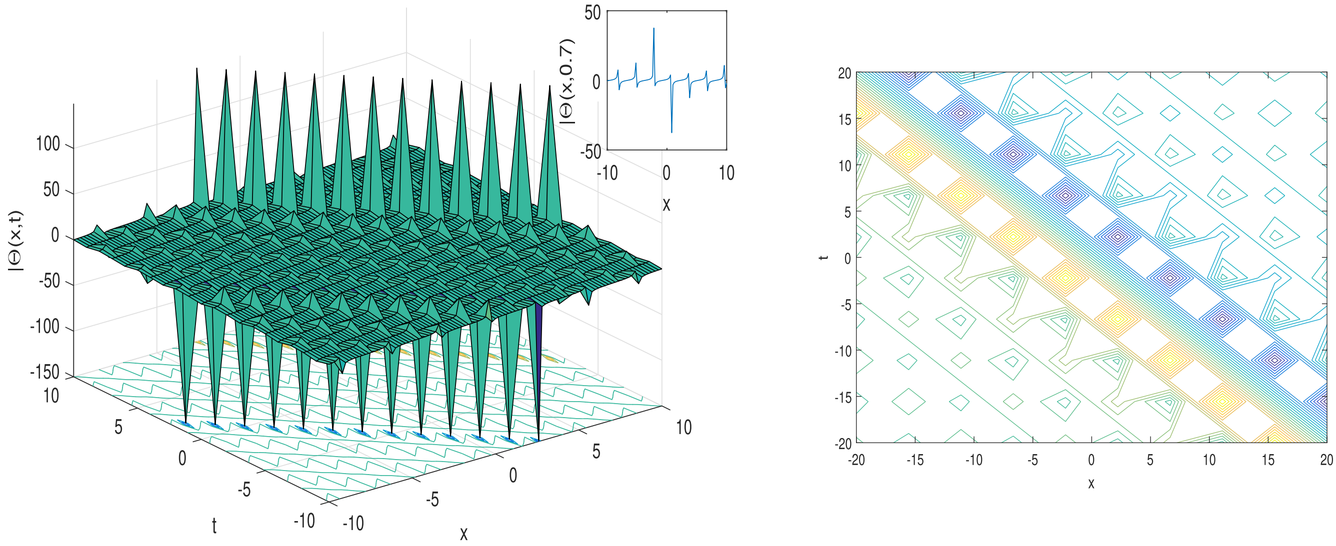
The (1 + 1)-dimensional coupled NLSE is given by [41]

$$\begin{aligned} &\mathrm{i}\Theta_t + \Theta_{xx} + \sigma(|\Theta|^2 + \alpha|\Phi|^2)\Theta = 0, \\ &\mathrm{i}\Phi_t + \Phi_{xx} + \sigma(\alpha|\Theta|^2 + |\Phi|^2)\Phi = 0, \end{aligned} \qquad (1)$$

where $\mathrm{i} = \sqrt{-1}$, σ and α are non-zero real numbers, and Θ and Φ are complex functions of x and t that stand for the amplitudes of circularly-polarized waves in a nonlinear optical fiber [42]. Equation (1) was developed by Boyd [42], and it is known to have a great impact on the pulse propagation through a two-mode optical fiber and the soliton wavelength division multiplexing.

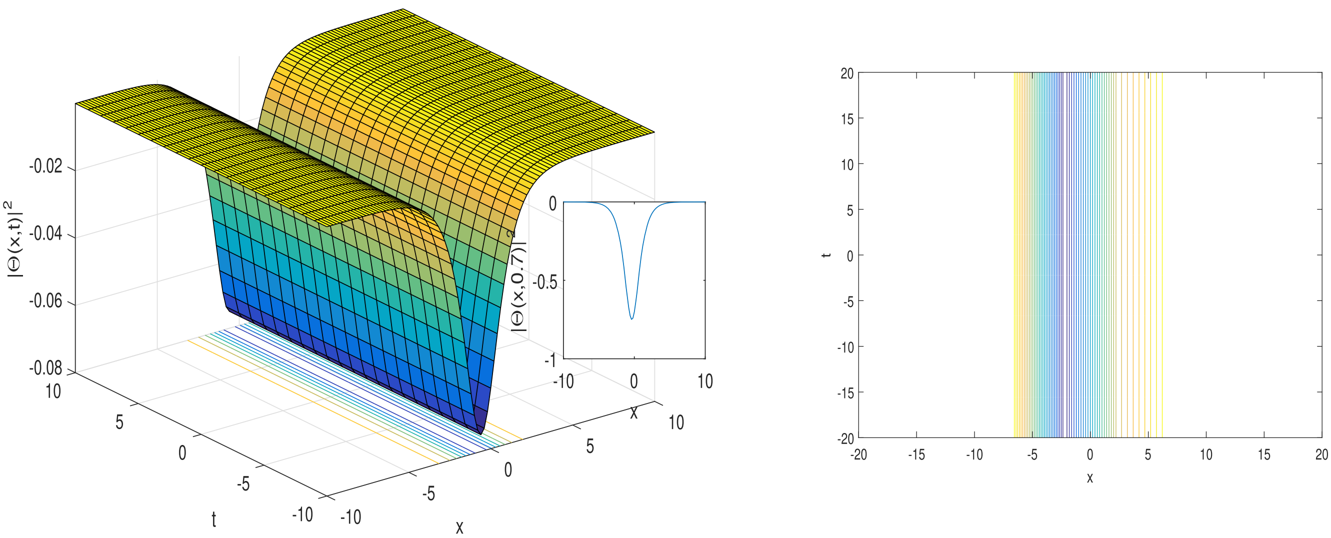
2. Analysis of the methods

In this section, we give the analysis of the extended rational sine-cosine and sinh-cosh methods [39].



(a) (b)

Figure 1. The (a) 2D, 3D and (b) contour surfaces of equation (13).



(a) (b)

Figure 2. The (a) 2D, 3D and (b) contour surfaces of equation (19).

Consider the general form of the nonlinear evolution equation and the wave transformation

$$P(\Theta_x,\ \Theta^2\Theta_{xx},\ \Theta_{xx},\ \dots) = 0, \tag{2}$$

$$\Theta(x,\ t) = \Theta(\zeta), \quad \zeta = \tau(x - vt), \tag{3}$$

respectively, where $\Theta(x, t)$ is the unknown function of x and t, v is the speed of the wave and μ is a non-zero real number.

Inserting equation (3) into equation (2) gives the following nonlinear ordinary differential equation:

$$D(\Theta,\ \Theta',\ \Theta'',\ \Theta^2\Theta',\ \dots) = 0, \tag{4}$$

where Θ is the unknown function of ζ, and the superscript indicates the derivative of the function Θ with respect to ζ.

2.1. Extended rational sine-cosine

In this subsection, the general steps of the extended rational sine-cosine are given.

Step I: Assume that equation (4) adopts the following forms of solution:

$$\Theta(\zeta) = \frac{b_0 \sin(\mu\zeta)}{b_2 + b_1 \cos(\mu\zeta)}, \quad \cos(\mu\zeta) \neq -\frac{b_2}{b_1}, \tag{5}$$

or

$$\Theta(\zeta) = \frac{b_0 \cos(\mu\zeta)}{b_2 + b_1 \sin(\mu\zeta)}, \quad \sin(\mu\zeta) \neq -\frac{b_2}{b_1}, \tag{6}$$

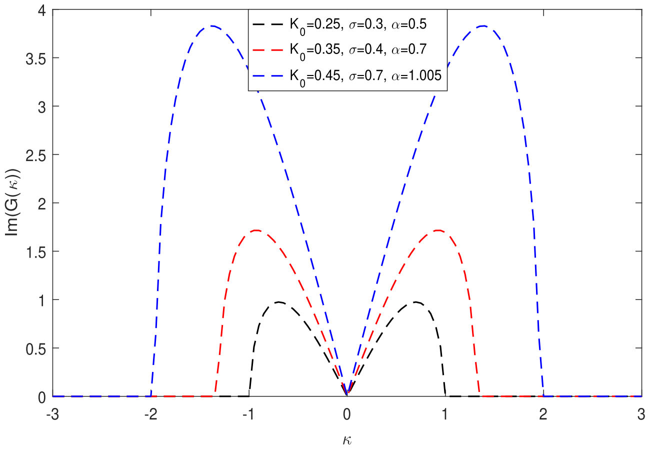


Figure 3. The gain spectrum of modulation instability for three different values of K_0, α and σ.

where b_j, $(j = o, 1, 2)$ are unknown parameters and μ is the wave number.

Step II: The unknown parameters are obtained by inserting equation (5) or equation (6) into equation (4). This yields a polynomial in powers of $\cos(\mu\zeta)$ or $\sin(\mu\zeta)$. Collecting the coefficients of $\cos(\mu\zeta)$ or $\sin(\mu\zeta)$ with the same power gives a group of algebraic equations after equating each summation to zero.

Step III: The solutions of equation (2) are secured by substituting the values of the unknown parameters into equation (5) or equation (6).

2.2. Extended rational sinh-cosh

In this subsection, the general steps of the extended rational sinh-cosh are given.

Step I: Assume that equation (4) adopts the following forms of solution:

$$\Theta(\zeta) = \frac{b_0 \sinh(\mu\zeta)}{b_2 + b_1 \cosh(\mu\zeta)}, \quad \cosh(\mu\zeta) \neq -\frac{b_2}{b_1}, \tag{7}$$

or

$$\Theta(\zeta) = \frac{b_0 \cosh(\mu\zeta)}{b_2 + b_1 \sinh(\mu\zeta)}, \quad \sinh(\mu\zeta) \neq -\frac{b_2}{b_1}, \tag{8}$$

where b_j, $(j = o, 1, 2)$ are unknown parameters and μ is the wave number.

Step II: The unknown parameters are obtained by inserting equation (7) or equation (8) into equation (4). This yields a polynomial in powers of $\cosh(\mu\zeta)$ or $\sinh(\mu\zeta)$. Collecting the coefficients of $\cosh(\mu\zeta)$ or $\sinh(\mu\zeta)$ with the same power gives a set of algebraic equations after equating each summation to zero.

Step III: The solutions of equation (2) are secured by inserting the values of the unknown parameters into equation (7) or equation (8).

3. Applications

In this section, we present the application of the extended rational sine-cosine/sinh-cosh method to the $(1 + 1)$-dimensional coupled NLSE.

3.1. Mathematical analysis of the governing model

Consider the following complex wave transformation:

$$\begin{aligned} &\Theta(x, t) = \Theta(\zeta)\mathrm{e}^{\mathrm{i}\varphi}, \quad \Phi(x, t) = \Phi(\zeta)\mathrm{e}^{\mathrm{i}\varphi}, \\ &\zeta = \tau(x - ct), \quad \varphi = -\kappa x + \phi t + q. \end{aligned} \tag{9}$$

Inserting equation (9) into equation (1) gives the following system of nonlinear ordinary differential equation:

$$\begin{aligned} &\tau^2\Theta'' + \sigma\Theta^3 + \sigma\alpha\phi^2\Theta - (\phi + k^2)\Theta = 0, \\ &\tau^2\Phi'' + \sigma\Phi^3 + \sigma\alpha\Theta^2\Phi - (\phi + k^2)\Phi = 0 \end{aligned} \tag{10}$$

from the real part of the transformation, and the relation; $c = -2\kappa$ from the imaginary part.

Assume that equation (10) possesses the solutions of the form

$$\begin{aligned} \Theta(\zeta) &= \frac{a_0 \sin(\mu\zeta)}{a_2 + a_1 \cos(\mu\zeta)}, \\ \Phi(\zeta) &= \frac{b_0 \sin(\mu\zeta)}{b_2 + b_1 \cos(\mu\zeta)}. \end{aligned} \tag{11}$$

Putting equation (11) into equation (10) yields a polynomial in trigonometric functions. Collecting the coefficients of the trigonometric functions of the same power of $\cos(\mu\zeta)$ yields a system of algebraic equations. Solving the system of algebraic equations gives the following set of values to the unknown parameters:

Set-1: When

$$a_0 = a_1\sqrt{-\frac{(\kappa^2 + \phi)}{\sigma(1 + \alpha)}}, \quad a_2 = a_1,$$
$$b_0 = b_1\sqrt{-\frac{(\kappa^2 + \phi)}{\sigma(1 + \alpha)}}, \quad b_2 = b_1, \quad \tau = \frac{\sqrt{2(\kappa^2 + \phi)}}{\mu},$$

we get the singular periodic wave solution

$$\begin{aligned} &\Theta_{1.1}(x, t) = \Phi_{1.1}(x, t) \\ &= \sqrt{-\frac{(\kappa^2 + \phi)}{\sigma(1 + \alpha)}} \frac{\sin[\sqrt{2(\kappa^2 + \phi)}(x + 2\kappa t)]}{(1 + \cos[\sqrt{2(\kappa^2 + \phi)}(x + 2\kappa t)])}\mathrm{e}^{\mathrm{i}\varphi}. \end{aligned} \tag{12}$$

Set-2: When

$$a_0 = a_1\sqrt{-\frac{(\kappa^2 + \phi)}{\sigma(1 + \alpha)}}, \quad a_2 = 0,$$
$$b_0 = -b_1\sqrt{-\frac{(\kappa^2 + \phi)}{\sigma(1 + \alpha)}}, \quad b_2 = 0, \quad \tau = \frac{\sqrt{\kappa^2 + \phi}}{\sqrt{2}\mu},$$

we get the singular periodic wave solution

$$\Theta_{1.2}(x,t)=\Phi_{1.2}(x,t)=\sqrt{-\frac{(\kappa^2+\phi)}{\sigma(1+\alpha)}}\tan\left[\frac{\sqrt{\kappa^2+\phi}}{\sqrt{2}}(x+2\kappa t)\right]\mathrm{e}^{\mathrm{i}\varphi}. \tag{13}$$

Assume that equation (10) possesses the solution of the form

$$\Theta(\zeta)=\frac{a_0\cos(\mu\zeta)}{a_2+a_1\sin(\mu\zeta)},\quad \Phi(\zeta)=\frac{b_0\cos(\mu\zeta)}{b_2+b_1\sin(\mu\zeta)}. \tag{14}$$

Putting equation (14) into equation (10) yields a polynomial in trigonometric functions. Collecting the coefficients of the trigonometric functions of the same power of $\sin(\mu\zeta)$ yields a system of algebraic equations. Solving the system of algebraic equations gives the following set of values to the unknown parameters:

Set-3: When

$$a_0=-a_1\sqrt{-\frac{(\kappa^2+\phi)}{\sigma(1+\alpha)}},\quad a_2=-a_1,$$
$$b_0=b_1\sqrt{\frac{(\alpha-1)(\kappa^2+\phi)}{\sigma(1-\alpha^2)}},$$
$$b_2=-b_1,\quad \tau=-\frac{\sqrt{2(\kappa^2+\phi)}}{\mu},$$

we get the singular periodic wave solution

$$\Theta_{2.1}(x,t)=\Phi_{2.1}(x,t)=\sqrt{-\frac{(\kappa^2+\phi)}{\sigma(1+\alpha)}}\frac{\cos[\sqrt{2(\kappa^2+\phi)}(x+2\kappa t)]}{(-1-\sin[\sqrt{2(\kappa^2+\phi)}(x+2\kappa t)])}\mathrm{e}^{\mathrm{i}\varphi}. \tag{15}$$

Set-4: When

$$a_0=-a_1\sqrt{-\frac{(\kappa^2+\phi)}{\sigma(1+\alpha)}},\quad a_2=0,$$
$$b_0=b_1\sqrt{\frac{-(\kappa^2+\phi)}{\sigma(1+\alpha)}},\quad b_2=0,\quad \tau=\frac{\sqrt{\kappa^2+\phi}}{\sqrt{2}\mu},$$

we get the singular periodic wave solution

$$\Theta_{2.2}(x,t)=\Phi_{2.2}(x,t)=\sqrt{-\frac{(\kappa^2+\phi)}{\sigma(1+\alpha)}}\cot\left[\frac{\sqrt{\kappa^2+\phi}}{\sqrt{2}}(x+2kt)\right]\mathrm{e}^{\mathrm{i}\varphi}. \tag{16}$$

Assume that equation (10) possesses the solution of the form

$$\Theta(\zeta)=\frac{a_0\sinh(\mu\zeta)}{a_2+a_1\cosh(\mu\zeta)},\quad \Phi(\zeta)=\frac{b_0\sinh(\mu\zeta)}{b_2+b_1\cosh(\mu\zeta)}. \tag{17}$$

Putting equation (17) into equation (10) yields a polynomial in trigonometric functions. Collecting the coefficients of the trigonometric functions of the same power of $\cosh(\mu\zeta)$ yields a system of algebraic equations. Solving the system of algebraic equations gives the following set of values to the unknown parameters:

Set-5: When

$$a_0=a_1\sqrt{\frac{\kappa^2+\phi}{\sigma(1+\alpha)}},\quad a_2=a_1,$$
$$b_0=b_1\sqrt{\frac{\kappa^2+\phi}{\sigma(1+\alpha)}},\quad b_2=b_1,\quad \tau=\frac{\sqrt{-2(\kappa^2+\phi)}}{\mu},$$

we get the periodic wave solution

$$\Theta_{3.1}(x,t)=\Phi_{3.1}(x,t)=\sqrt{\frac{\kappa^2+\phi}{\sigma(1+\alpha)}}\frac{\sinh[\sqrt{-2(\kappa^2+\phi)}(x+2\kappa t)]}{(1+\cosh[\sqrt{-2(\kappa^2+\phi)}(x+2\kappa t)])}\mathrm{e}^{\mathrm{i}\varphi}. \tag{18}$$

Set-6: When

$$a_0=a_1\sqrt{\frac{\kappa^2+\phi}{\sigma(1+\alpha)}},\quad a_2=0,$$
$$b_0=-b_1\sqrt{\frac{\kappa^2+\phi}{\sigma(1+\alpha)}},\quad b_2=0,\quad \tau=\frac{\sqrt{-(\kappa^2+\phi)}}{\sqrt{2}\mu},$$

we get the dark soliton

$$\Theta_{3.2}(x,t)=\Phi_{3.2}(x,t)=\sqrt{\frac{\kappa^2+\phi}{\sigma(1+\alpha)}}\tanh\left[\frac{\sqrt{-(\kappa^2+\phi)}}{\sqrt{2}}(x+2\kappa t)\right]\mathrm{e}^{\mathrm{i}\varphi}. \tag{19}$$

Assume that equation (10) possesses the solution of the form

$$\Theta(\zeta)=\frac{a_0\cosh(\mu\zeta)}{a_2+a_1\sinh(\mu\zeta)},\quad \Phi(\zeta)=\frac{b_0\cosh(\mu\zeta)}{b_2+b_1\sinh(\mu\zeta)}. \tag{20}$$

Putting equation (20) into equation (10), yields a polynomial in trigonometric functions. Collecting the coefficients of the trigonometric functions of the same power of $\sinh(\mu\zeta)$ gives a system of algebraic equations. Solving the system of algebraic equations gives the following set of values to the unknown parameters:

Set-7: When

$$a_0=a_1\sqrt{\frac{\kappa^2+\phi}{\sigma(1+\alpha)}},\quad a_2=\mathrm{i}a_1,$$
$$b_0=-b_1(1-\mathrm{i})\sqrt{\frac{(\alpha-1)(\kappa^2+\phi)}{2\mathrm{i}\sigma(1-\alpha^2)}},$$
$$b_2=\mathrm{i}b_1,\quad \tau=\frac{\sqrt{-2(\kappa^2+\phi)}}{\mu},$$

we get the periodic wave solution

$$\Theta_{4.1}(x,t)=\Phi_{4.1}(x,t)=\sqrt{\frac{\kappa^2+\phi}{\sigma(1+\alpha)}}\frac{\cosh[\sqrt{-2(\kappa^2+\phi)}(x+2\kappa t)]}{(\mathrm{i}+\sinh[\sqrt{-2(\kappa^2+\phi)}(x+2\kappa t)])}\mathrm{e}^{\mathrm{i}\varphi}. \tag{21}$$

Set-8: When

$$a_0 = a_1\sqrt{\frac{\kappa^2+\phi}{\sigma(1+\alpha)}},\ a_2 = 0,$$
$$b_0 = -b_1\sqrt{\frac{\kappa^2+\phi}{\sigma(1+\alpha)}},\ b_2 = 0,\ \tau = \frac{\sqrt{-(\kappa^2+\phi)}}{\sqrt{2}\mu},$$

we get the singular soliton

$$\Theta_{4.2}(x,t) = \Phi_{4.2}(x,t) = \sqrt{\frac{\kappa^2+\phi}{\sigma(1+\alpha)}}\coth\left[\frac{\sqrt{-(\kappa^2+\phi)}}{\sqrt{2}}(x+2\kappa t)\right]e^{i\varphi}. \tag{22}$$

Remark 1. Solutions in set-1 to -4 are valid for $\kappa^2+\phi \geqslant 0$, and solutions in set-5 to -8 are valid for $\kappa^2+\phi \leqslant 0$.

4. Modulation instability analysis

Several nonlinear phenomena display an instability that results in modulation of the steady state as a result of a coaction between the nonlinear and dispersive effects [43]. In this section, we derive the modulation instability of the $(1+1)$-dimensional coupled NLSE by employing the standard linear stability analysis [43, 20, 44].

Assume the steady-state solutions of the $(1+1)$-dimensional coupled NLSE to be of the form

$$\Theta(x,t) = (\sqrt{K_0} + U(x,t)e^{iK_0x},$$
$$\Phi(x,t) = (\sqrt{K_0} + V(x,t)e^{iK_0x}, \tag{23}$$

where K_0 is the normalized optical power.

Inserting equation (23) into equation (1) and linearizing yields

$$iU_t + U_{xx} + \alpha(\sigma+1)K_0(U+U^*) + \alpha(\sigma+1)\sqrt{K_0}(U+U^*) = 0,$$
$$iV_t + V_{xx} + \alpha(\sigma+1)K_0(V+V^*) + \alpha(\sigma+1)\sqrt{K_0}(V+V^*) = 0. \tag{24}$$

Remark 1. The modulation instability analysis of the parturition $V(x,t)$ is similar to $U(x,t)$. In this case, we only examine the evolution of the perturbation of $U(x,t)$. Figure 1 depicts the physical features of the singular periodic wave solution, figure 2 depicts the physical features of the dark optical soliton, and figure 3 depicts the physical features of the gain spectrum.

Assume the solution of the first equation in equation (24) to be of the form

$$U(x,t) = \delta_1 e^{i(\kappa x-\phi t)} + \delta_2 e^{-i(\kappa x-\phi t)}, \tag{25}$$

where κ and ϕ are the normalized wave number and frequency of perturbation, respectively.

Substituting equation (25) into the first equation in equation (24), splitting the coefficients of $e^{i(\kappa x-\phi t)}$ and $e^{-i(\kappa x-\phi t)}$, and solving the determinant of the coefficient matrix, we get the following dispersion relation:

$$\delta_1\alpha(1+\sigma)\sqrt{K_0}(1+\sqrt{K_0}) + \delta_2(-\kappa^2+\alpha(1+\sigma)(\sqrt{K_0}+K_0)-\phi) = 0. \tag{26}$$

Solving the dispersion relation (26) for ϕ yields

$$\phi = \sqrt{\kappa^2(\kappa^2 - 2\alpha(1+\sigma)\sqrt{K_0}(1+\sqrt{K_0})}. \tag{27}$$

In a situation whereby $\kappa^2(\kappa^2 - 2\alpha(1+\sigma)\sqrt{K_0}(1+\sqrt{K_0}) \geqslant 0$ or $\kappa^2 \geqslant 2\alpha(1+\sigma)\sqrt{K_0}(1+\sqrt{K_0})$, the wave number ϕ is real for all κ and the steady state is stable against small perturbations. Moreover, contrary to the above statement, the steady-state solution turns out to be unstable in the situation whereby $\kappa^2 < 2\alpha(1+\sigma)\sqrt{K_0}(1+\sqrt{K_0})$, the wave number ϕ becomes imaginary and the perturbation grows exponentially. Under this condition, the growth rate of modulation stability gain spectrum $G(\kappa)$ may be given as

$$G(\kappa) = 2\,\mathrm{Im}(\phi) = 2\,\mathrm{Im}(\sqrt{\kappa^2(\kappa^2 - 2\alpha(1+\sigma)\sqrt{K_0}(1+\sqrt{K_0})}). \tag{28}$$

5. Conclusions

In this study, the $(1+1)$-dimensional coupled NLSE is investigated by using the extended rational sine-cosine and rational sinh-cosh approaches. Dark singular solitons, periodic wave and singular periodic wave solutions are successfully revealed. The modulation instability analysis of the studied model is analyzed. It can be seen that the steady state of the studied model is unstable when $\kappa^2 < 2\alpha(1+\sigma)\sqrt{K_0}(1+\sqrt{K_0})$. The extended rational sine-cosine and rational sinh-cosh approaches are simple and efficient mathematical tools for investigating several complex nonlinear models in the fields of nonlinear sciences.

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