

A low cost validation method of finite element analysis on a thin walled vertical pressure vessels

Juan Pratama^{1*}, Deni Fajar Fitriyana^{2*}, Rusiyanto², Januar Parlaungan Siregar³, Wahyu Caesarendra⁴

¹ Department of Mechanical Engineering, Faculty of Engineering, Darma Persada University, Jl. Radin Inten II, Pondok Kelapa, Jakarta, 13450, Indonesia

² Department of Mechanical Engineering, Faculty of Engineering, Universitas Negeri Semarang, Kampus Sekaran, Gunungpati, Semarang, 50229, Indonesia

³ Faculty of Mechanical and Automotive Engineering Technology, Universiti Malaysia Pahang, 26600 Pekan, Pahang, Malaysia

⁴ Faculty of Integrated Technologies, Universiti Brunei Darussalam, Jalan Tungku Link, GadongBE1410, Brunei Darussalam

*Corresponding author email: juanprtm@gmail.com; deniifa89@mail.unnes.ac.id.

Abstract.

Pressure vessel is a container that has been used to contain a pressurized fluids, either oil, gas, or other chemical fluids. It is widely used for the oil, gas and other chemical based industries. Nowadays, finite element analysis is commonly used to reduce the high cost of testing a pressure vessel before manufacturing process. However, further validation is needed to ensure the results of the simulation and safety of the pressure vessels. In this study, theory of distortion energy is used as the tools of validation based on materials properties and behavior. And finally to ascertain whether the pressure vessel is possible for production or a refinement for safety is needed. The results of the study shown that theory of distortion energy can be used as validation tool for finite element analysis on a pressure vessels, however it cannot ensure the safety. Therefore other validation methods are needed to ascertain the safety of the pressure vessel discussed in this report. The cost analysis shown that failure theory combined with other calculation methods can save costs in pressure vessel testing, although some fairly expensive tests cannot be avoided.

1. Introduction

Pressure vessel is one of engineering structures that required to work under high mechanical deformations and elevated temperature [1]. also considered the most important and irreplaceable components in the oil and gas or petrochemical industries [2]. The high cost of testing and manufacturing pressure vessels triggers the development of science to find ways to reduce these costs through analysis without making actual objects. One of these analytical science is a finite element analysis. Finite element is a numerical method used to solve technical and mathematical problems of physical phenomena [3]. In present time, numerical calculations on finite elements are generally done using software such as ansys, abaqus, etc. However, the finite element method requires further validation to guarantee and ensure the truth of the results from the simulation that has been done [4]. Validation of finite element



method is generally performed using two ways validations such as numerical method (direct manual calculations) and experimental method by making simulated object. Validation by numerical method is the most difficult and requires a long time, while experimental validation requires high enough cost to make a prototype of the object to be made [4]. Therefore an accurate and inexpensive method of validation is needed to support the development of finite element research, especially in the field of pressure vessels for both university or wider research scale. Mechanics of materials as well as mechanical behavior of materials explain 3 failure theories which are the basis for finite element calculations, such as maximum normal stress theory, maximum shear stress theory, and theory of distortion energy [5] & [6]. This study aims how to determine what theories are used and how to apply them, especially in the case of vertical pressure vessels simulations. and finally to ascertain whether the use of failure theory alone is sufficient to validate and determine the safety of the simulation results.

2. Research methodology

The case used in this study was the result of simulation that has been carried out on a vertical pressure vessel with the following form, shown in fig.1, and specifications shown in table 1 & 2. The pressure vessel was used as a feed gas scrubber to separate oil and gas in the industries [7].

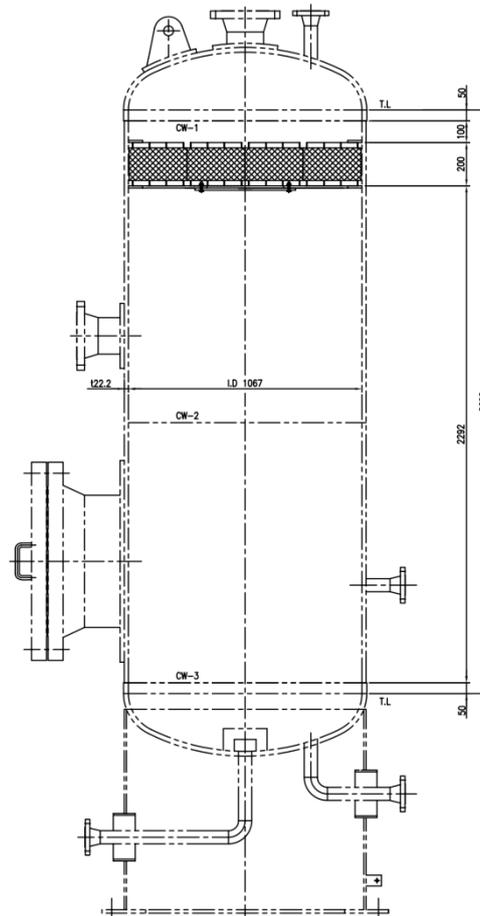


Figure 1. Vertical pressure vessel without the saddle

Table 1. Specification of the pressure vessel

Vessel Type	Vertical Feed Gas Scrubber - 2Phase
Internal Diameter	1067 mm
Length (T/T)	2692 mm
Volume	2.55 m ³
Wall Thickness	0.985 in
Working Pressure	629 psi
Maximum Allowable Working Pressure (MAWP)	818 psi
Working Temperature	70 °F
Corrosion Allowance	0.125 in

Table 2. Materials of Pressure Vessel

Shell (body)	ASTM SA 516-70	Carbon Steel
Head	ASTM SA 516-70	Carbon Steel
Man Hole	ASTM SA 516-70	Carbon Steel
Nozzles	ASTM A106 & A105 Gr.B	Seamless Carbon Steel

And the mechanical properties of material used are shown in table 3.

Table 3. Mechanical Properties of Materials [8]

	Head and Shell	Nozzles
Type	ASTM SA516 Grade 70	ASTM 105 & 106 Gr. B
Modulus of Elasticity	3×10^7 psi	2.9×10^7
Poisson's Ratio	0.3	0.3
Yield Strength	38000 psi	36259 psi
Ultimate Tensile Strength	70000 psi	66717 psi

After all data were known, the simulation then performed using ANSYS by following the standard of ASME [9]. The simulation was carried out using tetrahedral mesh and the pressure value of 818 psi (MAWP), while the force and moment load of the nozzle referred to the standard of Petronas and pipe stress analysis [10] & [11].

3. Results and Discussion

Based on failure theory, maximum normal stress theory is best used for loading on a brittle materials, maximum shear stress theory is best used for the cutting process of a ductile materials, and theory of distortion energy is best used for loading on a ductile materials [5]. According to Table 2 & 3. the materials of pressure vessel were made of carbon steel which has ductile properties and behavior. The function of a pressure vessel is to hold pressurized fluid in it and to withstand the load from the fluid. Therefore, the theory of distortion energy was chosen and used to support this simulation which the results are indicated by von mises stress. The result of the simulation shown in figure 2.

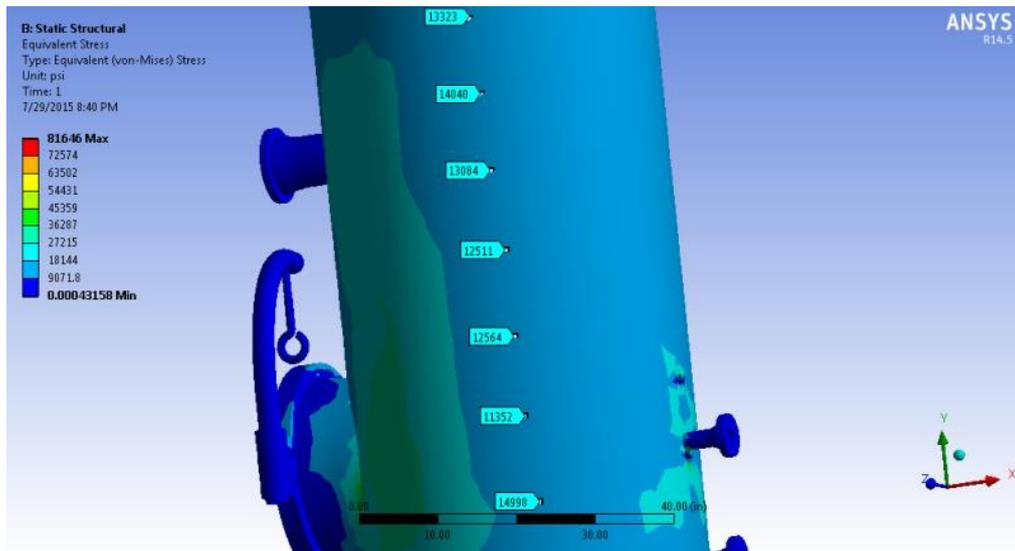


Figure 2. Results of simulation

Based on the results of von mises stress simulation on the body of the vessel, the stress results shown in table 4.

Table 4. Stress value of the body

Stress value (psi)
13323
14040
13084
12511
12564
11352
14998

However, there are 2 critical areas located around the manhole and c2 nozzle shown in figure 3.

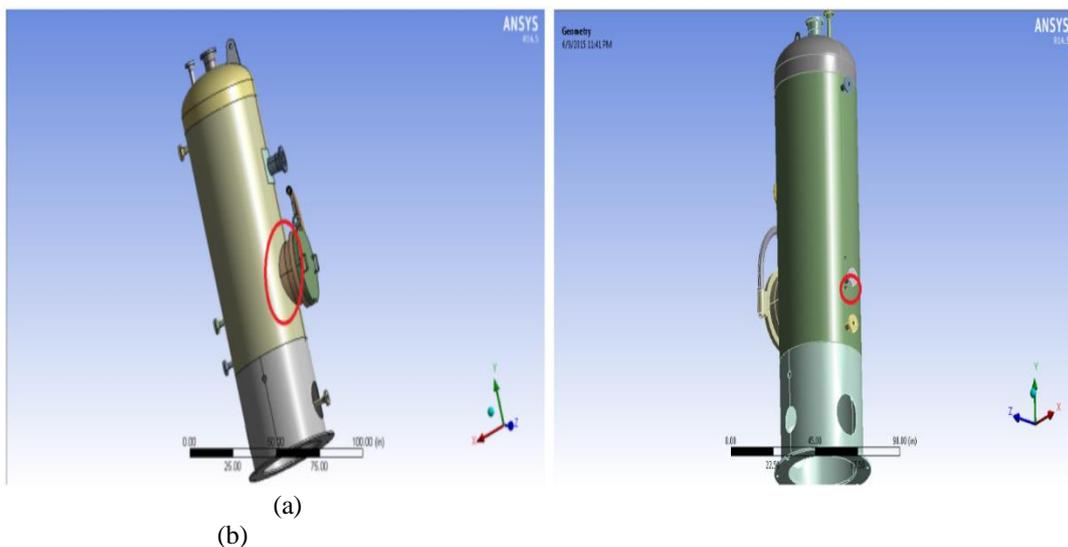


Figure 3. Critical areas of the pressure vessel, (a) Manhole and (b) C2 Nozzle

The stress value shown in Table 5.

Distance from the hole (in)	Von - mises stress (psi)	
	Manhole	C2
0	30106	44608
1	27922	39421
2	23928	31271
3	22817	24956
4	20932	18168
5	17990	15534

3.1. Basic Validation

Based on the theory of distortion energy, failure occurs when the von mises stress is equal to or exceeds the yield strength of the material [5], shown in equation 1

$$\sigma' \geq \sigma_y \text{ or } Y_s \quad (1)$$

And the theory of distortion energy graph becomes

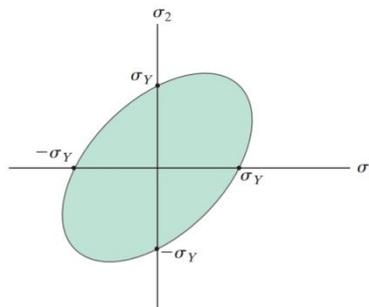


Figure 4. Maximum distortion energy theory [5]

After knowing that the yield stress value can be used as a limit for failure, it can be plotted into the graph of distortion energy theory as a maximum value, After that the value of the simulation results can be plotted into the graph as well. Shown in Fig 5. positive sign was defined as tensile load and negative sign was defined as compressive load [4].

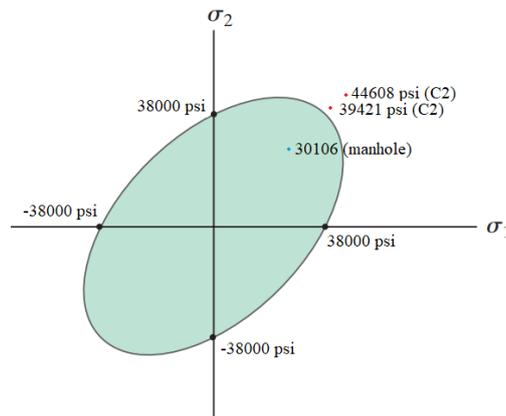


Figure 5. Validation result using theory of distortion energy

As shown in Fig 5. there are 2 values that are outside the graph, which mean there will be a failure in C2 area, while the highest value in the manhole area is already in the graph which means it's safe. Therefore a refinement was needed to prevent failure in C2 area [12].

3.2. Further Validation

Section 3.1 shown that failure theory can be used as a reference for simulation results. However, the validation results did not ensure the safety of the pressure vessels to be produced because it's too simple. In fact, a lot of testing procedure has to be done. Therefore, further validation was needed to support the results of validation.

3.2.1. Maximum Allowable Stress

There are two main stresses on a pressure vessel, hoop stress or circumferential stress (σ_H) and longitudinal stress (σ_L). Shown in Fig. 6. These main stresses need to be calculated to determine the safety factor.

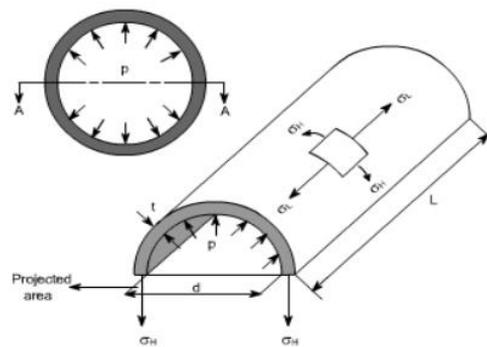


Figure 6. Hoop stress and longitudinal stress [13]

a. Hoop Stress

The hoop stress can be calculated using equation [14] :

$$\sigma_H = \frac{PD}{2t} \tag{2}$$

$$\sigma_H = \frac{818x(1067 + 25)}{2(25)}$$

$$\sigma_H = 17865 \text{ psi}$$

b. Longitudinal Stress

The longitudinal stress can be calculated using equation [14] :

$$\sigma_L = \frac{PD}{4t} \quad (3)$$

$$\sigma_L = \frac{818x(1067 + 25)}{4(25)}$$

$$\sigma_L = 8932.5 \text{ psi}$$

Where,

σ_H = Hoop stress

σ_L = Longitudinal stress

P = Pressure (MAWP)

D = Mean diameter (diameter + thickness)

t = thickness

After both main stresses were calculated, maximum allowable stress can be determined. The standardized maximum allowable stress or maximum shear stress used for 36000 psi yield carbon steel from ASME section II Part D (Table 1A, Cont'd) is 21500 psi [15] & [16].

3.2.2. Factor of safety

The factor of safety can be calculated using equation [16] :

$$FoS = \frac{\sigma_{\max}}{\sigma_H} \quad (4)$$

$$FoS = \frac{21500}{17865}$$

$$FoS = 1.20$$

3.2.3 Joint efficiency

Double butt welded was used at each pressure vessel joint, therefore joint efficiency was needed to determine initial thickness. The joint efficiency used was 1 [14]. Spot heating was used to remove distortion from the joint. Fig. 8 shown the stresses in the longitudinal direction.



Figure 7. Double butt welded joint

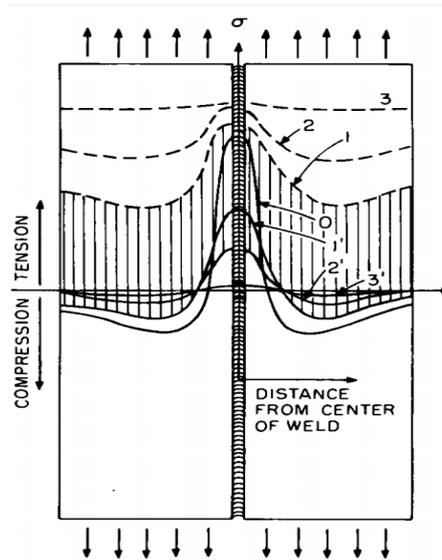


Figure 8. Schematics distributions of stresses on a butt weld when uniform tensile load are applied and of residual stresses after the loads are released [17]

“Curve 1 shows the stress distribution when a uniform tensile stress $\sigma = \sigma_1$ is applied. Yield stress is reached near the weld, and most of the stress increase in areas away from the weld. As the applied stress increased, the stress distribution across the weld evens out, that is, the effect of welding residual stress distribution decreases” [17]. Residual stress significantly affects only those phenomena that occur under a low applied stress, such as brittle fracture and stress corrosion cracking. As the level of applied stress increases, the effect of residual stress decreases. Welding sequence and welding direction also have an influence on the magnitude and distribution of the residual stress [18].

3.2.4. Initial & optimal thickness inspection

Pressure vessel has 3 types of wall thickness, which are shell, head and nozzle. Shell and head thickness is the most important to ensure the safety of the pressure vessel and to determine the lifetime of the vessel

a. Shell thickness

The thickness of the vessel wall that sustains internal pressure must not be thinner than the value calculated from the following formula [14] :

$$t = \frac{PR}{SE - 0.6P} \tag{5}$$

$$t = \frac{818 \times 21.00395}{(21500 \times 1) - (0.6 \times 818)}$$

$$t = 0.817''$$

Corrosion Allowance added $t = 0.817'' + 0.125'' (C.A)$

$$t = 0.942'' = 23.92mm$$

b. Head thickness

$$t = \frac{PD}{2SE - 0.2P} \quad (6)$$

$$t = \frac{818 \times 42.0079}{2(21500 \times 1) - (0.2 \times 818)}$$

$$t = 0.802''$$

Corrosion Allowance added $t = 0.802'' + 0.125''$

$$t = 0.927'' = 23.54 \text{ mm}$$

Where,

t = thickness

P = maximum allowable working pressure

R = inside radius

D = inside diameter

S = maximum allowable stress

E = joint efficiency

After the thickness of the shell and head is known, the next step is to determine the optimal thickness used. This step needs to be done because in reality there is no metal sheets with these dimensions. Therefore a standard thickness of produced metal sheets was needed. In this case, standard metal sheets in Indonesia [19] was used. Shown in Table 6.

Table 6. List of nominal thickness of commercial steel plates by SNI (Indonesian National Standards)

Nominal Thickness (mm)	Width (mm)			
	<1600	1600 ≤ W < 2000	2000 ≤ W < 2500	2500 ≤ W < 3500
19.00	± 0.65	± 0.75	± 0.75	± 0.95
21.00	± 0.65	± 0.75	± 0.75	± 0.95
22.00	± 0.65	± 0.75	± 0.75	± 0.95
24.00	± 0.65	± 0.75	± 0.75	± 0.95
25.00	± 0.70	± 0.80	± 0.80	± 1.00

Shell and head thickness was calculated as 23.92 mm and 23.54 mm, then 24.00 mm thickness was selected. The pressure vessel has length of 2692 mm, then 0.95 mm added to thickness. Therefore the optimal thickness become 24.95 mm or 25 mm or 0.985”.

3.2.5. Critical thickness

Critical thickness is the minimum thickness before the vessel fails. Critical thickness needs to be calculated to determine the working age of the vessel. Eq (2) can be used to calculate critical thickness by replacing σ_H with maximum allowable stress or σ_{max} [14]. Therefore the equation becomes.

$$\sigma_{\max} = \frac{PD}{2t} \quad (7)$$

$$21500 = \frac{818 \times (1067 + 25)}{2t}$$

$$t = 20.75 \text{ mm}$$

3.2.6. Working age

Pressure vessels with material SA516 Gr70 that contain LPG have the following chemical composition. Shown in Table 7.

Table 7. Typical chemical composition of ASTM SA516 Grade 70 (Percent %) [20]

Grade	C	Mn	Si	Al	P	S
70	0.20	1.05	0.32	0.04	0.015	0.008

The pressure vessel with material SA516 Gr 70 and chemical composition as shown in Table 7. have a corrosion rates of 0.46 mm/year [21]. Therefore the working age of the vessel can be calculated,

$$t = 2.5 - 20.75 = 4.25 \text{ mm} \quad (8)$$

$$\text{WorkingAge} = \frac{4.25}{0.46} = 9.2 \text{ years}$$

3.2.7. Corrosion analysis

Corrosion is a common phenomenon in the oil and gas industries and also other chemical based industries. Cracks can propagate either in length or depth. Surface crack can happen through thickness cracks, causing failure in the equipment [22]. The brittle fracture of the base metal is a sudden failure without exhibiting plasticity. Metal may become brittle following a decline in temperature; thus, the operation of the equipment should be carried within the permissible temperature range that will not result in the brittle fracture failure [22]. For the brittle fracture assessment, the fracture toughness of the material K_{mat} is required, which measures the ability of the material to resist crack growth propagation in the material after its initiation. Unfortunately, particular experiments and testing are still needed to ensure the same results as simulations because many factors can change the material behavior, such as temperature, environment, contained fluids, working age, etc. Therefore, in this case the simulation for corrosion was not carried out but rather a direct testing procedure using ultrasonic thickness measurements [23].

3.2.8. Buckling

Buckling analysis was performed to ensure structural stability. The pressure vessel has a spherical head or cap and 10 external stringers, also has uniform thickness in head and body to avoid reduction of carrying capacity [24]. Fig. 9 shown the eigenshape of dome and external stringer. Fig. 10 shown the buckling simulation results.

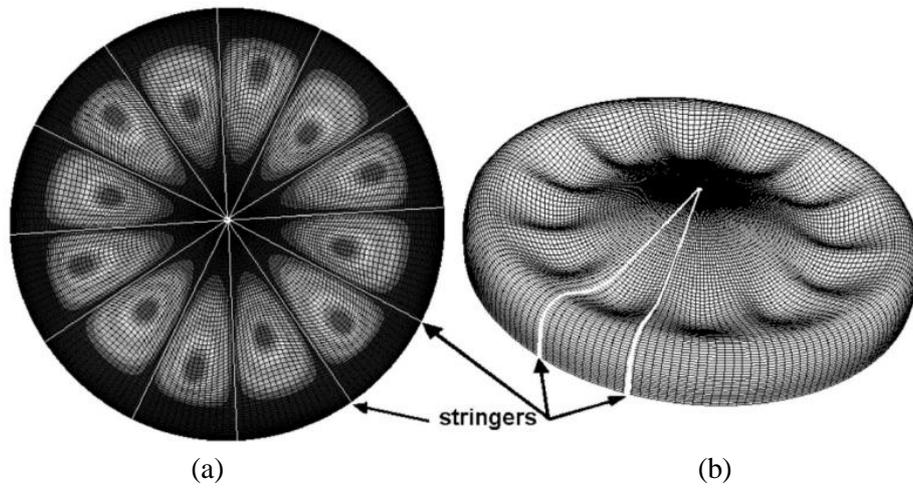


Figure 9. Eigenshape of dome 11 with twelve external stringers. (a) top view, (b) isometric view [25]

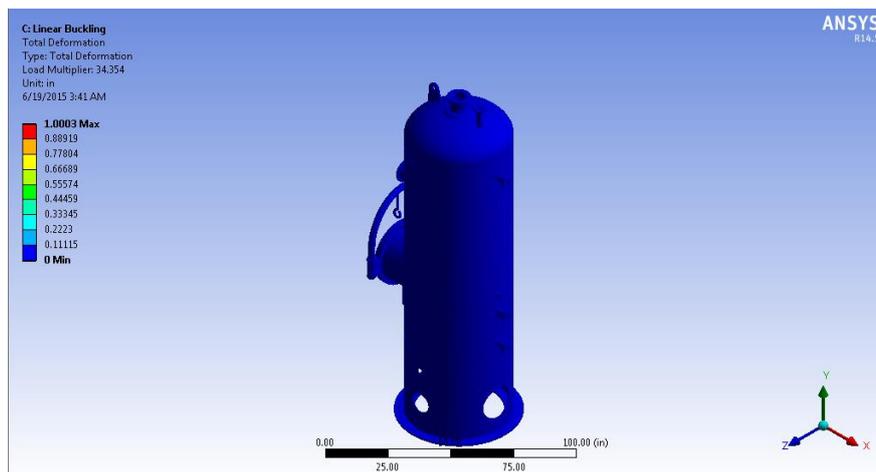


Figure 10. Buckling simulation results

From the buckling simulation results it is known that the vessel was able to hold 34.35 times its dead weight and considered safe. However, validation was needed to ensure the simulation results. Fig. 11 shown the influence of stringer to the magnitude of buckling pressure, in this case a deadweight.

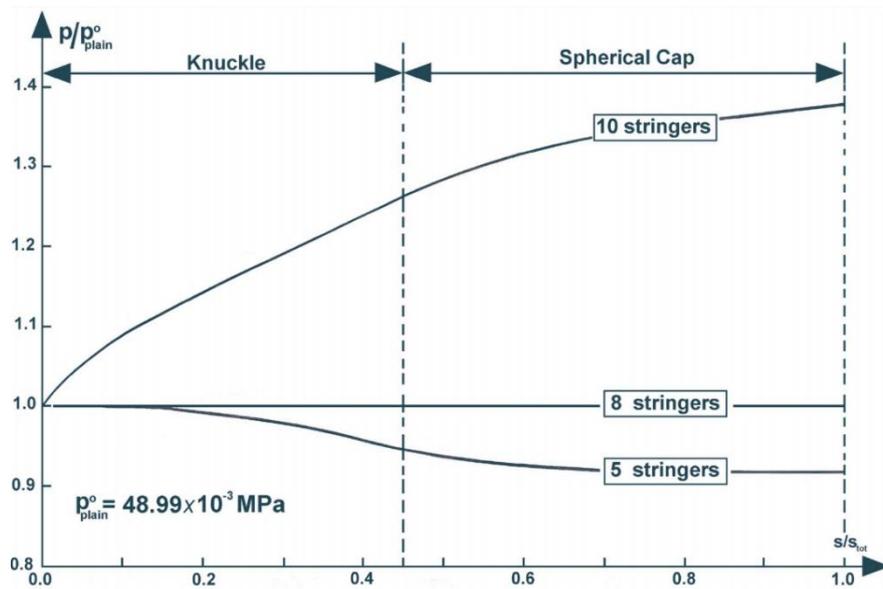


Figure 11. Influence of shorter stringers on the magnitude of buckling pressure [25]

Fig. 11 shown the comparison between pressure and pressure at plain (P/P^o). P was generally higher compare to plain. 10 stringers has 1.37 P/P^o . Therefore the pressure or deadweight can be divided by 1.37 which leads to 25.07 and still considered safe. The axial buckling load can also be decreased by increasing the external pressure [26].

3.2.9. Refinement

As shown in Fig.5, refinement was needed at C2 areas and Manhole areas were considered safe. However, if referred to the maximum allowable stress which is 21500 psi, the manhole area is beyond safe zone because it has stress value of 30106 psi and requires refinement, as well. The refinement has been done by following instruction from ASME. Fig. 12 & 13 shown the refinement of the critical areas. And Table 8. shown the stress value from the simulation results after refinement.

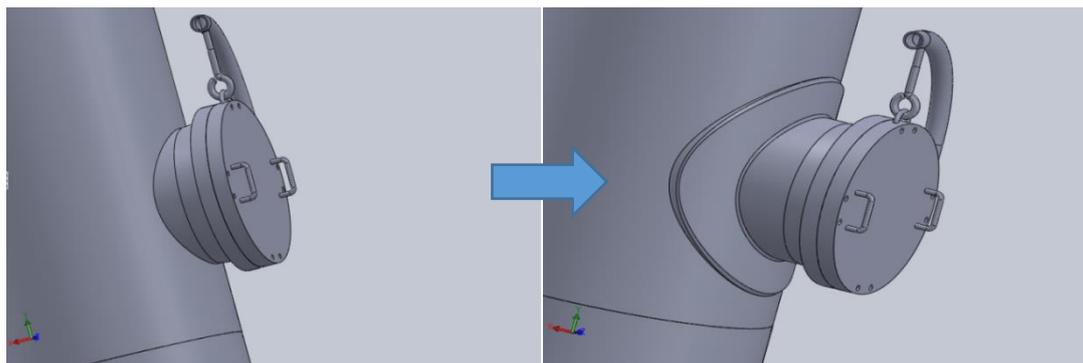


Figure 12. Refinement at manhole area

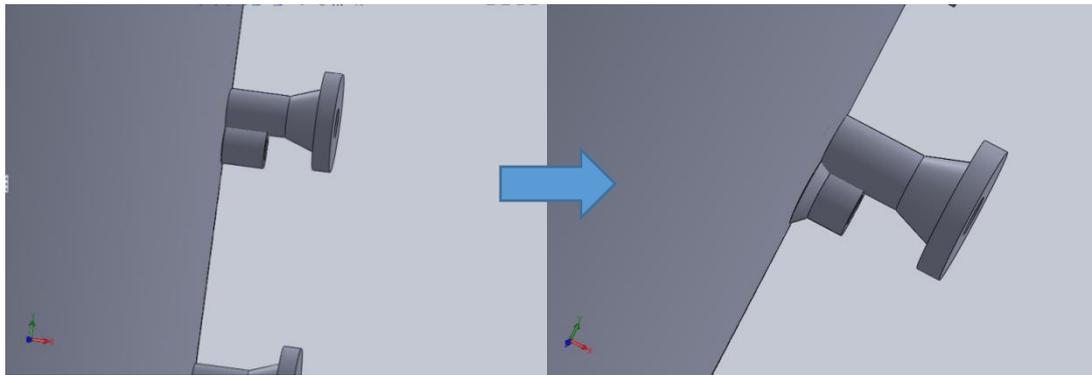


Figure 13. Refinement at C2 area

Table 8. Stress value of critical areas after refinement

Distance from the hole (in)	Von - mises stress (psi)	
	Manhole	C2
0	15436	15542
1	11150	13035
2	10354	11467
3	8874	14952
4	7585	14400
5	6437	14341

As shown in Table 8. von mises stress that occurs in critical areas has decreased. In the manhole area, the dimensions of the refinement exceed 5 inches, then after getting farther from the refinement area the stress value approaches the stress value in the body as shown in C2 area at 3, 4, and 5 inches.

3.2.10. Validation using theory of distortion energy after refinement

Using the same method as shown in Figure 5, maximum allowable stress (MAS) was plotted into the graph to be a more secure safety limit. Fig. 14 shown the validation results after refinement at three different areas (C2, ManHole or MH, and Body of vessel) using the highest value from each area.

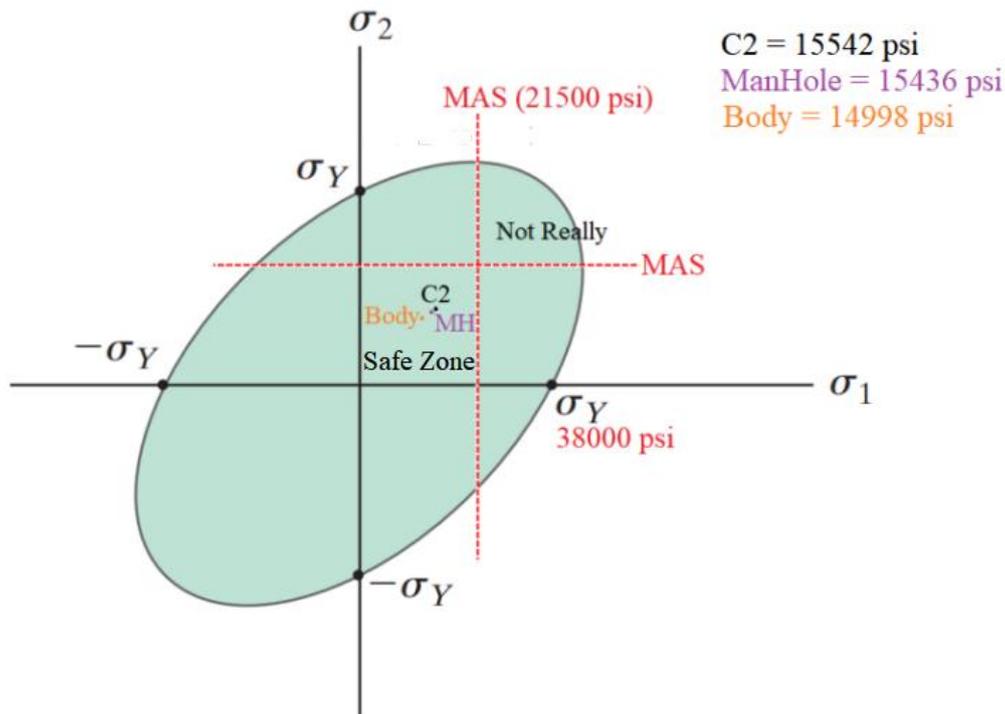


Figure 14. Validation results after refinement using theory of distortion energy

As shown in Fig. 14. Three of the highest stresses have been plotted into the graph. Even the highest stress did not exceed the safety limit or MAS. After the simulation and all validation criteria have been done, the results indicates that the pressure vessel is considered safe to be produced.

3.2.11. Cost Analysis

The purpose of this study was to demonstrate a low cost validation method on the vertical pressure vessel. After all validation methods have been done, a simple cost analysis can be performed to prove whether this method really requires a low cost. Table 9 shown the results of the simple cost analysis of the research. However, the testing, inspection, and also production cost for each vessel will be different depending on the dimension, material used, function, and necessity of the vessel or even company and country that made the vessel. Therefore, a simpler method will be used. Every test or inspection and validation that requires a fee will be marked "1" while avoidable costs are marked "0". The low cost validation method named LCV, while the actual procedure named AP. Testing procedure based on inspection company [27] & [28]. The cost analysis only focused on NDT, while destructive test such as mechanical properties test, hardness test, impact test, etc. were eliminated, because such testing procedure couldn't be avoided. As shown in table 9. LCV can save a lot of costs that can be done with finite elements method or other calculations method. Not all types of testing were included in the cost analysis. but at least it shows that the proposed method can be considered as a science that deserves to be developed.

Table 9. Simple cost analysis comparison between LCV and AP

No	Testing Procedure	LCV	AP
1	Visual testing (dimension, thickness, height, and weight inspections)	1	1
2	Working pressure determination (from amine process)	1	1
3	Maximum allowable stress determination	0	1
4	Factor of safety determination	0	1
5	Initial and optimal thickness determination	0	1
6	Critical thickness determination	0	1
7	Working age determination	0	1
8	Joint inspection	1	1
9	Corrosion inspection	1	1
10	Buckling inspection	0	1
11	Seismic testing	1	1
	TOTAL	5	11

4. Conclusions

Based on the results of simulations and validations that have been carried out, the following conclusions are obtained :

- a. Basic validation with failure theory can be used as a validation for simulation results and also can be reference and the benchmark for decision making such as reinforcement and others. However, it is not enough to ensure the safety of the vessel. Therefore, further validation and other method besides finite element analysis were needed to support the simulation results.
- b. Some simulations such as corrosion, fatigue, seismic, and wind load can actually be done in finite element analysis. However, there is no validation method to ensure the results of the simulation except by direct testing.
- c. Dynamic load was not the focus of the simulation, because it is clear that the load used is the maximum load that is allowed on the vessel. And the pressure vessel only function as a scrubber which works under constant load. Assuming that the vessel considered safe at maximum load, then the actual work load will also be safe, which is 629 psi.
- d. Basic validation alone is not recommended as a reference to determine the safety of pressure vessels. It would be better by doing calculations and analysis on all aspects then validated using failure theory after knowing the maximum allowable stress.
- e. Theory of distortion energy was used as a validation tool in this study, however that does not mean other failure theories cannot be used as a reference for validation. The choice of failure theory used must be based on material behavior and properties of material and the loading method of the structure.
- f. Fatigue and creep testing on the pressure vessel was not the focus of the study because the working age of the pressure vessel is relatively short, which is 9.2 years. While the project period using the

- pressure vessel is only 8 years, which means the pressure vessel will be replaced even before the vessel fails.
- g. Cost analysis has proven that the LCV method can save some expensive testing costs. Therefore an expensive testing to determine the maximum allowable stress, factor of safety, initial and optimal thickness, critical thickness, working age, and buckling inspection can be avoided.
 - h. There are still many developments that can be done for this method, the most important thing is to understand how the structure works and the loading method of the structure itself. This research can only be used in the case of a vertical 2 phase pressure vessel. If there is a change of material, load, function, or even type of vessel, it must be validated from the beginning depending on those aspects.

5. References

- [1] M. Khorsand, K. Fu, and Y. Tang, 2019. *Multi-Directional Functionally Graded Materials for Enhancing The Durability of Shell Structures*. International Journal of Pressure Vessels and Piping, vol 175. Elsevier Ltd.
- [2] Z. Xiao, J. Shi, X. Cao, Y. Xu, and Y. Hu, 2018. *Failure Probability Analysis of Pressure Vessels That Contain Defects Under The Coupling of Inertial Force and Internal Pressure*. International Journal of Pressure Vessels and Piping, vol 168. p 59-65. Elsevier Ltd.
- [3] D. V. Hutton, 2004. *Fundamentals of Finite Element Analysis, First Edition*. McGraw-Hill Inc, New York.
- [4] R. D. Cook, 1981. *Concept and Applications of Finite Element Analysis*. John Willey & Sons, New York.
- [5] R. C. Hibbeler, 2011. *Mechanics of Materials Eight Edition*. Pearson Prentice Hall, USA.
- [6] N. E. Dowling, 1993. *Mechanical Behavior of Materials*. Prentice Hall, USA.
- [7] *Piping & Instrument Diagram*. P.T Yudistira Haka Perkasa. Jakarta. Indonesia
- [8] S. Timoshenko, 1955. *Strength of Materials*. D. Van Nostrand Co. New York.
- [9] ASME, 2000. *Rules for Construction of Pressure Vessel Section VIII Division 1*. The American Society of Mechanical Engineers. New York.
- [10] *Minimum Design Values For Nozzle Forces and Moments*. Petroliam Nasional Berhad (PETRONAS).
- [11] P. E. Sam Kannappan, 1986. *Introduction to Pipe Stress Analysis*. John Wiley and Sons. New York.
- [12] P. E. John F. Harvey, 1985. *Theory and Design of Pressure Vessels*. Van Nostrand Reinhold Company, New York.
- [13] Z. Huda and M. H. Ajani, 2015. *Evaluation of Longitudinal and Hoop Stresses and a Critical Study of Factor of Safety (FoS) in Design of a Glass-Fiber Pressure Vessel*. International Journal of Materials and Metallurgical Engineering, vol 9. No. 1. World Academy of Science, Engineering and Technology
- [14] E. F. Megyesy and P. Buthod, 2001. *Pressure Vessel Handbook 12th edition*. Pressure Vessel Publishing, Inc., Oklahoma.
- [15] ASME, 2010. *Boiler and Pressure Vessel Code, Properties (Metric) Materials Section II Part D*. The American Society of Mechanical Engineers, New York.
- [16] D. R. Moss, 2004. *Pressure Vessel Design Manual Third Edition*. Gulf Professional Publishing, USA.
- [17] K. Masubuchi, 1980. *Analysis of Welded Structure first edition*. Pergamon Press Ltd, U.K, USA, Canada, Australia, France, Germany.
- [18] R. Bai, Z. Guo, C. Tian, Z. Lei, C. Yan, and W. Tao, 2018. *Study of Welding Sequence of Butt-Welded Structures Based on Equivalent Heat Source Parameter*. International Journal of Pressure Vessels and Piping, vol 163. p 15-22. Elsevier Ltd.
- [19] Indonesian National Standards. *Hot Rolled Sheets, Plates, and Coils*. Badan Standarisasi Nasional - BSN. Senayan, Jakarta, Indonesia.

- [20] *Pressure Vessel Steel ASTM SA516 Grade 60/65/70*. MACSTEEL VRN. Germany.
- [21] A. L. Kohan and M. Harry, 1981. *Boiler Construction, Operation, Inspection, Maintenance, and Repair, With Typical Boiler Questions and Answers for Plant, Operating, and Maintenance Engineers*. McGraw-Hill Book, New York.
- [22] E. Azzuni and S. Guzey, 2019. *Brittle Fracture Failure Assessment of API 12D Tanks with a Semicircular-topped Rectangular Cleanout and Stepped Shell Design*. International Journal of Pressure Vessels and Piping, vol 175. Elsevier Ltd, USA.
- [23] N. Kasai, S. Mori, K. Tamura, K. Sekine, T. Tsuchida, and Y. Serizawa, 2016. *Predicting Maximum Depth of Corrosion Using Extreme Value Analysis and Bayesian Inference*. International Journal of Pressure Vessels and Piping, vol 146. p 129-134. Elsevier Ltd.
- [24] H. L. T. Nguyen, I. Elishakoff, and V. T. Nguyen, 2009. *Buckling Under The External Pressure of Cylindrical Shells with Variable Thickness*. International Journal of Solids and Structures, vol 46, issue 24. p 4163-4168. Elsevier Ltd.
- [25] J. Blachut, 2019. *Elastic Buckling of Stringer Reinforced Torispheres Under External Pressure*. International Journal of Pressure Vessels and Piping, vol 175, Elsevier Ltd. UK.
- [26] F. M. Nasrekani and H. Eipakchi, 2019. *Analytical Solution for Buckling Analysis of Cylinders with Varying Thickness Subjected to Combined Axial and Radial Loads*. International Journal of Pressure Vessels and Piping, vol 172. p 220-226. Elsevier Ltd.
- [27] *Pressure Vessel Inspection Procedure*. Technical Inspection and Certification. P.T. Wismar Inspeksindo. Jakarta. Indonesia.
- [28] *Boiler Inspection and Testing Procedure*. Technical Inspection and Certification. P.T. Cahaya Mitra Inspeksi. Banten. Indonesia.