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Fringe visibility for two qubits interacting with a macroscopic medium*

Yu-Heng Li (李玉恒)[1], Yan-Jun Liu (刘彦军)[1,2], Jing Lu (卢竞)[1] and Lan Zhou (周兰)[1,3]

[1] Department of Physics and Key Laboratory for Matter Microstructure and Function of Hunan Province, Hunan Normal University, Changsha 410081, China
[2] College of Physics, Mechanical and Electronical College, Shijiazhuang University, Shijiazhuang, 050035, Hebei, China

E-mail: zhoulan@hunnu.edu.cn

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Abstract

We study both the two-particle and single-particle fringe visibility in the generalized version of the Nakazato–Pascazio model where two qubits interact with a finite length one-dimensional array. Both the two-particle and single-particle fringe visibilities are investigated with different initial states of the particles spin. For different initial states of the particles spin, the two-particle fringe visibility either decreases or increases over time, and may even decrease first and increase later. Due to the interaction between the particles and the one-dimensional array, the single-particle fringe visibility increases over time when the initial state of the two particles spin is independent. The single-particle fringe visibility is equal to 0 as the two-particle spin is initially in the completely entangled state or in the singlet state.

Keywords: two-particle fringe visibility, single-particle fringe visibility, Nakazato-Pascazio generalized model

(Some figures may appear in colour only in the online journal)

1. Introduction

The superposition principle of quantum states is a basic feature of quantum mechanics. Quantum coherence is originally derived from the superposition principle of quantum states. Therefore, quantum coherence is considered to be one of the most basic properties of quantum mechanics. Quantum coherence is not only deeply investigated in quantum mechanics, but also plays an important role in quantum optics [1, 2] and quantum information [3]. At the same time, quantum coherence has been widely used in many emerging fields such as quantum metrology [4–6], quantum thermodynamics [7–13] and quantum biology [14–18]. After Baumgratz *et al* quantitatively studied the quantum coherence from the perspective of resource theory [19], the coherence theory developed explosively [20–26].

Quantum decoherence and quantum measurement are closely related. A quantum measurement process consists of a quantum system and a measuring apparatus. The correlation between the quantum system and the measuring apparatus is established by the interaction between the quantum system and the measuring apparatus. The correlation between the quantum system and the measuring apparatus open up a new approach to understand the decoherence. For the sake of simplicity, Hepp and Coleman further considered the essential difference between the measuring apparatus and the quantum system, and proposed a dynamic model of quantum measurement, known as the Coleman–Hepp (CH) model [27]. The CH model is an exactly solvable model. In this model, the measuring apparatus is often regarded as a macroscopic object, and the evolution of each microscopic particle constituting the macroscopic object satisfies the Schiödinger equation. In this way, the quantum measurement process is actually the dynamic evolution process of the quantum system interacting with the macroscopic object. In the process of interaction, the off-diagonal element of the density matrix

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[3] Author to whom any correspondence should be addressed.

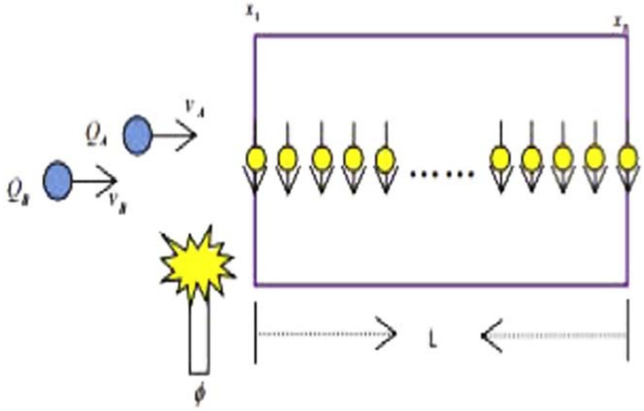

Figure 1. The schematic sketch of the generalized version of the NP model, where two particles Q_A and Q_B interact with the one-dimensional array. A pulse acts on the spin of the particle.

describing the state of the quantum system will disappear with time. This phenomenon is generally called the decoherence phenomenon caused by the quantum measurement. Later on, Nakazat and Pascazio proposed the Nakazato–Pascazio (NP) model, which is a modified version of the CH model [28, 29]. The model considers the energy exchange between the quantum system and the measuring apparatus. Both the CH model and NP model are exactly solvable models of quantum measurements. These two models not only provide insight into the physical essence of quantum measurement, but also better understand the quantum decoherence in the measurement process. In this paper, the fringe visibility is used to represent the coherence of quantum state. Both the two-particle and single-particle fringe visibility are investigated in the generalized version of the NP model where the two qubits interact with a finite length one-dimensional array. Here, we choose the different initial state of the particles spin. Under the different initial state of the particles spin, we studied the change of both the two-particle and single-particle fringe visibility with time, respectively.

This paper is organized as follows. In section 2, we introduce the generalized version of the NP model where the two particles with spin-1/2 interact with a finite length one-dimensional array. In section 3, we derive the dynamic evolution of the total system. In section 4, we study both the two-particle and single-particle fringe visibility. In section 5, we make our conclusion.

2. The model

The NP model describes the interaction between an ultra-relativistic particle and a one-dimensional array of N harmonic oscillators. The particles interact with the spins of their local array through a local potential of the spin flip. In this paper, we consider the interaction between two particles Q_A and Q_B and the same one-dimensional array, which is shown in figure 1. Each particle Q_j $(j = A, B)$ has a spin τ_j^z of magnitude 1/2. The spatial degrees of freedom of the particle is characterized by the position operators $\hat{x}_j$ and the momentum operator $\hat{p}_j$. Here, the one-dimensional array has a finite length $L = x_n - x_1$, where x_1 and x_n represent the position of the first spin and the final spin of the array, respectively. The particle interacts with the one-dimensional array if and only if the spin direction of the particle is up.

The Hamiltonian of the total system can be expressed as

$$H = H_Q + H_D + H_1. \tag{1}$$

The free Hamiltonian of two particles can be expressed as

$$H_Q = v_A\hat{p}_A + v_B\hat{p}_B + \sum_{j=A,B}\frac{1}{2}\hbar\omega_j(1 + \tau_j^z), \tag{2}$$

where the positive constant v_j $(j = A, B)$ is the velocity of the particle Q_j, and $\hat{p}_j$ represents the momentum operator of the particle Q_j. The Hamiltonian of the one-dimensional array is

$$H_D = \frac{1}{2}\hbar\omega\sum_{n=1}^{N}(1 + \sigma_n^z), \tag{3}$$

where σ_n^z is the Pauli spin operator of the nth spin of the one-dimensional spin array. When the two particles interact with the same one-dimensional array, the interaction Hamiltonian can be expressed as

$$H_1 = \sum_{j=A,B}\sum_{n=1}^{N}\frac{1 + \tau_j^z}{2}V(\hat{x}_j - x_n)(\sigma_n^+ e^{-i\frac{\omega}{v_j}\hat{x}_j} + \sigma_n^- e^{i\frac{\omega}{v_j}\hat{x}_j}), \tag{4}$$

where $V(\hat{x}_j - x_n)$ characterizes the strength of the interaction between the particle and each spin of the spin array. x_j and x_n represent the position of the jth of the particle and the nth spin of the one-dimensional array, respectively. σ_n^+ and σ_n^- are the creation and annihilation operators of the nth spin of the one-dimensional array.

For the convenience of the latter, the Hamiltonian of the total system reads

$$H = H_0 + H_1, \tag{5}$$

where $H_0 = H_Q + H_D$.

3. The dynamics of the total system

The initial state of the total system can be expressed as

$$|\Psi(0)\rangle = |\psi\rangle \otimes |\psi_s\rangle \otimes |\psi_D\rangle = |\psi, \psi_s, \psi_D\rangle. \tag{6}$$

The expansion of $|\psi\rangle$ in terms of the coordinate eigenstates is

$$|\psi\rangle = \int_{-\infty}^{\infty} dx_A \int_{-\infty}^{\infty} dx_B \psi_{AB}(x_A, x_B)|x_A, x_B\rangle, \tag{7}$$

where $|x_A, x_B\rangle = |x_A\rangle \otimes |x_B\rangle$ and $\hat{X}_l|x_l\rangle = x_l|x_l\rangle$. The initial state of the particles spin can be expressed as

$$\begin{aligned}|\psi_s\rangle = &p_{\uparrow_A\uparrow_B}|\uparrow_A\uparrow_B\rangle + p_{\uparrow_A\downarrow_B}|\uparrow_A\downarrow_B\rangle\\ &+ p_{\downarrow_A\uparrow_B}|\downarrow_A\uparrow_B\rangle + p_{\downarrow_A\downarrow_B}|\downarrow_A\downarrow_B\rangle,\end{aligned} \tag{8}$$

where $|\uparrow_A\uparrow_B\rangle = |\!\uparrow_A\rangle \otimes |\uparrow_B\rangle$, and $|\uparrow\rangle(|\downarrow\rangle)$ represents the spin-up (spin-down). We assume that the initial state of the one-dimensional array is in the ground state $|0\rangle$ (i.e., all spins down), which is given by

$$|\psi_D\rangle = |0\rangle = \prod_{n=1}^{N} |\downarrow_n\rangle. \quad (9)$$

Before the interaction between two particles and the one-dimensional array, a pulse performs a phase rotation

$$U_\phi = e^{-i\phi_A|\downarrow_A\rangle\langle\downarrow_A| - i\phi_B|\downarrow_B\rangle\langle\downarrow_B|} \otimes \hat{1}^Q \otimes \hat{1}^D, \quad (10)$$

on the spin of the particles. The parameters ϕ_A and ϕ_B represent the phase rotation angle of the spin of the particle Q_A and Q_B, respectively. For a given initial state of the total system $\rho(0) = |\Psi(0)\rangle\langle\Psi(0)|$. After the time t, the state of the total system evolves into

$$\rho(t) = U(t)U_\phi\rho(0)U_\phi^\dagger U^\dagger(t), \quad (11)$$

where $U(t) = e^{-iHt/\hbar}$. To exactly solve the current model, the unitary operator can be expressed as $U(t) = e^{-iH_0t/\hbar}U_{AB}(t)$ by introducing the interaction picture. The unitary operator $U_{AB}(t)$ on the right side of the formula satisfies the Schrödinger equation with the interaction Hamiltonian H_1. Since particles interact with the one-dimensional array only when the spin direction of the particle is up, the operator $U_{AB}(t)$ can be decomposed into

$$\begin{aligned} U_{AB}(t) = {} & U_A(t)U_B(t)|\uparrow_A\uparrow_B\rangle\langle\uparrow_A\uparrow_B| + |\downarrow_A\downarrow_B\rangle\langle\downarrow_A\downarrow_B| \\ & + U_A(t)|\uparrow_A\downarrow_B\rangle\langle\uparrow_A\downarrow_B| + U_B(t)|\downarrow_A\uparrow_B\rangle\langle\downarrow_A\uparrow_B|, \end{aligned} \quad (12)$$

where states $|\uparrow_j\rangle$ and $|\downarrow_j\rangle$ are the eigenstates of the operator τ_j^z. The operator U_j conform to the Schrödinger equation

$$\begin{aligned} i\hbar\partial_t U_j(t) &= e^{iH_0t/\hbar}H_1e^{-iH_0t/\hbar}U_j(t) \\ &= \textstyle\sum_{j=A,B}\sum_{n=1}^{N} V(\hat{x}_j + v_jt - x_n) \\ &\quad \times \frac{1+\tau_j^z}{2}(\sigma_n^+e^{-i\frac{\omega}{v_j}\hat{x}_j} + \sigma_n^-e^{i\frac{\omega}{v_j}\hat{x}_j})U_j(t). \end{aligned} \quad (13)$$

By solving the equation (13), we can obtain

$$\begin{aligned} U_j(x_j, t) = \exp\Bigg[& -\frac{i}{\hbar}\textstyle\sum_{n=1}^{N}\int_{t_0}^{t} dt'V(x_j + v_jt' - x_n) \\ & \times (\sigma_n^+e^{-i\frac{\omega}{v_j}\hat{x}_j} + \sigma_n^-e^{i\frac{\omega}{v_j}\hat{x}_j})\Bigg], \end{aligned} \quad (14)$$

in the coordinator representation. Here, we assume that the initial time $t_0 = 0$ and define the tipping angles of the nth spin as

$$\alpha_n^{[j]}(t) = \int_0^t dt'V(x_j + v_jt' - x_n)/\hbar. \quad (15)$$

The exponential of the equation (14) can be disentangled in the form

$$\begin{aligned} & U_j(x_j, t) \\ & = \prod_n e^{-i\sigma_n^+\tan\alpha_n^{[j]}(\hat{x}_j,t)}e^{-i\sigma_n^z\ln\cos\alpha_n^{[j]}(\hat{x}_j,t)}e^{-i\sigma_n^-\tan\alpha_n^{[j]}(\hat{x}_j,t)}, \end{aligned} \quad (16)$$

by making use of the SU(2) algebra [30] . After the interaction between the particle and the one-dimensional array, we can obtain the final state of the particles spin

$$\begin{aligned} \hat{\rho}_s^{out} &= \int_{-\infty}^{\infty} dx_A \int_{-\infty}^{\infty} dx_B Tr_D\langle x_A, x_B|\, e^{-(\frac{it}{\hbar})\widehat{H}}U_\phi \\ &\quad \times |\Psi(0)\rangle\langle\Psi(0)|\, U_\phi^\dagger e^{(\frac{it}{\hbar})\widehat{H}}|x_A, x_B\rangle \\ &= \int_{-\infty}^{\infty} dx_A \int_{-\infty}^{\infty} dx_B P_{AB}(x_A, x_B) \\ &\quad \times [|p_{\uparrow_A\uparrow_B}|^2|\uparrow_A\uparrow_B\rangle\langle\uparrow_A\uparrow_B| + |p_{\downarrow_A\downarrow_B}|^2|\downarrow_A\downarrow_B\rangle\langle\downarrow_A\downarrow_B| \\ &\quad + |p_{\uparrow_A\downarrow_B}|^2|\uparrow_A\downarrow_B\rangle\langle\uparrow_A\downarrow_B| + |p_{\downarrow_A\uparrow_B}|^2|\downarrow_A\uparrow_B\rangle\langle\downarrow_A\uparrow_B| \\ &\quad + p_{\uparrow_A\uparrow_B}p_{\downarrow_A\downarrow_B}^*|\uparrow_A\uparrow_B\rangle\langle\downarrow_A\downarrow_B|\, f_2(t) + (h.c.) \\ &\quad + p_{\downarrow_A\uparrow_B}p_{\uparrow_A\downarrow_B}^*|\downarrow_A\uparrow_B\rangle\langle\uparrow_A\downarrow_B|\, f_1(t) + (h.c.) \\ &\quad + p_{\uparrow_A\uparrow_B}p_{\uparrow_A\downarrow_B}^*|\uparrow_A\uparrow_B\rangle\langle\uparrow_A\downarrow_B|\; g_2(t) + (h.c.) \\ &\quad + p_{\uparrow_A\uparrow_B}p_{\downarrow_A\uparrow_B}^*|\uparrow_A\uparrow_B\rangle\langle\downarrow_A\uparrow_B|\; g_1(t) + (h.c.) \\ &\quad + p_{\uparrow_A\downarrow_B}p_{\downarrow_A\downarrow_B}^*|\uparrow_A\downarrow_B\rangle\langle\downarrow_A\downarrow_B|\; g_1(t) + (h.c.) \\ &\quad + p_{\downarrow_A\uparrow_B}p_{\downarrow_A\downarrow_B}^*|\downarrow_A\uparrow_B\rangle\langle\downarrow_A\downarrow_B|\; g_2(t) + (h.c.)], \end{aligned} \quad (17)$$

where $P_{AB}(x_Ax_B) = |\psi_{AB}(x_Ax_B)|^2$ is the probability that two particles are located at position x_A and position x_B, respectively. The time-dependent factors

$$\begin{aligned} f_1(t) &= e^{i(\omega_A-\omega_B)t}\textstyle\prod_n \cos[\alpha_n^{[A]}(t) - \alpha_n^{[B]}(t)], \\ f_2(t) &= e^{-i(\omega_A+\omega_B)t}\textstyle\prod_n \cos[\alpha_n^{[A]}(t) + \alpha_n^{[B]}(t)], \\ g_1(t) &= e^{-i\omega_At}\textstyle\prod_n \cos\alpha_n^{[A]}(t), \\ g_2(t) &= e^{-i\omega_Bt}\textstyle\prod_n \cos\alpha_n^{[B]}(t). \end{aligned} \quad (18)$$

It can be seen that the one-dimensional array has no effect on the diagonal elements of the spin-state of the particles, and the influence of the off-diagonal elements on the spin-state of the particles is related to the time factor.

For the sake of simplicity, we consider that the interaction local potential as the δ potential satisfying the relationship $V(x) = V_0\Omega\delta(x)$. The tipping angles in equation (15) are rewritten as

$$\alpha_n^{[j]}(x_j, t) = \frac{V_0\Omega}{\hbar v_j}\Theta(x_j + v_jt' - x_n), \quad (19)$$

where the $\Theta(y)$ is the Heaviside unit step function, i.e., $\Theta(y) = 1$ for $y > 0$, and $\Theta(y) = 0$ for $y < 0$. In equation (19), we have assumed that the initial positions x_A and x_B of the two particles satisfy the relationship $x_A, x_B < x_1$. Here, we introduce the parameter

$$q_j = \sin^2\frac{V_0\Omega}{\hbar v_j}, \quad (20)$$

to represent the 'spin-flip' probability. Under the condition of the weak-coupling macroscopic limit, the 'spin-flip' probability can be expressed as

$$q_j \approx \left(\frac{V_0\Omega}{\hbar v_j}\right)^2. \quad (21)$$

In this paper, we assume that two particles propagate at the same velocity, i.e., $v = v_A = v_B$ and $q = q_A = q_B$. The length of the one-dimensional array is

$$L = x_n - x_1 = (N-1)\Delta, \tag{22}$$

where N is the number of spins of the one-dimensional array, and Δ represents the distance between two adjacent spins in the one-dimensional array. The time-dependent factors of equation (18) are approximately calculated as

$$\begin{aligned}
f_1(t) &= e^{i(\omega_A-\omega_B)t} e^{-\frac{\bar{n}}{2}\left[\frac{x_A+vt-x_1}{L}\Theta(x_n-x_A-vt)\Theta(x_A+vt-x_1)+\Theta(x_A+vt-x_n)\right]} \\
&\quad \times e^{\frac{\bar{n}}{2}\left[\frac{x_B+vt-x_1}{L}\Theta(x_n-x_B-vt)\Theta(x_B+vt-x_1)+\Theta(x_B+vt-x_n)\right]}, \\
f_2(t) &= e^{-i(\omega_A+\omega_B)t} \\
&\quad \times e^{-\frac{\bar{n}}{2}\left[\frac{x_A+vt-x_1}{L}\Theta(x_n-x_A-vt)\Theta(x_A+vt-x_1)+\Theta(x_A+vt-x_n)\right]} \\
&\quad \times e^{-\frac{3\bar{n}}{2}\left[\frac{x_B+vt-x_1}{L}\Theta(x_n-x_B-vt)\Theta(x_B+vt-x_1)+\Theta(x_B+vt-x_n)\right]}, \\
g_1(t) &= e_n^{-i\omega_A t} e^{-\frac{\bar{n}}{2}\left[\frac{x_A+vt-x_1}{L}\Theta(x_n-x_A-vt)\Theta(x_A+vt-x_1)+\Theta(x_A+vt-x_n)\right]}, \\
g_2(t) &= e^{-i\omega_B t} e^{-\frac{\bar{n}}{2}\left[\frac{x_B+vt-x_1}{L}\Theta(x_n-x_B-vt)\Theta(x_B+vt-x_1)+\Theta(x_B+vt-x_n)\right]},
\end{aligned} \tag{23}$$

where $\bar{n} = qN$ represents the average number of excitation and is required to be finite.

4. Fringe visibility of the two-particle and single-particle

To study the coherence of the particles, a pulse that detects the spin direction of the particles acts on the spin state of the particles. The joint probabilities of detecting the spin direction of particles as $(\Uparrow_a, \Uparrow_b)$, $(\Uparrow_a, \Downarrow_b)$, $(\Downarrow_a, \Uparrow_b)$, $(\Downarrow_a, \Downarrow_b)$ are

$$\begin{aligned}
P(\Uparrow_a, \Uparrow_b) &= \langle \Uparrow_a \Uparrow_b | \ \hat{\rho}_s^{out} | \Uparrow_a \Uparrow_b \rangle, \\
P(\Uparrow_a, \Downarrow_b) &= \langle \Uparrow_a \Downarrow_b | \ \hat{\rho}_s^{out} | \Uparrow_a \Downarrow_b \rangle, \\
P(\Downarrow_a, \Uparrow_b) &= \langle \Downarrow_a \Uparrow_b | \ \hat{\rho}_s^{out} | \Downarrow_a \Uparrow_b \rangle, \\
P(\Downarrow_a, \Downarrow_b) &= \langle \Downarrow_a \Downarrow_b | \ \hat{\rho}_s^{out} | \Downarrow_a \Downarrow_b \rangle,
\end{aligned} \tag{24}$$

where $| \Uparrow_k \rangle = (| \uparrow_j \rangle + | \downarrow_j \rangle)/\sqrt{2}$ and $| \Downarrow_k \rangle = (| \uparrow_j \rangle - | \downarrow_j \rangle)/\sqrt{2}$, and the subscript $k = a$ $(k = b)$ when $j = A$ $(j = B)$. The $P(\Uparrow_a, \Uparrow_b)$ indicates the probability that both the particle a and the particle b are detected in the direction of spin-up. The two-particle fringe visibility, which documents the coherence of the two-particle, is defined as [31]

$$V_{AB} = \frac{\max \bar{P}(\Uparrow_a, \Uparrow_b) - \min \bar{P}(\Uparrow_a, \Uparrow_b)}{\max \bar{P}(\Uparrow_a, \Uparrow_b) + \min \bar{P}(\Uparrow_a, \Uparrow_b)}, \tag{25}$$

with

$$\bar{P}(\Uparrow_a, \Uparrow_b) = P(\Uparrow_a, \Uparrow_b) - P(\Uparrow_a)P(\Uparrow_b) + \frac{1}{4}, \tag{26}$$

where the marginal probabilities $P(\Uparrow_a)$ and $P(\Uparrow_b)$ are denoted by

$$\begin{aligned}
P(\Uparrow_a) &= P(\Uparrow_a, \Uparrow_b) + P(\Uparrow_a, \Downarrow_b), \\
P(\Uparrow_b) &= P(\Uparrow_a, \Uparrow_b) + P(\Downarrow_a, \Uparrow_b).
\end{aligned} \tag{27}$$

The fringe visibility V_j of the single-particle can be expressed as

$$V_j = \frac{\max P(\Uparrow_k) - \min P(\Uparrow_k)}{\max P(\Uparrow_k) + \min P(\Uparrow_k)}. \tag{28}$$

4.1. Two particles located initially at different positions

In this section, we assume that the particles a and b are initially well located around the positions of $x_A = 0$ and $x_B < 0$. Two particles located initially at different positions can be divided into two situations: the distance between two particles is less than the length of the one-dimensional array and the distance between two particles is greater than the length of the one-dimensional array.

First, we consider that the initial state of the two particles spin is completely entangled

$$|\psi_s\rangle = \frac{|\uparrow_A \uparrow_B\rangle + |\downarrow_A \downarrow_B\rangle}{\sqrt{2}}. \tag{29}$$

At this point, equation (17) can be written as

$$\begin{aligned}
\hat{\rho}_s^{out} &= \frac{1}{2} |\uparrow_A \uparrow_B\rangle \langle \uparrow_A \uparrow_B| + \frac{1}{2} |\downarrow_A \downarrow_B\rangle \langle \downarrow_A \downarrow_B| \\
&\quad + \frac{1}{2} e^{i(\phi_A + \phi_B - (\omega_A + \omega_B)t)} \\
&\quad \times |\uparrow_A \uparrow_B\rangle \langle \downarrow_A \downarrow_B| \ e^{-\frac{\bar{n}}{2} f_A} e^{-\frac{3\bar{n}}{2} f_B} + (h.c.),
\end{aligned} \tag{30}$$

where

$$\begin{aligned}
f_A &= \frac{vt - x_1}{L} \Theta(x_N - vt)\Theta(vt - x_1) + \Theta(vt - x_N), \\
f_B &= \frac{x_B + vt - x_1}{L} \Theta(x_N - x_B - vt) \\
&\quad \times \Theta(x_B + vt - x_1) + \Theta(x_B + vt - x_N).
\end{aligned} \tag{31}$$

Via equation (24), the joint probabilities read

$$\begin{aligned}
P(\Uparrow_a, \Uparrow_b) = P(\Downarrow_a, \Downarrow_b) &= \frac{1}{4}[1 + \Gamma(\phi_A + \phi_B)], \\
P(\Uparrow_a, \Downarrow_b) = P(\Downarrow_a, \Uparrow_b) &= \frac{1}{4}[1 - \Gamma(\phi_A + \phi_B)],
\end{aligned} \tag{32}$$

where

$$\Gamma(\phi_A + \phi_B) = \cos(\phi_A + \phi_B - (\omega_A + \omega_B)t) e^{-\frac{\bar{n}}{2} f_A} e^{-\frac{3\bar{n}}{2} f_B}. \tag{33}$$

The marginal probabilities are obtained as

$$P(\Uparrow_a) = P(\Downarrow_a) = P(\Uparrow_b) = P(\Downarrow_b) = \frac{1}{2}. \tag{34}$$

One can easily obtain the two-particle fringe visibility

$$V_{AB_1} = e^{-\frac{\bar{n}}{2} f_A} e^{-\frac{3\bar{n}}{2} f_B}, \tag{35}$$

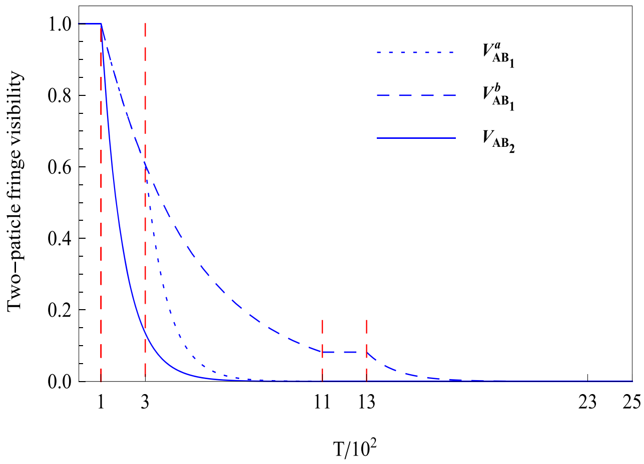

Figure 2. The two-particle fringe visibility as a function of T for a given $x_B = 0$ (solid line), $x_B = -1200$ (dash line), $x_B = -200$ (dotted line). Moreover, we have set $q = 0.005$, $x_1 = 100$, $\triangle = 1$, $L = 1000$, $v = 1$.

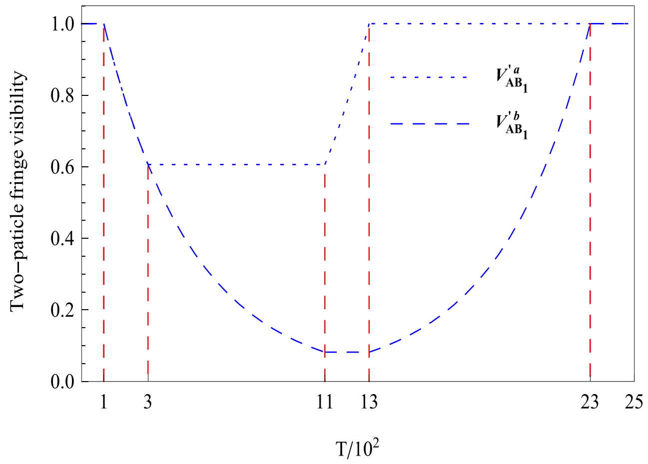

Figure 3. The two-particle fringe visibility as a function of T for a given $x_B = -1200$ (dash line), $x_B = -200$ (dotted line). The other parameters are the same as those in figure 2.

by adjusting the ϕ_j of equation (32). In figure 2, we plot the two-particle fringe visibility V_{AB_1} as a function of time. $V^a_{AB_1}$ and $V^b_{AB_1}$ represent the two-particle fringe visibility when the distance between two particles is less than the length of the one-dimensional array and when the distance between two particles is greater than the length of the one-dimensional array, respectively. It can be observed that the two-particle fringe visibility $V^a_{AB_1}$ remains the initial value before the two particles interact with the one-dimensional array. Since the particles exchange energy with the one-dimensional array, the two-particle fringe visibility $V^a_{AB_1}$ begins to decrease when the particle A enters the one-dimensional array. After the particle B enters the one-dimensional array, both the two particles interact with the one-dimensional array. We find that the two-particle fringe visibility $V^a_{AB_1}$ decreases faster than before and decay to zero asymptotically. From figure 2, we can obtain that the two-particle fringe visibility $V^b_{AB_1}$ remains the initial value when particle A does not enter the one-dimensional array. When the particle A propagates in the one-dimensional array, the two-particle fringe visibility $V^b_{AB_1}$ starts to decrease. Then, particle A leaves the one-dimensional array, and particle B does not enter the one-dimensional array. At this point, the value of the $V^b_{AB_1}$ does not change with time because the two particles do not interact with the one-dimensional array. The two-particle fringe visibility $V^b_{AB_1}$ will decrease faster than before and decay to zero asymptotically after particle B enters the one-dimensional array.

Then the fringe visibility of the single-particle is calculated as

$$V_{A_1} = V_{B_1} = 0. \tag{36}$$

Equation (36) shows that when the initial state of the two particles spin is completely entangled, the coherent term of the final state of the single-particle spin is 0.

Next, we consider that the two-particle spin is initially in the singlet state

$$|\psi_s\rangle = \frac{|\uparrow_A\downarrow_B\rangle - |\downarrow_A\uparrow_B\rangle}{\sqrt{2}}. \tag{37}$$

The final state of the particle spin reads

$$\begin{aligned}\hat{\rho}_s^{out} &= \frac{1}{2}|\uparrow_A\downarrow_B\rangle\langle\uparrow_A\downarrow_B| + \frac{1}{2}|\downarrow_A\uparrow_B\rangle\langle\downarrow_A\uparrow_B| \\ &- \frac{1}{2}e^{-i(\phi_A-\phi_B-(\omega_A-\omega_B)t)} \\ &\times |\downarrow_A\uparrow_B\rangle\langle\uparrow_A\downarrow_B|\, e^{-\frac{\pi}{2}f_A} e^{\frac{\pi}{2}f_B} - (h.c.).\end{aligned} \tag{38}$$

The joint probabilities

$$\begin{aligned}P(\uparrow_a, \uparrow_b) = P(\downarrow_a, \downarrow_b) &= \frac{1}{4}[1 - \gamma(\phi_A - \phi_B)], \\ P(\uparrow_a, \downarrow_b) = P(\downarrow_a, \uparrow_b) &= \frac{1}{4}[1 + \gamma(\phi_A - \phi_B)],\end{aligned} \tag{39}$$

are derived from equation (24), where

$$\gamma(\phi_A - \phi_B) = \cos(\phi_A - \phi_B - (\omega_A - \omega_B)t)e^{-\frac{\pi}{2}f_A}e^{\frac{\pi}{2}f_B}. \tag{40}$$

Using equation (27), we can obtain the marginal probabilities

$$P(\uparrow_a) = P(\downarrow_a) = P(\uparrow_b) = P(\downarrow_b) = \frac{1}{2}. \tag{41}$$

The two-particle fringe visibility is calculated to be

$$V'_{AB_1} = e^{-\frac{\pi}{2}f_A}e^{\frac{\pi}{2}f_B}. \tag{42}$$

The two-particle fringe visibility V'_{AB_1} as a function of time is shown in figure 3. $V'^a_{AB_1}$ and $V'^b_{AB_1}$ represent the two-particle fringe visibility when the distance between two particles is less than the length of the one-dimensional array and when the distance between two particles is greater than the length of the one-dimensional array, respectively. The change of the two-particle fringe visibility $V'^a_{AB_1}$ and $V'^b_{AB_1}$ may be divided into five periods. The interaction between the particle A and the 1st spin of the one-dimensional array indicates the end of the first period. In this period, neither $V'^{a}_{AB_1}$ nor $V'^{b}_{AB_1}$ has changed because the particles do not interact with the one-dimensional array. When particle A enters the one-dimensional array, the second period begins. At this point, both $V'^{a}_{AB_1}$ and $V'^{b}_{AB_1}$ begin to decrease. In the third period: (1) the two-particle fringe visibility $V'^{a}_{AB_1}$ is a constant. At this point, both particle A and particle B interact with the one-dimensional array. (2) The two-particle fringe visibility $V'^{b}_{AB_2}$ remains unchanged. At this point, particle A leaves the one-dimensional array, and particle B does not enter the one-dimensional array. When the one-dimensional array only interacts with the particle B, the fourth period begins. In the process, both $V'^{a}_{AB_1}$ and $V'^{b}_{AB_1}$ increases as T increases. In the fifth period, particle B leaves the one-dimensional array. Both $V'^{a}_{AB_1}$ and $V'^{b}_{AB_1}$ reach their initial value. Such a process can be regarded as particle A encoding the which-path information in the one-dimensional array and particle B erasing the which-path information encoded in the one-dimensional array. From figure 3, we also find that the change of $V'^{b}_{AB_1}$ is larger than $V'^{a}_{AB_1}$.

The fringe visibility of the single-particle given by equation (28) becomes

$$V'_{A_1} = V'_{B_1} = 0. \tag{43}$$

Equation (43) indicates that when the two particles spin are initially in the singlet state, the coherent term of the final state of the single-particle spin is 0.

Finally, we consider that the initial state of the two-particle spin is independent

$$|\psi_s\rangle = \frac{|\Uparrow_A\rangle + |\Downarrow_A\rangle}{\sqrt{2}} \otimes \frac{|\Uparrow_B\rangle + |\Downarrow_B\rangle}{\sqrt{2}}. \tag{44}$$

The final state of the particles spin can be written as

$$\begin{aligned}
\hat{\rho}_s^{out} = &\frac{1}{4}|\Uparrow_A\Uparrow_B\rangle\langle\Uparrow_A\Uparrow_B| + e^{i(\phi_A+\phi_B-(\omega_A+\omega_B)t)}\\
&\times \frac{1}{4}|\Uparrow_A\Uparrow_B\rangle\langle\Downarrow_A\Downarrow_B|\; e^{-\frac{\pi}{2}f_A}e^{-\frac{3\pi}{2}f_B} + (h.c.)\\
&+ \frac{1}{4}|\Downarrow_A\Downarrow_B\rangle\langle\Downarrow_A\Downarrow_B| + e^{-i(\phi_A-\phi_B-(\omega_A-\omega_B)t)}\\
&\times \frac{1}{4}|\Downarrow_A\Uparrow_B\rangle\langle\Uparrow_A\Downarrow_B|\; e^{-\frac{\pi}{2}f_A}e^{\frac{\pi}{2}f_B} + (h.c.)\\
&+ e^{i(\phi_A-\omega_A t)}\frac{1}{4}|\Uparrow_A\Downarrow_B\rangle\langle\Downarrow_A\Downarrow_B|\; e^{-\frac{\pi}{2}f_A} + (h.c.)\\
&+ e^{i(\phi_B-\omega_B t)}\frac{1}{4}|\Downarrow_A\Uparrow_B\rangle\langle\Downarrow_A\Downarrow_B|\; e^{-\frac{\pi}{2}f_B} + (h.c.)\\
&+ \frac{1}{4}|\Uparrow_A\Downarrow_B\rangle\langle\Uparrow_A\Downarrow_B| + e^{i(\phi_A-\omega_A t)}\\
&\times \frac{1}{4}|\Uparrow_A\Uparrow_B\rangle\langle\Downarrow_A\Uparrow_B|\; e^{-\frac{\pi}{2}f_A} + (h.c.)\\
&+ \frac{1}{4}|\Downarrow_A\Uparrow_B\rangle\langle\Downarrow_A\Uparrow_B| + e^{i(\phi_B-\omega_B t)}\\
&\times \frac{1}{4}|\Uparrow_A\Uparrow_B\rangle\langle\Uparrow_A\Downarrow_B|\; e^{-\frac{\pi}{2}f_B} + (h.c.).
\end{aligned} \tag{45}$$

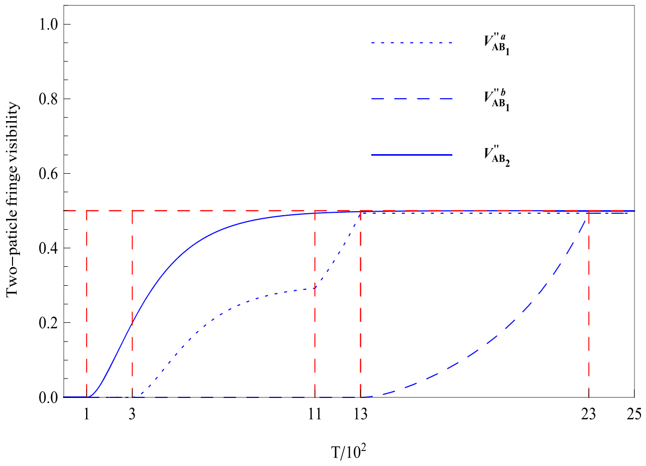

Figure 4. The two-particle fringe visibility as a function of T for a given $x_B = 0$ (solid line), $x_B = -1200$ (dash line), $x_B = -200$ (dotted line). The other parameters are the same as those in figure 2.

The joint probabilities become

$$\begin{aligned}
P(\Uparrow_a, \Uparrow_b) &= \frac{1}{4}[1 + \Upsilon(\phi_A) + \Upsilon(\phi_B) + \Upsilon(\phi_A, \phi_B)],\\
P(\Uparrow_a, \Downarrow_b) &= \frac{1}{4}[1 + \Upsilon(\phi_A) - \Upsilon(\phi_B) - \Upsilon(\phi_A, \phi_B)],\\
P(\Downarrow_a, \Uparrow_b) &= \frac{1}{4}[1 - \Upsilon(\phi_A) + \Upsilon(\phi_B) - \Upsilon(\phi_A, \phi_B)],\\
P(\Downarrow_a, \Downarrow_b) &= \frac{1}{4}[1 - \Upsilon(\phi_A) - \Upsilon(\phi_B) + \Upsilon(\phi_A, \phi_B)]
\end{aligned} \tag{46}$$

with

$$\begin{aligned}
\Upsilon(\phi_A) &= \cos(\phi_A - \omega_A t)e^{-\frac{\pi}{2}f_A},\\
\Upsilon(\phi_B) &= \cos(\phi_B - \omega_B t)e^{-\frac{\pi}{2}f_B},\\
\Upsilon(\phi_A, \phi_B) &= \frac{1}{2}\cos(\phi_A + \phi_B - (\omega_A + \omega_B)t)e^{-\frac{\pi}{2}f_A}e^{-\frac{3\pi}{2}f_B}\\
&+ \frac{1}{2}\cos(\phi_A - \phi_B - (\omega_A - \omega_B)t)e^{-\frac{\pi}{2}f_A}e^{\frac{\pi}{2}f_B}.
\end{aligned} \tag{47}$$

The marginal probabilities can be obtained

$$\begin{aligned}
P(\Uparrow_A) &= \frac{1}{2}[1 + \Upsilon(\phi_A)],\; P(\Downarrow_A) = \frac{1}{2}[1 - \Upsilon(\phi_A)],\\
P(\Uparrow_B) &= \frac{1}{2}[1 + \Upsilon(\phi_B)],\; P(\Downarrow_B) = \frac{1}{2}[1 - \Upsilon(\phi_B)].
\end{aligned} \tag{48}$$

The two-particle fringe visibility is given by

$$V''_{AB_1} = -e^{-\frac{\pi}{2}f_A}e^{-\frac{\pi}{2}f_B} + \frac{e^{-\frac{\pi}{2}f_A}e^{-\frac{3\pi}{2}f_B}}{2} + \frac{e^{-\frac{\pi}{2}f_A}e^{\frac{\pi}{2}f_B}}{2}. \tag{49}$$

In figure 4, we plot the two-particle fringe visibility V''_{AB_1} as a function of time. $V''^{a}_{AB_1}$ and $V''^{b}_{AB_1}$ represent the two-particle fringe visibility when the distance between two particles is less than the length of the one-dimensional array and when the distance between two particles is greater than the length of the one-dimensional array, respectively. From figure 4, we find that the values of the $V''^{a}_{AB_1}$ and $V''^{b}_{AB_1}$ increases when the

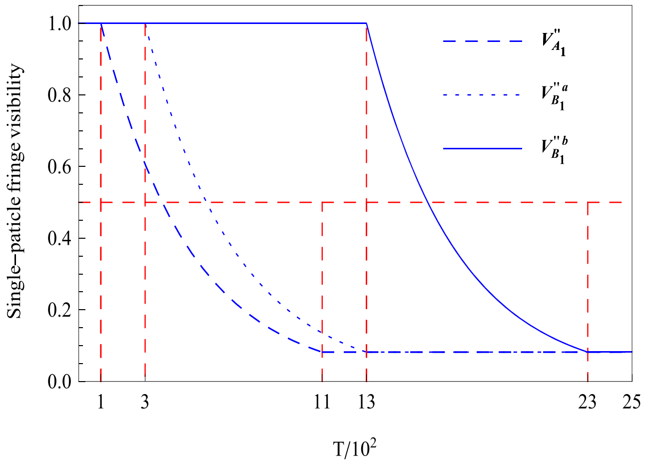

Figure 5. The fringe visibility of particle A as a function of T (dash line). The fringe visibility of particle B as a function of T for a given $x_B = -1200$ (solid line), $x_B = 0$ (dash line), $x_B = -200$ (dotted line). The other parameters are the same as those in figure 2.

particle B interact with the one-dimensional array. One also finds that the two-particle fringe visibility $V''^{a}_{AB_1}$ increases faster when the one-dimensional array interacts only with particle B. Finally, both the $V''^{a}_{AB_1}$ and $V''^{b}_{AB_1}$ will increase to 0.5 asymptotically as T increases. The asymptotic value of 0.5 is determined by the coefficient $1/\sqrt{2}$ of the singlet state $(|\uparrow_A\downarrow_B\rangle - |\downarrow_A\uparrow_B\rangle)/\sqrt{2}$.

The fringe visibility of the single-particle via the equation (28) reads

$$V''_{A_1} = e^{-\frac{\pi}{2}f_A},\ V''_{B_1} = e^{-\frac{\pi}{2}f_B}. \tag{50}$$

In figure 5, we plot the single-particle fringe visibility as a function of time. The fringe visibility of particle A is denoted by $V''^{a}_{A_1}$. $V''^{a}_{B_1}$ and $V''^{b}_{B_1}$ represent the fringe visibility of the particle B of the distance between two particles is less than the length of the one-dimensional array and the distance between two particles is greater than the length of the one-dimensional array, respectively. It is shown that the single-particle fringe visibility decreases when the particle interacts with the one-dimensional array in figure 5.

4.2. Two particles located initially at the same position

For the sake of simplicity, we assume that the two particles are initially located at the origin, i.e., $x_A = x_B = 0$, this moment $f_A = f_B = \frac{vt - x_1}{L}\Theta(x_N - vt)\Theta(vt - x_1) + \Theta(vt - x_N)$.

(1) For the completely entangled state $(|\uparrow_A\uparrow_B\rangle + |\downarrow_A\downarrow_B\rangle)/\sqrt{2}$, the two-particle fringe visibility of equation (35) becomes

$$V_{AB_2} = e^{-2\bar{\pi}f_A}, \tag{51}$$

when the two particles are initially located in the same position. We plot the two-particle fringe visibility V_{AB_2} as a function of time in figure 2. The two-particle fringe visibility V_{AB_2} changes only when the particles interact with the one-dimensional array. After the particles enter the one-dimensional array, two-particle fringe visibility V_{AB_2} decreases monotonically as T increases and decays to zero asymptotically. Form figure 2, we also found that when the two particles interact with the one-dimensional array at the same time, the decay of the two-particle fringe visibility is accelerated. In this case, the fringe visibility of the single-particle is also equal to 0.

(2) Now, we consider that the particles are initially in the singlet state $(|\uparrow_A\downarrow_B\rangle - |\downarrow_A\uparrow_B\rangle)/\sqrt{2}$. At this time, the final state of the particle spin is only related to the time-dependent factors $f_1(t)$, and satisfies $|f_1(t)| = 1$. Since there is no energy exchange between the one-dimensional array and the particles, the coherence of the two-particle does not change. In this case, the two-particle fringe visibility is equal to its initial value of 1. Here, the single-particle fringe visibility is equal to 0.

(3) When the particles are initially in the state $\frac{|\uparrow_A\rangle + |\downarrow_A\rangle}{\sqrt{2}} \otimes \frac{|\uparrow_B\rangle + |\downarrow_B\rangle}{\sqrt{2}}$. The two-particle fringe visibility can be expressed as

$$V''_{AB_2} = -e^{-\bar{\pi}f_A} + \frac{e^{-2\bar{\pi}f_A}}{2} + \frac{1}{2}. \tag{52}$$

We plot the two-particle fringe visibility V''_{AB_2} as a function of time in figure 4. It can be observed in figure 4 that when the particles interact with the one-dimensional array, the two-particle fringe visibility V''_{AB_2} increases monotonically as T increases and increases to 0.5 asymptotically. We also calculate the single-particle fringe visibility

$$V'' = V''_{A_2} = V''_{B_2} = e^{-\frac{\pi}{2}f_A}. \tag{53}$$

We find that the single-particle fringe visibility V'' satisfies the relationship $V'' = V''_{A_1}$.

5. Conclusion

We have investigated both the two-particle and single-particle fringe visibility of different initial states of the particle spin. It is found that different initial state of the particle spin can lead to the difference of both the two-particle and single-particle fringe visibility with time. (1) For the completely entangled state $(|\uparrow_A\uparrow_B\rangle + |\downarrow_A\downarrow_B\rangle)/\sqrt{2}$, the two-particle fringe visibility decreases as time increases and decays to zero asymptotically. The fringe visibility of the single-particle is equal to 0. (2) If the particles spin are initially in the singlet state $(|\uparrow_A\downarrow_B\rangle - |\downarrow_A\uparrow_B\rangle)/\sqrt{2}$. The inter-distance of the two particles is absent or does not result in different two-particle fringe visibility. When the internal distance of the two particles disappears, the value of the two-particle fringe visibility is 1. For a nonzero internal distance, the two-particle fringe visibility not only undergoes a constant process, but also experiences first decrease and then increase during the time evolution. Here, the fringe visibility of the single-particle is also equal to 0. (3) When the particles are initially in the state $\frac{|\uparrow_A\rangle + |\downarrow_A\rangle}{\sqrt{2}} \otimes \frac{|\uparrow_B\rangle + |\downarrow_B\rangle}{\sqrt{2}}$, the two-particle fringe visibility increases as time increases and increases to 0.5 asymptotically. In this case, the single-particle fringe visibility decreases, due to the interaction between the particle and the one-dimensional array.

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