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Present value of the Universe's acceleration

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Abstract – The present value of the Universe's acceleration is deduced using the uncertainty principle of Heisenberg and the vacuum fluctuations that cause the appearance of a particle-antiparticle pair close to cosmic or Hubble horizon. Moreover using the generalized uncertainty principle the first correction to the present value of the Universe's acceleration is provided.

Introduction. – In this letter we present a simple deduction of the present value of the Universe's acceleration. The derivation is obtained using the vacuum fluctuations that produce an Unruh effect in the cosmic horizon and the Heisenberg uncertainty principle. In fact these vacuum fluctuations produce observable macroscopic consequences, see for instance [1,2].

First we recall that the slow expansion of the Universe is in any direction from the observer. In such a way the observer sees all the objects go far away from his/her position. In this image someone can think that the observer is at the center of the expansion but this vision is valid for any observer at any point of the Universe. In order to find the exact value of the Universe's acceleration we set our sights in a very distant object that moves away from us with the acceleration that we try to determine. For not far away objects the local movements can perturb the Universe's acceleration that we see for all the receding objects far enough. Of course these far away objects are very close to the cosmic horizon or Hubble horizon, the horizon defining the boundary between particles that are moving slower and faster than the speed of light with respect to the observer. It is clear that all the objects outside this horizon have not causal connection with the observer.

Once the object is fixed, this object determines a direction in the space and now we apply the relativity principle of the movement and we can think that the object is at rest and the observer is with an acceleration in the opposite direction. Thinking in this way the observer is not in an inertial frame. As it really is since the observer is submerged in an expanding universe. On the contrary the observer is in a uniformly accelerated frame in the flat space-time and by a detector placed in this frame the observer can measure what is named an Unruh temperature by the Unruh effect. In fact any observer in our Universe in accelerated expansion is affected by this acceleration. We can recall that the Unruh effect is due to the quantum fluctuations near the Rindler event horizon. In this work we consider that this Rindler event horizon is in fact the cosmic or Hubble horizon and from here we deduce the present value of the Universe's acceleration using the uncertainty principle. The deduction does not depend on the fixed object neither on the direction in the space.

The Unruh effect revisited. – The Fulling-Davies-Unruh effect [3–5], predicts that an accelerating observer will observe blackbody radiation while an inertial observer would observe none. A detector subjected to a uniform acceleration in a flat spacetime has a response as if the detector were put in a thermal bath with a temperature given by

$$T = \frac{\hbar a}{2\pi c k_B}, \tag{1}$$

where a is the acceleration of the detector, see also [6]. In fact if the observer is accelerated to its right a Rindler event horizon appear to its left.

The explanation of the Unruh radiation is due to the vacuum fluctuations that cause the appearance of a particle-antiparticle pair close to the Rindler event horizon. One particle crosses the Rindler event horizon while the other is perceived by the observer. The Rindler event horizon is at distance c^2/a from the observer in any direction to its left (see fig. 1). In [7] the exact expressions of the Hawking effect [8] and Unruh effect [5] are derived

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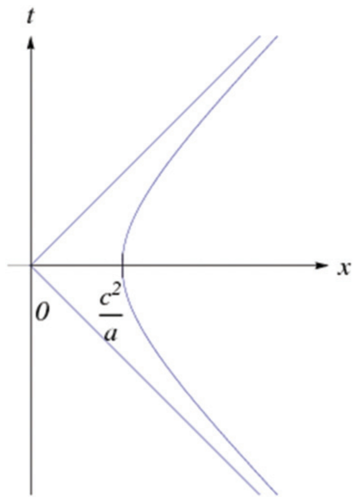

Fig. 1: The Rindler chart plotted on a Minkowski diagram. The bisectors are the Rindler horizons at a distance c^2/a away and the worldline of a body in hyperbolic motion having constant proper acceleration a.

from the uncertainty principle. In previous works some other heuristic derivations were presented, however only approximations of the exact values were obtained, see for instance [9,10]. Moreover in [11] the expression for vacuum energy was deduced through the uncertainty of the energy of the universe.

We first recall here the deduction of the Unruh temperature from the principle of uncertainty applied to the photon that has crossed the Rindler event horizon. We recall that the Unruh temperature associated to the Rindler event horizon is always associated to an observer which is in a non-inertial frame with certain acceleration a. Following [7] the uncertainty in the position Δx of the particle that crosses the Rindler event horizon is given by $\Delta x = \pi r_R$, where r_R is the Rindler radius of the Rindler event horizon that coincides with the distance from the horizon to the observer $r_R = c^2/a$. Therefore we obtain

$$\Delta x = \frac{\pi c^2}{a}. \tag{2}$$

Now we consider the momentum-position form of the Heisenberg principle

$$\Delta p \Delta x \simeq \frac{\hbar}{2}. \tag{3}$$

From (3) and (2) we have

$$\Delta p \simeq \frac{\hbar a}{2\pi c^2}. \tag{4}$$

Since the energy of the photon is given by $E = pc$, we have that the uncertainty in the energy is $\Delta E = c\Delta p$ and

$$\Delta E \simeq \frac{\hbar a}{2\pi c}. \tag{5}$$

Interpreting the energy fluctuation in terms of a classical thermal bath we have $\Delta E = k_B \Delta T$, where k_B is the Boltzmann constant and eq. (5) becomes

$$\Delta T \simeq \frac{\hbar a}{2\pi c k_B}, \tag{6}$$

which coincides with eq. (1) and is the expression given by Unruh in [5]. If there is not acceleration we have not a Rindler event horizon and eq. (6) is not valid.

The Unruh effect at the cosmological horizon. – In this section and following the same procedure we compute the Unruh temperature at the cosmological horizon. Hence we consider the situation described in the introduction. In this case the uncertainty in the position Δx is given by $\Delta x = \pi r_{HS}$, where r_{HS} is the Hubble radius of the Hubble sphere or cosmic horizon. For distances close to the radius r_{HS} of the Hubble sphere, objects recede close to the speed of light and according to the Hubble law, we have $r_{HS} = c/H_0$, where H_0 is the present value of the Hubble constant today. We recall the Hubble constant is a constant only in space, but not in time. The radius r_{HS} of the Hubble sphere may increase or decrease over various time intervals. Consequently from (3) we have

$$\Delta p \simeq \frac{\hbar}{2\pi r_{HS}}, \tag{7}$$

and the uncertainty in the energy is

$$\Delta E \simeq \frac{c\hbar}{2\pi r_{HS}}. \tag{8}$$

Now we consider the slow accelerate expansion of the Universe which implies that the cosmic horizon is in fact a Rindler event horizon (following the relativity principle mentioned in the introduction in any direction) and this Rindler event horizon has the Unruh associated temperature,

$$\Delta T \simeq \frac{c\hbar}{2\pi r_{HS} k_B} = \frac{\hbar H_0}{2\pi k_B}. \tag{9}$$

Now in order to deduce the present value of the Universe's acceleration we can compare eq. (6) with eq. (9) and we obtain the value of the acceleration

$$a_H = cH_0 \sim 10^{-9}\,\mathrm{m/s^2}, \tag{10}$$

in agreement with the observations, see [12] and references therein. In fact the same result can be obtained by requiring that the radius of the Hubble horizon r_{HS} will be equal to the distance from the observer to the Rindler horizon r_R, which implies $c/H_0 = c^2/a$ and we get eq. (10).

Generalized uncertainty principle. – In the last decades several works have studied the possibility of a generalization of the Heisenberg uncertainty principle in order to take into account the gravitation. In ordinary quantum level gravity can be neglected if we compare it with the other fundamental forces. However at large scales like the cosmic scales the gravity has a fundamental role.

In consequence the gravity must also affect the formulation of the Heisenberg's principle and several proposals have been made, see for instance [13].

In this section we consider a generalized uncertainty principle (GUP) that corresponds to a deformation of the fundamental commutator obtained by adding a quadratic term in the momentum, that is

$$\Delta p \Delta x \geq \frac{\hbar}{2}\left[1+\beta\left(\frac{\Delta p}{m_p c}\right)^2\right], \tag{11}$$

with β a dimensionless parameters and where m_p is the Planck mass given by $m_p = \varepsilon_p/c^2 \approx 10^{-8}\,\mathrm{kg}$, where the Planck energy ε_p satisfies $\varepsilon_p \ell_p = \hbar c/2$ and the Planck length $\ell_p = \sqrt{G\hbar/c^3}$, see for instance [13]. This inequality for symmetric states with $\langle \hat{p} \rangle = 0$ is equivalent to the commutator

$$[\hat{x}, \hat{p}] = i\hbar\left[1+\beta\left(\frac{\hat{p}}{m_p c}\right)^2\right]. \tag{12}$$

From the uncertainty relation (11) we can arrive at the first correction in the parameter β of the present value of the Universe's acceleration. In order to do that we divide expression (11) by Δp and we get

$$\Delta x \sim \frac{\hbar}{2}\left[\frac{1}{\Delta p}+\beta\frac{\Delta p}{m_p^2 c^2}\right]. \tag{13}$$

We now take into account that the Hubble horizon is in fact a Rindler horizon with respect to an observer, consequently we have

$$\frac{\pi c^2}{a} \sim \frac{\hbar}{2}\left[\frac{c}{\Delta E}+\beta\frac{\Delta E}{m_p^2 c^3}\right], \tag{14}$$

where we have substitute $\Delta p = \Delta E/c$. Next we take into account that the Unruh energy is thermalised so we have $\Delta E = k_B \Delta T$ and substituting into (14) we obtain

$$\frac{\pi c^2}{a} \sim \frac{\hbar}{2}\left[\frac{c}{k_B \Delta T}+\beta\frac{k_B \Delta T}{m_p^2 c^3}\right]. \tag{15}$$

Isolating the acceleration and substituting ΔT by its first approximation $\Delta T = \hbar H_0/(2\pi k_B)$ we have

$$a \sim \frac{\pi c^2}{\frac{\pi c}{H_0}+\beta\frac{\hbar^2 H_0}{4\pi m_p^2 c^3}}. \tag{16}$$

Next we divide by πc^2 and multiply by cH_0 and we get

$$a \sim \frac{cH_0}{1+\beta\frac{\hbar^2 H_0^2}{4\pi^2 m_p^2 c^4}}. \tag{17}$$

Finally the term $\hbar^2 H_0^2/(4\pi^2 m_p^2 c^4)$ can be written as $k_B^2 \Delta T^2/(m_p^2 c^4) = \Delta E^2/(m_p^2 c^4) = m^2/m_p^2$ and it is reasonable to assume that it is very small for any fundamental particle with $m \ll m_p$. Therefore we can expand in this term (17) and we obtain

$$a \sim cH_0\left(1-\beta\frac{\ell_p^2 H_0^2}{\pi^2 c^2}\right) = cH_0\left(1-\beta\frac{G\hbar H_0^2}{\pi^2 c^5}\right), \tag{18}$$

and this is the first correction to the present value of the Universe's acceleration that can be checked by the new astronomical data. We recall that the Planck length ℓ_P is the length around which the quantum fluctuations become essential, see [14–16]. A heuristic derivation of the Planck length is also given in [9].

Conclusion. – The derivations of the present value of the Universe's acceleration is obtained based on the vacuum fluctuations and the Heisenberg uncertainty principle. Although we have made the approximation assuming a half spherical Rindler horizon, the expressions derived is in agreement with the astronomical data. In the last section we find, using the generalized uncertainty principle, taking into account gravity, the first correction in the parameter β to the present value of the Universe's acceleration.

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