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Non-fragile $\mathscr{L}_2 - \mathscr{L}_\infty$ synchronization for chaotic time-delay neural networks with semi-Markovian jump parameters

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Abstract

The issue of the non-fragile $\mathscr{L}_2 - \mathscr{L}_\infty$ synchronization for chaotic time-delay neural networks subject to semi-Markovian jump parameters is addressed in this paper. Unlike the Markovian jump process, the sojourn time of the semi-Markovian jump process allows to be non-exponential distributed and the transition rate allows to be time-varying. By utilizing the discretized Lyapunov–Krasovskii functional method and introducing two free-weighting matrices, a sufficient condition is proposed to ensure the synchronization error system to be stochastically stable with an $\mathscr{L}_2 - \mathscr{L}_\infty$ performance index. Then, by means of a matrix congruence transformation and some inequality techniques, an approach to the non-fragile $\mathscr{L}_2 - \mathscr{L}_\infty$ controller design is developed. Finally, two illustrative examples are employed to show the usefulness of the proposed results.

Keywords: chaos, time delay, semi-Markovian process, non-fragile control, synchronization, neural networks

(Some figures may appear in colour only in the online journal)

1. Introduction

Over the past few decades, neural networks (NNs) have attracted considerable research interest due to their applications in various fields including target tracking [1], trajectory prediction [2], secure communication [3], and signal classification [4]. These applications are closely linked to the dynamic behavior of NNs. For example, it has been recognized that a NN is suitable for secure communication via the well-known master-slave synchronization configuration provided it possesses a chaotic behavior. Time delays are often unavoidable in both biological and artificial NNs and shown to be able to make a chaotic NN yield more complex strange chaotic attractors. Therefore, a great deal of efforts have been dedicated to the synchronization issue of chaotic time-delay NNs. In [5], by designing an adaptive controller, the global asymptotic synchronization of two classes of different chaotic time-delay NNs was guaranteed. Time-delay NNs with mismatched parameters were researched in [6], where an integral sliding mode control method was developed for ensuring the global asymptotic synchronization. When the

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burden of network bandwidth is considered, it was shown in [7] that the exponential synchronization of chaotic time-delay NNs can be reached by using a hybrid event trigger approach. For recent developments on the subject of synchronization for chaotic time-delay NNs, one can refer to [8–10].

In engineering practice, the NNs may show the behavior of finite-state expressions, which is called as the information latching [11]. In this circumstance, the network states are able to jump between different modes in light of a Markov chain and the NN can be modeled as a switched system with Markovian jump parameters. During the past twenty years, many researchers are committed to Markovian jump NNs and a large number of achievements have been made [12–18]. In most literature, however, the sojourn time between the jumping subsystems is required to obey the exponential distribution, which is quite restrictive. Some attention has therefore been paid to the semi-Markovian jump process, in which the sojourn time allows to be non-exponential distributed and the transition rate allows to be time-varying and, thus, it is more general than the usual Markovian jump process. Very recently, the synchronization problem of time-delay NNs with semi-Markovian jump parameters has been investigated in [19–22], where the Lyapunov theory and several integral inequality techniques were used to develop sufficient conditions for the stochastic synchronization, passivity-based synchronization, and the event-triggered synchronization, respectively.

Although substantial progress has been achieved in the study of synchronization for time-delay NNs with semi-Markovian jump parameters, there are some basic issues need to be addressed further. An important observation is that [19–22] did not take into account the external interference, which, in reality, is generally inevitable in the environment of neurons and might exert a great influence on the synchronization dynamics of the NNs. This leads to the need to introduce the $\mathscr{L}_2-\mathscr{L}_\infty$ control approach, which aims at ensuring that the peak value of the controlled output to be within a certain range, and its effectiveness has been well established in the control community for various disturbance-affected systems; see, e.g. [23–25]. On the other hand, it is observed that the existing controller designs were not concerned with the possibility of gain perturbations. In the engineering implementation of a control system, however, the controller usually has parameter inaccuracy to some degree as a result of the digital rounding failures, the demand for extra tuning of parameters, the imprecision of analog-digital conversions, etc [26]. As pointed out in [27], even a relatively slight controller gain perturbation can severely reduce the control effect. In view of the above findings, a question arises: how to determine non-fragile $\mathscr{L}_2-\mathscr{L}_\infty$ controllers to ensure the synchronization for chaotic time-delay NNs with semi-Markovian jump parameters?

In this paper, we try to solve such an issue within the master-slave synchronization framework. The slave system is assumed to be affected by external disturbances and the controller gain is allowed to be subject to norm-bounded perturbations. The main contributions are as follows: (1) The issue of non-fragile $\mathscr{L}_2-\mathscr{L}_\infty$ synchronization for chaotic time-delay NNs with semi-Markovian jump parameters is investigated for the first time; (2) By utilizing the discretized Lyapunov–Krasovskii functional (LKF) method and introducing suitable free-weighting matrices, a sufficient condition is proposed for ensuring the synchronization error system to be stochastically stable with an $\mathscr{L}_2-\mathscr{L}_\infty$ performance index γ; (3) By means of a matrix congruence transformation as well as some inequality techniques, an approach is developed for the non-fragile design. It is shown that the required controller gain can be obtained via the feasible solution of a number of linear matrix inequities (LMIs), which are computationally tractable. The rest of this paper is organized as follows: in section 2, we give the models of master and slave chaotic time-delay NNs and prepare necessary preliminaries. In section 3, we propose our analysis and synthesis results for the non-fragile $\mathscr{L}_2-\mathscr{L}_\infty$ synchronization. In section 4, we offer two numerical examples to show the usefulness of the results obtained. In the last section, we summarize our conclusions.

Notation: In the paper, the notations used is listed in table 1.

Table 1. List of notations.

Notations	Representations
X^T	The transpose of matrix X
$\mathrm{sym}\{X\}$	$X+X^T$
I (respectively, 0)	A unity (respectively, zero) matrix with appropriate dimension
$\mathrm{diag}\{\cdots\}$	A diagonal matrix
$*$	A symmetric block
$\lambda_{\min}(X)$	The smallest eigenvalue of matrix X
$\mathcal{E}\{\cdot\}$	The mathematical expectation
$X>0$ (respectively, $X\geqslant 0$)	Matrix X is symmetric positive definite (respectively, semi-definite)
$\mathcal{R}^{n_1}$	The n_1-dimensional Euclidean space

2. Preliminaries

Consider the non-fragile $\mathscr{L}_2-\mathscr{L}_\infty$ synchronization of chaotic time-delay NNs with semi-Markovian jump parameters, where the master system is given by the following chaotic NN model:

$$\begin{cases}\dot{x}(t)=-\mathcal{A}(\varsigma(t))x(t)+\mathcal{B}_1(\varsigma(t))\hat{g}(x(t))+\mathcal{B}_2(\varsigma(t))\hat{g}(x(t-\varrho))\\ \qquad +I(t),\\ \hat{z}(t)=L(\varsigma(t))x(t),\end{cases} \tag{1}$$

where $x(t)=[x_1^T(t),\cdots,x_i^T(t),\cdots,x_{n_1}^T(t)]^T\in\mathcal{R}^{n_1}$ with $x_i(t)$ being the state variable of the ith neuron at time t; $\mathcal{A}(\varsigma(t))=\mathrm{diag}\{a_1(\varsigma(t)),\cdots,a_i(\varsigma(t)),\cdots,a_{n_1}(\varsigma(t))\}$ with $a_i(\varsigma(t))$ being the rate at which the ith neuron resets the potential to the rest state in isolation when isolated from the system and external input; $\mathcal{B}_1(\varsigma(t))$ and $\mathcal{B}_2(\varsigma(t))$ are connection weight matrices of the NN; $\hat{g}(x(t))=[\hat{g}_1^T(x_1(t)),\cdots,\hat{g}_{n_1}^T(x_{n_1}(t))]^T\in\mathcal{R}^{n_1}$ stands for activation function of neurons; $I(t)=[I_1^T(t),\cdots,I_{n_1}^T(t)]^T\in\mathcal{R}^{n_1}$ and $\hat{z}(t)\in\mathcal{R}^{n_1}$ represent the external input and the controlled output, respectively; $L(\varsigma(t))\in\mathcal{R}^{n_1\times n_1}$ is a real constant matrix; ϱ stands for the time delay. Note that the time delay considered herein is supposed to be time-invariant for brevity. For systems with time-varying delay, one might refer to [28–31]. In (1), we use $(\varsigma(t), t \geqslant 0)$ to represent a continuous-time and discrete-state semi-Markovian process, which takes values in a fixed set $\Gamma = (1, 2, \ldots, \mathcal{H})$. The transition probability matrix $\Psi(\kappa) = \{\pi_{mn}(\kappa)\}$ is given as

$$\begin{aligned}&\Pr\{\varsigma(t+\kappa) = n|\varsigma(t) = m\}\\&= \begin{cases}\pi_{mn}(\kappa)\kappa + o(\kappa), & m \neq n,\\ 1 + \pi_{mm}(\kappa)\kappa + o(\kappa), & m = n,\end{cases}\end{aligned} \tag{2}$$

in which $\kappa > 0$ represents the sojourn time, $\lim_{\kappa\to 0}\left(\frac{o(\kappa)}{\kappa}\right) = 0$, $\pi_{mn}(\kappa) \geqslant 0$ stands for the transition rate from mode m at time t to mode n at time $t + \kappa$ for $m \neq n$, and $\pi_{mm}(\kappa) = -\sum_{n\in\Gamma, m\neq n}\pi_{mn}(\kappa)$.

Remark 1. Unlike the Markovian jump systems, the sojourn time of the semi-Markovian jump process allows to be non-exponential distributed and the transition rate allows for changes over time. Consequently, the NN model with semi-Markovian jump parameters considered in this paper is more general.

In this paper, the slave system is given by the following perturbed NN model

$$\begin{cases}\dot{y}(t) = -\mathcal{A}(\varsigma(t))y(t) + \mathcal{B}_1(\varsigma(t))\hat{g}(y(t)) + \mathcal{B}_2(\varsigma(t))\hat{g}(y(t-\varrho))\\ \qquad +I(t) + D(\varsigma(t))\omega(t) + u(t),\\ \check{z}(t) = L(\varsigma(t))y(t),\end{cases} \tag{3}$$

where $y(t) \in \mathcal{R}^{n_1}$, and $\check{z}(t) \in \mathcal{R}^{n_1}$ are the state, the control input, and the output, respectively; $D(\varsigma(t)) \in \mathcal{R}^{n_1\times n_1}$ is a real constant matrix; $\omega(t) \in \mathcal{R}^m$ is external disturbance belongs to $\mathcal{L}_2[0, \infty)$ [32–34].

Considering the possible gain perturbations, the controller to be used is given as

$$u(t) = (K(\varsigma(t)) + \Delta K(\varsigma(t)))(y(t) - x(t)), \tag{4}$$

where $K(\varsigma(t))$ represents a real constant matrix denoting the gain, $\Delta K(\varsigma(t))$ denotes the gain uncertainty, which is required to meet the following assumption:

Assumption 1. [35–37] Gain perturbation ΔK_m possesses the norm-bounded form as

$$\Delta K_m = H_m \mathscr{F} E_m, \tag{5}$$

where $m \in \Gamma$, H_m and E_m are certain real constant matrices, and $\mathscr{F}$ stands for an uncertain matrix meeting $\mathscr{F}^T\mathscr{F} \leqslant I$.

Define $e(t) = y(t) - x(t)$, $z(t) = \check{z}(t) - \hat{z}(t)$. The following synchronization error system can be acquired:

$$\begin{cases}\dot{e}(t) = -\mathcal{A}(\varsigma(t))e(t) + \mathcal{B}_1(\varsigma(t))g(e(t)) + \mathcal{B}_2(\varsigma(t))g(e(t-\varrho))\\ \qquad +D(\varsigma(t))\omega(t) + (K(\varsigma(t)) + \Delta K(\varsigma(t)))e(t),\\ z(t) = L(\varsigma(t))e(t),\end{cases} \tag{6}$$

where $g(e(\cdot)) = \hat{g}(y(\cdot)) - \hat{g}(x(\cdot))$.

For the convenience of the notation, set $\mathcal{A}(\varsigma(t)) = \mathcal{A}_m$, $\mathcal{B}_1(\varsigma(t)) = \mathcal{B}_{1m}$, $\mathcal{B}_2(\varsigma(t)) = \mathcal{B}_{2m}$, $D(\varsigma(t)) = D_m$, $K(\varsigma(t)) + \Delta K(\varsigma(t)) = K_m + \Delta K_m$, and $L(\varsigma(t)) = L_m$. Then system (6) is able to be rewritten as

$$\begin{cases}\dot{e}(t) = -\mathcal{A}_m e(t) + \mathcal{B}_{1m}g(e(t)) + \mathcal{B}_{2m}g(e(t-\varrho)) + D_m\omega(t)\\ \qquad +(K_m + \Delta K_m)e(t),\\ z(t) = L_m e(t).\end{cases} \tag{7}$$

For the derivation of our main results, let us prepare the following Assumption, Definition, and Lemma:

Assumption 2. [38] The activation function $\hat{g}_\alpha(x_\alpha(t))$ is continuous and bounded, and meets the form as follows

$$\upsilon_\alpha^- \leqslant \frac{\hat{g}_\alpha(s_2) - \hat{g}_\alpha(s_1)}{s_2 - s_1} \leqslant \upsilon_\alpha^+,\ \alpha = 1, 2, \ldots, n_1, \tag{8}$$

where $s_2, s_1 \in R$, $s_2 \neq s_1$, and υ_α^- and υ_α^+ are known scalars. The value of scalars $\upsilon_\alpha^- + \upsilon_\alpha^+$ and $\upsilon_\alpha^- - \upsilon_\alpha^+$ can be set as zero, negative or positive.

Note that the condition of the activation function in Assumption 2 is more general than the Lipschitz condition in [39–42]. In order to simplify the sign, we set

$$\phi^- = \mathrm{diag}\{\upsilon_1^+\upsilon_1^-, \upsilon_2^+\upsilon_2^-, \ldots, \upsilon_{n_1}^+\upsilon_{n_1}^-\},$$

$$\phi^+ = \mathrm{diag}\left\{\frac{\upsilon_1^+ + \upsilon_1^-}{2}, \frac{\upsilon_2^+ + \upsilon_2^-}{2}, \ldots, \frac{\upsilon_{n_1}^+ + \upsilon_{n_1}^-}{2}\right\}.$$

Definition 1. [43] System (7) is stochastically stable with an $\mathscr{L}_2 - \mathscr{L}_\infty$ performance index $\gamma > 0$ if the following conditions hold:

(1) It is stochastically stable with $\omega(t) = 0$;
(2) For any $\omega(t) \in \mathcal{L}_2[0, \infty)$,

$$\sup_{t\geqslant 0}\mathcal{E}\{(z^T(t)z(t)\} \leqslant \gamma^2\int_0^\infty \omega^T(t)\omega(t)\mathrm{d}t \tag{9}$$

is satisfied under the zero initial condition.

Lemma 1. *[44] Given a symmetric matrix $\mathbb{J}$, matrices $\mathbb{M}$, and $\mathbb{N}$ with compatible dimensions, then*

$$\mathbb{J} + \mathbb{M}\Delta\mathbb{N} + \mathbb{N}^T\Delta^T\mathbb{M}^T < 0$$

holds for any matrix Δ with $\Delta^T\Delta \leqslant I$ if there exists a scalar $\beta > 0$ satisfying

$$\mathbb{J} + \beta^{-1}\mathbb{M}\mathbb{M}^T + \beta\mathbb{N}\mathbb{N}^T < 0.$$

At the end of this section, let us clearly state the issue of non-fragile $\mathscr{L}_2 - \mathscr{L}_\infty$ synchronization to be investigated: for the master-slave chaotic systems in (1) and (3), determine the non-fragile controller in (4) ensuring the synchronization error system in (7) to be stochastically stable with a predefined $\mathscr{L}_2 - \mathscr{L}_\infty$ performance index $\gamma > 0$.

3. Main results

In this section, we first give a sufficient condition for the $\mathscr{L}_2 - \mathscr{L}_\infty$ synchronization analysis of system (7).

Theorem 1. *Suppose Assumption 2 holds. For a given scalar $\gamma > 0$ and a positive integer N, set $h = \varrho/N$. Then system (7) is stochastically stable with $\mathscr{L}_2 - \mathscr{L}_\infty$ performance index γ, if there are matrices $P_m > 0$ for each $m \in \Gamma$, Λ_1, Λ_2, $\tilde{Q}_i$, diagonal matrices $M_1 > 0$, $M_2 > 0$, and symmetric matrices $\tilde{S}_i > 0$, $\tilde{R}_{ij}$ $(i, j = 0, 1,\ldots,N)$ such that the following matrix inequalities hold:*

$$L_m^T L_m - P_m < 0, \tag{10}$$

$$\begin{bmatrix} P_m & \breve{Q} \\ * & \breve{R} + \breve{S} \end{bmatrix} > 0, \tag{11}$$

$$\Omega = \begin{bmatrix} \Sigma_1 & F^s & F^a & \Sigma_2 \\ * & -\bar{R}_d - \bar{S}_d & 0 & 0 \\ * & * & -3\bar{S}_d & 0 \\ * & * & * & \Sigma_3 \end{bmatrix} < 0, \tag{12}$$

in which

$$\breve{Q} = [\tilde{Q}_0 \tilde{Q}_1 \ldots \tilde{Q}_N],\ \breve{S} = \mathrm{diag}\left\{\frac{1}{h}\tilde{S}_0 \frac{1}{h}\tilde{S}_1 \ldots \frac{1}{h}\tilde{S}_N\right\},$$

$$\breve{R} = \begin{bmatrix} \tilde{R}_{00} & \tilde{R}_{01} & \ldots & \tilde{R}_{0N} \\ \tilde{R}_{10} & \tilde{R}_{11} & \ldots & \tilde{R}_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{R}_{N0} & \tilde{R}_{N1} & \ldots & \tilde{R}_{NN} \end{bmatrix},$$

$$\Sigma_1 = \begin{bmatrix} \Sigma_1^{00} & \Sigma_1^{01} & \Sigma_1^{02} \\ * & \Sigma_1^{11} & 0 \\ * & * & \Sigma_1^{22} \end{bmatrix},$$

$$\Sigma_1^{00} = \mathrm{sym}\{\tilde{Q}_0 - \Lambda_1 \mathcal{A}_m + \Lambda_1(K_m + \Delta K_m)\} + \tilde{S}_0 + \sum_{n\in\Gamma} \tilde{\pi}_{mn} P_n - \phi^- M_1,$$

$$\Sigma_1^{01} = -\tilde{Q}_N,\ \Sigma_1^{02} = P_m^T - \Lambda_1 - \mathcal{A}_m^T \Lambda_2^T + (K_m + \Delta K_m)^T \Lambda_2^T,$$
$$\Sigma_1^{11} = -\tilde{S}_N - \phi^- M_2,\ \Sigma_1^{22} = -\mathrm{sym}\{\Lambda_2\},$$

$$\Sigma_2 = \begin{bmatrix} \Sigma_2^{00} & \Sigma_2^{01} & \Sigma_2^{02} \\ 0 & \phi^+ M_2 & 0 \\ \Sigma_2^{20} & \Sigma_2^{21} & \Sigma_2^{22} \end{bmatrix},$$
$$\Sigma_2^{00} = \Lambda_1 \mathcal{B}_{1m} + \phi^+ M_1,\ \Sigma_2^{01} = \Lambda_1 \mathcal{B}_{2m},\ \Sigma_2^{02} = \Lambda_1 D_m,$$
$$\Sigma_2^{20} = \Lambda_2 B_{1m},\ \Sigma_2^{21} = \Lambda_2 \mathcal{B}_{2m},\ \Sigma_2^{22} = \Lambda_2 D_m,$$

$$\bar{R}_d = \begin{bmatrix} \tilde{R}_{d11} & \tilde{R}_{d12} & \ldots & \tilde{R}_{d1N} \\ \tilde{R}_{d21} & \tilde{R}_{d22} & \ldots & \tilde{R}_{d2N} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{R}_{dN1} & \tilde{R}_{dN2} & \ldots & \tilde{R}_{dNN} \end{bmatrix},$$
$$\bar{R}_{dij} = h(\tilde{R}_{i-1,j-1} - \tilde{R}_{ij}),$$
$$\bar{S}_d = \mathrm{diag}\{\tilde{S}_{d1}\tilde{S}_{d2} \ldots \tilde{S}_{dN}\},\ \bar{S}_{di} = \tilde{S}_{i-1} - \tilde{S}_i,$$

$$F^s = [F_1^s F_2^s \ldots F_N^s],\ F_i^s = \begin{bmatrix} F_{0i}^s \\ F_{1i}^s \\ F_{2i}^s \end{bmatrix},$$
$$F_{0i}^s = \frac{h}{2}(\tilde{R}_{0,i-1} + \tilde{R}_{0,i}) - (\tilde{Q}_{i-1} - \tilde{Q}_i),$$
$$F_{1i}{}^s = \frac{h}{2}(\tilde{Q}_{i-1} + \tilde{Q}_i),\ F_{2i}{}^s = -\frac{h}{2}(\tilde{R}_{N,i-1} + \tilde{R}_{N,i}),$$
$$F^a = [F_1^a F_2^a \ldots F_N^a],\ F_i^a = \begin{bmatrix} F_{0i}^a \\ F_{1i}^a \\ F_{2i}^a \end{bmatrix},$$
$$F_{0i}^a = -\frac{h}{2}(\tilde{R}_{0,i-1} - \tilde{R}_{0,i}),$$
$$F_{1i}{}^a = -\frac{h}{2}(\tilde{Q}_{i-1} - \tilde{Q}_i),\ F_{2i}{}^a = \frac{h}{2}(\tilde{R}_{N,i-1} - \tilde{R}_{N,i}),$$
$$\Sigma_3 = \begin{bmatrix} \Sigma_3^{00} & 0 & 0 \\ * & \Sigma_3^{11} & 0 \\ * & * & \Sigma_3^{22} \end{bmatrix},$$
$$\Sigma_3^{00} = -M_1,\ \Sigma_3^{11} = -M_2,\ \Sigma_3^{22} = -\gamma^2 I,$$
$$\tilde{\pi}_{mn} = E\{\pi_{mn}(\kappa)\} = \int_0^\infty \pi_{mn}(\kappa) f_m(\kappa)\mathrm{d}\kappa,$$

with $f_m(\kappa)$ being the probability density function of sojourn time κ.

Proof. Consider an LKF candidate as

$$V(e(t), \varsigma(t), t) = V_1(e(t), \varsigma(t), t) + V_2(e(t), t) + V_3(e(t), t) + V_4(e(t), t), \tag{13}$$

where

$$V_1(e(t), \varsigma(t), t) = \mathrm{e}^T(t)P(\varsigma(t))e(t),$$
$$V_2(e(t), t) = 2\mathrm{e}^T(t)\int_{-\varrho}^0 \tilde{Q}(\zeta)e(t+\zeta)\mathrm{d}\zeta,$$
$$V_3(e(t), t) = \int_{-\varrho}^0 \int_{-\varrho}^0 \mathrm{e}^T(t+\zeta)\tilde{R}(\zeta)e(t+\eta)\mathrm{d}\zeta\mathrm{d}\eta,$$
$$V_4(e(t), t) = \int_{-\varrho}^0 \mathrm{e}^T(t+\zeta)\tilde{S}(\zeta)e(t+\eta)\mathrm{d}\zeta.$$

Denote by $\mathfrak{L}$ be the infinitesimal generator. Then, by the definition of $\mathfrak{L}$ in [45], one has

$$\mathfrak{L}V_1(e(t), \varsigma(t), t) = \lim_{\sigma\to 0}\frac{1}{\sigma}[\mathcal{E}\{V_1(e(t+\sigma), \varsigma(t+\sigma), t + \sigma|e(t), \varsigma(t), t\} - V_1(e(t), \varsigma(t), t)],$$

where σ is a small positive number. For $\varsigma(t)=m$, one can write

$$\begin{aligned}
&\mathfrak{L}V_1(e(t), m, t)\\
&= \lim_{\sigma\to 0}\frac{1}{\sigma}\left[\mathcal{E}\left\{\sum_{n=1,n\neq m}^{\Gamma}\Pr\{\varsigma(t+\sigma)=n|\varsigma(t)=m\}\mathrm{e}^T\right.\right.\\
&\times(t+\sigma)P_n e(t+\sigma)\\
&\left.+\Pr\{\varsigma(t+\sigma)=m|\varsigma(t)=m\}\mathrm{e}^T(t+\sigma)P_m e(t+\sigma)\right\}-\mathrm{e}^T\\
&\left.\times(t)P_m e(t)\right]\\
&= \lim_{\sigma\to 0}\frac{1}{\sigma}\left[\mathcal{E}\left\{\sum_{n=1,n\neq m}^{\Gamma}\frac{\Pr\{\varsigma(t+\sigma)=n,\varsigma(t)=m\}}{\Pr\{\varsigma(t)=m\}}\mathrm{e}^T\right.\right.\\
&\times(t+\sigma)P_n e(t+\sigma)\\
&\left.+\frac{\Pr\{\varsigma(t+\sigma)=m,\varsigma(t)=m\}}{\Pr\{\varsigma(t)=m\}}\mathrm{e}^T(t+\sigma)P_m e(t+\sigma)\right\}-\mathrm{e}^T\\
&\left.\times(t)P_m e(t)\right]\\
&= \lim_{\sigma\to 0}\frac{1}{\sigma}\left[\mathcal{E}\left\{\sum_{n=1,n\neq m}^{\Gamma}\frac{q_{mn}(G_m(\chi+\sigma)-G_m(\chi))}{1-G_m(\chi)}\mathrm{e}^T\right.\right.\\
&\times(t+\sigma)P_n e(t+\sigma)\\
&\left.+\frac{1-G_m(\chi+\sigma)}{1-G_m(\chi)}\mathrm{e}^T(t+\sigma)P_m e(t+\sigma)\right\}-\mathrm{e}^T\\
&\left.\times(t)P_m e(t)\right],
\end{aligned}$$

where χ denotes the time elapsed when the system remains at mode m from the last jump; $G_m(t)$ denotes the cumulative distribution function of the sojourn time; q_{mn} stands for the probability intensity of the system jumping from mode m to mode n. Following the same line as the proof of the analysis result in [46], one gets

$$\begin{aligned}
&\mathfrak{L}V_1(e(t), m, t)\\
&\quad=\dot{\mathrm{e}}^T(t)\mathrm{sym}\{P_m\}e(t)+\sum_{n\in\Gamma}\tilde{\pi}_{mn}\mathrm{e}^T(t)P_n e(t).
\end{aligned}\tag{14}$$

In addition, it can be acquired that

$$\begin{aligned}
\mathfrak{L}V_2(e(t), t) &= 2\dot{\mathrm{e}}^T(t)\int_{-\varrho}^{0}\tilde{Q}(\zeta)\\
&\times e(t+\zeta)\mathrm{d}\zeta-2\mathrm{e}^T(t)\int_{-\varrho}^{0}\dot{\tilde{Q}}(\zeta)e(t+\zeta)\mathrm{d}\zeta\\
&+2\mathrm{e}^T(t)\tilde{Q}(0)e(t)-2\mathrm{e}^T(t)\tilde{Q}(-\varrho)e(t-\varrho),
\end{aligned}\tag{15}$$

$$\begin{aligned}
&\mathfrak{L}V_3(e(t), t)\\
&=2\mathrm{e}^T(t)\int_{-\varrho}^{0}\tilde{R}(0,\zeta)e(t+\zeta)\mathrm{d}\zeta\\
&-2\mathrm{e}^T(t-\varrho)\int_{-\varrho}^{0}\tilde{R}(-\varrho,\zeta)e(t+\zeta)\mathrm{d}\zeta\\
&-\int_{-\varrho}^{0}\int_{-\varrho}^{0}\mathrm{e}^T(t+\zeta)[\tilde{R}_\zeta(\zeta,\eta)+\tilde{R}_\eta(\zeta,\eta)]e(t+\eta)\mathrm{d}\zeta\mathrm{d}\eta,
\end{aligned}\tag{16}$$

$$\begin{aligned}
&\mathfrak{L}V_4(e(t), t)\\
&=\mathrm{e}^T(t)\tilde{S}(0)e(t)-\mathrm{e}^T(t-\varrho)\tilde{S}(-\varrho)e(t-\varrho)\\
&-\int_{-\varrho}^{0}\mathrm{e}^T(t+\zeta)\dot{\tilde{S}}(\zeta)e(t+\zeta)\mathrm{d}\zeta.
\end{aligned}\tag{17}$$

Divide $[-\varrho, 0]$ into N equal parts $[\delta_i, \delta_{i-1}](i=1, 2,\ldots,N)$ of length $h=\varrho/N$, where $\delta_i=-ih$. Also, divide $[-\varrho, 0]\times[-\varrho, 0]$ into $N\times N$ small squares $[\delta_i, \delta_{i-1}]\times[\delta_j, \delta_{j-1}]$ $(i, j=1, 2,\ldots,N)$. Then, divide further each square into two triangles. Matrix $\tilde{Q}(\zeta)$ and $\tilde{S}(\zeta)$ are set to be linear within each $[\delta_i, \delta_{i-1}]$, and $\tilde{R}(\zeta, \eta)$ is set to be linear with each triangle. Then, using the linear interpolation formula, it can be written that:

$$\begin{aligned}
&\tilde{Q}(\delta_i+\varepsilon h)=(1-\varepsilon)\tilde{Q}_i+\varepsilon\tilde{Q}_{i-1},\ \tilde{S}(\delta_i+\varepsilon h)\\
&=(1-\varepsilon)\tilde{S}_i+\varepsilon\tilde{S}_{i-1},\ \tilde{R}(\delta_i+\varepsilon h, \delta_j+\epsilon h)\\
&=\begin{cases}(1-\varepsilon)\tilde{R}_{i,j}+\epsilon\tilde{R}_{i-1,j-1}+(\varepsilon-\epsilon)\tilde{R}_{i-1,j}, & \varepsilon\geqslant\epsilon,\\ (1-\epsilon)\tilde{R}_{i,j}+\varepsilon\tilde{R}_{i-1,j-1}+(\epsilon-\varepsilon)\tilde{R}_{i,j-1}, & \varepsilon<\epsilon,\end{cases}
\end{aligned}$$

where $0\leqslant\varepsilon\leqslant1$, $0\leqslant\epsilon\leqslant1$ $(i, j=1,\ldots,N)$. Therefore, the LKF in (13) is entirely relying on P_m, $\tilde{Q}_i$, $\tilde{S}_i$ and $\tilde{R}_{ij}$, $(i, j=1, 2,\ldots,N)$. Note that $V(e(t), \varsigma(t), t)\geqslant\mu\|e_1(t)\|^2$ is satisfied with $\mu\geqslant\lambda_{\min}(P_1)$ if $\tilde{S}_i>0$ $(i=1, 2,\ldots,N)$ and (11) hold true. In this regard, for $\delta_i<\zeta<\delta_{i-1}$, $\delta_j<\eta<\delta_{j-1}$, one can write [47]:

$$\dot{\tilde{Q}}(\zeta)=\frac{1}{h}(\tilde{Q}_{i-1}-\tilde{Q}_i),\ \dot{\tilde{S}}(\zeta)=\frac{1}{h}(\tilde{S}_{i-1}-\tilde{S}_i),$$

$$\tilde{R}_\zeta(\zeta,\eta)+\tilde{R}_\eta(\zeta,\eta)=\frac{1}{h}(\tilde{R}_{i-1,j-1}-\tilde{R}_{i,j}).$$

Then, from (13) and (14)–(17) one has

$$\begin{aligned}
&\mathfrak{L}V(e(t), m, t)\\
&=\varphi^T(t)\bar{\Sigma}_1\varphi(t)\\
&+\mathrm{sym}\left\{\varphi^T(t)\sum_{i=1}^{N}\int_0^1[F_i^s+(1-2\varepsilon)F_i^a]e(t+\delta_i+\varepsilon h)\mathrm{d}\varepsilon\right\}\\
&-\sum_{i=1}^{N}\int_0^1\mathrm{e}^T(t+\delta_i+\varepsilon h)\tilde{S}_{di}e(t+\delta_i+\varepsilon h)\mathrm{d}\varepsilon\\
&-\sum_{i=1}^{N}\sum_{j=1}^{N}\int_0^1\left[\int_0^1\mathrm{e}^T(t+\delta_i+\varepsilon h)\tilde{R}_{dij}e(t+\delta_i+\epsilon h)\mathrm{d}\varepsilon\right]\mathrm{d}\epsilon\\
&=\varphi^T(t)\bar{\Sigma}_1\varphi(t)\\
&+\mathrm{sym}\left\{\varphi^T(t)\int_0^1[F^s+(1-2\varepsilon)F^a]\bar{e}(t,\varepsilon)\mathrm{d}\varepsilon\right\}\\
&-\int_0^1\bar{\mathrm{e}}^T(t,\varepsilon)\overline{S}_d\bar{e}(t,\varepsilon)\mathrm{d}\varepsilon-\int_0^1\int_0^1\bar{\mathrm{e}}^T(t,\varepsilon)\overline{R}_d\bar{e}(t,\epsilon)\mathrm{d}\varepsilon\mathrm{d}\epsilon,
\end{aligned}$$

where

$$\varphi(t)=[\mathrm{e}^T(t), \mathrm{e}^T(t-\varrho), \dot{\mathrm{e}}^T(t)]^T,$$

$$\bar{\Sigma}_1=\begin{bmatrix}\bar{\Sigma}_1^{00} & \bar{\Sigma}_1^{01} & \bar{\Sigma}_1^{02}\\ * & \bar{\Sigma}_1^{11} & 0\\ * & * & 0\end{bmatrix},$$

$$\bar{e}(t,\varepsilon)=[\mathrm{e}^T(t+\delta_1+\varepsilon h)\ \ldots\ \mathrm{e}^T(t+\delta_N+\varepsilon h)]^T,$$

$$\bar{\Sigma}_1^{00}=\mathrm{sym}\{\tilde{Q}_0\}+\tilde{S}_0+\sum_{n\in\Gamma}\tilde{\pi}_{mn}P_n,$$

$$\bar{\Sigma}_1^{02}=P_m^T,\ \bar{\Sigma}_1^{11}=-\tilde{S}_N.$$

Set $\xi^T(t,\varepsilon) = \varphi^T(t)[F^s + (1-2\varepsilon)F^a]$. Then for any matrix $U > 0$ satisfying

$$\begin{bmatrix} U & -I \\ -I & \bar{S}_d \end{bmatrix} > 0,$$

by using Jensen inequality one can write

$$\begin{aligned} &\mathfrak{L}V(e(t), m, t) \\ &\leqslant \varphi^T(t)\overline{\Sigma}_1\varphi(t) + \int_0^1 \xi^T(t,\varepsilon)U\xi(t,\varepsilon)\mathrm{d}\varepsilon \\ &- \int_0^1 \bar{\xi}^T(t,\varepsilon)\mathrm{d}\varepsilon \begin{bmatrix} U & -I \\ -I & \bar{S}_d \end{bmatrix} \int_0^1 \bar{\xi}(t,\varepsilon)\mathrm{d}\varepsilon \\ &- \int_0^1 \bar{\mathrm{e}}^T(t,\varepsilon)\mathrm{d}\varepsilon \overline{R}_d \int_0^1 \bar{e}(t,\varepsilon)\mathrm{d}\varepsilon, \end{aligned} \tag{18}$$

in which $\bar{\xi}(t,\varepsilon) = [\xi^T(t,\varepsilon) \quad \bar{\mathrm{e}}^T(t,\epsilon)]^T$. By simple calculation, one is able to get that

$$\begin{aligned} &\int_0^1 \xi^T(t,\varepsilon)\mathrm{d}\varepsilon = \varphi^T(t)F^s, \\ &\int_0^1 \xi^T(t,\varepsilon)U\xi(t,\varepsilon)\mathrm{d}\varepsilon \\ &= \varphi^T(t)\left\{F^sUF^{sT} + \frac{1}{3}F^aUF^{aT}\right\}\varphi(t). \end{aligned}$$

Substituting the above two equalities into (18) gives rise to

$$\begin{aligned} &\mathfrak{L}V(e(t), m, t) \\ &\leqslant \left[\varphi^T(t) \quad \int_0^1 \bar{\mathrm{e}}^T(t,\varepsilon)\mathrm{d}\varepsilon\right]\Phi_1\begin{bmatrix} \varphi(t) \\ \int_0^1 \bar{e}(t,\varepsilon)\mathrm{d}\varepsilon \end{bmatrix}, \end{aligned} \tag{19}$$

in which

$$\Phi_1 = \begin{bmatrix} \bar{\Sigma}_1 + \frac{1}{3}F^aUF^{aT} & F^s \\ * & -\bar{R}_d - \bar{S}_d \end{bmatrix}.$$

By Assumption 2 one has

$$\begin{bmatrix} e(t) \\ e(t-\varrho) \\ g(e(t)) \\ g(e(t-\varrho)) \end{bmatrix}^T \bar{M} \begin{bmatrix} e(t) \\ e(t-\varrho) \\ g(e(t)) \\ g(e(t-\varrho)) \end{bmatrix} \geqslant 0, \tag{20}$$

where

$$\bar{M} = \begin{bmatrix} -\phi^- M_1 & 0 & \phi^+ M_1 & 0 \\ * & -\phi^- M_2 & 0 & \phi^+ M_2 \\ * & * & -M_1 & 0 \\ * & * & * & -M_2 \end{bmatrix}.$$

Furthermore, by (7), for any compatible free-weighting matrices Λ_1 and Λ_2, one can write

$$\begin{aligned} &[\mathrm{e}^T(t)\Lambda_1 + \dot{\mathrm{e}}^T(t)\Lambda_2][-\dot{e}(t) - \mathcal{A}_m e(t) \\ &+ \mathcal{B}_{1m}g(e(t)) + \mathcal{B}_{2m}g(e(t-\varrho)) \\ &+ D_m\omega(t) + (K_m + \Delta K_m)e(t)] = 0. \end{aligned} \tag{21}$$

Now, with the help of (19)–(21), it is easy to get

$$\mathfrak{L}V(e(t), m, t) - \gamma^2\omega^T(t)\omega(t) \leqslant \tilde{\varphi}^T(t)\Phi_2\tilde{\varphi}(t),$$

where

$$\begin{aligned} \tilde{\varphi}(t) &= \left[\varphi^T(t) \quad \int_0^1 \bar{\mathrm{e}}^T(t,\varepsilon)\mathrm{d}\varepsilon \quad \hat{\varphi}^T(t)\right]^T, \\ \hat{\varphi}(t) &= \left[g^T(e(t)) \quad g^T(e(t-\varrho)) \quad \omega^T(t)\right]^T, \\ \Phi_2 &= \begin{bmatrix} \Sigma_1 + \frac{1}{3}F^aUF^{aT} & F^s & \Sigma_2 \\ * & -\bar{R}_d - \bar{S}_d & 0 \\ * & * & \Sigma_3 \end{bmatrix}. \end{aligned}$$

By Schur's complement, (12) ensures $\Phi_2 < 0$, and thus

$$\mathfrak{L}V(e(t), m, t) - \gamma^2\omega^T(t)\omega(t) < 0 \tag{22}$$

holds for any $\tilde{\varphi}^T(t) \neq 0$.

When $\omega(t) = 0$, one obtains from (22) that $E\{\mathfrak{L}V(e(t), m, t)\} < 0$, which means that system (7) is stochastically stable [48].

Next, one concentrates on the $\mathscr{L}_2 - \mathscr{L}_\infty$ performance of system (7) with $\omega(t) \neq 0$. Denote

$$J = \mathcal{E}\{z^T(t)z(t)\} - \gamma^2\int_0^t \omega^T(t)\omega(t)\mathrm{d}t.$$

Then, under the zero initial condition, one has

$$\begin{aligned} J &= E\{z^T(t)z(t) - \mathrm{e}^T(t)P_m e(t)\} \\ &\quad -\gamma^2\int_0^t \omega^T(t)\omega(t)\mathrm{d}t + E\{\mathrm{e}^T(t)P_m e(t)\} \\ &\leqslant E\{\mathrm{e}^T(t)(L_m^T L_m - P_m)e(t)\} \\ &\quad +E\left\{\int_0^t (\mathfrak{L}V(e(t), m, t) - \gamma^2\omega^T(t)\omega(t))\mathrm{d}t\right\}. \end{aligned}$$

From (10) and (22) it is easy to see that $J \leqslant 0$ for any $t > 0$, which means (9). That is, synchronization error system (7) possesses $\mathscr{L}_2 - \mathscr{L}_\infty$ performance index γ. The proof is completed.

Remark 2. Theorem 1 gives a criterion for the stochastic stability and $\mathscr{L}_2 - \mathscr{L}_\infty$ performance of system (7). It can be seen that the condition depends on the number of segments (i.e. N), which specifies the degree of discretization. The change of N can give rise to different consequences. In general, the larger N is selected, the less conservative result can be obtained.

When there are no parameter switches and controllers, system (7) reduces to

$$\begin{cases} \dot{e}(t) = -\mathcal{A}e(t) + \mathcal{B}_1 g(e(t)) + \mathcal{B}_2 g(e(t-\varrho)) + D\omega(t) \\ z(t) = Le(t). \end{cases} \tag{23}$$

Then, we can propose the following result.

Corollary 1. *Suppose Assumption 2 holds. For a given scalar $\gamma > 0$ and a positive integer N, set $h = \varrho/N$. Then system (23) is asymptotically stable with $\mathscr{L}_2 - \mathscr{L}_\infty$ performance index γ, if there are matrices $P > 0$, Λ_1, Λ_2, $\tilde{Q}_i$, diagonal* *matrices* $M_1 > 0$, $M_2 > 0$, *and symmetric matrices* $\tilde{S}_i > 0$, $\tilde{R}_{ij}$ $(i, j = 0, 1,\ldots,N)$ *such that*

$$L^T L - P < 0, \tag{24}$$

$$\begin{bmatrix} P & \breve{Q} \\ * & \breve{R} + \breve{S} \end{bmatrix} > 0, \tag{25}$$

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\Sigma}_1 & F^s & F^a & \tilde{\Sigma}_2 \\ * & -\bar{R}_d - \bar{S}_d & 0 & 0 \\ * & * & -3\bar{S}_d & 0 \\ * & * & * & \Sigma_3 \end{bmatrix} < 0, \tag{26}$$

in which

$$\tilde{\Sigma}_1 = \begin{bmatrix} \tilde{\Sigma}_1^{00} & \Sigma_1^{01} & \tilde{\Sigma}_1^{02} \\ * & \Sigma_1^{11} & 0 \\ * & * & \Sigma_1^{22} \end{bmatrix},$$

$$\tilde{\Sigma}_1^{00} = \mathrm{sym}\{\tilde{Q}_0 - \Lambda_1 \mathcal{A}\} + \tilde{S}_0 - \phi^- M_1,$$

$$\tilde{\Sigma}_1^{02} = P^T - \Lambda_1 - \mathcal{A}^T \Lambda_2^T,$$

$$\tilde{\Sigma}_2 = \begin{bmatrix} \tilde{\Sigma}_2^{00} & \tilde{\Sigma}_2^{01} & \tilde{\Sigma}_2^{02} \\ 0 & \phi^+ M_2 & 0 \\ \tilde{\Sigma}_2^{20} & \tilde{\Sigma}_2^{21} & \tilde{\Sigma}_2^{22} \end{bmatrix},$$

$$\tilde{\Sigma}_2^{00} = \Lambda_1 \mathcal{B}_1 + \phi^+ M_1,\ \tilde{\Sigma}_2^{01} = \Lambda_1 \mathcal{B}_2,\ \tilde{\Sigma}_2^{02} = \Lambda_1 D,$$

$$\tilde{\Sigma}_2^{20} = \Lambda_2 \mathcal{B}_1,\ \tilde{\Sigma}_2^{21} = \Lambda_2 \mathcal{B}_2,\ \tilde{\Sigma}_2^{22} = \Lambda_2 D.$$

The other symbols are the same as those in Theorem 1.

Remark 3. In Corollary 1, a novel criterion for the $\mathscr{L}_2 - \mathscr{L}_\infty$ synchronization analysis of chaotic time-delay NNs is proposed. As shall be shown in Example 2, the criterion is less conservative compared with that in [49].

In the following, our attention is paid on solving of the non-fragile $\mathscr{L}_2 - \mathscr{L}_\infty$ synchronization issue. An approach for the required non-fragile $\mathscr{L}_2 - \mathscr{L}_\infty$ control design is given as follows:

Theorem 2. *Suppose Assumptions 1 and 2 hold. For given scalars* $\gamma > 0$, $l > 0$, *and positive integer* N, *set* $h = \varrho/N$. *Then the non-fragile* $\mathscr{L}_2 - \mathscr{L}_\infty$ *synchronization issue is solvable, if, there are matrices* $P_m > 0$, Υ_{1m} *for each* $m \in \Gamma$, $\Lambda_1 > 0$, $\tilde{Q}_i$, *diagonal matrices* $M_1 > 0 > 0$, $M_2 > 0$, *symmetric matrices* $\tilde{S}_i > 0$, $\tilde{R}_{ij}$ $(i, j = 0,1, \ldots,N)$, *and scalar* $\beta > 0$ *such that conditions (10), (11) and the following LMI*

$$\bar{\Omega} = \begin{bmatrix} \Theta_1 & F^s & F^a & \Theta_2 \\ * & -\bar{R}_d - \bar{S}_d & 0 & 0 \\ * & * & -3\bar{S}_d & 0 \\ * & * & * & \Theta_3 \end{bmatrix} < 0 \tag{27}$$

hold, where

$$\Theta_1 = \begin{bmatrix} \Theta_1^{00} & \Sigma_1^{01} & \Theta_1^{02} \\ * & \Sigma_1^{11} & 0 \\ * & * & \Theta_1^{22} \end{bmatrix},$$

$$\Theta_2 = \begin{bmatrix} \Sigma_2^{00} & \Sigma_2^{01} & \Sigma_2^{02} & \Theta_2^{03} & \Theta_2^{04} \\ 0 & \phi^+ M_2 & 0 & 0 & 0 \\ \Theta_2^{20} & \Theta_2^{21} & \Theta_2^{22} & 0 & 0 \end{bmatrix},$$

$$\Theta_1^{00} = \mathrm{sym}\{\tilde{Q}_0 - \Lambda_1 \mathcal{A}_m + \Upsilon_{1m}\} + \tilde{S}_0 + \sum_{n\in\Gamma} \tilde{\pi}_{mn} P_n - \phi^- M_1,$$

$$\Theta_1^{02} = P_m^T - \Lambda_1 - l\mathcal{A}_m^T \Lambda_1^T + l\Upsilon_{1m}^T,\ \Theta_1^{22} = -\mathrm{sym}\{l\Lambda_1\},$$

$$\Theta_2^{03} = \Lambda_1 H_m,\ \Theta_2^{04} = E_m^T \Lambda_1,\ \Theta_2^{20} = l\Lambda_1 \mathcal{B}_{1m},\ \Theta_2^{21} = l\Lambda_1 \mathcal{B}_{2m},\ \Theta_2^{22} = l\Lambda_1 D_m,$$

$$\Theta_3 = \mathrm{diag}\{\Sigma_3^{00}, \Sigma_3^{11}, \Sigma_3^{22}, \Theta_3^{33}, \Theta_3^{44}\},\ \Theta_3^{33} = -\beta I,\ \Theta_3^{44} = \beta - \Lambda_1 - \Lambda_1^T.$$

The other symbols are the same as those in Theorem 1. In this case, the gains of non-fragile controller (4) is able to be given by $K_m = \Lambda_1^{-1} \Upsilon_{1m}$ $(m \in \Gamma)$.

Proof. Set

$$\mathbb{M}^T = [\Lambda_1^T, 0, \Lambda_2^T, 0, 0, 0, 0],\ \mathbb{N} = [K_m, 0, 0, 0, 0, 0, 0, 0].$$

Then, according to Lemma 1, (12) can be ensured by

$$\hat{\Omega} = \begin{bmatrix} \hat{\Theta}_1 & F^s & F^a & \hat{\Theta}_2 \\ * & -\bar{R}_d - \bar{S}_d & 0 & 0 \\ * & * & -3\bar{S}_d & 0 \\ * & * & * & \hat{\Theta}_3 \end{bmatrix} < 0, \tag{28}$$

where

$$\hat{\Theta}_1 = \begin{bmatrix} \bar{\Theta}_1^{00} & \Sigma_1^{01} & \bar{\Theta}_1^{02} \\ * & \Sigma_1^{11} & 0 \\ * & * & \bar{\Theta}_1^{22} \end{bmatrix},$$

$$\hat{\Theta}_2 = \begin{bmatrix} \Sigma_2^{00} & \Sigma_2^{01} & \Sigma_2^{02} & \bar{\Theta}_2^{03} & \bar{\Theta}_2^{04} \\ 0 & \phi^+ M_2 & 0 & 0 & 0 \\ \bar{\Theta}_2^{20} & \bar{\Theta}_2^{21} & \bar{\Theta}_2^{22} & 0 & 0 \end{bmatrix},$$

$$\bar{\Theta}_1^{00} = \mathrm{sym}\{\tilde{Q}_0 - \Lambda_1 \mathcal{A}_m + \Lambda_1 K_m\} + \tilde{S}_0 + \sum_{n\in\Gamma} \tilde{\pi}_{mn} P_n - \phi^- M_1,$$

$$\bar{\Theta}_1^{02} = P_m^T - \Lambda_1 - \mathcal{A}_m^T \Lambda_2^T + K_m^T \Lambda_2^T,\ \bar{\Theta}_1^{22} = -\mathrm{sym}\{\Lambda_2\},$$

$$\bar{\Theta}_2^{03} = H_m,\ \bar{\Theta}_2^{04} = E_m^T,\ \bar{\Theta}_2^{20} = \Lambda_2 \mathcal{B}_{1m},$$

$$\bar{\Theta}_2^{21} = \Lambda_2 \mathcal{B}_{2m},\ \bar{\Theta}_2^{22} = \Lambda_2 D_m,$$

$$\hat{\Theta}_3 = \mathrm{diag}\{\Sigma_3^{00}, \Sigma_3^{11}, \Sigma_3^{22}, \bar{\Theta}_3^{33}, \bar{\Theta}_3^{44}\},$$

$$\bar{\Theta}_3^{33} = -\beta I,\ \bar{\Theta}_3^{44} = -\beta^{-1} I.$$

Table 2. Minimum allowable $\mathscr{L}_2 - \mathscr{L}_\infty$ performance index γ for different N.

N	1	3	5	7	9
$\gamma_{\min}$	0.79	0.78	0.76	0.74	0.71

Table 3. Maximum allowable time-delay bound ϱ for different N.

N	1	2	3	4	5
$\varrho_{\max}$	1.17	1.29	1.39	1.50	1.66

On the other hand, let us set

$$\Upsilon_{1m} = \Lambda_1 K_m, \;\; \Lambda_2 = l\Lambda_1,$$

and pre- and post-multiple the obtained matrix by $\mathrm{diag}\{I, I, I, I, I, I, I, I, \Lambda_1^{-1}\}$ and its transpose. Then, according to the inequality

$$-\Lambda_1^T \beta^{-1} \Lambda_1 \leqslant \beta - \Lambda_1 - \Lambda_1^T,$$

one can obtain (28) from (27). In this way, the proof is completed.

Remark 4. With the aid of a matrix congruence transformation and some inequality techniques, an approach to the design of non-fragile controller (4) for ensuring synchronization error system (7) to be stochastically stable with an $\mathscr{L}_2 - \mathscr{L}_\infty$ performance index γ is developed in Theorem 2. It is shown that the desired control gains can be obtained via solving a number of LMIs, which are computationally tractable via the well-known software MATLAB.

4. Numerical example

In this section, two numerical examples are given to demonstrate the usefulness of the proposed analysis and design results, respectively.

Example 1. Consider system (1) with the following parameters [49]:

$$\mathcal{A} = \begin{bmatrix} 2.2 & 0 \\ 0 & 3.5 \end{bmatrix}, \; \mathcal{B}_1 = 0, \; \mathcal{B}_2 = \begin{bmatrix} -1 & 0.4 \\ 0 & -0.1 \end{bmatrix}, \; L = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}. \tag{29}$$

Take $\varrho = 1$. It is found that $\mathscr{L}_2 - \mathscr{L}_\infty$ stability criterion in [49] is not applicable for $\gamma < 1.26$. However, when setting $N = 1$, it can be verified the LMIs in (24)–(26) possess feasible solutions for $\gamma > 0.79$, which means that system (23) is asymptotically stable with minimum allowable $\mathscr{L}_2 - \mathscr{L}_\infty$ performance index $\gamma_{\min} = 0.79$. Table 2 gives the minimum allowable $\mathscr{L}_2 - \mathscr{L}_\infty$ performance index γ for different N. As can be seen from the table, when N increases, the results obtained become less conservative. A similar phenomenon is shown in table 3.

Example 2. The parameters of the NNs in (1) and (3), and the controller in (4) are chosen as follows:

$$\hat{g}(x) = \begin{bmatrix} \tanh(x_1) \\ \tanh(x_2) \end{bmatrix}, \; \omega(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \frac{1}{e^t + 1}, \; I(t) = 0, \; \varrho = 1;$$

Mode 1:

$$\mathcal{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathcal{B}_{11} = \begin{bmatrix} 2.0 & -0.1 \\ -5.0 & 2.0 \end{bmatrix},$$
$$\mathcal{B}_{21} = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -1.5 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.2 & -0.1 \\ -0.1 & 0.2 \end{bmatrix}, \quad H_1 = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.1 \end{bmatrix}, \quad L_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix};$$

Mode 2:

$$\mathcal{A}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathcal{B}_{12} = \begin{bmatrix} 2.0 & -0.1 \\ -5.0 & 4.5 \end{bmatrix},$$
$$\mathcal{B}_{22} = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -4.0 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} 0.21 & -0.15 \\ -0.1 & 0.19 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$E_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.2 & -0.3 \\ -0.3 & 0.2 \end{bmatrix}.$$

According to the chosen activation function, one can get

$$\phi^- = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \phi^+ = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}. \tag{30}$$

The probability density function of the sojourn time is described by

$$f_m(\kappa) = (\varpi/\vartheta)(\kappa/\vartheta)^{\varpi-1}\exp(-(\kappa/\vartheta)^{\varpi}), \; \kappa > 0.$$

And the transition rate functions rely on the Weibull distribution with ($\vartheta = 1$, $\varpi = 4$) for m = 1 and ($\vartheta = 1, \varpi = 3$) for m = 2, respectively. Then the transition probability matrix of $\varsigma(t)$ is calculated as follows:

$$\Psi(\kappa) = \begin{bmatrix} \theta_{11}(\kappa) & \theta_{12}(\kappa) \\ \theta_{21}(\kappa) & \theta_{22}(\kappa) \end{bmatrix} = \begin{bmatrix} -4\kappa^3 & 4\kappa^3 \\ 3\kappa^2 & -3\kappa^2 \end{bmatrix}.$$

Consequently, the mathematical expectation of matrix $\Psi(\kappa)$ can be obtained

$$\mathcal{E}\{\Psi(\kappa)\} = \begin{bmatrix} -3.6763 & 3.6763 \\ 2.7082 & -2.7082 \end{bmatrix}. \tag{31}$$

Then, the mode transitions are shown in figure 1. Under the mode change in figure 1, the chaotic attractor of the master system with initial value $x(s) = [-3; -0.4]^T$ ($s \in [-\varrho, \; 0]$) is described in figure 2.

Next, we show the usefulness of the controller designed. Choose $\gamma = 0.1$ and $l = 1$. Then, solving the LMIs in (10), (11), and (27), the corresponding controller gains can be

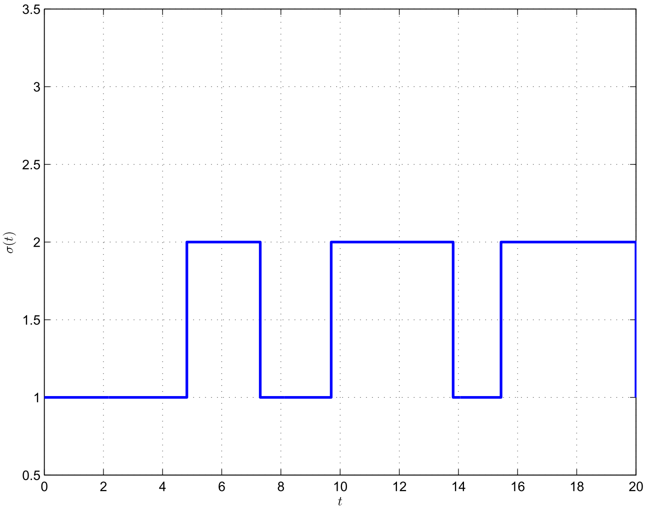

Figure 1. Mode transitions.

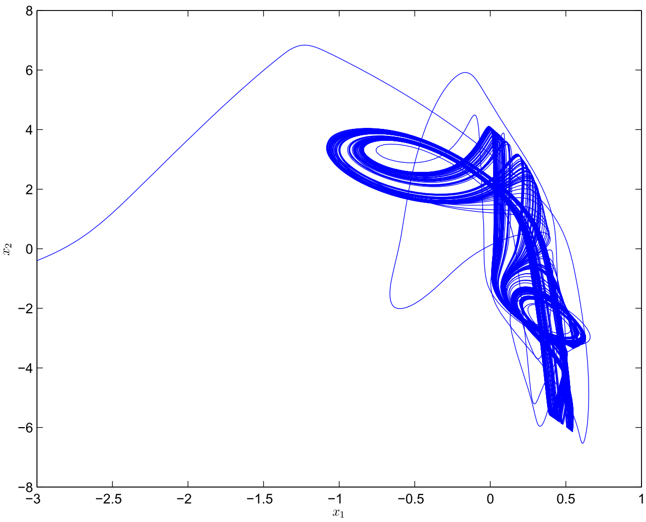

Figure 2. Phase-plane plot of the master system.

obtained as follows:

$$K_1 = \begin{bmatrix} -14.4506 & 8.2655 \\ 9.4769 & -48.5009 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -16.9373 & 9.2686 \\ 10.3032 & -56.6638 \end{bmatrix}.$$

Then we divide the time-delay interval $[-\varrho, 0]$ into 3 segments; that is, $N = 3$. Set the initial conditions of the master-slave systems to be $x(s) = [-3; -0.4]^T$ $(s \in [-\varrho, 0])$ and $y(s) = [-8; 2]^T$ $(s \in [-\varrho, 0])$, respectively. Then, the state evolution of the master-slave systems and that of the synchronization error system are depicted in figures 3 and 4, respectively. It can be found that, under the effect of the non-fragile control, the state trajectories of the master-slave systems tend to coincide rapidly as time goes, which means that the synchronization is achieved. The plot of $\gamma(t) = \sqrt{\mathcal{E}\{(z^T(t)z(t)\} \Big/ \int_0^\infty \omega^T(t)\omega(t)\mathrm{d}t}$ versus time under the zero initial condition is shown in figure 5, from which one can see

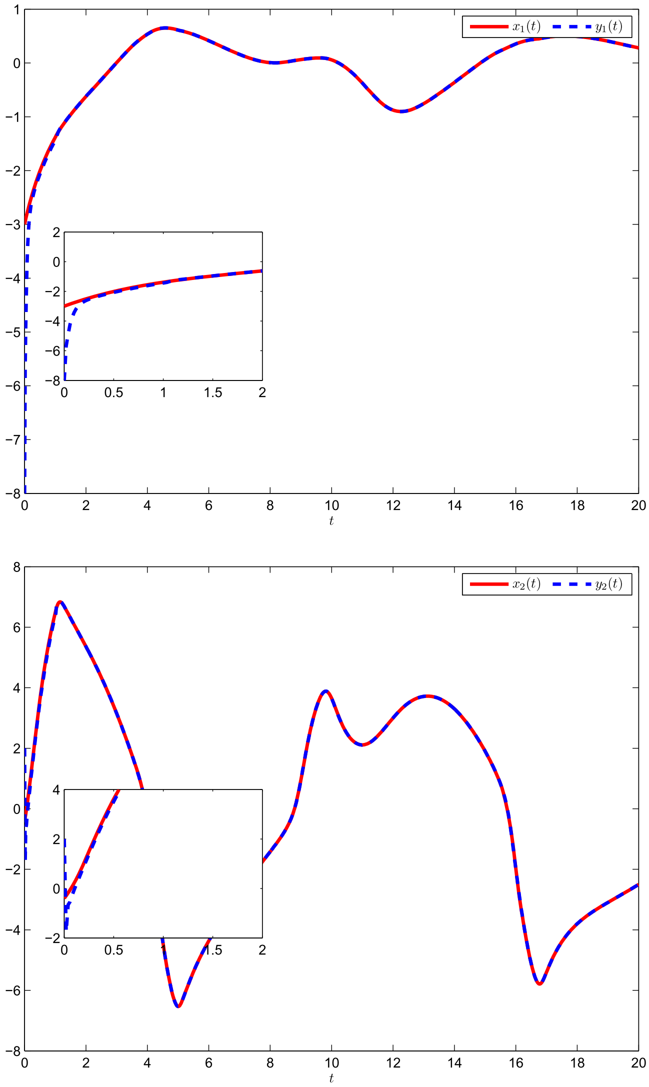

Figure 3. State trajectories of the master-slave systems.

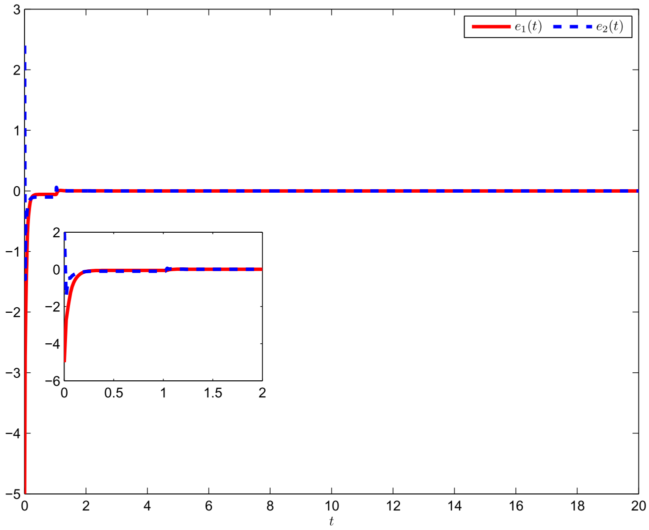

Figure 4. State trajectories of the synchronization error system.

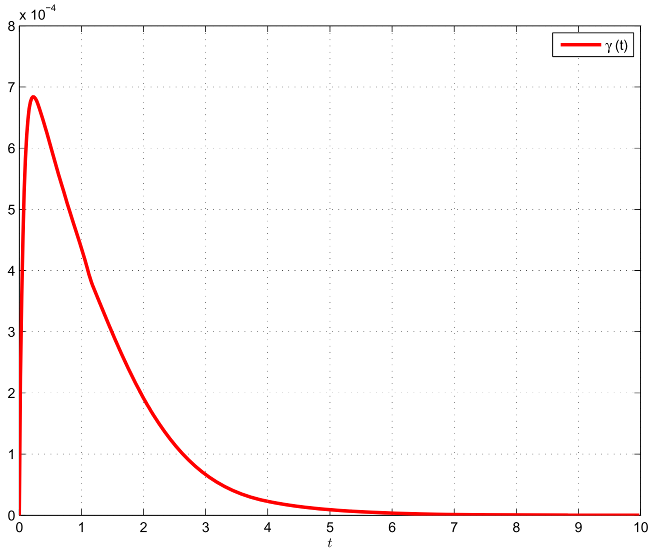

Figure 5. $\gamma\ (t)$.

that $\sup_t \gamma(t) = 6.5981 \times 10^{-4}$. It is less than the prescribed $\mathscr{L}_2 - \mathscr{L}_\infty$ performance index γ (i.e. 0.1).

5. Conclusions

The issue of non-fragile $\mathscr{L}_2 - \mathscr{L}_\infty$ synchronization for chaotic NNs with semi-Markovian jump parameters has been addressed in this paper. By utilizing the discretized LKF method and introducing two free-weighting matrices, a sufficient condition has been obtained to guarantee the stochastic stability and $\mathscr{L}_2 - \mathscr{L}_\infty$ performance of the error system (7). Then, by means of the matrix congruence transformation and some inequality techniques, an approach to the non-fragile $\mathscr{L}_2 - \mathscr{L}_\infty$ design has been developed for which the desired controller can be acquired by the feasible solution of several LMIs. In the end, two illustrative examples have been employed to show the usefulness of the proposed analysis and design results.

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