IOP Publishing Classical and Quantum Gravity
Class. Quantum Grav. 37 (2020) 079501 (4pp) https://doi.org/10.1088/1361-6382/ab74f7

Corrigendum

Corrigendum: Covariant BSSN formulation in bimetric relativity (2020 *Class. Quantum Grav.* 37 025013)

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Received 14 January 2020
Accepted for publication 11 February 2020
Published 5 March 2020

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We provide corrections to a typo and an error in the original paper.

1. Correction to the typo

The correct version of equations (3.1) and (3.12) in the main text is,

$$\overline{\gamma}_{ij} := \mathrm{e}^{-4\phi}\gamma_{ij}, \qquad \overline{\gamma}^{ij} := \mathrm{e}^{4\phi}\gamma^{ij}, \tag{1.1a}$$

$$\widehat{\varphi}_{ij} := \mathrm{e}^{-4\psi}\varphi_{ij}, \qquad \widehat{\varphi}^{ij} := \mathrm{e}^{4\psi}\varphi^{ij}. \tag{1.1b}$$

2. Correction to the error in appendix A.1

This section replaces the first part of appendix A.1, up to (A.25).

Consider the spatial parts $\gamma = e^{\intercal}\boldsymbol{\delta}e$ and $\varphi = m_{\circ}{}^{\intercal}\boldsymbol{\delta}m_{\circ}$ of two Lorentzian metrics g, f. In [1], it is established that the existence of the *real* square root $\left(g^{-1}f\right)^{1/2}$ implies,

$$\beta := q + \alpha n = q + \alpha e^{-1}\mathbf{p}\lambda^{-1}, \tag{2.1a}$$

$$\widetilde{\beta} := q - \widetilde{\alpha}\widetilde{n} = q - \widetilde{\alpha}m^{-1}\mathbf{p}\lambda^{-1}, \tag{2.1b}$$

$$\chi = e^{\intercal}\boldsymbol{\delta}\boldsymbol{\Lambda}_{\mathrm{s}}\mathbf{R}m_{\circ} = \chi^{\intercal}. \tag{2.1c}$$

To be more precise, the freely specifiable spatial vielbein $m_{\circ}$ is used to compute the vielbein $\mathbf{R}m_{\circ}$ such that the spatial part χ of the geometric mean metric $h = g\#f$ is given by $\chi = e^{\intercal}\boldsymbol{\delta}\boldsymbol{\Lambda}_{\mathrm{s}}(\mathbf{R}m_{\circ})$. This is obtained by imposing (2.1c) and solving it for the Euclidean orthogonal transformation $\mathbf{R}$ in terms of $\mathbf{\Lambda}_{\mathrm{s}}$ and the vielbeins e, m_{o}. Such a solution always exists, as it is part of the polar decomposition of the invertible matrix $\mathbf{R}_{\mathrm{o}}$ [1, 2] (see (3.7) in the main text). For the sake of simplicity, we define the new vielbein of φ to be $m := \mathbf{R}m_{\mathrm{o}}$; we have the freedom to do that since $m_{\mathrm{o}}{}^{\mathsf{T}}\mathbf{R}^{\mathsf{T}}\boldsymbol{\delta}\mathbf{R}m_{\mathrm{o}} = m_{\mathrm{o}}{}^{\mathsf{T}}\boldsymbol{\delta}m_{\mathrm{o}}$, implying that φ is blind to this choice. The interaction terms are not affected as well, since they always contain both $\mathbf{\Lambda}_{\mathrm{s}}$ and $\mathbf{R}$, irrespective of this choice. The matrix $\mathbf{\Lambda}_{\mathrm{s}}$ explicitly appears in them. On the contrary, $\mathbf{R}$ does not appear explicitly, but it is taken into account implicitly inside m.

We define the bimetric interactions as [3],

$$\mathsf{n} := e^{-1}\mathbf{v}, \qquad \widetilde{\mathsf{n}} := m^{-1}\mathbf{v}, \tag{2.2a}$$

$$\mathcal{Q} := e^{-1}\mathbf{\Lambda}_{\mathrm{s}}{}^{2}e, \qquad \widetilde{\mathcal{Q}} := m^{-1}\mathbf{\Lambda}_{\mathrm{s}}{}^{2}m, \tag{2.2b}$$

$$\mathcal{D} := m^{-1}\mathbf{\Lambda}_{\mathrm{s}}{}^{-1}e, \qquad \widetilde{\mathcal{D}} := e^{-1}\mathbf{\Lambda}_{\mathrm{s}}{}^{-1}m, \tag{2.2c}$$

$$\mathcal{B} := \mathcal{D}^{-1} = e^{-1}\mathbf{\Lambda}_{\mathrm{s}}m, \qquad \widetilde{\mathcal{B}} := \widetilde{\mathcal{D}}^{-1} = m^{-1}\mathbf{\Lambda}_{\mathrm{s}}e, \tag{2.2d}$$

$$\mathcal{V} := -\,m^{d}\sum_{n=0}^{d}\beta_{n}e_{n}(\widetilde{\mathcal{D}}), \qquad \widetilde{\mathcal{V}} := -\lambda^{-1}\,m^{d}\sum_{n=0}^{d}\beta_{n}e_{n-1}(\mathcal{B}), \tag{2.2e}$$

$$\mathcal{U} := -\lambda^{-1}\,m^{d}\sum_{n=0}^{d}\beta_{n}Y_{n-1}(\mathcal{B}), \qquad \widetilde{\mathcal{U}} := -\widetilde{\mathcal{D}}\,m^{d}\sum_{n=0}^{d}\beta_{n}Y_{n-1}(\widetilde{\mathcal{D}}), \tag{2.2f}$$

$$(\mathcal{Q}\tilde{\mathcal{u}}) := \mathcal{Q}\widetilde{\mathcal{U}} = -\mathcal{B}\,m^{d}\sum_{n=0}^{d}\beta_{n}Y_{n-1}(\widetilde{\mathcal{D}}), \qquad (\widetilde{\mathcal{Q}}\mathcal{u}) := \widetilde{\mathcal{Q}}\mathcal{U} = -\lambda^{-1}\widetilde{\mathcal{Q}}\,m^{d}\sum_{n=0}^{d}\beta_{n}Y_{n-1}(\mathcal{B}), \tag{2.2g}$$

where $e_n(X)$ are the elementary symmetric polynomials of the linear operator X,

$$e_{n}(X) = X^{[a_1}{}_{a_1}X^{a_2}{}_{a_2}\dots X^{a_n]}{}_{a_n}, \tag{2.3}$$

and $Y_n(X)$ is defined as,

$$Y_{n}(X) := \sum_{k=0}^{n}(-1)^{n+k}e_{k}(X)X^{n-k}. \tag{2.4}$$

See [3] for more details about the properties of $e_n(X)$ and $Y_n(X)$. Note that d is the dimension of the spacetime, that is, $d = N + 1$. Hence, some terms in the summations will be zero. The $\beta_{(n)}$ parameters are $d + 1$ real dimensionless constants appearing in the bimetric interaction potential, together with the energy scale m [4]. We define the bimetric sources (respectively, the bimetric energy densitites, the bimetric currents and the bimetric spatial stress–energy tensors) as [3],

$$\rho^{\mathrm{b}} = -e_{n}(\mathcal{B}), \qquad j^{\mathrm{b}}{}_{i} = -\gamma_{ik}(\mathcal{Q}\tilde{\mathcal{u}})^{k}{}_{j}\mathsf{n}^{j}, \quad J^{\mathrm{b}}{}_{ij} = \gamma_{ik}\left[\mathcal{V}\delta^{k}{}_{j} - (\mathcal{Q}\tilde{\mathcal{u}})^{k}{}_{j} + W^{-1}\mathcal{U}^{k}{}_{j}\right], \tag{2.5a}$$

$$\widetilde{\rho}^{\mathrm{b}} = -\frac{\lambda e_{n-1}(\mathcal{B})}{\det(me^{-1})}, \quad \widetilde{j}^{\mathrm{b}}{}_{i} = -\frac{j^{\mathrm{b}}{}_{i}}{\det(me^{-1})}, \quad \widetilde{J}^{\mathrm{b}}{}_{ij} = \frac{\varphi_{ik}\left[\widetilde{\mathcal{V}}\delta^{k}{}_{j} - (\widetilde{\mathcal{Q}}\mathcal{u})^{k}{}_{j} + W\,\widetilde{\mathcal{U}}^{k}{}_{j}\right]}{\det(me^{-1})}, \tag{2.5b}$$

where the summation $-m^d\sum_{n=0}^{d}\beta_{(n)}$ is understood in front of all the bimetric sources. Note the relation between the two bimetric currents $j^{\mathrm{b}}{}_i, \widetilde{j}^{\mathrm{b}}{}_i$, which implies the relation (A.35) between the momentum constraints in the main text.

Here we compute the expressions for the bimetric interaction and sources in the (c)BSSN formalism. We require that the symmetrization condition (2.1) holds for the BSSN variables as well. Since the shifts are the same in the BSSN formalism, we require conditions (2.1*a*) and (2.1*b*) to stay the same. The condition (2.1*c*) should instead lead to its analog in the BSSN formalism,

$$\mathring{\chi} = \bar{e}^{\mathsf{T}}\boldsymbol{\delta}\overset{*}{\boldsymbol{\Lambda}}_{\mathrm{s}}\overset{*}{\mathbf{R}}\widehat{m}_{\mathrm{o}} = \mathring{\chi}^{\mathsf{T}}, \tag{2.6}$$

where $\overset{*}{\boldsymbol{\Lambda}}_{\mathbf{s}}, \overset{*}{\mathbf{R}}$ are the BSSN counterparts of the spatial part of the Lorentz boost (3.8) and the orthogonal transformation in (3.7) in the main text, whose expression is unknown yet.

We start by computing the conformal decomposition of the objects in the Lorentz frame. The requirement that (2.1*a*) stays the same implies,

$$\beta = q + \alpha e^{-1}\mathbf{p}\lambda^{-1} = q + \alpha e^{-1}\xi\overset{*}{\mathbf{p}}\overset{*}{\lambda}^{-1} \iff \mathbf{p}\lambda^{-1} = \xi\overset{*}{\mathbf{p}}\overset{*}{\lambda}^{-1}, \tag{2.7}$$

where the scalar ξ accounts for the conformal decomposition of $\mathbf{p}\lambda^{-1}$. It follows that,

$$\frac{\mathbf{p}}{(1+\mathbf{p}^{\mathsf{T}}\boldsymbol{\delta}\mathbf{p})^{1/2}} = \frac{\xi\overset{*}{\mathbf{p}}}{(1+\overset{*}{\mathbf{p}}{}^{\mathsf{T}}\boldsymbol{\delta}\overset{*}{\mathbf{p}})^{1/2}} \iff \mathbf{p} = \xi\left(\frac{1+\mathbf{p}^{\mathsf{T}}\boldsymbol{\delta}\mathbf{p}}{1+\overset{*}{\mathbf{p}}{}^{\mathsf{T}}\boldsymbol{\delta}\overset{*}{\mathbf{p}}}\right)^{1/2}\overset{*}{\mathbf{p}}. \tag{2.8}$$

We apply $\mathbf{p}^{\mathsf{T}}\boldsymbol{\delta}$ to (2.8) and obtain,

$$\frac{\mathbf{p}^{\mathsf{T}}\boldsymbol{\delta}\mathbf{p}}{(1+\mathbf{p}^{\mathsf{T}}\boldsymbol{\delta}\mathbf{p})^{1/2}} = \frac{\xi\mathbf{p}^{\mathsf{T}}\boldsymbol{\delta}\overset{*}{\mathbf{p}}}{(1+\overset{*}{\mathbf{p}}{}^{\mathsf{T}}\boldsymbol{\delta}\overset{*}{\mathbf{p}})^{1/2}} = \xi^2\left(\frac{1+\mathbf{p}^{\mathsf{T}}\boldsymbol{\delta}\mathbf{p}}{1+\overset{*}{\mathbf{p}}{}^{\mathsf{T}}\boldsymbol{\delta}\overset{*}{\mathbf{p}}}\right)^{1/2}\frac{\overset{*}{\mathbf{p}}{}^{\mathsf{T}}\boldsymbol{\delta}\overset{*}{\mathbf{p}}}{(1+\overset{*}{\mathbf{p}}{}^{\mathsf{T}}\boldsymbol{\delta}\overset{*}{\mathbf{p}})^{1/2}}, \tag{2.9}$$

which is equivalent to

$$\frac{\mathbf{p}^{\mathsf{T}}\boldsymbol{\delta}\mathbf{p}}{1+\mathbf{p}^{\mathsf{T}}\boldsymbol{\delta}\mathbf{p}} = \frac{\xi^2\overset{*}{\mathbf{p}}{}^{\mathsf{T}}\boldsymbol{\delta}\overset{*}{\mathbf{p}}}{1+\overset{*}{\mathbf{p}}{}^{\mathsf{T}}\boldsymbol{\delta}\overset{*}{\mathbf{p}}} \iff \overset{*}{\mathbf{p}}{}^{\mathsf{T}}\boldsymbol{\delta}\overset{*}{\mathbf{p}} = \frac{\mathbf{p}^{\mathsf{T}}\boldsymbol{\delta}\mathbf{p}}{\xi^2(1+\mathbf{p}^{\mathsf{T}}\boldsymbol{\delta}\mathbf{p}) - \mathbf{p}^{\mathsf{T}}\boldsymbol{\delta}\mathbf{p}}. \tag{2.10}$$

Hence, in general, we can rescale $\mathbf{p}$ as in (2.8) with a generic ξ when we recast the equations into the (c) BSSN formulation, as long as we satisfy (2.10). However, there is no need to rescale it since this is an unnecessary complication. Indeed, we can always satisfy (2.8) and (2.10) by choosing $\xi = 1$, which implies $\mathbf{p} = \overset{*}{\mathbf{p}}$. It immediately follows,

$$\mathbf{p} = \overset{*}{\mathbf{p}} \implies \lambda = (1+\mathbf{p}^{\mathsf{T}}\boldsymbol{\delta}\mathbf{p})^{1/2} = \left(1+\overset{*}{\mathbf{p}}{}^{\mathsf{T}}\boldsymbol{\delta}\overset{*}{\mathbf{p}}\right)^{1/2} = \overset{*}{\lambda} \tag{2.11a}$$

$$\implies \mathbf{v} = \mathbf{p}\lambda^{-1} = \overset{*}{\mathbf{p}}\overset{*}{\lambda}^{-1} = \overset{*}{\mathbf{v}} \tag{2.11b}$$

$$\implies \boldsymbol{\Lambda}_{\mathrm{s}} = (1+\mathbf{p}\mathbf{p}^{\mathsf{T}}\boldsymbol{\delta})^{1/2} = \left(1+\overset{*}{\mathbf{p}}\overset{*}{\mathbf{p}}{}^{\mathsf{T}}\boldsymbol{\delta}\right)^{1/2} = \overset{*}{\boldsymbol{\Lambda}}_{\mathrm{s}}, \tag{2.11c}$$

which implies (see (A.23) in the main text),

$$\mathbf{R} := \left(\boldsymbol{\delta}^{-1}\mathbf{R}_{\mathrm{o}}{}^{\mathsf{T}}\boldsymbol{\delta}\mathbf{R}_{\mathrm{o}}\right)^{1/2}\mathbf{R}_{\mathrm{o}}{}^{-1} = \mathrm{e}^{2(\phi-\psi)}\left(\boldsymbol{\delta}^{-1}\overset{*}{\mathbf{R}}_{\mathbf{o}}{}^{\mathsf{T}}\boldsymbol{\delta}\overset{*}{\mathbf{R}}_{\mathbf{o}}\right)^{1/2}\mathrm{e}^{-2(\phi-\psi)}\overset{*}{\mathbf{R}}_{\mathbf{o}}{}^{-1} = \overset{*}{\mathbf{R}}. \tag{2.12}$$

Using (3.11) in the main text and (2.11*c*), (2.12) and $\widehat{m}_{\mathrm{o}} = \mathrm{e}^{-2\psi}m_{\mathrm{o}}$ (which follows from (3.11) in the main text), the spatial part of the symmetrization condition (2.1*c*) can be written as,

$$\begin{aligned}\chi &= e^{\mathsf{T}}\boldsymbol{\delta}\boldsymbol{\Lambda}_{\mathrm{s}}\mathbf{R}m_{\circ} = \mathrm{e}^{2(\phi+\psi)}\bar{e}^{\mathsf{T}}\boldsymbol{\delta}\boldsymbol{\Lambda}_{\mathrm{s}}\mathbf{R}\widehat{m}_{\circ} =: \mathrm{e}^{2(\phi+\psi)}\mathring{\chi} \\ &= \chi^{\mathsf{T}} = (e^{\mathsf{T}}\boldsymbol{\delta}\boldsymbol{\Lambda}_{\mathrm{s}}\mathbf{R}m_{\circ})^{\mathsf{T}} = \mathrm{e}^{2(\phi+\psi)}\,(\bar{e}^{\mathsf{T}}\boldsymbol{\delta}\boldsymbol{\Lambda}_{\mathrm{s}}\mathbf{R}\widehat{m}_{\circ})^{\mathsf{T}} =: \mathrm{e}^{2(\phi+\psi)}\mathring{\chi}^{\mathsf{T}},\end{aligned} \tag{2.13}$$

that is, if χ is symmetric, its BSSN counterpart $\mathring{\chi}$ is also symmetric, as desired.

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