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Weak magnetic field corrections to pion and constituent quarks form factors

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Abstract
A quark–quark interaction due to nonperturbative one gluon exchange is considered for the derivation of leading weak magnetic field induced anisotropic corrections to usual pion—constituent quarks form factors. Besides that, few chiral and isospin symmetry breaking effective couplings which emerge only due to the weak magnetic field are also found. Numerical estimations for these magnetic field corrections are presented. All of these corrections are ultraviolet finite and their relative values are found to be of order of $(eB_0/M^{*2})^n$ being $n = 1$ for the vector and axial pion couplings and $n = 2$ for the pseudoscalar and scalar ones. The corresponding anisotropic corrections to the constituent quark and pion strong averaged quadratic radii in the plane perpendicular to the magnetic field are also calculated as functions of the quark effective mass.

Keywords: pion vector and scalar form factors, axial and pseudoscalar constituent quark form factor, global color model, auxiliary field method, averaged quadratic radius, constituent quark model

1. Introduction

In the last decade a high interest on the effect of magnetic fields on hadron properties and dynamics [1–7] appeared due to estimates of intense magnetic fields expected to be found in peripheral heavy ions collisions, supernovae and in magnetars [1, 2, 8]. Large magnetic fields, of the order of $(eB_0) \simeq 10^{17} - 10^{19}\mathrm{G} \simeq (0.1 - 15)m_\pi^2$, would not be so large as compared to an hadron mass scale such as the nucleon or constituent quark mass, although they would only appear for a short time interval in the case of non central heavy ions collisions [8]. One cannot

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expect large magnetic fields in low/intermediary energies hadron collisions in which the usual pion dynamics is expected to be dominant, however pion interactions with photons and their behavior under weak magnetic field might eventually provide observable effects at intermediary energies. Moreover, it has been envisaged nuclear structure changes due to external magnetic fields [9, 10] and it becomes important to understand further the magnetic field effects on each part of the nucleon and nuclear potentials. In [11, 12], Electromagnetic and Strong form factors of light vector and axial mesons coupled to constituent quarks were derived and anisotropic corrections to their quadratic radii due to a weak magnetic field were found.

Hadrons electromagnetic and strong form factors make possible comparisons of many important observables calculated theoretically from different approaches [14–28] with experiments [29–33]. In particular hadron charge distribution, spin structure and electroweak interaction properties can be understood in terms of electromagnetic and axial form factors. Due to the enormous difficulties in solving QCD in the low energy non perturbative regime, effective models have been developed or articulated based on phenomenology and general QCD symmetries and properties. Among these, the approximated chiral symmetry and its dynamical symmetry breaking (DChSB) have wide consequences. The constituent quark model describes many aspects of phenomenology and it is usually considered to incorporate the pion cloud [34–36]. A further proposal along these lines is the Weinberg's large N_c effective field theory (EFT) that copes constituent quark picture with the large N_c expansion [37]. This large Nc-EFT has been derived in [38, 39] without and with electromagnetic interactions by starting from a quark–quark interaction due to a dressed one gluon exchange. Background quarks, dressed by a sort of gluon cloud described by a non perturbative gluon propagator, yield constituent quarks. Vector and axial pion couplings to constituent quarks are part of the large Nc EFT and the leading contributions to these coupling constants have also been calculated in these works. The corresponding leading pion and constituent quark Strong form factors, with their averaged quadratic radii, were presented in [13]. In the present work, an investigation of the leading weak magnetic field corrections to those pion and constituent quark form factors and to their averaged quadratic radii will be presented.

The non perturbative one gluon exchange quark–quark interaction is one of the leading terms of QCD quark- effective action. With the minimal coupling to a background electromagnetic field, it is given by the following generating functional [40–42]:

$$Z = N\int \mathcal{D}[\bar{\psi}, \psi] \exp \mathrm{i} \int_x \left[\bar{\psi}(\mathrm{i}\not{D} - m)\psi - \frac{g^2}{2}\int_y j_\mu^b(x)\tilde{R}_{bc}^{\mu\nu}(x-y)j_\nu^c(y) + \bar{\psi}J + J^*\psi \right], \tag{1}$$

where $\int_x$ stands for $\int \mathrm{d}^4x$, $\mathcal{D}[\bar{\psi}, \psi]$ is the functional measure of integration, J, J^* are the quark sources, g^2 is the quark gluon coupling constant, indices $a, b... = 1, \ldots(N_c^2 - 1)$ stand color in the adjoint representation, being $N_c = 3$, and the quark sources are written in the last terms. Along the work indices $i, j, k = 0, \ldots(N_f^2 - 1)$, being the number of flavors N_f, that will be used for isospin indices, with $N_f = 2$. The color quark current is given by $j_a^\mu = \bar{\psi}\lambda_a\gamma^\mu\psi$, and the sums in color, flavor and Dirac indices are implicit. $D_\mu = \partial_\mu - \mathrm{i}eQA_\mu$ is the covariant quark derivative with the minimal coupling to a background electromagnetic field, with the diagonal matrix $\hat{Q} = \mathrm{diag}(2/3, -1/3)$ for up and down electromagnetic couplings. The quark current masses m and effective masses M^* will be considered to be the same for u and d quarks. The non perturbative gluon propagator is an external input and it is written as $\tilde{R}_{ab}^{\mu\nu}(x-y)$. It must be a non perturbative one by incorporating to some extent the gluonic non Abelian character and, in particular, it will be required that, with a corrected quark-gluon coupling, it has enough strength to yield dynamical chiral symmetry breaking (DChSB), as it has been found in several approaches [16, 43–48]. In several gauges this kernel can be written in terms of transversal and longitudinal components, for partial derivatives as momentum operators in coordinate space as:$\tilde{R}_{ab}^{\mu\nu}(x-y) \equiv \tilde{R}_{ab}^{\mu\nu} = \delta_{ab}\left[\left(g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{\partial^2}\right)R_T(x-y) + \frac{\partial^\mu\partial^\nu}{\partial^2}R_L(x-y)\right]$. The approach adopted in the present manuscript keeps similarities with the Schwinger Dyson equations (SDE) framework at the rainbown ladder approximation [43, 49].

This work is organized as follows. In the next section the method employed is described shortly with the relevant steps that yield the sea quark determinant. The method, that involves the background field and the auxiliary field methods, was described with details in [7, 11, 12, 38, 39, 50] and therefore it will be very succinctly reminded. The determinant is finally presented for structureless pion field in the presence of constituent quark currents and of a background electromagnetic field. Section 3 is devoted to present the leading resulting terms of the expansion for large quark effective mass and small electromagnetic field. The momentum dependence of the leading electromagnetic effective couplings of pion and constituent quarks are presented as momentum integrals of components of the quark and gluon propagators. In section 4 the limit of magnetic field which is weak with respect to the quark effective mass is considered. Corrections to pion-constituent quarks interactions induced by magnetic fields are obtained. In section 4.1 two non perturbative effective gluon propagators are considered to provide numerical results, the Tandy–Maris propagator [51] and an effective confining one [45], being that both of them produce DChSB. The corresponding anisotropic corrections to the axial and pseudoscalar constituent quark Strong averaged quadratic radii (a.q.r.) and pion vector and scalar a.q.r. due to a relatively weak magnetic field are presented in section 4.2 as functions of the quark effective mass. In the final section there is a summary.

2. Constituent quarks and quark–antiquark light mesons

To make possible a more complete investigation of all the flavor channels, a Fierz transformation for the quark–quark interaction (1) is performed and only the color singlet terms are considered. The color non singlet ones are reduced at least due to a factor $1/N_c$. The quark field must be responsible for the formation of mesons and baryons and these different possibilities are envisaged by considering the background field method (BFM). Background quark component will be eventually associated to constituent quark (ψ_1) and the sea quark can be integrated out (ψ_2). For the one loop BFM it is enough to perform a shift of each of the quark currents obtained from the Fierz transformation [39, 52]. Whereas the one loop BFM would simply neglect the sea quark ψ_2 self interaction, an improvement is considered by integrating out fully these quarks with the introduction of light quark–antiquark states, mesons and excitations, by means of bilocal auxiliary fields [53]. In this way, DChSB can be incorporated by means of the scalar (quark–antiquark) state that might have a non trivial vacuum expectation value if the quark-gluon coupling constant g^2 is strong enough. The bilocal auxiliary fields for the different flavors can be expanded in an infinite orthogonal basis with all the excitations in the corresponding channel. For the lower energies regime, the lowest energy and lightest modes will be kept, i.e. for the pseudoscalar mesons π_i that are the pions. The (heavier) vector and axial mesons with their couplings to the constituent quarks can be neglected in the lower energy regime indeed. With that, by integrating out the sea quarks, the background photon couplings to light mesons and constituent quarks arise. The auxiliary fields are undetermined and the corresponding saddle point equations can be used for this. In the mean field they can be written from the conditions:

$$\frac{\partial S_{\rm af}^{\rm eff}}{\partial \phi_\alpha} = 0, \tag{2}$$

where $S_{\rm af}^{\rm eff}$ is the effective action obtained with the integration of the sea quark with the auxiliary fields and ϕ_α stands for each of the (constant) auxiliary fields. These gap equations for the Nambu–Jona Lasinio (NJL) model and for the model (1) have been analyzed in many works in the vacuum or under finite energy densities. In the vacuum, the scalar auxiliary field is the only one whose gap equation has a non trivial solution corresponding to a scalar quark–antiquark condensate as the order parameter of DChSB. At non zero constant magnetic fields a contribution to the quark effective mass arises associated to the so called magnetic catalysis that is well established from NJL-type and other similar models and also lattice QCD [2, 54, 55]. The scalar quark–antiquark field does not necessarily correspond to a light meson and a chiral rotation can be performed by freezing this degree of freedom. With this, the Goldstone bosons field will be described by means of the collective variables: $U = \mathrm{e}^{\mathrm{i}\vec{\sigma}\cdot\vec{\pi}}$ and $U^\dagger = \mathrm{e}^{-\mathrm{i}\vec{\sigma}\cdot\vec{\pi}}$.

The sea quark determinant yields the dynamics of the mesons fields with their couplings to constituent quarks, and it can be written as [38, 39]:

$$S_{\rm det} = -\mathrm{i}\,\mathrm{Tr}\ln\{-\mathrm{i}S_{c,q}^{-1}(x-y)\}, \tag{3}$$

$$S_{c,q}^{-1}(x-y) \equiv S_{0,c}^{-1}(x-y) + \Xi_s(x-y) + \sum_q a_q \Gamma_q j_q(x,y), \tag{4}$$

where Tr stands for traces of all discrete internal indices and integration of spacetime coordinates and $\Xi_s\,(x-y)$ stands for the coupling of sea quark to the fluctuations of the pseudoscalar field. It can be written as:

$$\Xi_s(x-y) = F[P_R(U-\tilde{U}) + P_L(U^\dagger - \tilde{U}^\dagger)]\delta(x-y), \tag{5}$$

where $P_{R/L} = (1 \pm \gamma_5)/2$ are the chiral right/left hand projectors and F the pion field normalization constant that emerge from the local limit of the bilocal auxiliary field [38] and it must be identified to $F = f_\pi$ as the pion decay constant. The quark kernel, with an implicit regularization, can be written in terms of the effective quark mass $\tilde{M}^* = m + <S> (P_R\tilde{U} + P_L\tilde{U}^\dagger)$ generated by the scalar field gap equation. In this expression, the degeneracy of the vacuum is exhibited in quark–antiquark scalar condensate that drives DChSB, being parameterized by constant $\tilde{U} = \mathrm{e}^{\mathrm{i}\vec{\pi}\cdot\sigma}$.

The quark propagator can be written as:

$$S_{0,c}^{-1}(x-y) = (\mathrm{i}\not{D} - \tilde{M}^*)\delta(x-y). \tag{6}$$

In expression (4) the following quantity with the color singlet chiral constituent quark currents has been defined:

$$\begin{aligned}\frac{\sum_q a_q \Gamma_q j_q(x,y)}{\alpha g^2} &= 2R(x-y)[\bar{\psi}(y)\psi(x) + \mathrm{i}\gamma_5\sigma_i\bar{\psi}(y)\mathrm{i}\gamma_5\sigma_i\psi(x)]\\ &\quad - \bar{R}^{\mu\nu}(x-y)\gamma_\mu\sigma_i[\bar{\psi}(y)\gamma_\nu\sigma_i\psi(x) + \gamma_5\bar{\psi}(y)\gamma_5\gamma_\nu\sigma_i\psi(x)],\end{aligned} \tag{7}$$

In this expression $\alpha = 2/9$ from the Fierz transformation, σ_i are the isospin Pauli matrices and combinations of the longitudinal and transversal parts of the gluon propagator were defined as:

$$R(x-y) = 3R_T(x-y) + R_L(x-y),$$
$$\bar{R}^{\mu\nu}(x-y) = g^{\mu\nu}(R_T(x-y) + R_L(x-y)) + 2\frac{\partial^\mu\partial^\nu}{\partial^2}(R_T(x-y) - R_L(x-y)). \tag{8}$$

With the background quark currents, different quark-couplings were found to emerge in the large quark effective mass expansion of the above determinant.

Before exhibiting the leading resulting interaction terms from the determinant expansion it is appropriate to describe the emerging picture of the procedure described so far. The BFM made possible to introduce (baryons) constituent quark currents differently from the quarks that were considered to form light mesons and the chiral condensate. Constituent quark currents have a closed gluon line, gluon cloud, attached by means of the gluon kernels in expression (7). These constituent quarks interact with light mesons, in particular to pions that might form a pion cloud if the complete leading terms from large N_c limit are considered [13, 38, 39].

3. Leading electromagnetic form factors

The large effective quark mass expansion of the determinant within the zero order derivative expansion [56] is performed in the following. The leading ($U(1)$ gauge invariant) momentum dependent couplings with the background electromagnetic field are the following:

$$\begin{aligned}
\mathcal{L}_A^{q-\pi} = {} & F_{s,\gamma}(K, Q, Q_1, Q_3) F_{\mu\nu}(Q_1) F^{\mu\nu}(Q_3) \pi_i(q_a) \pi_i(q_b) \bar{\psi}(K) \psi(K + Q + Q_1 + Q_3) \\
& + \mathrm{i}\epsilon_{ij3} F_{\mathrm{ps},\gamma}(K, Q, Q_1, Q_3) F_{\mu\nu}(Q_1) F^{\mu\nu}(Q_3) \pi_i(Q_1) \bar{\psi}(K) \sigma_j \gamma_5 \psi(K + Q + Q_1 + Q_3) \\
& + \mathrm{i}\epsilon_{jki} F_{V,\gamma}(K, Q, Q_1) F^{\mu\nu}(Q_1) \pi_j(q_a) (\partial_\mu \pi_k(q_b)) \bar{\psi}(K) \gamma_\nu \sigma_i \psi(K + Q + Q_1) \\
& + \mathrm{i}\epsilon_{ij3} F_{A,\gamma}(K, Q, Q_1) F^{\mu\nu}(Q_1) \partial_\mu \pi_i(Q) \bar{\psi}(K) \gamma_5 \gamma_\nu \sigma_j \psi(K + Q + Q_1),
\end{aligned} \tag{9}$$

where (K, Q, Q_1, Q_3) are, respectively, the incoming constituent quark momentum K, incoming or outgoing pion momentum Q and incoming or outgoing electromagnetic field momenta Q_1, Q_3, being that in the couplings with two pions $Q = q_a + q_b$. σ_i are the Pauli matrices. The resulting form factors will be written below in such a way to not exhibit the particular electromagnetic coupling to pions or constituent quarks but in terms of an *averaged* electromagnetic coupling of charged pions and quarks in each of the resulting effective coupling. This is clear by noting that the charge operator $\hat{Q}$ extracts both the pion and the quark electric charges in the traces taken in the expressions below. In particular: $\hat{Q}\sigma_i\sigma_j\pi_i j_j = ([\hat{Q}, \sigma_i]\sigma_j + \sigma_i\hat{Q}\sigma_j)\pi_i j_j$, where $[\hat{Q}, \sigma_i]$ yields pion charges and $\hat{Q}\sigma_j$ the quark charge.

The form factors in this expression are given by:

$$F_{s,\gamma}(K, Q, Q_1, Q_3) = \frac{80}{9} d_1 N_c (\alpha g^2) e^2 F F_4^t(K, Q, Q_1, Q_3), \tag{10}$$

$$F_{\mathrm{ps},\gamma}(K, Q, Q_1, Q_3) = \frac{64}{3} d_1 N_c (\alpha g^2) e^2 F F_4^t(K, Q, Q_1, Q_3), \tag{11}$$

$$F_{V,\gamma}(K, Q, Q_1) = \frac{64}{3} d_1 N_c F (\alpha g^2) e F_5^t(K, Q, Q_1), \tag{12}$$

$$F_{A,\gamma}(K, Q, Q_1) = \frac{64}{3} d_1 N_c F(\alpha g^2) e F_5^t(K, Q, Q_1). \tag{13}$$

In these expressions $d_n = \frac{(-1)^{n+1}}{2n}$ are the coefficients from the expansion of the determinant, $F = 92\,\mathrm{MeV}$ and $N_c = 3$.

Besides the couplings above there are unusual couplings to the electromagnetic field that also break chiral and isospin symmetries explicitly. These couplings, when associating the constituent quark currents to a corresponding quark–antiquark meson with the same quantum numbers, correspond to sort of mixing couplings induced by the photon. The leading ones can be written as:

$$\begin{aligned}\mathcal{L}_{Aj} = &-\mathrm{i}\epsilon_{ij3} F F_6(K, Q, Q_1) A_\mu (\partial^\mu \pi_i(q_a)) \pi_j(q_b) \overline{\psi}(K) \psi(Q_T)\\ &-2\mathrm{i}\epsilon_{ij3} F F_8(K, Q, Q_1) A_\mu \partial^\mu \pi_i(Q) \overline{\psi}(K) \mathrm{i}\gamma_5 \sigma_j \psi(Q_T)\\ &+\mathrm{i}J_{ijk} F F_7(K, Q, Q_1) A_\mu \pi_i(q_a) \pi_j(q_b) \overline{\psi}(K) \mathrm{i}\gamma^\mu \sigma_k \psi(Q_T)\\ &+2\mathrm{i}\epsilon_{ji3} F F_9(K, Q, Q_1) A_\mu \pi_j(Q) \overline{\psi}(K) \gamma^\mu \gamma_5 \sigma_i \psi(Q_T),\end{aligned} \tag{14}$$

where $Q_T = K + Q + Q_1$ is the total momentum of the outgoing constituent quark, $J_{ijk} = \delta_{ij\neq 3}\delta_{k3} + \delta_{jk\neq 3}\delta_{i3} - \delta_{ik\neq 3}\delta_{j3} + \frac{\mathrm{i}}{3}\epsilon_{ijk}$. These four resulting coupling constants in $\mathcal{L}_{Aj}$ are not explicitly gauge invariant although they may become gauge invariant by considering higher order momentum dependent terms such as to rewrite them in terms of a covariant derivative $\bar{D}_\mu = \partial_\mu + \mathrm{i}e\bar{g}A_\mu(Q_1)$ instead of the photon field A_μ. In this expression $\bar{g}$ an averaged electromagnetic coupling of charged quarks and pions. Below only the photon couplings of these terms will be of interest. These last form factors were defined in terms of functions $H_{6,8}$ and $H_{7,9}$:

$$F_6(K, Q, Q_1) = 4 d_1 e N_c K_0 H_6^t(K, Q, Q_1), \tag{15}$$

$$F_8(K, Q, Q_1) = 4 d_1 e N_c K_0 H_8^t(K, Q, Q_1), \tag{16}$$

$$F_7(K, Q, Q_1) = 4 d_1 e N_c K_0 H_7^t(K, Q, Q_1), \tag{17}$$

$$F_9(K, Q, Q_1) = 4 d_1 e N_c K_0 H_9^t(K, Q, Q_1), \tag{18}$$

where $K_0 = \alpha g^2$.

The diagrams for expressions (9) and (14) are presented in figure 1. The incoming constituent quark has momentum K and $K + Q_T$ is the outgoing constituent quark momentum, where Q_T is the total momentum transferred by the pion(s) and photon(s) to the constituent quark. Q denotes the total transferred momentum from the pion(s) and Q_1, Q_3 are each of the photon momenta. To describe the pion form factors the appropriate incoming and outgoing pion momenta for figures 1(a) and (c) must be considered such that $q_a + q_b = Q$.

The functions $F_i^t(K, Q, Q_1)$, $H_i^t(K, Q, Q_j)$ used above were defined as:

$$F_3^t(K, Q, Q_1) = \frac{1}{2}(F_3(K, Q, Q_1) + F_3(k, Q_1, Q), \tag{19}$$

$$F_4^t(K, Q, Q_1, Q_3) = \frac{1}{2}(F_4(K, Q, Q_1, Q_3) + \tilde{F}_4(K, Q_3, Q_1, Q), \tag{20}$$

$$F_5^t(K, Q, Q_1) = \frac{1}{2}(F_5(K, Q, Q_1) + F_5(K, Q_1, Q), \tag{21}$$

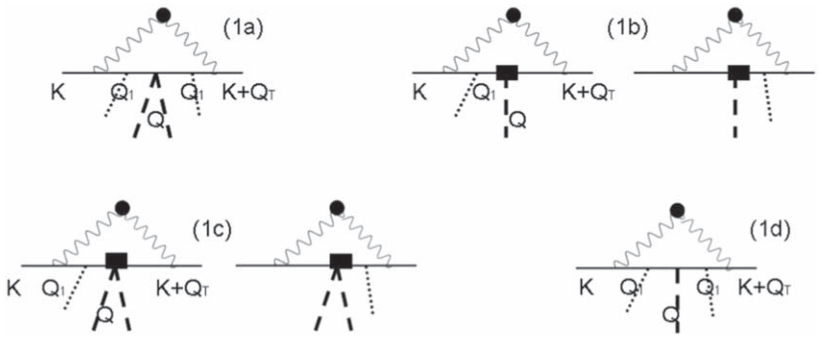

Figure 1. Diagrams (1a),(1b),(1c),(1d) correspond to the couplings of expressions (9, 14). The wavy line with a full dot is a (dressed) non perturbative gluon propagator, the solid lines stand for quarks, dashed lines for pions and the dotted line stands for the photon strength tensor. A full square in a vertex represent momentum dependent pion coupling. The momenta of each of the particles are indicated by K (quarks), Q (pion(s)) and Q_1 (photons).

$$H_6^t(K, Q, Q_1) = \frac{1}{2}(F_6(K, Q, Q_1) + F_6(K, Q_1, Q), \tag{22}$$

$$H_8^t(K, Q, Q_1) = \frac{1}{2}(F_6(K, Q, Q_1) - F_6(K, Q_1, Q), \tag{23}$$

$$H_7^t(K, Q, Q_1) = \frac{1}{2}(F_7(K, Q, Q_1) + F_7(K, Q_1, Q), \tag{24}$$

$$H_9^t(K, Q, Q_1) = \frac{1}{2}(F_7(K, Q, Q_1) - F_7(K, Q_1, Q). \tag{25}$$

In these expressions the loop momentum integrals of each of the form factors can be written in Euclidean momentum space as:

$$\begin{aligned} F_3(K, Q, Q_1) = \int_k [k \cdot (k + Q_1) - M^{*2}]\tilde{S}_0(k)\tilde{S}_0(k + Q_1) \\ \times \tilde{S}_0(k + Q + Q_1)\bar{\bar{R}}(-k - K), \end{aligned} \tag{26}$$

$$\begin{aligned} F_4(K, Q_1, Q, Q_3) = \int_k [-k \cdot (k + Q_1 + Q) + M^{*2}]\tilde{S}_0(k)\tilde{S}_0(k + Q_1 + Q) \\ \times \tilde{S}_0(k + Q_1)\tilde{S}_0(k + Q_4)R(-k - K), \end{aligned} \tag{27}$$

$$\begin{aligned} \tilde{F}_4(K, Q, Q_1, Q_3) = \int_k [-(k^2 + k \cdot Q) + M^{*2}]\tilde{S}_0(k)\tilde{S}_0(k + Q)\tilde{S}_0(k + Q + Q_1) \\ \times \tilde{S}_0(k + Q_4)R(-k - K), \end{aligned} \tag{28}$$

$$F_5(K, Q, Q_1) = \int_k M^*\tilde{S}_0(k)\tilde{S}_0(k + Q_1)\tilde{S}_0(k + Q + Q_1)\bar{\bar{R}}(-k - K), \tag{29}$$

$$\begin{aligned} F_6(K, Q, Q_1) = \int_k [k \cdot (k + Q + Q_1) - M^{*2}]\tilde{S}_0(k)\tilde{S}_0(k + Q_1) \\ \times \tilde{S}_0(k + Q + Q_1)R(-k - K), \end{aligned} \tag{30}$$

$$F_7(K, Q, Q_1) = \int_k [3k^2 + k\cdot(4Q_1 + 2Q) + Q_5^2 - M^{*2}]\tilde{S}_0(k)\tilde{S}_0(k+Q_1) \times \tilde{S}_0(k+Q+Q_1)\bar{R}(-k-K), \tag{31}$$

where $M^* = m + < S >$, the momenta $Q_4 = Q_1 + Q + Q_3$, $Q_5^2 = Q_1^2 + Q\cdot Q_1$, $\int_k = \int \frac{d^4k}{(2\pi)^4}$ and $\bar{\bar{R}}(k) = 2R(k)$. The following function was defined: $\tilde{S}_0 = \frac{1}{k^2 + M^{*2}}$.

The complete momentum structures of form factors such as F_3 and F_4 present a non monotonic behavior that yield negative averaged quadratic radii in the vacuum in the case of constant effective mass M^* [11, 13]. To mend this behavior the resulting momentum dependence of the quark kernel for constant effective mass will be truncated so that the full quark propagator is exchanged by a truncated one given by:

$$S_0^{\rm tr}(k) \sim M^*\tilde{S}_0(k). \tag{32}$$

This truncation might be seen as to mimic a momentum dependent effective mass $\tilde{M}^*(k)$ that is typically obtained from Schwinger Dyson equations approach although it is not possible however to guarantee its behavior is quantitatively equivalent to results from SDE calculations. For the expressions above, the form factors F_3 and F_4 or $\tilde{F}_4$ will be considered in both exact and truncated forms. If one considers a momentum dependent effective mass $\tilde{M}^*(k)$ for the complete quark propagator, the truncation is equivalent to the following approximation for the corresponding part of functions F_3 and F_4 in Euclidean momentum space:

$$\frac{\not{k}^2 - (\tilde{M}^*(k)^2}{(k^2 + \tilde{M}^*(k)^2} \sim \frac{M^{*2}}{(k^2 + M^{*2})^2}. \tag{33}$$

It yields truncated form factors with monotonic behavior with pion momenta and it makes possible to obtain always positive quadratic mean radii with the correct order of magnitude. Numerical results for both the complete and the truncated expressions will be shown in sections (4.1) and (4.2).

4. Weak magnetic field

It is shown now that corrections induced by a weak background magnetic field to usual form factors are obtained by considering two different effects. Firstly the leading correction to the quark kernel and also by assuming the weak magnetic field (with respect to the constituent quark mass) is strong enough to show up in the electromagnetic couplings of the previous section. For that, the Landau gauge will be considered in which $A^\mu = -B_0\,(0, 0, x, 0)$.

The leading quark propagator dependence on the weak magnetic field, for equal up and down quark effective masses M^*, will not be derived in this work and it can be written as [57, 58]:

$$G(k) = S_0(k) + S_1(k)(eB_0) = \frac{\not{k} + M^*}{k^2 - M^{*2}} + i\gamma_1\gamma_2\frac{(\gamma_0 k^0 - \gamma_3 k^3 + M^*)}{(k^2 - M^{*2})^2}\hat{Q}(eB_0). \tag{34}$$

By substituting the vacuum quark propagator by a $G(k)$ different anisotropic weak magnetic field-dependent corrections to pion constituent quark couplings appear. For some of the leading pion-constituent quark couplings found in [13] however the first order correction to the quark propagator $S_1(k)$ does not contribute in the leading terms, i.e. linear in (eB_0). In the second order in $(eB_0)^2$ more terms arise but these will not be considered below.

The correction $S_1(k)$ to the quark propagator yield the following anisotropic contributions from the leading order determinant for the axial constituent quark current expansion:

$$\mathcal{L}_{S,B} = G^B_{A,1}(Q)\epsilon^{0\rho\mu 3}\partial_\rho\pi_i(Q)\bar{\psi}(K)\gamma_5\gamma_\mu\sigma_i\psi(K, Q) + \mathrm{i}\epsilon_{ij3}G^B_{V,2}(Q)\partial_0\pi_i(Q)\bar{\psi}(K)\gamma_3\sigma_j\psi(K, Q) + G^B_{V,1}(Q)\epsilon^{12\mu\rho}\left(\delta_{ij}\delta_{k3} + \frac{\mathrm{i}}{3}\epsilon_{ijk}\right)\partial_\mu(\pi_i(q_a)\pi_j(q_b))\bar{\psi}(K)\gamma_5\gamma_\rho\sigma_k\psi(K, Q), \tag{35}$$

where $\epsilon^{\nu\sigma\mu\rho}$ is the Levi Civita tensor, ∂_0 stands for $\partial/\partial t$, and the effective coupling constants were defined with the trace of internal indices with (off shell) zero external momenta as:

$$\frac{G^B_{A,1}}{\left(\frac{eB_0}{M^{*2}}\right)} = \frac{2}{3}\frac{G^B_{V,1}}{\left(\frac{eB_0}{M^{*2}}\right)} = \frac{1}{6}\frac{G^B_{V,2}}{\left(\frac{eB_0}{M^{*2}}\right)} \simeq \frac{8}{3}d_1 N_c F M^{*2}(\alpha g^2)F^t_5(0, 0, 0), \tag{36}$$

where $F^t_5(K, Q, 0)$ is given above in expressions (21) and (29). There are terms depending on momenta in the direction transversal to the magnetic field, a term along the magnetic field and terms dependent rather on the pion energy $\partial_0\pi_i$.

Other leading contributions, however, arise from the zeroth order quark propagator of expression (34), $S_0(k)$, and a background photon. By considering the pion momentum Q, or $Q = q_a + q_b$ for the two-pion couplings, the resulting corrections to pion-constituent quark form factors are obtained from expressions (9). They are given by:

$$\begin{aligned}\mathcal{L}^{q-\pi}_B &= F^B_s(K, Q)F\pi_i(q_a)\pi_i(q_b)\bar{\psi}(K)\psi(K + Q)\\ &+ \epsilon_{ij3}F^B_{\mathrm{ps}}(K, Q)F\pi_i(Q)\bar{\psi}(K)\sigma_j\mathrm{i}\gamma_5\psi(K + Q)\\ &+ \mathrm{i}\epsilon_{jki}F^B_V(K, Q)\pi_j(q_a)(\partial_x\pi_k(q_b))\bar{\psi}(K)\gamma_2\sigma_i\psi(K, Q)\\ &+ \mathrm{i}\epsilon_{ij3}F^B_A(K, Q)\partial_x\pi_i(Q)\bar{\psi}(K)\mathrm{i}\gamma_5\gamma_2\sigma_j\psi(K, Q),\end{aligned} \tag{37}$$

where:

$$F^B_{\mathrm{ps}}(K, Q) = \frac{12}{5}F^B_s(K, Q) = \left(\frac{eB_0}{M^{*2}}\right)^2\frac{64}{3}d_1 N_c M^{*4}(\alpha g^2)F_4(K, Q, Q_1 = Q_3 = 0), \tag{38}$$

$$F^B_A(K, Q) = \frac{F^B_V(K, Q)}{3} = \left(\frac{eB_0}{M^{*2}}\right)\frac{64}{3}d_1 N_c F M^{*2}(\alpha g^2)F_5(K, Q, Q_1 = 0). \tag{39}$$

In these expressions, note there are weak magnetic field induced corrections to the pion scalar and vector form factors, $F^B_s(K, Q)$ and $F^B_V(K, Q)$. In spite of the direct apparent proportionality to the pseudoscalar and axial form factors, $F^B_{\mathrm{ps}}(K, Q)$ and $F^B_A(K, Q)$, respectively, they must be considered with the appropriated momentum dependence of each of the external lines. In terms of the external momenta K, Q they are the same functions however, and the scalar and vector ones will not be drawn in the figures in this work.

However the quark effective mass M^* also receives corrections from the weak magnetic field in the scalar gap equation and, although this will not addressed with details in the present work, for the numerical estimates this was taken into account by means of two values of the effective mass M^*. These expressions provide numerical values one or two orders of magnitude smaller than of the original pion—constituent quark couplings because they have multiplicative extra factors B_0 or B_0^2 that can be factorized in dimensionless constants such as as eB_0/M^{*2} or $(eB_0)^2/M^{*4}$ within the current large quark effective mass regime. These make explicit that the B_0 induced corrections are considerably smaller than the original coupling constants and form factors.

The unusual isospin and chiral symmetry breaking electromagnetic pion-quark couplings from expression (14) generate magnetic field induced mixing pion couplings breaking explicitly chiral and isospin symmetries. For pion momentum Q, or $Q = q_a + q_b$ in the two pion couplings, it yields:

$$\begin{aligned}\mathcal{L}_{Aj} = &-\mathrm{i}\epsilon_{ij3}F_{6B}(K,\, Q)(\partial^y\pi_i(q_a))\pi_j(q_b)\bar{\psi}(K)\psi(K+Q)\\ &- 2\mathrm{i}\epsilon_{ij3}F_{8B}(K,\, Q)\partial^y\pi_i(Q)\bar{\psi}(K)\mathrm{i}\gamma_5\sigma_j\psi(K+Q)\\ &+ \mathrm{i}J_{ijk}F_{7B}(K,\, Q)\pi_i(q_a)\pi_j(q_b)\bar{\psi}(K)\mathrm{i}\gamma^2\sigma_k\psi(K+Q)\\ &+ 2\mathrm{i}\epsilon_{ji3}F_{9B}(K,\, Q)\pi_j(Q)\bar{\psi}(K)\mathrm{i}\gamma^2\gamma_5\sigma_j\psi(K+Q).\end{aligned} \tag{40}$$

To write down these expressions, the magnetic field from the Landau gauge must be written for $\hat{x} = \mathrm{i}\partial_{Q_{1,x}} = \mathrm{i}\partial/\partial Q_1^x$ at zero transferred electromagnetic field momentum $Q_1 = 0$.

The following functions were used in expressions (40), written in terms of the momentum derivative:

$$F_{6B}(K,\, Q) = \left(\frac{eB_0}{M^{*2}}\right)4d_1FN_cM^{*2}K_0(\partial_{Q_{1,x}}H_6^t(K,\, Q,\, Q_1)_{Q_1=0}, \tag{41}$$

$$F_{8B}(K,\, Q) = \left(\frac{eB_0}{M^{*2}}\right)4d_1FN_cM^{*2}K_0(\partial_{Q_{1,x}}H_8^t(K,\, Q,\, Q_1)_{Q_1=0}, \tag{42}$$

$$F_{7B}(K,\, Q) = \left(\frac{eB_0}{M^{*2}}\right)4d_1FN_cM^{*2}K_0(\partial_{Q_{1,x}}H_7^t(K,\, Q,\, Q_1)_{Q_1=0}, \tag{43}$$

$$F_{9B}(K,\, Q) = \left(\frac{eB_0}{M^{*2}}\right)4d_1FN_cM^{*2}K_0(\partial_{Q_{1,x}}H_9^t(K,\, Q,\, Q_1)_{Q_1=0}. \tag{44}$$

By comparing expressions (36) and (39) the following exact ratios for off shell momenta are obtained:

$$\frac{G_{A,1}^B(0,\, 0)}{F_A^B(0,\, 0)} = 2\frac{G_{V,1}^B(0,\, 0)}{F_V^B(0,\, 0)} = \frac{G_{V,1}^B(0,\, 0)}{G_{V,2}^B(0,\, 0)} = \frac{1}{8}. \tag{45}$$

These ratios are simple numerical factors. The momentum dependence of only one of these form factors, $F_A^B(0,\, Q)$, will be explicitly shown below.

4.1. Numerical results

In the following, numerical estimations for the above form factors will be shown. Two gluon propagators will be considered, firstly a transversal one from Tandy–Maris $D_I(k)$ [51] and the other is an effective longitudinal confining one by Cornwall $D_{II}(k)$ [45]. Both of them yield DChSB in the gap equation for the scalar auxiliary field being that:

$$g^2\tilde{R}^{\mu\nu}(k) \equiv h_aD_a^{\mu\nu}(k), \tag{46}$$

where $D_a^{\mu\nu}(k)$ $(a = I,\, II)$ are the chosen gluon propagators whose expressions are the following:

$$D_I(k) = \frac{8\pi^2}{\omega^4}D\mathrm{e}^{-k^2/\omega^2} + \frac{8\pi^2\gamma_m E(k^2)}{\ln[\tau + (1 + k^2/\Lambda_{\mathrm{QCD}}^2)^2]}, \tag{47}$$

$$D_{II}(k) = K_F/(k^2 + M_k^2)^2, \tag{48}$$

where for the first expression $\gamma_m = 12/(33 - 2\bar{N}_f)$, $\bar{N}_f = 4$, $\Lambda_{\rm QCD} = 0.234$ GeV, $\tau = e^2 - 1$, $E(k^2) = [1 - \exp(-k^2/[4m_t^2])/k^2$, $m_t = 0.5\,{\rm GeV}$, $\omega = 0.5\,{\rm GeV}$, $D = 0.55^3/\omega$ (GeV^2); and for the second expression $K_F = (2\pi\, M_k/(3k_e))^2$ where $k_e = 0.15$ and $M_k = 220\,{\rm MeV}$. In expression (46) h_a is a constant factor already considered in previous works [39, 12] to fix the quark gluon (running) coupling constant such as to reproduce a particular value for one of the resulting effective coupling constant, for example the pion axial coupling constant $g_A\ h_a = 1$ or the coupling constant of the ρ-vector meson to constituent quark $g_\rho h_a \simeq 12$. In the present work they were fixed with the same convention of most part of [13] for the pion constituent quark form factors, i.e. $h_I = 0.82$ and $h_{II} = 0.3$. As a consequence of this choice, some of the resulting normalization values, for example for $< r^2 >_A$, might be different from values exhibited elsewhere. The magnetic field was chosen to be such that $eB_0/M*^2 = 0.1$ or 0.2 in terms of a constituent quark mass $M^* = 0.31$ GeV or $M^* = 0.35\,{\rm GeV}$. This yields $eB_0 \sim 10^{18}{\rm G} \sim m_\pi^2$. All the form factors will be plotted for an incoming constituent quark with off shell zero momentum, $K = 0$. Averaged quadratic radii and coupling constants are usually defined in such limit or close to it. A more complete investigation on form factors would involve more extensive analysis of their momentum distributions and this is outside the scope of this work.

Complete expressions for the axial and pseudoscalar form factors, including both their values in the vacuum and the weak B_0 correction, might be written as:

$$G_A^B(Q) = G_A^{M^*}(Q) + \left(\frac{eB_0}{M*^2}\right)\frac{F_A^B(0, Q)}{(eB_0/M*^2)}. \tag{49}$$

$$G_{\rm ps}^B(Q) = G_{\rm ps}^{M^*}(Q) + \left(\frac{eB_0}{M*^2}\right)^2\frac{F_{\rm ps}^B(0, Q)}{(eB_0/M*^2)^2}, \tag{50}$$

where $G_A^{M^*}(Q)$ is the form factor presented and investigated in [13], $G_A^{M^*}(Q) = G_A^U(0, Q)$, and similarly $G_{\rm ps}^{M^*}(Q) = G_{\rm ps}^U(0, Q)$. In figure (2) the axial and non truncated pseudoscalar form factors in the vacuum—from [13]—and with a weak magnetic field $eB_0/M*^2 = 0.2$ from expressions (49) and (50) are presented for the two gluon propagators with an unique value of the quark effective mass $M^* = 310\,{\rm MeV}$. It is seen the non monotonic behavior of the pseudoscalar form factor in the vacuum, from [13], that also appears for its magnetic field correction by means of $F_4(K, Q{,}0{,}0)$, equation (38). This behavior will be shown in the next section to be responsible for the negative pseudoscalar a.q.r. and, to mend that, the truncation of the quark propagator for constant quark effective mass was proposed. Note however the B_0 correction to the pseudoscalar form factor has a factor $\left(\frac{eB_0}{M*^2}\right)^2$ that is considerably smaller than $\left(\frac{eB_0}{M*^2}\right)$ in the B_0 induced correction to the axial form factor.

In figure (3) the magnetic field induced correction to the axial form factor $F_A^B(K, Q)/(eB_0/M*^2)$ is shown as function of the pion momentum $Q = |Q|$ for two different quark effective masses M^*. The contribution $F_A^B(K, Q)$ for the form factor in the figure is divided by a factor $(eB_0/M*^2)$ (=0.1) to make easier the interpretation and the comparison of each of the contributions for any small value of $(eB_0/M*^2)$. The effective mass M^* is, however, kept constant. The following fits of the normalized axial form factor $g_A(Q)/g_A(0)$ [20] are plotted in dotted lines:

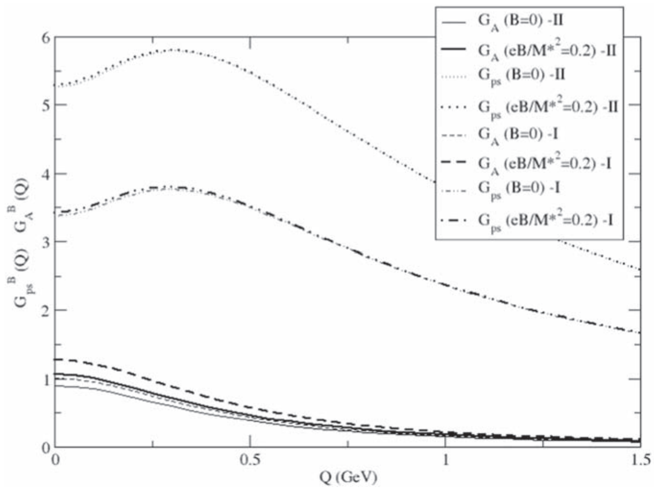

Figure 2. Axial and (non truncated) pseudoscalar form factors for $B_0 = 0$, from [13], and for $(eB_0/M{*}^2) = 0.2$ with different contributions: from the quark mass dependence on the weak magnetic field and from the correction to the form factor from expressions (49) and (50) for the two gluon propagators $D_I(k)$ and $D_{II}(k)$ and with $M^* = 310$ MeV. $h_I = 0.82$ and $h_{II} = 0.3$.

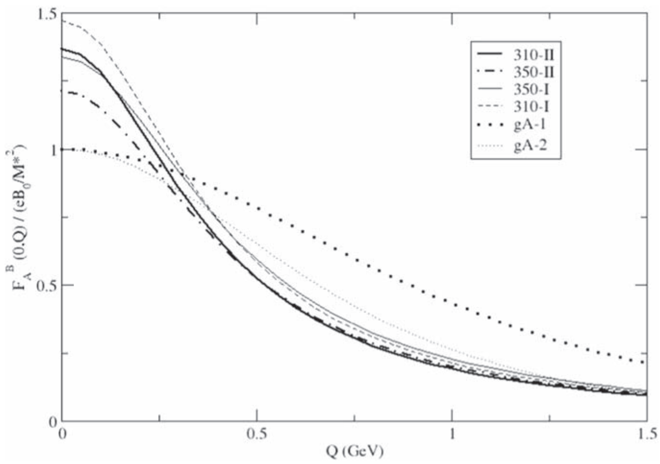

Figure 3. In this figure the leading magnetic field correction to the axial form factor $F_A^B(0, Q)/(eB_0/M{*}^2)$ as a function of the pion momentum for the usual pion field is presented for the two gluon propagators D_I (k) (thin lines) and $D_{II}(k)$ (thick lines). Different values of the quark effective mass are considered $M^* = 350$ MeV in dotted–dashed lines, $M^* = 310$ MeV in dashed lines. $h_I = 0.82$ and $h_{II} = 0.3$. The (thick and thin) dotted lines correspond to two different fits for the normalized axial form factor $g_A(Q)$ [20] as discussed in the text.

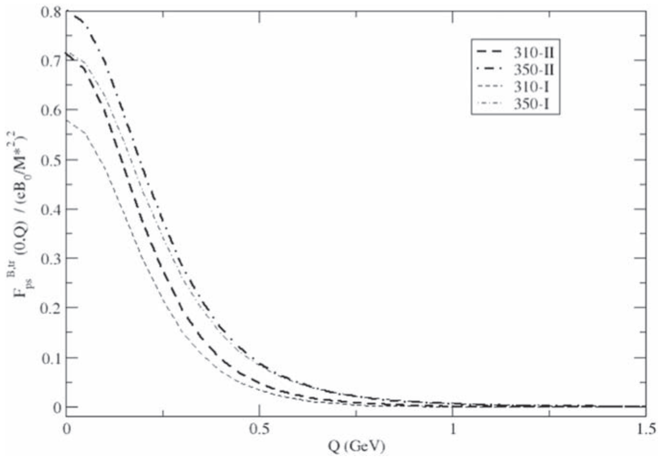

Figure 4. In this figure the leading—truncated—magnetic field correction to the pseudoscalar form factor $F_{ps}^{B}(0,\, Q)/(eB_0/M^{*2})^2$ as a function of the pion momentum for the usual pion field is presented for the two gluon propagators $D_I(k)$ (thin lines) and $D_{II}(k)$ (thick lines). Different values of the quark effective mass are considered $M^* = 350$ MeV in dotted–dashed lines, $M^* = 310$ MeV in dashed lines. $h_I = 0.82$ and $h_{II} = 0.3$.

$$g_A(Q) = \frac{1}{\left(1 + \frac{Q^2}{M_A^2}\right)^2},$$

being $M_A = 1.36$ GeV for thick dotted line and $M_A = 1.03$ GeV for thin dotted line. The axial pion coupling constant to constituent quark is usually considered to be $g_A = 0.8$ or 1 [19, 37]. Without further assumptions about the quark-gluon coupling constant, the weak magnetic field correction to the axial form factor is smaller for the gluon propagator $D_{II}(k)$ than for $D_I(k)$ and this is simply due to the overall strength of the corresponding propagator and quark-gluon coupling constant.

In figure (4) the weak magnetic field anisotropic truncated correction to the pseudoscalar form factor, divided by $(eB_0/M^{*2})^2$, is presented for the two gluon propagators as a function of the pion momentum and for different values of M^*. Although the value of the pseudoscalar coupling constant is of the order of 10 times the value of the axial coupling constant the weak magnetic field corrections calculated in these figures are basically of the same order of magnitude.

The unusual weak magnetic field induced anisotropic form factors, $F_{6B}(0,\, Q)$ and $F_{7B}(0,\, Q)$, are exhibited in figure (5) as function of the pion momentum , $Q = |Q_x|$, for the two gluon propagators with $M^* = 310$ MeV. They disappear in the zero pion momentum limit. The dependence of the form factor $F_{7B}(0,\, Q)$ on the gluon propagator is seemingly larger than for the previous form factors analysed in the present work. Although the order of magnitude might be larger than the corresponding electromagnetic form factors (14) these values must be multiplied by $(eB_0)/M^{*2}$.

4.2. Averaged quadratic radii

From the above form factors, weak-B_0 induced anisotropic corrections to the axial and pseudoscalar constituent quark a.q.r. can be obtained. The magnetic field along the $\hat{z}$ direction

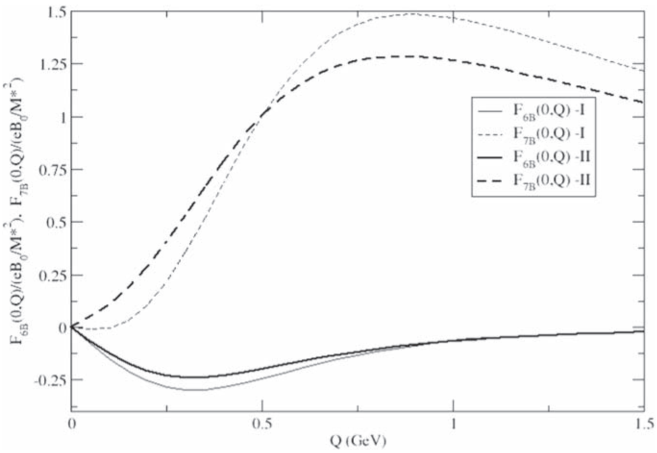

Figure 5. The leading magnetic field induced form factors for the pion coupling to quark currents $F_{6P-B}(0, Q)$, $F_{7P-B}(0, Q)$, divided by $(eB_0)/M^{*2}$, as functions of the pion momentum, $Q = |Q_x|$, are presented for the two gluon propagators $D_I(k)$ (thin lines) and $D_{II}(k)$ (thick lines) and $M^* = 310$ MeV. $h_I = 0.82$ and $h_{II} = 0.3$.

can be chosen to be $A_\mu = -B_0(0, y, x, 0)/2$ for which it can be obtained a symmetrized result. Because the B_0 corrections to the form factors are dimensionless as defined above, the a.q.r. can be defined simply as:

$$\Delta < r^2>_A = -6 \frac{\mathrm{d}F_A^B(0, Q_\pi)}{\mathrm{d}Q_\pi^2}\Bigg|_{Q_\pi=0},$$
$$\Delta < r^2>_{\mathrm{ps}} = -6 \frac{\mathrm{d}F_{\mathrm{ps}}^B(0, Q_\pi)}{\mathrm{d}Q_\pi^2}\Bigg|_{Q_\pi=0}. \tag{51}$$

that correspond to corrections in the plane $x - y$ perpendicular to the constant weak magnetic field. The corresponding a.q.r. in the vacuum by considering the same method have been presented in [13]. The resulting value for the axial and pseudoscalar square radii are obtained by adding their values in the vacuum to the magnetic field correction. The quark effective mass M however is kept constant in spite of its eventual magnetic field dependence. These values are obtained by:

$$<r^2>_A^B = <r^2>_A + \Delta < r^2>_A|_{x-y}, \tag{52}$$

$$<r^2>_{\mathrm{ps}}^B = <r^2>_{\mathrm{ps}} + \Delta < r^2>_{\mathrm{ps}}|_{x-y}, \tag{53}$$

where it was emphasized the magnetic field corrections stand only in the plane perpendicular to the weak constant magnetic field.

In figure (6) the axial quadratic radii extracted from [13] are compared with the anisotropic B_0 induced contributions above, equation (51), as functions of the quark effective mass for each of the gluon propagators. Note however that the numerical values presented in [13] were obtained by considering different values for h_I and h_{II}. Therefore there is a simple multiplicative constant to relate the (vacuum) values for $< r^2 >_A$ of figure 6 to the values exhibited in the corresponding figure for $< r^2 >_A^U$ from [13]. The values of the magnetic field induced anisotropic contribution exhibited in these figures must be multiplied by eB_0/M^{*2} to

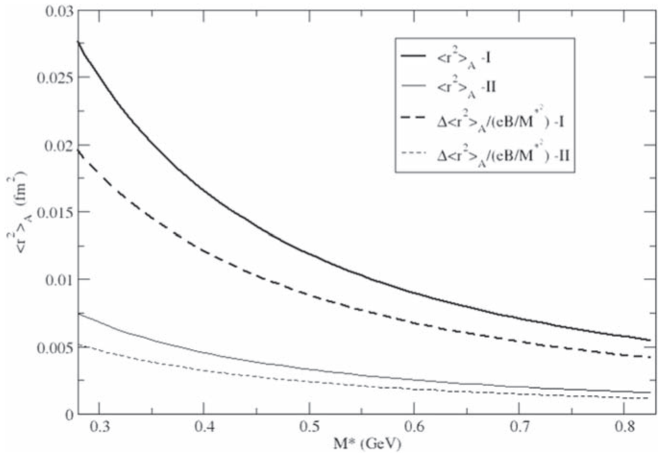

Figure 6. The axial squared radius from [13] (thin and thick continuous lines) and anisotropic weak magnetic field induced corrections (dashed lines) for the gluon propagators $D_I(k)$ and $D_{II}(k)$ as functions of the effective quark mass from the gap equation. $h_{II} = 0.3$. The weak magnetic field corrections from expressions (51) are shown divided by the factor eB/M^{*2}.

be added to the values $< r^2 >_A$ in the vacuum as shown in expression (52). For the sake of comparison it is interesting to quote previous estimations of the constituent quark quadratic radii to be $\sqrt{<r^2>} \simeq 0.2 - 0.3\,\mathrm{fm}$ [19, 59], being the relative weak B_0 correction is of the relative order of magnitude of eB_0/M^{*2} with respect to the corresponding value in the vacuum. The axial a.q.r. is, as expected, considerably smaller than the corresponding nucleon's a.q.r. $\sqrt{<r^2>_A} = 0.639 - 0.68$ fm [20].

The weak B_0 pseudoscalar square radius, $< r^2 >_{\mathrm{ps}}$ from [13] for the truncated and non truncated expressions, and the anisotropic B_0 induced correction, $\Delta < r^2 >_{\mathrm{ps}}$, from expression (51), are exhibited in figure (7) as functions of the quark gap effective mass M^* for $D_I(k)$ and $D_{II}(k)$. Note the sign minus in front of the non-truncated pseudoscalar curves in thin continuous and dashed lines. This behavior is due to the non monotonic behavior of the form factor at low momenta shown in figure 2. The truncated expressions correct this (low momenta) behavior. The anisotropic weak magnetic field corrections $\Delta_B < r^2 >_{\mathrm{ps}}$ yield very small values with respect to their values in the vacuum because of the factor $(eB_0/M^{*2})^2$.

Finally the scalar and vector pion-constituent quark couplings provide the corresponding pion form factors and averaged quadratic radii, $<r^2>_\pi^s$ and $<r^2>_\pi^V$. There are anisotropic corrections to the pion (strong) vector and scalar square radii that are similar and proportional to these axial and pseudoscalar a.q.r. shown above as obtained from the expressions (38) and (39). By taking into account the different pion external momenta, they are related by:

$$\Delta < r^2>_V^\pi|_{x-y} = 3\Delta < r^2>_A|_{x-y},$$
$$\Delta < r^2>_s^\pi|_{x-y} = \frac{5}{12}\Delta < r^2>_{\mathrm{ps}}|_{x-y}. \tag{54}$$

These quantities have been investigated extensively in the last years [26, 27]. There still are large uncertainties in the theoretical descriptions/values as listed from several different estimations that are in the following ranges [26]:

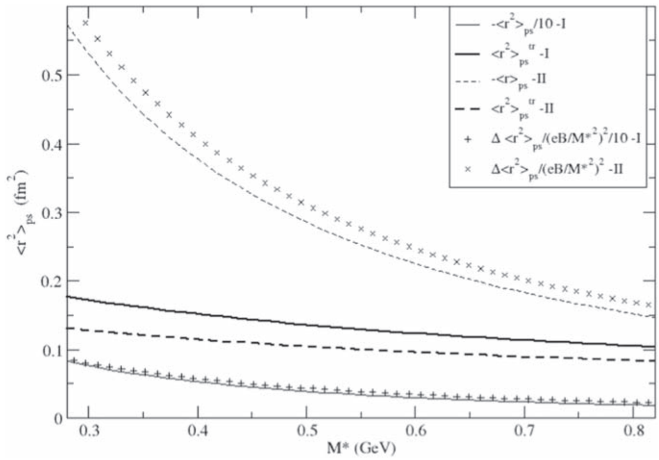

Figure 7. The pseudoscalar squared radius, untruncated and truncated expressions, from [13] (continuous and dashed lines) and the anisotropic magnetic field induced correction for the two gluon propagators, with $+$ and $\times$, as functions of the effective quark mass from the gap equation. The case of untruncated $<r^2>_{\rm ps}$ have a sign minus and the case of $D_I(k)$ is divided by a factor 10. $h_I = 0.82$ and $h_{II} = 0.3$ The weak magnetic field corrections are shown divided by the factor $(eB/M^{*2})^2$.

$$
\begin{aligned}
&<r^2>_\pi^s \sim 0.481 - 0.637 \text{ fm}^2,\\
&<r^2>_\pi^V \sim 0.310 - 0.494 \text{ fm}^2.
\end{aligned}
\tag{55}
$$

The form factors momentum dependence are explicit in expressions (9) and (37). Numerical values are read from figures (6) and (7). The absolute values and magnetic field induced corrections for the vector and scalar a.q.r. read from the figures (6) and (7) are smaller than lattice estimations (55). As noted above, the weak magnetic field induced correction to the scalar a.q.r. receives a contribution one order of magnitude smaller than the correction to the vector a.q.r. due to the corresponding factors $(eB_0/M^{*2})^2$ and (eB_0/M^{*2}). Some of the unusual couplings presented in expressions (14) and (35) might also contribute although with considerably weaker strength.

5. Summary

The leading weak magnetic field induced corrections to pion and constituent quark Strong form factors were derived from the quark–quark interaction mediated by a nonperturbative one gluon exchange. Weak magnetic field means weak with respect to a hadron mass scale, $eB_0/M^{*2} < 1$, that in fact may correspond to large absolute values, $eB_0 \sim m_\pi^2$. Magnetic field induced corrections to axial and pseudoscalar constituent quark averaged quadratic radii and to scalar and vector pion a.q.r. were also calculated. The (relatively) weak magnetic field expansion allows to extract analytical expressions for the corresponding form factors and effective coupling constants. This makes explicit the involved physical effects behind observables. Two types of weak magnetic field corrections were investigated. Firstly, the usual linear term from the quark propagator for a magnetic field along the $\hat{z}$ direction.

Secondly, the photon coupling to pion and constituent quark vertices might give rise to a magnetic field correction. It has been shown that this second effect provides larger contributions. In both cases, magnetic field effects were found to appear in two ways. In one hand, the induced corrections to the form factors and coupling constants were found to factorize in multiplicative constant factors eB_0/M^{*2}. On the other hand the magnetic field correction to the quark effective mass obtained from the gap equation also was taken into account. This second contribution was taken into account in the numerical estimations by considering two values of the quark effective mass, one for the absence of magnetic field and the other for finite B_0, $M^* = 310$ and 350 MeV. Besides the magnetic field induced corrections to usual scalar, pseudoscalar, vector and axial pion interactions with constituent quarks, two unusual photon couplings were also found, those proportional to the functions $F_6\,(K, Q)$ and $F_7\,(K, Q)$. These couplings break chiral and isospin symmetry accordingly and they yield magnetic field induced anisotropies in the pion momentum or in the constituent quark-currents seen in expression (40).

Relations between the resulting form factors and their weak magnetic field corrections for off shell zero momenta were also presented. Numerical results for the form factors were shown with the complete quark kernel momentum structure and also with a truncated propagator. The main reason for truncating some of the expressions is to avoid the negative a.q.r. obtained in some cases for the complete quark kernel with constant quark effective mass. Besides the correct sign, the truncation of the quark propagator provides form factors (F_3^{tr} and F_4^{tr}) and a.q.r. of the expected order of magnitude when compared to the non truncated ones, to nucleons form factors obtained from a Schwinger Dyson calculation at the rainbown ladder level or experimental values. The corrections to vector and axial form factors due to the weak B_0 are not equal to each other as it could be expected because of the explicit isospin and chiral symmetry breakings. Because there are neither current experimental measurements nor other theoretical estimations of the magnetic field contribution to the hadrons form factors no further comparisons were possible. Finally, weak magnetic field anisotropic corrections to axial and pseudoscalar constituent quarks averaged quadratic radii were also calculated as functions of the quark effective mass. An increase of the quark effective mass value might occur due to usual magnetic field contribution. These quadratic radii decrease considerably with the values of M from 280 MeV to 780 MeV. The magnetic field induced corrections for the a.q.r. are however always smaller than the a.q.r. themselves by one or two orders of magnitude, by factors eB_0/M^{*2} or $(eB_0/M^{*2})^2$. The same relative contributions, with factors eB_0/M^{*2} and $(eB_0/M^{*2})^2$, were found for the vector and scalar pion a.q.r. being their values for $B_0 = 0$ very close to values quoted in the literature. The different gluon propagators were also found to provide different normalizations and behaviors. The difficulties of establishing unbambiguous or precise values and behavior for the quark gluon running coupling constant and non perturbative gluon propagator manifest mainly in the ambiguity of fixing the normalization values, for example the zero momentum values of the form factors by fixing the parameter h_a. The magnetic field effects on gluon propagator and on quark-gluon interaction were not considered in the present work. Note that in all the expressions of the form factors and momentum integrals, eg. expressions (26–31) or (38) and (39), there appears the quark effective mass $M^* = m + < S >$ for which the current quark mass m is a small contribution. Therefore the chiral limit, $m = 0$, will not produce meaningful differences in numerical values. The more general calculation for strong magnetic fields is intended to be investigated in a different work.

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References

[1] Andersen J O, Naylor W R and Tranberg A 2016 Phase diagram of QCD in a magnetic field: a review *Rev. Mod. Phys.* **88** 025001

[2] Miransky V A and Shovkovy I A 2015 Quantum field theory in a magnetic field: from quantum chromodynamics to graphene and Dirac semimetals *Phys. Rep.* **576** 1

[3] Ferrer E J, de la Incera V and Sanchez A 2011 Paraelectricity in magnetized massless QED *Phys. Rev. Lett.* **107** 041602

Ferrer E J, de la Incera V, Portillo I and Quiroz M 2014 New look at the QCD ground state in a magnetic field *Phys. Rev.* D **89** 085034

[4] Li C-F, Yang L, Wen X J and Peng G X 2016 Magnetized quark matter with a magnetic-field dependent coupling *Phys. Rev.* D **93** 054005

[5] Andreichikov M A, Orlovsky V D and Simonov Y A 2013 Asymptotic freedom in strong magnetic fields *Phys. Rev. Lett.* **110** 162002

[6] Rabhi A and Providencia C 2011 Quark matter under strong magnetic fields in chiral models *Phys Rev.* C **83** 055801

[7] Braghin F L 2016 SU(2) low energy quark effective couplings in weak external magnetic field *Phys. Rev* D **94** 074030

[8] Bzdak A and Skokov V 2012 Event-by-event fluctuations of magnetic and electric fields in heavy ion collisions *Phys. Lett.* B **710** 171

Skokov V V, Illarionov A Y and Toneev V D 2009 Estimate of the magnetic field strength in heavy-ion collision *Int. J. Mod. Phys.* A **24** 5925

[9] Kondratyev V N, Maruyama T and Chiba S 2001 Magnetic field effect on masses of atomic nuclei *Astrophys. J.* **546** 1137

[10] Pena Arteaga D, Grasso M, Khan E and Ring P 2011 Nuclear structure in strong magnetic fields: nuclei in the crust of a magnetar *Phys. Rev.* C **84** 045806

[11] Braghin F L 2018 Light vector and axial mesons effective couplings to constituent quarks *Phys. Rev.* D **97** 054025

[12] Braghin F L 2018 Constituent quark-light vector mesons effective couplings in a weak background magnetic field *Phys. Rev.* D **97** 014022

[13] Braghin F L 2019 Pion constituent quark coupling strong form factors: a dynamical approach *Phys. Rev.* D **99** 014001

[14] Beise E J 2005 The axial form factor of the nucleon *Eur.Phys. J.* A **24** 43

[15] Drechsel D and Walcher T 2008 Hadron structure at low Q^2 *Rev. Mod. Phys.* **80** 731

[16] Maris P and Roberts C D 2003 Dyson–Schwinger equations: a tool for hadron physics *Int. J. Mod. Phys.* E **12** 297

Tandy P 1997 Hadron physics from the global color model of QCD *Prog. Part. Nucl. Phys.* **39** 297

[17] Yamazaki T *et al* 2009 Nucleon form factors with 2 + 1 flavor dynamical domain-wall fermions *Phys. Rev.* D **79** 114505

[18] Alexandrou C *et al* 2017 Nucleon axial form factors using $Nf = 2$ twisted mass fermions with a physical value of the pion mass *Phys. Rev.* D **96** 054507

[19] Vogl U and Weise W 1991 The Nambu and Jona-Lasinio model: its implications for hadrons and nuclei *Prog. Part. Nucl. Phys.* **27** 195

[20] Gupta R, Jang Y-C, Lin H-W, Yoon B and Bhattacharya T 2017 Axial-vector form factors of the nucleon from lattice QCD *Phys. Rev.* D **96** 114503

[21] Hoferichter M, Ditsche C, Kubis B and Meissner U-G 2012 Dispersive analysis of the scalar form factor of the nucleon *JHEP* **06** 063
[22] Ramalho G and Tsushima K 2016 Holographic estimate of the meson cloud contribution to nucleon axial form factor *Phys. Rev.* D **94** 014001
[23] Eichmann G and Fischer C S 2012 Nucleon axial and pseudoscalar form factors from the covariant Faddeev equation *Eur. Phys. J.* A **48** 9
[24] Bernard V, Elouadrihiri L and Meissner U-G 2002 Axial structure of the nucleon *J. Phys. G: Nucl. Part. Phys.* **28** R1
[25] Bratt J D *et al* 2010 Nucleon structure from mixed action calculations using 2 + 1 flavors of asqtad sea and domain wall valence fermions *Phys. Rev.* D **82** 094502
[26] Aoki S *et al* 2017 Review of Lattice results concerning low-energy particle physics *Eur. Phys. J.* C **77** 112
[27] Alexandrou C *et al* 2018 Pion vector form factor from lattice QCD at the physical point *Phys. Rev.* D **97** 014508
[28] Miller G A 2019 Defining the proton radius: a unified treatment *Phys. Rev.* C **99** 035202
[29] Miller K L *et al* 1982 Study of the reaction $\nu_\mu d \to \mu^- p p_s$ *Phys. Rev.* D **26** 537
Kitagaki T *et al* 1983 High-energy quasielastic $\nu_\mu n \to \mu^- p$ scattering in deuterium *Phys. Rev.* D **28** 436
[30] Gaillard J-M and Sauvage G 1984 Hyperon Beta Decays *Ann. Rev. Nucl. Part. Sci* **34** 351
[31] Choi S *et al* 1993 Axial and pseudoscalar nucleon form factors from low energy pion electroproduction *Phys. Rev. Lett.* **71** 3927
[32] Bardin G *et al* 1981 Measurement of the ortho para transition rate in the pμp molecule and deduction of the pseudoscalar coupling constant g_p^μ *Phys. Lett.* B **104** 320
[33] Andreev V A *et al* (MuCap Collaboration) 2007 Measurement of the Muon capture rate in hydrogen gas and determination of the proton pseudoscalar coupling g_P *Phys. Rev. Lett.* **99** 032002
[34] Godfrey S and Isgur N 1985 Mesons in a relativized quark model with chromodynamics *Phys. Rev.* D **32** 189
Capstick S and Isgur N 1986 Baryons in a relativized quark model with chromodynamics *Phys. Rev.* D **34** 2809
[35] Lavelle M and McMullan D 1997 Constituent quarks from QCD *Phys. Rep.* **279** 1
de Rafael E 2011 The constituent chiral quark model revisited *Phys. Lett.* B **703** 60
[36] Thomas A W 2003 *Nucl. Phys. Proc. Suppl.* B **119** 50
Young R D, Leinweber D B and Thomas A W 2003 *Prog. Part. Nucl. Phys.* **50** 399 and references therein
[37] Weinberg S 2010 Pions in Large N quantum chromodynamics *Phys. Rev. Lett.* **105** 261601
[38] Braghin F L 2016 Quark and pion effective couplings from polarization effects *Eur. Phys. J.* A **52** 134
[39] Braghin F L 2018 Low energy constituent quark and pion effective couplings in a weak external magnetic field *Eur. Phys. J.* A **54** 45
[40] Roberts C D, Cahill R T and Praschifka J 1988 The effective action for the Goldstone modes in a global colour symmetry model of QCD *Ann. Phys.* **188** 20
[41] Ebert D, Reinhardt H and Volkov M K 1994 Effective hadron theory of QCD *Prog. Part. Nucl. Phys* **33** 1
[42] Meissner U G 1988 Low energy hadron physics from effective chiral Lagrangians with vector mesons *Phys. Rep.* **161** 213
[43] Binosi D, Chang L, Papavassiliou J and Roberts C D 2015 Bridging a gap between continuum-QCD and *ab initio* predictions of hadron observables *Phys. Lett.* B **742** 183 and references therein
[44] Kondo K-I 1998 Abelian-projected effective gauge theory of QCD with asymptotic freedom and quark confinement *Phys. Rev.* D **57** 7467
[45] Cornwall J M 2011 Entropy, confinement, and chiral symmetry breaking *Phys. Rev.* D **83** 076001
[46] Higashijima K 1984 Dynamical chiral-symmetry breaking *Phys. Rev.* D **29** 1228
[47] Holdom B 1992 Approaching low-energy QCD with a gauged, nonlocal, constituent-quark model *Phys. Rev.* D **45** 2534
[48] Wang Q, Kuang Y-P, Wang X-L and Xiao M 2000 Derivation of the effective chiral Lagrangian for pseudoscalar mesons from QCD *Phys. Rev.* D **61** 054011

Ren K, Fu H-F and Wang Q 2017 Derivation of the effective chiral Lagrangian for pseudoscalar, scalar, vector, and axial-vector mesons from QCD *Phys. Rev.* D **95** 074012
[49] Roberts C D and Williams A G 1994 Dyson-Schwinger equations and their application to hadronic physics *Prog. Part. Nucl. Phys.* **33** 477
[50] Paulo A Jr and Braghin F L 2014 Vacuum polarization corrections to low energy quark effective couplings *Phys. Rev.* D **90** 014049
[51] Maris P and Tandy P C 1999 Bethe–Salpeter study of vector meson masses and decay constants *Phys. Rev.* C **60** 055214
[52] Abbott L F 1982 Introduction to the background field method *Acta Phys. Pol.* B **13** 33
[53] Kleinert H 1978 Understanding the fundamental constituents of matter (*Erice Summer Institute 1976* vol 289) ed A Zichichi (New York: Plenum Press)
[54] Bali G S, Bruckmann F, Endrodi G, Fodor Z, Katz S D and Schafer A 2012 *Phys. Rev.* D **86** 071502
[55] Bali G S *et al* 2012 *JHEP* **2** 44
Endrodi G 2013 *JHEP* **04** 023
[56] Mosel U 2004 Path integrals in field theory *An Introduction* (Berlin: Springer)
[57] Chyi T-K *et al* 2000 Weak-field expansion for processes in a homogeneous background magnetic field *Phys. Rev.* D **62** 105014
[58] Gorbar E V, Miransky V A, Shovkovy I A and Wang X 2013 Radiative corrections to chiral separation effect in QED *Phys. Rev.* D **88** 025025
[59] Petronzio R, Simula S and Ricco G 2003 Possible evidence of extended objects inside the proton *Phys. Rev.* D **67** 094004
Petronzio R, Simula S and Ricco G 2003 *Phys. Rev.* D **68** 099901 (erratum)