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Parametric excitation surface waves at plasma boundary under action of *p*-polarized laser radiation

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Abstract

The parametric excitation of surface waves at the plasma boundary under the action of a *p*-polarized electromagnetic pump wave is analyzed. The instability, at which the pump wave decays into two surface electromagnetic TM modes, is considered. The dependence of the instability growth rate on the plasma electron density, the angle of incidence of the pump wave, and the direction of propagation of the excited surface waves is investigated. It is shown that the instability growth rate has a maximum value in the case when short-wave quasistatic surface waves are excited.

Keywords: surface waves, electromagnetic pump wave, decay instability, growth rate of instability

(Some figures may appear in colour only in the online journal)

1. Introduction

Surface electromagnetic waves (SEWs), propagating along the interface between two media, have some unusual physical properties and can be used for a number of practical applications [1–3]. SEWs are used to diagnose the surface, study the properties of thin films and interfaces of various media, study the spectra of surface excited states, and transmit signals over long distances. Linear dispersion properties of surface waves and methods for their excitation are described in sufficient detail in [1–3]. The excitation of SEWs at the boundary of solid-state targets on exposure to short pulses of intense laser radiation, when nonlinear effects become significant, was observed in a number of experiments [4–8]. As for the theory of nonlinear methods for excitation of SEWs, the generation of low-frequency SEWs by ponderomotive forces and drag currents in plasma and conducting media when exposed to laser radiation is discussed in publications [9–12]. Another nonlinear mechanism for excitation of the surface waves is associated with the development of parametric instabilities under the action of laser radiation on the plasma and conducting media. The theory of parametric excitation of surface waves at the plasma boundary under the action of an electromagnetic pump wave was first formulated in [13], where a hydrodynamic approach was used. The kinetic theory of the excitation of surface waves under the action of a high-frequency electromagnetic field was developed somewhat later in the publication [14], where not only the growth rate, but also the instability threshold was calculated. The parametric decay of the pump wave into two SEWs of the same frequency in a transparent plasma was considered in [15]. In [16, 17], the role of a plasmon localized in a narrow surface layer in the decay of a *p*-polarized pump wave into two SEWs was investigated. The excitation of two surface waves during the development of decay instability in a semi-infinite plasma when exposed to a *p*-polarized electromagnetic wave is considered in [18], where the growth rates and instability thresholds are calculated. At the same time, the authors of [18] erroneously neglected the nonlinear surface current, which makes the same contribution to the instability growth rate as the ponderomotive nonlinearity taken into account by them. The excitation of surface waves due to the surface current and ponderomotive effects when an *s*-polarized pump wave falls on a semi-infinite plasma was first considered in [19]. We note that in [18, 19], in contrast to [15], the parametric excitation of SEWs was studied in not transparent plasma, when the skin effect for the pump wave occurs.

It should be noted that the development of instability and the excitation of surface waves, considered in publications [13, 14], is associated with the motion of ions, which can be realized for laser radiation of picosecond duration. During the action of femtosecond laser pulses, the instability considered in [13, 14] do not have time to develop. The parametric instability with respect to the excitation of SEWs, associated exclusively with the motion of electrons, was previously considered in [15, 18] for p-polarization and in [19] for s-polarization of the pump wave. In contrast to [15, 18], in this paper, for the first time, the dependence of the decay instability growth rate on plasma density, the angle of incidence of a p-polarized pump wave, and the angle between the wave vectors of the excited surface waves is studied in detail. It is established that the instability growth rate has a maximum value in near-critical plasma, when excitation of short-wave quasistatic surface waves occurs, propagating almost across to the wave vector of the pump wave. It is shown that the instability growth rate for p-polarization is significantly larger than in the case of a s-polarized pump wave [19], which is explained by the appearance of an additional current flowing along the plasma boundary due to the appearance of a surface charge in the ground state. Note that below, as in [19], a sufficiently high concentration of electrons is considered, when the pump wave penetrates into the plasma only to the depth of the skin layer.

The article has the following structure: in the second section, the ground state is described in the framework of the hydrodynamic approach and the equations and boundary conditions for the perturbations of the electromagnetic field are presented. It is shown that the presence of the p-component of the pump wave leads to the appearance in the ground state of the surface charge density oscillating at the frequency of the incident radiation. In the third section, the dispersion equation is obtained and the instability growth rate associated with the decay of the pump wave into two surface TM modes is found. The dependence of the rate of the decay instability growth on the electron density is analyzed and it is shown that the instability growth rate is maximum in a plasma with an electron density close to the critical value. The growth rate of parametric decay instability is investigated as a function of the angle of incidence of the laser radiation and the angle between the wave vectors of the excited surface waves. It is shown that the instability growth rate has a maximum value in the short-wave limit, when the excited surface waves propagate almost across the wave vector of the pump wave. The instability threshold is calculated and it is shown that it has a minimum value at excitation of short-wave surface waves. In conclusion, the obtained results are discussed and estimates are given for the characteristic parameters of modern laser-plasma experiments.

The parametric instability considered in this work, associated with the decay of an electromagnetic wave into two surface plasmons, is of interest for laser-fusion studies, the generation of infrared and THz radiation, and laser acceleration of electrons at the interaction of high-power laser radiation with a dense bounded plasma.

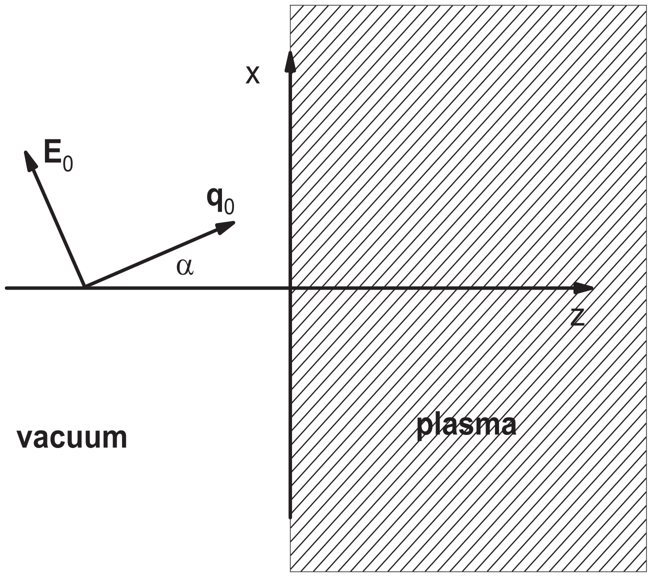

Figure 1. The incidence of a p-polarized electromagnetic wave with frequency ω_0, wave vector $\mathbf{q}_0 = (\omega_0/c)(\mathbf{e}_x \sin\alpha + \mathbf{e}_z \cos\alpha)$ and vector amplitude of the electric field $\mathbf{E}_0 = E_0(\mathbf{e}_x \cos\alpha - \mathbf{e}_z \sin\alpha)$ at an angle α from vacuum to the boundary of a dense plasma occupying half-space $z > 0$.

2. Ground state and equations for perturbations

Let a p-polarized electromagnetic wave with frequency ω_0, wave vector $\mathbf{q}_0 = (\omega_0/c)(\mathbf{e}_x \sin\alpha + \mathbf{e}_z \cos\alpha)$ and amplitude E_0 fall at an angle α from a vacuum onto a non transparent plasma boundary ($\omega_0 \cos\alpha \leqslant \omega_p$) occupying a half-space $z > 0$, where ω_p is the Langmuir frequency of electrons (see figure 1). Then the electric and magnetic field of the pump wave in vacuum ($z < 0$) can be written in the following form

$$
\begin{aligned}
\mathbf{E}_L^{\mathrm{inc}}(\mathbf{r}, t) &= (\mathbf{e}_x \cos\alpha - \mathbf{e}_z \sin\alpha) \\
&\quad \times E_0 \cos\left[\omega_0 t - \frac{\omega_0}{\mathrm{c}}(x \sin\alpha + z\cos\alpha)\right], \\
\mathbf{B}_L^{\mathrm{inc}}(\mathbf{r}, t) &= \mathbf{e}_y E_0 \cos\left[\omega_0 t - \frac{\omega_0}{\mathrm{c}}(x \sin\alpha + z\cos\alpha)\right],
\end{aligned}
\tag{2.1}
$$

where $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ are the basis vectors of the Cartesian coordinate system.

Let us consider the plasma stability with respect to the excitation of SEWs under the action of the radiation field (2.1). To do this, we use the Maxwell equations for the electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields, as well as the equations of collisionless hydrodynamics for the velocity $\mathbf{v}_e$ and density n_e of electrons

$$
\begin{aligned}
&\mathrm{rot}\mathbf{B} = \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E} + \frac{4\pi}{c} e n_e \mathbf{v}_e, \quad \mathrm{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{B}, \\
&\mathrm{div}\mathbf{E} = 4\pi e(n_e - N_e), \\
&\frac{\partial}{\partial t} n_e + \mathrm{div}(n_e \mathbf{v}_e) = 0, \quad \frac{\partial}{\partial t}\mathbf{v}_e = \frac{e}{m_e}\mathbf{E} - \frac{1}{2}\nabla \mathbf{v}_e^2,
\end{aligned}
\tag{2.2}
$$

where e, m_e is the charge and mass of the electron, c is the speed of light, $N_e = N_{0e}\theta(z)$ is the coordinate-dependent electron density in the equilibrium state, which is equal N_{0e} in the plasma and vanishes in vacuum, $\theta(z)$ is the unit Heaviside step function. The last equation in set (2.2) was obtained using the well-known relationship of vector analysis $(\mathbf{v}_e\nabla)\mathbf{v}_e = (1/2)\nabla\mathbf{v}_e^2 - [\mathbf{v}_e \times \mathrm{rot}\mathbf{v}_e]$ and the conservation law of the generalized vortex $\boldsymbol{\Omega} = \mathrm{rot}\mathbf{v}_e + (e/m_e c)\mathbf{B} = 0$ [20]. The set of equation (2.2) is applicable under the condition $\omega_0 \gg \nu_{ei}$ when the frequency of the incident wave ω_0 significantly exceeds the frequency of electron–ion collisions ν_{ei}.

Let's present all physical quantities in the form of small deviations δN, $\delta\mathbf{V}$, $\delta\mathbf{E}$, $\delta\mathbf{B}$ from the ground state $N_e + N_L$, $\mathbf{V}_L$, $\mathbf{E}_L$, $\mathbf{B}_L$

$$n_e = N_e + N_L + \delta N,\ \ \mathbf{v}_e = \mathbf{V}_L + \delta\mathbf{V},\ \ \mathbf{E} = \mathbf{E}_L + \delta\mathbf{E},\ \ \mathbf{B} = \mathbf{B}_L + \delta\mathbf{B}. \tag{2.3}$$

Taking into account the representation (2.3) from (2.2), we have the following set of equations in the ground state

$$\mathrm{rot}\mathbf{B}_L = \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E}_L + \frac{4\pi}{c}\mathbf{j}_L,\quad \mathrm{rot}\mathbf{E}_L = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{B}_L,$$
$$\frac{\partial}{\partial t}N_L + \mathrm{div}\left(\frac{\mathbf{j}_L}{e}\right),\quad \frac{\partial}{\partial t}\mathbf{V}_L = \frac{e}{m_e}\mathbf{E}_L - \frac{1}{2}\nabla\mathbf{V}_L^2,$$
$$\mathbf{j}_L = e(N_e + N_L)\mathbf{V}_L. \tag{2.4}$$

For small deviations from the ground state, the equations are valid

$$\mathrm{rot}\delta\mathbf{B} = \frac{1}{c}\frac{\partial}{\partial t}\delta\mathbf{E} + \frac{4\pi}{c}\delta\mathbf{j},\quad \mathrm{rot}\delta\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\delta\mathbf{B},$$
$$\frac{\partial}{\partial t}\delta N + \mathrm{div}\left(\frac{\delta\mathbf{j}}{e}\right) = 0,\quad \frac{\partial}{\partial t}\delta\mathbf{V} = \frac{e}{m_e}\delta\mathbf{E} - \nabla(\mathbf{V}_L\delta\mathbf{V}),$$
$$\delta\mathbf{j} = e[(N_e + N_L)\delta\mathbf{V} + \delta N\mathbf{V}_L]. \tag{2.5}$$

Since the instability growth rate is proportional to the first power of the incident wave amplitude E_0, in the ground state we restrict ourselves to a linear approximation. Let us represent all physical quantities in the following form

$$\{\mathbf{E}_L, \mathbf{B}_L, \mathbf{V}_L, N_L\} = \frac{1}{2}\{\mathbf{E}_1, \mathbf{B}_1, \mathbf{V}_1, N_1\}\exp\left(-\mathrm{i}\omega_0 t + \mathrm{i}\frac{\omega_0}{c}x\sin\alpha\right) + \mathrm{c.c.}, \tag{2.6}$$

where $\mathbf{E}_1$, $\mathbf{B}_1$, $\mathbf{V}_1$, N_1—are the complex amplitudes of the electric and magnetic fields, as well as the speed and density of electrons. Then from the set of equation (2.4) follows the equation for the component of the magnetic field and the expression for the components of the electric field in the plasma

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}B_{1,y} - \frac{\omega_0^2}{c^2}[\sin^2\alpha - \varepsilon(\omega_0)]B_{1,y} = 0,$$
$$E_{1,x} = -\frac{\mathrm{i}c}{\omega_0\varepsilon(\omega_0)}\frac{\mathrm{d}}{\mathrm{d}z}B_{1,y},\quad E_{1,z} = -\frac{\sin\alpha}{\varepsilon(\omega_0)}B_{1,y}, \tag{2.7}$$

here $\varepsilon(\omega_0) = 1 - \omega_p^2/\omega_0^2$–is the plasma dielectric constant at the frequency of the pump wave. For the fields in a vacuum, it should be assumed that in formulas (2.7) the dielectric constant is equal to unity. Solving equation (2.7) in plasma and in vacuum and using the continuity conditions for the tangential components of the electromagnetic field at the plasma boundary, we get the next solution of the boundary problem in the ground state for magnetic and electric fields in vacuum $z < 0$

$$\mathbf{E}_1 = E_0\left\{(\mathbf{e}_x\cos\alpha - \mathbf{e}_z\sin\alpha)\exp\left(\mathrm{i}\frac{\omega_0}{c}z\cos\alpha\right) - R(\mathbf{e}_x\cos\alpha + \mathbf{e}_z\sin\alpha)\exp\left(-\mathrm{i}\frac{\omega_0}{c}z\cos\alpha\right)\right\},$$
$$\mathbf{B}_1 = \mathbf{e}_y E_0\left[\exp\left(\mathrm{i}\frac{\omega_0}{c}z\cos\alpha\right) + R\exp\left(-\mathrm{i}\frac{\omega_0}{c}z\cos\alpha\right)\right], \tag{2.8}$$

where the reflection coefficient R is determined by the ratio

$$R = \frac{\varepsilon(\omega_0)\cos\alpha - \mathrm{i}\sqrt{\sin^2\alpha - \varepsilon(\omega_0)}}{\varepsilon(\omega_0)\cos\alpha + \mathrm{i}\sqrt{\sin^2\alpha - \varepsilon(\omega_0)}}. \tag{2.9}$$

In plasma $(z > 0)$ the distribution of electromagnetic fields is as follows

$$\mathbf{E}_1 = \frac{2E_0\cos\alpha[\mathrm{i}\mathbf{e}_x\sqrt{\sin^2\alpha - \varepsilon(\omega_0)} - \mathbf{e}_z\sin\alpha]}{\varepsilon(\omega_0)\cos\alpha + \mathrm{i}\sqrt{\sin^2\alpha - \varepsilon(\omega_0)}}\exp(-\kappa_L z),$$
$$\mathbf{B}_1 = \frac{2E_0\varepsilon(\omega_0)\cos\alpha\,\mathbf{e}_y}{\varepsilon(\omega_0)\cos\alpha + \mathrm{i}\sqrt{\sin^2\alpha - \varepsilon(\omega_0)}}\exp(-\kappa_L z), \tag{2.10}$$

where the coefficient $\kappa_L = (\omega_0/c)\sqrt{\sin^2\alpha - \varepsilon(\omega_0)}$ characterizes the depth of penetration of the field into a dense plasma $\delta_L = 1/\kappa_L = (c/\omega_0)[\sin^2\alpha - \varepsilon(\omega_0)]^{-1/2}$. With an increase in the angle of incidence α, the depth of the skin layer δ_L monotonously decreases from the value $\delta_L = c\Big/\sqrt{\omega_p^2 - \omega_0^2}$ at normal incidence, when $\alpha = 0$, to the minimum value $\delta_L = c/\omega_p$ at grazing incidence $\alpha \to \pi/2$. In the field of an electromagnetic wave (2.10) the speed and density of electrons in a plasma in accordance with equation (2.4) at linear approximation are determined by the expressions

$$\mathbf{V}_1 = -\frac{2V_E\cos\alpha[\mathrm{i}\mathbf{e}_x\sqrt{\sin^2\alpha - \varepsilon(\omega_0)} - \mathbf{e}_z\sin\alpha]}{\varepsilon(\omega_0)\cos\alpha + \mathrm{i}\sqrt{\sin^2\alpha - \varepsilon(\omega_0)}}\exp(-\kappa_L z),$$
$$N_1 = -\frac{2r_E\sin\alpha\cos\alpha}{\varepsilon(\omega_0)\cos\alpha + \mathrm{i}\sqrt{\sin^2\alpha - \varepsilon(\omega_0)}}N_{0e}\delta(z), \tag{2.11}$$

where $V_E = eE_0/(m_e\omega_0)$, $r_E = V_E/\omega_0 = eE_0/(m_e\omega_0^2)$, $\delta(z)$ is the Dirac's delta function. Note that, in accordance with formulas (2.6), (2.11), the surface charge density N_L oscillating at the frequency of the pump wave ω_0 occurs in the ground state. We note that such oscillations of the surface charge were not taken into account by the authors of [18].

We now consider small deviations from the ground state (2.10), (2.11), which are described by equation (2.5). We will use the following expansion in the Fourier series for perturbations of the electromagnetic field and electric current

$$\{\delta\mathbf{E}(\mathbf{r},t),\ \delta\mathbf{B}(\mathbf{r},t),\ \delta\mathbf{j}(\mathbf{r},t)\} = \sum_{n=-\infty}^{+\infty}\{\delta\mathbf{E}^{(n)}(z),\ \delta\mathbf{B}^{(n)}(z),\ \delta\mathbf{j}^{(n)}(z)\} \times\exp(-\mathrm{i}\omega_n t+\mathrm{i}\mathbf{k}_n\boldsymbol{\rho}), \tag{2.12}$$

where $\boldsymbol{\rho}=x\mathbf{e}_x+y\mathbf{e}_y$. Restricting ourselves to a linear approximation in the amplitude of the pump wave, after simple calculations we find the following set of equations for electromagnetic field perturbations at a frequency $\omega_n=\omega+n\omega_0$ with a wave vector $\mathbf{k}_n=\mathbf{k}+n\mathbf{k}_0$ lying in the plane XOY

$$\delta\mathbf{B}^{(n)}=\frac{c}{\omega_n}[(\mathbf{k}_n-\mathrm{i}\mathbf{e}_z\nabla_z)\times\delta\mathbf{E}^{(n)}],$$
$$[(\mathbf{k}_n-\mathrm{i}\mathbf{e}_z\nabla_z)\times\delta\mathbf{B}^{(n)}]=-\frac{\omega_n}{c}\varepsilon_n\delta\mathbf{E}^{(n)}-\frac{4\pi\mathrm{i}}{c}\delta\mathbf{J}^{(n)}, \tag{2.13}$$

where $\mathbf{k}_0=\mathbf{e}_x(\omega_0/c)\sin\alpha$, $\varepsilon_n\equiv\varepsilon(\omega_n)=1-{\omega_p}^2/{\omega_n}^2$—is the dielectric constant at frequency ω_n, and the expression for perturbations of nonlinear electric current can be represented as the sum of three terms (see also [15])

$$\delta\mathbf{J}^{(n)}=\delta\mathbf{J}_1^{(n)}+\delta\mathbf{J}_2^{(n)}+\delta\mathbf{J}_3^{(n)}, \tag{2.14}$$

where

$$\delta\mathbf{J}_1^{(n)}=\frac{\mathrm{i}\omega_p^2}{8\pi\omega_n}(\mathbf{k}_n-\mathrm{i}\mathbf{e}_z\nabla_z)\left[\frac{\mathbf{V}_1\delta\mathbf{E}^{(n-1)}}{\omega_{n-1}}+\frac{\mathbf{V}_1^*\delta\mathbf{E}^{(n+1)}}{\omega_{n+1}}\right],$$
$$\delta\mathbf{J}_2^{(n)}=\frac{\mathrm{i}\mathbf{V}_1}{8\pi}\left[(\mathbf{k}_{n-1}-\mathrm{i}\mathbf{e}_z\nabla_z)\frac{\omega_p^2}{\omega_{n-1}^2}\delta\mathbf{E}^{(n-1)}\right]+\frac{\mathrm{i}\mathbf{V}_1^*}{8\pi}\left[(\mathbf{k}_{n+1}-\mathrm{i}\mathbf{e}_z\nabla_z)\frac{\omega_p^2}{\omega_{n+1}^2}\delta\mathbf{E}^{(n+1)}\right],$$
$$\delta\mathbf{J}_3^{(n)}=\frac{\mathrm{i}\omega_p^2}{8\pi}\left[\frac{N_1}{N_{0e}}\frac{\delta\mathbf{E}^{(n-1)}}{\omega_{n-1}}+\frac{N_1^*}{N_{0e}}\frac{\delta\mathbf{E}^{(n+1)}}{\omega_{n+1}}\right]. \tag{2.15}$$

Potential nonlinear current $\delta\mathbf{J}_1\propto\nabla(\mathbf{V}_L\delta\mathbf{V}_1)$ due to ponderomotive effects follows from the first term in the formula for the current $eN_{0e}\delta\mathbf{V}$ and the second term on the right side of the equation for speed $-\nabla(\mathbf{V}_L\delta\mathbf{V})$ in the set of equation (2.5) and exists only in the plasma volume. Currents $\delta\mathbf{J}_2$, $\delta\mathbf{J}_3$ are the surface ones, as they flow along the plasma boundary. In this case, the surface current $\delta\mathbf{J}_2=e\delta N\mathbf{V}_L$ is proportional to the product of the electric field of the pumping wave and the surface charge perturbation, and the current $\delta\mathbf{J}_3=eN_L\delta\mathbf{V}$ flowing along the boundary is determined by the product of the surface charge density in the ground state and the electric field perturbation. The expression for nonlinear current (2.14), (2.15) differs from the case of an s-polarized pump wave [19] precisely by the presence of the last term associated with the appearance in the ground state of a rapidly oscillating (at frequency ω_0) surface charge density [15].

Considering the excitation of surface TM modes, we represent the electromagnetic field and nonlinear current $\delta\mathbf{J}^{(n)}$ in the following form

$$\delta\mathbf{E}^{(n)}=\frac{\mathbf{k}_n}{k_n}\delta E_\parallel^{(n)}+\mathbf{e}_z\delta E_z^{(n)},\quad \delta\mathbf{B}^{(n)}=\left[\mathbf{e}_z\times\frac{\mathbf{k}_n}{k_n}\right]\delta B_\perp^{(n)},\quad \delta\mathbf{J}^{(n)}=\frac{\mathbf{k}_n}{k_n}\delta J_\parallel^{(n)}+\mathbf{e}_z\delta J_z^{(n)}. \tag{2.16}$$

It should be noted that the contribution of TE modes to the instability growth rate can be neglected, since it turns out to be proportional to the square of the amplitude of the pump wave. Then the equations for the components of the perturbations of the electromagnetic field follow from (2.13), (2.16)

$$k_n\delta B_\perp^{(n)}=-\frac{\omega_n}{c}\varepsilon_n\delta E_z^{(n)}-\frac{4\pi\mathrm{i}}{c}\delta J_z^{(n)},$$
$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}z}\delta B_\perp^{(n)}=-\frac{\omega_n}{c}\varepsilon_n\delta E_\parallel^{(n)}-\frac{4\pi\mathrm{i}}{c}\delta J_\parallel^{(n)},$$
$$\delta B_\perp^{(n)}=-\frac{c}{\omega_n}\left(k_n\delta E_z^{(n)}+\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}z}\delta E_\parallel^{(n)}\right), \tag{2.17}$$

where the nonlinear current components $\delta\mathbf{J}^{(n)}$ (2.14), (2.15) are defined by the following expressions

$$\delta J_{1,z}^{(n)}=\frac{\omega_p^2}{8\pi\omega_n}\frac{\mathrm{d}}{\mathrm{d}z}\left\{\frac{\mathbf{k}_{n-1}\mathbf{V}_1}{k_{n-1}\omega_{n-1}}\delta E_\parallel^{(n-1)}+\frac{\mathbf{e}_z\mathbf{V}_1}{\omega_{n-1}}\delta E_z^{(n-1)}+\frac{\mathbf{k}_{n+1}\mathbf{V}_1^*}{k_{n+1}\omega_{n+1}}\delta E_\parallel^{(n+1)}+\frac{\mathbf{e}_z\mathbf{V}_1^*}{\omega_{n+1}}\delta E_z^{(n+1)}\right\},$$
$$\delta J_{2,z}^{(n)}=\frac{\mathrm{i}\mathbf{e}_z\mathbf{V}_1}{8\pi}\left\{\frac{\omega_p^2}{\omega_{n-1}^2}k_{n-1}\delta E_\parallel^{(n-1)}-\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\omega_p^2}{\omega_{n-1}^2}\delta E_z^{(n-1)}\right)\right\}+\frac{\mathrm{i}\mathbf{e}_z\mathbf{V}_1^*}{8\pi}\left\{\frac{\omega_p^2}{\omega_{n+1}^2}k_{n+1}\delta E_\parallel^{(n+1)}-\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\omega_p^2}{\omega_{n+1}^2}\delta E_z^{(n+1)}\right)\right\},$$
$$\delta J_{3,z}^{(n)}=\frac{\mathrm{i}\omega_p^2}{8\pi}\left\{\frac{N_1}{N_{0e}}\frac{\delta E_z^{(n-1)}}{\omega_{n-1}}+\frac{N_1^*}{N_{0e}}\frac{\delta E_z^{(n+1)}}{\omega_{n+1}}\right\}, \tag{2.18}$$

$$\delta J_{1,\parallel}^{(n)}=\frac{\mathrm{i}\omega_p^2k_n}{8\pi\omega_n}\left\{\frac{\mathbf{k}_{n-1}\mathbf{V}_1}{k_{n-1}\omega_{n-1}}\delta E_\parallel^{(n-1)}+\frac{\mathbf{e}_z\mathbf{V}_1}{\omega_{n-1}}\delta E_z^{(n-1)}+\frac{\mathbf{k}_{n+1}\mathbf{V}_1^*}{k_{n+1}\omega_{n+1}}\delta E_\parallel^{(n+1)}+\frac{\mathbf{e}_z\mathbf{V}_1^*}{\omega_{n+1}}\delta E_z^{(n+1)}\right\},$$
$$\delta J_{2,\parallel}^{(n)}=\frac{\mathrm{i}\mathbf{k}_n\mathbf{V}_1}{8\pi k_n}\left\{\frac{\omega_p^2}{\omega_{n-1}^2}k_{n-1}\delta E_\parallel^{(n-1)}-\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\omega_p^2}{\omega_{n-1}^2}\delta E_z^{(n-1)}\right)\right\}+\frac{\mathrm{i}\mathbf{k}_n\mathbf{V}_1^*}{8\pi k_n}\left\{\frac{\omega_p^2}{\omega_{n+1}^2}k_{n+1}\delta E_\parallel^{(n+1)}-\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\omega_p^2}{\omega_{n+1}^2}\delta E_z^{(n+1)}\right)\right\},$$
$$\delta J_{3,\parallel}^{(n)}=\frac{\mathrm{i}\omega_p^2}{8\pi k_n}\left\{\frac{N_1}{N_{0e}}\frac{\mathbf{k}_n\mathbf{k}_{n-1}}{k_{n-1}\omega_{n-1}}\delta E_\parallel^{(n-1)}+\frac{N_1^*}{N_{0e}}\frac{\mathbf{k}_n\mathbf{k}_{n+1}}{k_{n+1}\omega_{n+1}}\delta E_\parallel^{(n+1)}\right\}, \tag{2.19}$$

here $\mathbf{V}_1$, N_1 are the amplitudes of the oscillation velocity and the electron density in the ground state (2.11). From the set of equation (2.17) we find the equation for magnetic field perturbations

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{\varepsilon_n}\frac{\mathrm{d}}{\mathrm{d}z}\delta B_{\perp}^{(n)}\right) - \frac{\kappa_n^2}{\varepsilon_n}\delta B_{\perp}^{(n)} = \frac{4\pi \mathrm{i}}{c}\frac{k_n}{\varepsilon_n}\delta J_z^{(n)} - \frac{4\pi}{c}\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\delta J_{\parallel}^{(n)}}{\varepsilon_n}\right). \tag{2.20}$$

The boundary conditions are obtained by integrating equation (2.17) over a thin transition boundary layer and have the form

$$\delta B_{\perp}^{(n)}|_{z=+0} - \delta B_{\perp}^{(n)}|_{z=-0} = -\frac{4\pi}{c}\int_{-0}^{+0}\mathrm{d}z(\delta J_{2,\parallel}^{(n)} + \delta J_{3,\parallel}^{(n)}),$$
$$\frac{1}{\varepsilon_n}\frac{\mathrm{d}}{\mathrm{d}z}\delta B_{\perp}^{(n)}\Big|_{z=+0} - \frac{1}{\varepsilon_n}\frac{\mathrm{d}}{\mathrm{d}z}\delta B_{\perp}^{(n)}\Big|_{z=-0} = -\frac{4\pi}{c}\frac{1}{\varepsilon_n}\delta J_{1,\parallel}^{(n)}\Big|_{z=+0}, \tag{2.21}$$

where $\kappa_n = \sqrt{k_n^2 - (\omega_n^2/c^2)\varepsilon_n}$, $\varepsilon_n = 1 - \omega_p^2/\omega_n^2$ for $n = \pm 1$, ± 2..., and $\kappa = \sqrt{k^2 - (\omega^2/c^2)\varepsilon}$, $\varepsilon = 1 - \omega_p^2/\omega^2$ for $n = 0$. Note that the jump in the magnetic field strength of the SEWs at the plasma boundary in formula (2.21) is associated with the appearance of a nonlinear surface current. Taking into account the expression (2.19), the boundary conditions (2.21) can be written explicitly

$$\delta B_{\perp}^{(n)}|_{z=+0} - \delta B_{\perp}^{(n)}|_{z=-0} = -\left\{\frac{\mathbf{k}_n\mathbf{V}_1}{2c\ k_n}\frac{\omega_p^2}{\omega_{n-1}^2}\delta E_z^{(n-1)} + \frac{\mathbf{k}_n\mathbf{V}_1^*}{2ck_n}\frac{\omega_p^2}{\omega_{n+1}^2}\delta E_z^{(n+1)}\right\}\Bigg|_{z=+0}$$
$$+ \frac{\mathrm{i}\omega_p^2}{2ck_n}\left\{n_1\frac{\mathbf{k}_n\mathbf{k}_{n-1}}{k_{n-1}\omega_{n-1}}\delta E_{\parallel}^{(n-1)} + n_1^*\frac{\mathbf{k}_n\mathbf{k}_{n+1}}{k_{n+1}\omega_{n+1}}\delta E_{\parallel}^{(n+1)}\right\}\Bigg|_{z=+0}, \tag{2.22}$$

$$\frac{1}{\varepsilon_n}\frac{\mathrm{d}}{\mathrm{d}z}\delta B_{\perp}^{(n)}\Big|_{z=+0} - \frac{\mathrm{d}}{\mathrm{d}z}\delta B_{\perp}^{(n)}\Big|_{z=-0}$$
$$= -\frac{\mathrm{i}\omega_p^2 k_n}{2c\ \varepsilon_n\omega_n}\left\{\frac{\mathbf{k}_{n-1}\mathbf{V}_1}{k_{n-1}\omega_{n-1}}\delta E_{\parallel}^{(n-1)} + \frac{\mathbf{e}_z\mathbf{V}_1}{\omega_{n-1}}\delta E_z^{(n-1)} + \frac{\mathbf{k}_{n+1}\mathbf{V}_1^*}{k_{n+1}\omega_{n+1}}\delta E_{\parallel}^{(n+1)} + \frac{\mathbf{e}_z\mathbf{V}_1^*}{\omega_{n+1}}\delta E_z^{(n+1)}\right\}\Bigg|_{z=+0}, \tag{2.23}$$

where

$$n_1 = \frac{2r_E\sin\alpha\cos\alpha}{\varepsilon(\omega_0)\cos\alpha + \mathrm{i}\sqrt{\sin^2\alpha - \varepsilon(\omega_0)}}. \tag{2.24}$$

According to formula (2.22), in addition to the term proportional to the product of surface charge perturbations and the amplitude of the electric field of the pumping wave, as was the case for *s*-polarization [19], the surface current also includes the product of electrical field perturbations and surface charge density in the ground state. Besides, an additional contribution in comparison with *s*-polarized pumping arises in the second boundary condition (2.23) (terms containing factors $\mathbf{e}_z\mathbf{V}_1$ and $\mathbf{e}_z\mathbf{V}_1^*$), which is due to the presence of the *z*-component in the electric field of the incident *p*-polarized wave. It should also be noted that the right-hand side of the second boundary condition is determined by the contribution of the nonlinear term $\nabla(\mathbf{V}_L\delta\mathbf{V})$ in the equation for the electron velocity perturbations in set (2.5). Only this term was kept by the authors of [18], who erroneously neglected nonlinear surface currents, which contribute to the boundary condition (2.22) and lead to a jump in the magnetic field strength of the SEWs at the plasma boundary.

3. Dispersion relation and instability growth rate

We represent the components of the electromagnetic field in the entire space as follows:

$$\delta E_{\parallel}^{(n)} = \theta(z)A_n\exp(-\kappa_n z) + \theta(-z)A_n\exp(\kappa_{n,V}z),$$
$$\delta E_z^{(n)} = \theta(z)C_n\exp(-\kappa_n z) + \theta(-z)C_{n,V}\exp(\kappa_{n,V}z),$$
$$\delta B_{\perp}^{(n)} = \theta(z)B_n\exp(-\kappa_n z) + \theta(-z)B_{n,V}\exp(\kappa_{n,V}z), \tag{3.1}$$

where $\kappa_{n,V} = \sqrt{k_n^2 - \omega_n^2/c^2}$ for $n = \pm 1, \pm 2$... and $\kappa_V = \sqrt{k^2 - \omega^2/c^2}$ for $n = 0$, and the indefinite coefficients A_n, B_n, $B_{n,V}$, C_n, $C_{n,V}$ can be found from boundary conditions (2.22), (2.23). In formulas (3.1), it is taken into account that the tangential component of the electric field is continuous at the plasma boundary, while the normal component of the electric field and tangential component of the magnetic field have a jump.

We note that in the case of a semi-bounded plasma considered here, only surface waves are electromagnetic eigenmodes. Therefore, below we will consider the decay instability with the excitation of two surface waves with the numbers $n = 0$ and $n = -1$, by analogy with the theory of decay instability of unlimited plasma (see for example [21]). Then from equations (2.22), (2.23), taking into account expressions (3.1), we find the following dispersion relation

$$D(\omega, \mathbf{k})D_{-1}(\omega_{-1}, \mathbf{k}_{-1}) = \frac{\omega_p^2}{2\omega^2}\frac{\omega_p^2}{2\omega_{-1}^2}\left\{\frac{(\mathbf{k}_{-1}\mathbf{V}_1^*)}{k_{-1}\omega}\frac{k}{\kappa} + \frac{(\mathbf{k}\mathbf{V}_1^*)}{k\omega_{-1}}\frac{k_{-1}}{\kappa_{-1}} + \mathrm{i}(\mathbf{e}_z\mathbf{V}_1^*)\left[\frac{kk_{-1}}{\omega\kappa\kappa_{-1}} + \frac{\mathbf{k}\mathbf{k}_{-1}}{\omega_0 kk_{-1}}\right]\right\}$$
$$\times\left\{\frac{(\mathbf{k}_{-1}\mathbf{V}_1)}{k_{-1}\omega}\frac{k}{\kappa} + \frac{(\mathbf{k}\mathbf{V}_1)}{k\omega_{-1}}\frac{k_{-1}}{\kappa_{-1}} + \mathrm{i}(\mathbf{e}_z\mathbf{V}_1)\left[\frac{kk_{-1}}{\omega_{-1}\kappa\kappa_{-1}} - \frac{\mathbf{k}\mathbf{k}_{-1}}{\omega_0 kk_{-1}}\right]\right\}, \tag{3.2}$$

where the functions $D(\omega, \mathbf{k})$ and $D_{-1}(\omega_{-1}, \mathbf{k}_{-1})$ have the form

$$D(\omega, \mathbf{k}) = \left(\frac{\varepsilon}{\kappa} + \frac{1}{\kappa_V}\right),$$
$$D_{-1}(\omega_{-1}, \mathbf{k}_{-1}) = \left(\frac{\varepsilon_{-1}}{\kappa_{-1}} + \frac{1}{\kappa_{-1,V}}\right), \tag{3.3}$$

and determine the dispersion properties of surface waves with the corresponding frequencies and wave numbers. Note that two of the four terms in each curly bracket of the right-hand side of expression (3.2) are the contribution of the nonlinear surface current, which was neglected by the authors of [18], and the other two are related to the ponderomotive effect of the pump wave. Taking into account formula (2.11) equation (3.2) takes the form

$$
\begin{aligned}
D(\omega,\mathbf{k})D_{-1}(\omega_{-1},\mathbf{k}_{-1}) &= \frac{\omega_p^2}{\omega^2}\frac{\omega_p^2}{\omega_{-1}^2} \\
&\times \frac{k^2V_E^2\cos^2\alpha}{|\varepsilon(\omega_0)\cos\alpha + \mathrm{i}\sqrt{\sin^2\alpha-\varepsilon(\omega_0)}\,|^2} \\
&\times \left\{\sqrt{\sin^2\alpha-\varepsilon(\omega_0)}\left[\frac{k_{-1}\cos\theta}{\omega_{-1}\kappa_{-1}k} + \frac{k\cos\theta-k_0}{\omega\kappa k_{-1}}\right]\right. \\
&\left. + \sin\alpha\left[\frac{k-k_0\cos\theta}{\omega_0 kk_{-1}} - \frac{k_{-1}}{\omega\kappa\kappa_{-1}}\right]\right\} \\
&\times \left\{\sqrt{\sin^2\alpha-\varepsilon(\omega_0)}\left[\frac{k_{-1}\cos\theta}{\omega_{-1}\kappa_{-1}k} + \frac{k\cos\theta-k_0}{\omega\kappa k_{-1}}\right]\right. \\
&\left. + \sin\alpha\left[\frac{k-k_0\cos\theta}{\omega_0 kk_{-1}} + \frac{k_{-1}}{\omega_{-1}\kappa_{-1}\kappa}\right]\right\},
\end{aligned}
\tag{3.4}
$$

where θ is the angle between the vectors $\mathbf{k}$ and $\mathbf{k}_0$.

We look for the solution of the dispersion equation (3.4) in the form $\omega + \mathrm{i}\gamma$, where ω and γ are the real quantities. Assuming that the instability growth rate γ is small compared to the frequency ω of the excited waves ($\omega \gg |\gamma|$) and keeping only linear terms on γ, we will use the expansion

$$
\begin{aligned}
&D(\omega+\mathrm{i}\gamma,k) = D(\omega,k) + \mathrm{i}\gamma\frac{\partial}{\partial\omega}D(\omega,k), \\
&D_{-1}(\omega_{-1}+\mathrm{i}\gamma,k_{-1}) = D_{-1}(\omega_{-1},k_{-1}) \\
&\quad +\mathrm{i}\gamma\frac{\partial}{\partial\omega_{-1}}D_{-1}(\omega_{-1},k_{-1}).
\end{aligned}
\tag{3.5}
$$

We believe that the conditions $D(\omega,k)=0$, $D_{-1}(\omega_{-1},k_{-1})=0$ are satisfied which corresponds to the excitation of two SEWs. Using (3.5) we find the expression for the instability growth rate γ

$$
\begin{aligned}
\gamma^2 &= -\frac{\omega_p^2}{\omega^2}\frac{\omega_p^2}{\omega_{-1}^2}\frac{k^2V_E^2\cos^2\alpha}{|\varepsilon(\omega_0)\cos\alpha + \mathrm{i}\sqrt{\sin^2\alpha-\varepsilon(\omega_0)}\,|^2} \\
&\times \frac{1}{\frac{\partial}{\partial\omega}D(\omega,k)\frac{\partial}{\partial\omega_{-1}}D_{-1}(\omega_{-1},k_{-1})} \\
&\times \left\{\sqrt{\sin^2\alpha-\varepsilon(\omega_0)}\left[\frac{k_{-1}\cos\theta}{\omega_{-1}\kappa_{-1}k} + \frac{k\cos\theta-k_0}{\omega\kappa k_{-1}}\right]\right. \\
&\left. + \sin\alpha\left[\frac{k-k_0\cos\theta}{\omega_0 kk_{-1}} - \frac{k_{-1}}{\omega\kappa\kappa_{-1}}\right]\right\} \\
&\times \left\{\sqrt{\sin^2\alpha-\varepsilon(\omega_0)}\left[\frac{k_{-1}\cos\theta}{\omega_{-1}\kappa_{-1}k} + \frac{k\cos\theta-k_0}{\omega\kappa k_{-1}}\right]\right. \\
&\left. + \sin\alpha\left[\frac{k-k_0\cos\theta}{\omega_0 kk_{-1}} + \frac{k_{-1}}{\omega_{-1}\kappa_{-1}\kappa}\right]\right\}.
\end{aligned}
\tag{3.6}
$$

For analysis of the growth rate (3.6), we find the solutions of dispersion equations $D(\omega,k)=0$ and $D_{-1}(\omega_{-1},k_{-1})=0$, and calculate derivatives $\partial D(\omega,k)/\partial\omega$ and $\partial D_{-1}(\omega_{-1},k_{-1})/\partial\omega_{-1}$. The solution of equations $D(\omega,k)=0$ and $D_{-1}(\omega_{-1},k_{-1})=0$ has a well-known form [1–3]

$$
k^2 = \frac{\omega^2}{c^2}\frac{\varepsilon}{\varepsilon+1}, \quad k_{-1}^2 = \frac{\omega_{-1}^2}{c^2}\frac{\varepsilon_{-1}}{\varepsilon_{-1}+1}.
\tag{3.7}
$$

Using formulas (3.7), we reduce the expressions for the coefficients κ, κ_V, κ_{-1}, $\kappa_{-1,V}$ to

$$
\begin{aligned}
&\kappa = \frac{|\omega|}{c}\sqrt{\frac{\varepsilon^2}{-\varepsilon-1}}, \quad \kappa_V = \frac{|\omega|}{c}\frac{1}{\sqrt{-\varepsilon-1}}, \\
&\kappa_{-1} = \frac{|\omega_{-1}|}{c}\sqrt{\frac{\varepsilon_{-1}^2}{-\varepsilon_{-1}-1}}, \quad \kappa_{-1,V} = \frac{|\omega_{-1}|}{c}\frac{1}{\sqrt{-\varepsilon_{-1}-1}}.
\end{aligned}
\tag{3.8}
$$

Taking into account formulas (3.8), derivatives of dispersion functions are expressed in terms of dielectric constants as follows

$$
\begin{aligned}
&\frac{\partial}{\partial\omega}D(\omega,k) = \frac{c\sqrt{(-1-\varepsilon)}\,(1-\varepsilon)(1+\varepsilon^2)}{\omega|\omega|\,\varepsilon^2}, \\
&\frac{\partial}{\partial\omega_{-1}}D_{-1}(\omega_{-1},k_{-1}) = \frac{c\sqrt{(-1-\varepsilon_{-1})}\,(1-\varepsilon_{-1})(1+\varepsilon_{-1}^2)}{\omega_{-1}|\omega_{-1}|\,\varepsilon_{-1}^2}.
\end{aligned}
\tag{3.9}
$$

Since the instability occurs when $\omega_{-1}<0$, we make replacing $\omega_{-1}\to$-ω_{-1}, $\mathbf{k}_{-1}\to$-$\mathbf{k}_{-1}$. Substituting relations (3.9) into formula (3.6) while taking into account the decay conditions $\omega_0=\omega+\omega_{-1}$ and $\mathbf{k}_0=\mathbf{k}+\mathbf{k}_{-1}$, we find the expression for the square of the instability growth rate

$$
\begin{aligned}
\gamma^2 &= \frac{k^2V_E^2\cos^2\alpha}{|\varepsilon(\omega_0)\cos\alpha + \mathrm{i}\sqrt{\sin^2\alpha-\varepsilon(\omega_0)}\,|^2} \\
&\times \frac{\omega^2\omega_{-1}^2\varepsilon^2\varepsilon_{-1}^2}{c^2\sqrt{(-1-\varepsilon)}\sqrt{(-1-\varepsilon_{-1})}(1+\varepsilon_{-1}^2)(1+\varepsilon^2)} \\
&\times \left\{\sqrt{\sin^2\alpha-\varepsilon(\omega_0)}\left[\frac{k\cos\theta-k_0}{\omega\kappa k_{-1}} - \frac{k_{-1}\cos\theta}{\omega_{-1}\kappa_{-1}k}\right]\right. \\
&\left. + \sin\alpha\left[\frac{k-k_0\cos\theta}{\omega_0 kk_{-1}} - \frac{k_{-1}}{\omega\kappa\kappa_{-1}}\right]\right\} \\
&\times \left\{\sqrt{\sin^2\alpha-\varepsilon(\omega_0)}\left[\frac{k\cos\theta-k_0}{\omega\kappa k_{-1}} - \frac{k_{-1}\cos\theta}{\omega_{-1}\kappa_{-1}k}\right]\right. \\
&\left. + \sin\alpha\left[\frac{k-k_0\cos\theta}{\omega_0 kk_{-1}} - \frac{k_{-1}}{\omega_{-1}\kappa_{-1}\kappa}\right]\right\}.
\end{aligned}
\tag{3.10}
$$

In this case, the conditions (3.7) should be satisfied for the excitation of two SEWs, which can be represented in the following form

$$
\begin{aligned}
&k^2 = \frac{\omega^2}{c^2}\frac{\omega_p^2-\omega^2}{\omega_p^2-2\omega^2}, \quad k_{-1}^2 = k^2 + k_0^2 - 2kk_0\cos\theta \\
&\quad = \frac{(\omega_0-\omega)^2}{c^2}\frac{\omega_p^2-(\omega_0-\omega)^2}{\omega_p^2-2(\omega_0-\omega)^2}.
\end{aligned}
\tag{3.11}
$$

Substituting the expression for the wave number k from the first relation in formula (3.11) into the second, we obtain the relationship between frequency ω and angles α, θ.

$$\frac{\omega^2}{\omega_0^2}\frac{\omega_p^2-\omega^2}{\omega_p^2-2\omega^2}+\sin^2\alpha-2\frac{\omega}{\omega_0}\sqrt{\frac{\omega_p^2-\omega^2}{\omega_p^2-2\omega^2}}\sin\alpha\cos\theta$$
$$=\frac{(\omega_0-\omega)^2}{\omega_0^2}\frac{\omega_p^2-(\omega_0-\omega)^2}{\omega_p^2-2(\omega_0-\omega)^2}. \tag{3.12}$$

In the expression for the growth rate (3.10), each curly bracket contains two square brackets, in which there are several terms. One part of them is determined by the contribution of the surface current and follows from the boundary condition (2.22), and the other part is related to the ponderomotive effect of the incident radiation and is determined by the boundary condition (2.23). Neglecting the surface current, the authors of the publication [18] do not actually take into account a number of terms that give the same contribution to the growth rate of decay instability.

As follows from the boundary condition (2.22) in the expression for the instability growth rate (3.10), several terms are connected exclusively with the contribution of the nonlinear surface current. In the first curly brackets, this is the second term in the first square bracket and the first term in the second square bracket. In the second curly brackets, the contribution of the nonlinear surface current is related to the first terms in both square brackets. Further analysis shows that the nonlinear current must be taken into account, since it makes the same contribution to the instability growth rate as the ponderomotive nonlinearity. At the same time, electromagnetic fields increase from the initial value δB_0 according to the exponential law $\delta B_\perp = \delta B_0 \exp(\gamma t)$.

Consider first excitation SEWs in a dense plasma, when the inequality $N=\omega_p^2/\omega_0^2=N_{0e}/N_{cr}\gg 1$ is satisfied, where $N_{cr}=m_e\omega_0^2/(4\pi e^2)$—the critical density for a given frequency ω_0. In this case, the expression for the square of the instability growth rate (3.10) takes the form

$$\gamma^2=\frac{\omega_0^2}{N^3}\frac{V_E^2}{c^2}x^3(1-x)\{(1-2x)\cos\theta+(1-x)\sin\alpha$$
$$+\sin^2\alpha\cos\theta\}^2\,, \tag{3.13}$$

where the notation $x=\omega/\omega_0$ is introduced. From equation (3.12) on condition $N\gg 1$ we find the dependence of the dimensionless frequency x on the angles α, θ

$$x=\frac{\cos^2\alpha}{2(1-\sin\alpha\cos\theta)}. \tag{3.14}$$

Substituting the expression for the dimensionless frequency (3.14) into formula (3.13), we obtain the dependence of the square of the instability growth rate in a dense plasma on the angles α,θ and concentration of electrons

$$\gamma^2=\frac{\omega_0^2}{N^3}\frac{V_E^2}{64c^2}\frac{\sin^2\alpha\cos^6\alpha}{(1-\sin\alpha\cos\theta)^6}$$
$$\times(1+\sin^2\alpha-2\sin\alpha\cos\theta)$$
$$\times\{(1-2\cos^2\theta)(1+\sin^2\alpha)+2\sin\alpha\cos\theta\}^2\,, \tag{3.15}$$

From formula (3.15) it follows that the maximum instability growth rate in an overdense plasma is determined by the expression

$$\gamma_{\max}=\frac{\omega_0}{N^{3/2}}\frac{3|V_E|}{8c}\approx 0.375\frac{\omega_0}{N^{3/2}}\frac{|V_E|}{c}, \tag{3.16}$$

and takes place for condition $\cos\theta=\sin\alpha=1$, when $\alpha\to\pi/2$, $\theta\to 0$. In this case, the dispersion laws of surface waves have the form $\omega\approx ck$, $\omega_{-1}\approx ck_{-1}$, and the frequencies of the excited SEWs coincide and are equal to half the frequency of the pump wave $\omega=\omega_{-1}=\omega_0/2$. Thus, in an overdense plasma, under the condition $N\gg 1$, the excitation of surface waves occurs in the electromagnetic spectral region and is most effective at grazing angles of the pump wave incidence. In this case, the excited SEWs have wave vectors of the same size $k=k_{-1}=k_0/2$ and propagate in the direction of the wave vector $\mathbf{k}_0$ of the pump wave. From the expression (3.16), it follows that for the plasma densities significantly above the critical value the instability growth rate is sufficiently small because of the dependence on the density in the form $\gamma_{\max}\propto N_{0e}^{-3/2}$. It can be concluded that the maximum value of the instability growth rate will occur in the near-critical plasma. Comparison the result (3.16) with the instability growth rate for the s-polarization of the pump wave [19]

$$\gamma_{\max}^{s}\approx 0.043\frac{\omega_0}{N^{3/2}}\frac{|V_E|}{c}, \tag{3.17}$$

shows that for the p-polarized incident electromagnetic wave the growth rate of decay instability is almost by one order of magnitude greater.

Consider the excitation of SEWs in plasma with an electron density that coincides with the critical value $N=\omega_p^2/\omega_0^2=N_{0e}/N_{cr}=1$. In this case, the expression for the instability growth rate takes the form

$$\gamma=\omega_0\frac{|V_E|}{c}G_1(x,\alpha,\theta), \tag{3.18}$$

where the function $G_1(x,\alpha,\theta)$ is determined by the relation

$$G_1^2(x,\alpha,\theta)=\frac{x^3(1-x)(1-x^2)[1-(1-x)^2]\cos^2\alpha}{(1-2x^2)^{3/2}[1-2(1-x)^2]^{1/2}}\times$$
$$\times\frac{[g_1(x,\alpha,\theta)-x(1-x)\sqrt{1-2x^2}\,][g_1(x,\alpha,\theta)-(1-x)^2\sqrt{1-2x^2}\,]}{[x^4+(1-x^2)^2]\{(1-x)^4+[1-(1-x)^2]^2\}}, \tag{3.19}$$

here

$$g_1(x, \alpha, \theta) = \sqrt{1 - x^2}\sqrt{1 - 2(1 - x)^2} \times [x\sqrt{1 - x^2} - \sqrt{1 - 2x^2}\sin\alpha\cos\theta] - (1 - x)\sqrt{1 - x^2}\sqrt{1 - 2x^2}\cos\theta - \sqrt{1 - 2(1 - x)^2} \times [\sqrt{1 - 2x^2}\sin\alpha - x\sqrt{1 - x^2}\cos\theta]. \tag{3.20}$$

The condition of joint excitation of two surface waves (ω, $\mathbf{k}$) and (ω_{-1}, $\mathbf{k}_{-1}$) (3.12) in dimensionless variables for $N = \omega_p^2/\omega_0^2 = N_{0e}/N_{cr} = 1$ takes the form

$$(1 - x)^2\frac{1 - (1 - x)^2}{1 - 2(1 - x)^2} = x^2\frac{1 - x^2}{1 - 2x^2} + \sin^2\alpha - 2x\sqrt{\frac{1 - x^2}{1 - 2x^2}}\sin\alpha\cos\theta. \tag{3.21}$$

To find the maximum of the function (3.19) upon condition (3.21), we used the direct search method for the absolute penalty function [22]. Numerical analysis of formulas (3.19)–(3.21) shows that the function G_1^2 has an absolute maximum when a pump wave decays into two SEWs with identical frequencies ($x = 1/2$), for angles sin $\alpha = 0.783226...$, $\cos\theta = 0.6395013...$, which is equal

$$G_{1,\,\max}^2 = 0.0376448.... \tag{3.22}$$

From here we find the maximum value of the instability growth rate

$$\gamma_{\max} \approx 0.2\omega_0\frac{|V_E|}{c}. \tag{3.23}$$

This value (3.23) is realized when the pump wave falls at an angle $\alpha \approx 52^\circ$ when the excited surface waves have equal frequencies $\omega = \omega_{-1} = \omega_0/2$ and propagate with wave vectors equal in magnitude $k = k_{-1} = \sqrt{3}\,\omega_0/(2\sqrt{2}\,c) \approx 0.612\omega_0/c$ to an angle $\theta \approx 50^\circ$ relative to the direction of the wave vector of the pump wave (see figure 2). In this case, the SEWs is excited with the dispersion law lying in the transition region between the fast electromagnetic ($\omega \approx ck$, $\omega_{-1} \approx ck_{-1}$) and short-wave quasistatic ($\omega = \omega_{-1} \approx \omega_p/\sqrt{2}$) modes. Comparison of expression (3.23) with instability growth rate for s-polarized pump wave [19]

$$\gamma_{\max}^s \approx 0.037\omega_0\frac{|V_E|}{c}, \tag{3.24}$$

shows that when a p-polarized wave is incident on a semi-infinite plasma, the decay instability growth rate is almost five times higher.

Let us now consider the excitation of SEWs when the frequency of the pump wave exceeds the Langmuir frequency and lies in the range $\omega_p < \omega_0 \leqslant \omega_p/\cos\alpha$ and the real part of the dielectric constant of the plasma is positive. Of special interest is the fall of an external electromagnetic wave on a plasma with the smallest of the above stated range density, when the condition is fulfilled

$$\sin^2\alpha = \varepsilon'(\omega_0), \tag{3.25}$$

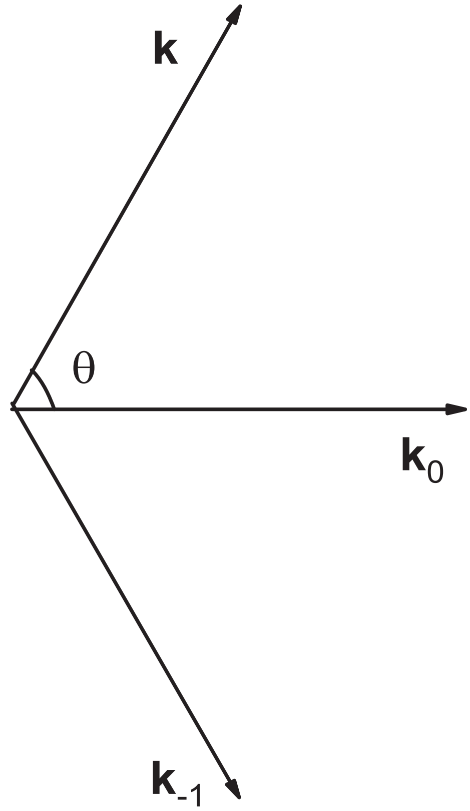

Figure 2. The direction of the wave vectors $\mathbf{k}$ and $\mathbf{k}_{-1}$ of the excited surface waves at the plasma boundary under the condition $\omega_p^2 = \omega_0^2$. The angle θ between the wave vectors $\mathbf{k}$ and $\mathbf{k}_0$ and between $\mathbf{k}_{-1}$ and $\mathbf{k}_0$, equals approximately 50°.

where $\varepsilon'(\omega_0)$— a real part of dielectric constant $\varepsilon(\omega_0) = \varepsilon'(\omega_0) + \mathrm{i}\varepsilon''(\omega_0)$. Below , we will also take into account its imaginary part $\varepsilon''(\omega_0) = \nu_{ei}\omega_p^2/\omega_0^3$, which is determined by the frequency of electron–ion collisions ν_{ei}. Angle α (3.25) is the angle of total internal reflection. The real part of the dielectric constant $\varepsilon'(\omega_0)$ is always less than unity $0 < \varepsilon'(\omega_0) < 1$. In this sense, vacuum with $\varepsilon(\omega_0) = 1$ is optically a more dense medium than plasma with a refractive index smaller than unity. Therefore, when an electromagnetic wave is incident at angles that satisfy the condition $\sin\alpha > \sqrt{\varepsilon'(\omega_0)}$, its full reflection occurs, and the field penetrates the plasma only to the depth of the skin layer δ_L.

When condition (3.25) is fulfilled, the electric field amplitude of the pump wave in a plasma is determined by the following formula

$$\mathbf{E}_1 = \frac{2E_0\cos\alpha[\mathbf{e}_x(\mathrm{i} + 1)\sqrt{\varepsilon''(\omega_0)/2} - \mathbf{e}_z\sin\alpha]}{\sin^2\alpha\cos\alpha + (\mathrm{i} + 1)\sqrt{\varepsilon''(\omega_0)/2}} \times \exp\left[(\mathrm{i} - 1)\frac{\omega_0}{c}z\sqrt{\varepsilon''(\omega_0)/2}\right], \tag{3.26}$$

and the expression for the instability growth rate has the form similar to (3.18)

$$\gamma = \omega_0\frac{|V_E|}{c}G_2(x, \alpha, \theta), \tag{3.27}$$

where the function $G_2^2(x,\ \alpha,\ \theta)$ is determined by the formula

$$G_2^2(x,\ \alpha,\ \theta) = \left| \frac{\sin^2\alpha\cos^2\alpha}{(\sin^2\alpha\cos\alpha + \sqrt{\varepsilon''(\omega_0)/2})^2 + \varepsilon''(\omega_0)/2} \right. \times \frac{x^3(1-x)(\cos^2\alpha - x^2)[\cos^2\alpha - (1-x)^2]}{[\cos^2\alpha - 2x^2 - \mathrm{i}x^2\varepsilon''(\omega_0 x)]^{3/2}\,[\cos^2\alpha - 2(1-x)^2 + \mathrm{i}(1-x)^2\varepsilon''(\omega_0[1-x])]^{1/2}} \times \frac{g_2(x,\ \alpha,\ \theta) - x(1-x)\sqrt{\cos^2\alpha - 2x^2 - \mathrm{i}x^2\varepsilon''(\omega_0 x)}}{x^4 + (\cos^2\alpha - x^2)^2} \times \left. \frac{g_2(x,\ \alpha,\ \theta) - (1-x)^2\sqrt{\cos^2\alpha - 2x^2 - \mathrm{i}x^2\varepsilon''(\omega_0 x)}}{(1-x)^4 + [\cos^2\alpha - (1-x)^2]^2} \right|, \tag{3.28}$$

surface waves in this case propagate in almost opposite directions at a right angle relative to the wave vector $\mathbf{k}_0$

here

$$g_2(x,\ \alpha,\ \theta) = |\sqrt{\cos^2\alpha - x^2}\sqrt{\cos^2\alpha - 2(1-x)^2 + \mathrm{i}(1-x)^2\varepsilon''(\omega_0[1-x])} \times [x\sqrt{\cos^2\alpha - x^2} - \sqrt{\cos^2\alpha - 2x^2 - \mathrm{i}x^2\varepsilon''(\omega_0 x)}\ \sin\alpha\cos\theta]|. \tag{3.29}$$

The condition of joint excitation of two surface waves (ω, $\mathbf{k}$) and (ω_{-1}, $\mathbf{k}_{-1}$) (3.12) in dimensionless variables when condition (3.25) or $\omega_p^2/\omega_0^2 = \cos^2\alpha$ is satisfied takes the form

$$(1-x)^2\left|\frac{1-(1-x)^2}{\cos^2\alpha - 2(1-x)^2 + \mathrm{i}(1-x)^2\varepsilon''(\omega_1)}\right| = x^2\left|\frac{1-x^2}{\cos^2\alpha - 2x^2 - \mathrm{i}x^2\varepsilon''(\omega)}\right| + \sin^2\alpha - 2x\sqrt{\left|\frac{1-x^2}{\cos^2\alpha - 2x^2 - \mathrm{i}x^2\varepsilon''(\omega)}\right|}\ \sin\alpha\cos\theta. \tag{3.30}$$

Note that in formulas (3.28)–(3.30), the imaginary part of the dielectric constant takes into account only in those factors and terms that can go to zero.

The analysis shows that the maximum value of the function (3.28) is equal to

$$G_2^2(x,\ \alpha,\ \theta) = \frac{1}{32\varepsilon''(\omega_0/2)} = \frac{\omega_0}{128\nu_{ei}} \tag{3.31}$$

and is realized when short-wave ($k \gg \omega_0/c$, ω_p/c) quasistatic surface waves are excited with coinciding frequencies $\omega = \omega_{-1} \approx \omega_p/\sqrt{2}$. In this case, the decay condition $\omega_0 = \omega + \omega_{-1}$ implies the relation $\omega_0 = \sqrt{2}\,\omega_p$ or $\varepsilon'(\omega_0) = 1/2$, from which, taking into account (3.25), we find the optimal angle of incidence of the pump wave on the plasma boundary $\alpha = \pi/4$. From the formula (3.30) it follows that the excited surface waves in this case propagate in almost opposite directions at a right angle relative to the wave vector $\mathbf{k}_0$ (figure 3). The maximum possible growth rate value in accordance with (3.27), (3.31) is

$$\gamma_{\alpha,\max} = \frac{\omega_0}{8\sqrt{2}}\sqrt{\frac{\omega_0}{\nu_{ei}}}\frac{|V_E|}{c} \approx 0.1\ \omega_0\sqrt{\frac{\omega_0}{\nu_{ei}}}\frac{|V_E|}{c}, \tag{3.32}$$

and because of the inequality $\omega_0 \gg \nu_{ei}$ significantly exceeds the value (3.23), which takes place under the condition $\omega_p^2 = \omega_0^2$. Note that in [18], the authors assumed that surface waves propagate along the direction of the wave vector of the pump wave. However, our results show that the maximum growth rate occurs when the excited SEWs propagate almost in the transverse direction relative to the $\mathbf{k}_0$—wave vector of the incident radiation.

To illustrate the results obtained, we consider the dependence of the instability growth rate (3.10) on the angle of incidence and electron density at the coincident frequencies of the excited surface waves. After simple transformations, the expression for the instability growth rate takes the form

$$\gamma = \omega_0\frac{|V_E|}{c}H(\alpha,\ N), \tag{3.33}$$

where the dependence on the angle of incidence and plasma density is described by the function $H(\alpha,\ N)$

$$H(\alpha,\ N) = \frac{1}{8}\frac{(N-1/4)\sin\alpha\cos\alpha}{[(N-1/2)^2 + (2N\nu)^2]^{1/4}} \times \frac{|N(1-2\mathrm{i}\nu)[1-2\cos^2\alpha] - 1/4 + \cos^2\alpha + 2\sqrt{[N(1-2\mathrm{i}\nu) - 1/2][N(1-2\mathrm{i}\nu) - \cos^2\alpha]}|}{[(N-1/4)^2 + (1/4)^2]|[1 - N(1-\mathrm{i}\nu)\cos\alpha + \mathrm{i}\sqrt{N(1-\mathrm{i}\nu) - \cos^2\alpha}]|}, \tag{3.34}$$

where $\nu = \nu_{ei}/\omega_0$. If the plasma electron density far exceeds the critical value $N \gg 1$, then the expansion in powers of a small parameter $1/N$ can be used. Then from formula (3.34)

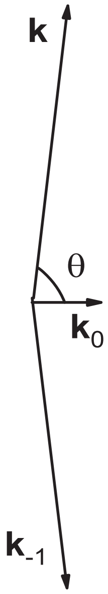

Figure 3. The direction of the wave vectors $\mathbf{k}$ and $\mathbf{k}_{-1}$ of the excited surface waves at the plasma boundary under the condition $\omega_p^2 = \omega_0^2 \cos^2\alpha$. The angle θ between the wave vectors $\mathbf{k}$ and $\mathbf{k}_0$ and between $\mathbf{k}_{-1}$ and $\mathbf{k}_0$, equals approximately 90°.

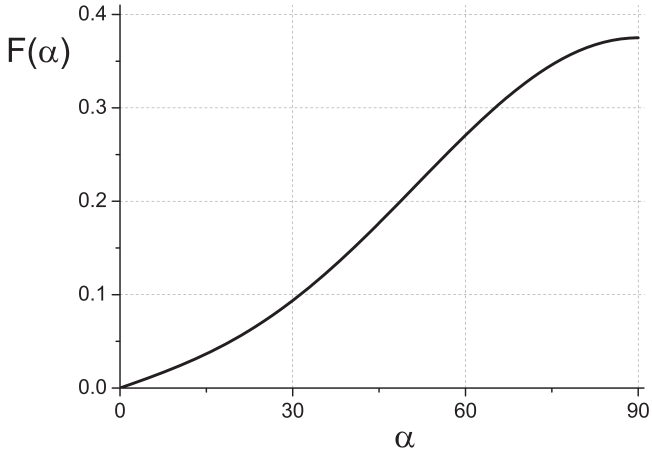

Figure 4. Dependence of the function (3.36) on the angle of incidence of the pump wave for a dense plasma $N \gg 1$.

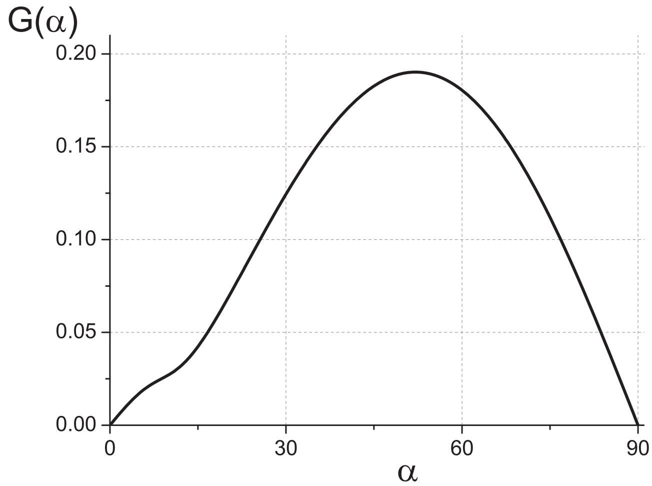

Figure 5. The dependence of the function (3.38) on the angle of incidence of the pump wave for near critical plasma at $N = 1$ for $\nu_{ei}/\omega_0 = 0.03$.

we obtain the dependence of the instability growth rate on the angle of incidence of the pump wave

$$\gamma = \frac{\omega_0}{N^{3/2}} \frac{|V_E|}{c} F(\alpha), \tag{3.35}$$

where the function $F(\alpha)$ has the form

$$F(\alpha) = \frac{1}{8} \sin\alpha (1 + 2\sin^2\alpha), \tag{3.36}$$

presented in figure 4. The maximum value of the instability growth rate takes place at the grazing incidence of the pump wave $\alpha \to \pi/2$, when $F(\alpha) = 3/8$, and coincides with the previously obtained expression (3.16). If the plasma density is two orders of magnitude higher than the critical value, then the accuracy of the approximate formula (3.35) due to expansion in a small parameter $1/N \ll 1$ is about one percent.

For the electron density that coincides with the critical value $N = 1$, from formula (3.34) we find the dependence of the instability growth rate on the angle of incidence of the pump wave

$$\gamma = \omega_0 \frac{|V_E|}{c} G(\alpha), \tag{3.37}$$

where the function $G(\alpha)$ has the form

$$G(\alpha) = \frac{3\sqrt{2}}{80} \sin\alpha \cos\alpha \times \frac{|3 - 4\cos^2\alpha - 8i\nu(1 - 2\cos^2\alpha) + 4\sqrt{2}\sqrt{(1 - 4i\nu)(\sin^2\alpha - 2i\nu)}|}{|\nu\cos\alpha + \sqrt{\sin^2\alpha - i\nu}|}, \tag{3.38}$$

and is shown in figure 5. From figure 5 it follows that the maximum value of the instability growth rate coincides with the value (3.23) and is realized for the angle of incidence $\alpha \approx 52^\circ$, which corresponds to the above results of numerical analysis. When condition (3.25) is fulfilled, from formula (3.34) we find that the maximum value of the instability growth rate coincides with the previously obtained expression (3.32) when the pump wave is incident at an angle $\alpha = \pi/4$ on the plasma with electron density $N = 1/2$, which corresponds to half the critical value.

We find the instability threshold under conditions when the growth rate has a maximum value (3.32). To do this, we take into account collisions in the expressions for the dielectric constant

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}\left(1 - \mathrm{i}\frac{\nu_{ei}}{\omega}\right), \quad \varepsilon_{-1} = 1 - \frac{\omega_p^2}{\omega_{-1}^2}\left(1 - \mathrm{i}\frac{\nu_{ei}}{\omega_{-1}}\right). \tag{3.39}$$

Then, for the instability growth rate, instead of expression (3.27), we have the following equation

$$\left[\gamma + \frac{\nu_{ei}}{2}\frac{1-\varepsilon}{1+\varepsilon^2}\right]\left[\gamma + \frac{\nu_{ei}}{2}\frac{1-\varepsilon_{-1}}{1+\varepsilon_{-1}^2}\right] = \omega_0^2\frac{V_E^2}{c^2}G_2^2. \tag{3.40}$$

The threshold value of the electric field of the pump wave is determined from the condition $\gamma = 0$ and has the form

$$\frac{|V_E|}{c} = \frac{\nu_{ei}}{2\omega_0}\frac{1}{G_2}, \tag{3.41}$$

since at the electrostatic limit $\varepsilon = \varepsilon_{-1} = -1$. The minimum value of the threshold field is

$$\left(\frac{|V_E|}{c}\right)_{\text{th}} = 4\sqrt{2}\left(\frac{\nu_{ei}}{\omega_0}\right)^{3/2}, \tag{3.42}$$

and takes place at the maximum value of the function G_2 (3.31). From the obtained expression (3.42) it follows that the threshold value of the electric field of the pump wave is small, since the condition $\nu_{ei} \ll \omega_0$ is usually satisfied. That is, the decay of the pump wave into two short-wave quasistatic plasmons can occur at sufficiently low intensities of the incident electromagnetic radiation. For example, when $\nu_{ei} = 10^{-2}\omega_0$ for a wavelength $\lambda_0 = 1\ \mu$m, the threshold value of the intensity of laser radiation in accordance with formula (3.42) is equal to $(I_L)_{\text{th}} \approx 4 \times 10^{13}\ \text{W cm}^{-2}$. Similarly, you can calculate the threshold for the case when the electron density is equal to the critical value. For this purpose, in equation (3.40) it is necessary to make a replacement $G_2 \to G_1$ and take into account that when $\omega_p^2 = \omega_0^2$ and $x = 1/2$ dielectric constants at frequencies of surface waves equal $\varepsilon = \varepsilon_{-1} = -3$. Then for the threshold field we get the following value

$$\left(\frac{|V_E|}{c}\right)_{\text{th}} = \frac{0.2}{G_{1,\max}}\frac{\nu_{ei}}{\omega_0} \approx \frac{\nu_{ei}}{\omega_0}, \tag{3.43}$$

which at condition $32(\nu_{ei}/\omega_0) < 1$ exceeds the value (3.42). For the above mentioned parameters $\nu_{ei} = 10^{-2}\omega_0$ and $\lambda_0 = 1\ \mu$m from the formula (3.43) for the threshold intensity of laser radiation we get $(I_L)_{\text{th}} \approx 1.4 \times 10^{14}\ \text{W cm}^{-2}$. In dense plasma, when the electron concentration significantly exceeds the critical value, the instability threshold in accordance with formulas (3.40), (3.15) has the form

$$\left(\frac{|V_E|}{c}\right)_{\text{th}} = \frac{1}{3}\frac{\omega_p}{\omega_0}\frac{\nu_{ei}}{\omega_0}, \tag{3.44}$$

and exceeds the corresponding values (3.42), (3.43) for near-critical plasma.

4. Conclusion

In the present work, the parametric excitation of TM surface waves at the boundary of a dense plasma under the action of a *p*-polarized electromagnetic wave is investigated. The ground state is considered and it is shown that the presence of the *p*-component in the pump wave leads to the appearance of a surface charge density oscillating at the pump wave frequency. Equations and boundary conditions for perturbation of the electromagnetic field are obtained. The dispersion equation is derived and the growth rate of the decay instability, at which two SEWs are excited, is calculated. The dependence of the instability growth rate on the electron density, the angle of incidence of the pump wave, and the angle between the wave vectors of the excited surface waves is investigated. It is shown that the growth rate has a maximum value in a plasma with an electron concentration close to the critical value when the excitation of short-wave quasistatic surface waves occurs. It is found that for *p*-polarization of the incident electromagnetic wave growth rate of decay instability significantly exceeds the growth rate of instability in case of *s*-polarized pump wave. The instability threshold is calculated and it is shown that it has a minimum value when exciting short-wave surface plasmons.

We discuss the conditions when the collisions of electrons in the equation for the speed of electrons in the system of equation (2.2) can be neglected . In accordance with the formula for the frequency of electron–ion collisions [23], we write its relation to the frequency of the incident wave in the form

$$\frac{\nu_{ei}}{\omega_0} = Z\frac{\sqrt{8}}{3}\frac{\omega_p}{\omega_0}\eta^{3/2}\ln\Lambda, \tag{4.1}$$

where Z is the charge number of ions, $\ln\Lambda = \ln([\eta^{3/2}\sqrt{8\pi}]^{-1})$ is the Coulomb logarithm, and the parameter η is equal to the ratio of the electron interaction energy to their temperature T_e

$$\eta = \frac{e^2 N_{0e}^{1/3}}{T_e}. \tag{4.2}$$

Since the frequencies of the excited surface waves are equal to half the frequency of the pump wave and the dielectric constant at these frequencies do not vanish, collisions of electrons in calculating the instability growth rate can be neglected if the condition $\nu_{ei} \ll \omega_0$ is satisfied. An exception is the case of a pump wave incident at an angle of total internal reflection (3.25), when the instability growth rate is completely determined by the frequency of electron collisions, for which the imaginary part is taken into account in the expression for dielectric constant at the pump frequency.

If the temperature of the electrons is equal to $T_e \approx 200$ eV, then we find from formula (4.1) $\nu_{ei} \approx 0.2\omega_0$ for the density $N_{0e} \approx 10^{23}$ cm^{-3} at $Z = 1$. When the electron density is close to the critical value $N_{0e} \approx 10^{21}$ cm^{-3}, the frequency of electron collisions at the same temperature is much lower $\nu_{ei} \approx 3 \times 10^{-3}\omega_0$. These estimates show that when calculating the instability growth rate, the frequency of electron collisions can be neglected even in the case of a dense plasma.

In conclusion, we estimate the instability growth rate for the characteristic parameters of the laser-plasma experiments. Let a laser pulse with the wavelength $\lambda_0 = 1\ \mu$m (frequency $\omega_0 \approx 1.88 \times 10^{15}$ s^{-1}), duration $\tau = 500$ fs, and intensity $I_L = 10^{16}$ W cm^{-2} be incident on the boundary of a plasma formed by the ionization of a solid target, the electron density in which, $N_{0e} \approx 10^{23}$ cm^{-3}, is two orders of magnitude higher than the critical density. In this case, according to formula (3.16), the growth rate is $\gamma \approx 3 \times 10^{-5}\omega_0$ and the instability has no time to develop during the laser pulse, because the gain factor $\Gamma = \gamma\tau \approx 3 \times 10^{-2}$ is too low. An appreciable effect can be achieved by using an aerogel target, which, being ionized, has an electron density close to the critical density. For an electron density of $N_{0e} \approx 10^{21}$ cm^{-3} and collision frequency $\nu_{ei} \approx 10^{-2}\omega_0$ we find from formula (3.32) that $\gamma \approx 10^{-1}\omega_0$. Then the magnetic field of the surface waves increases in time from the initial value δB_0 according to the following law $\delta B_\perp = \delta B_0 \exp(\gamma t)$, where the instability growth rate in this case is equal to $\gamma \approx 0.1\omega_0$. In this case, the threshold intensity value (3.43) is exceeded by almost two orders of magnitude, and the gain factor has a value $\Gamma = \gamma\tau \approx 100$. Therefore, the initial seed amplitudes of surface waves may well grow to noticeable values.

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