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A trimmed moving total least-squares method for curve and surface fitting

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Abstract

The moving least-squares (MLS) method has been developed for fitting measurement data contaminated with errors. The local approximants of the MLS method only take the random errors of the dependent variable into account, whereas the independent variables of measurement data always contain errors. To consider the influence of errors of dependent and independent variables, the moving total least-squares (MTLS) method is a better choice. However, both MLS and MTLS methods are sensitive to outliers, greatly affecting fitting accuracy and robustness. This paper presents an improved method, the trimmed MTLS (TrMTLS) method, in which the total least-squares method with a truncation procedure is adopted to determine the local coefficients in the influence domain. This method can deal with outliers and random errors of all variables without setting the threshold or adding small weights subjectively. The results of numerical simulation and experimental measurements indicate that the proposed algorithm has better fitting accuracy and robustness than the MTLS and MLS methods.

Keywords: moving least squares, random errors, outliers, local approximants

(Some figures may appear in colour only in the online journal)

1. Introduction

Various methods of approximation or interpolation of measurement data have been researched in past decades [1]. The moving least-squares (MLS) method is one popular method for approximating a function from a set of scattered data [2, 3]. The MLS method for smoothing and approximating scattered data was first introduced by Shepard [4] in the lowest-order case and generalized to a higher degree by Lancaster and Salkauska [5]. The principle of the MLS method is to start with the weighted least-squares (WLS) [6] estimation in the influence domain at an arbitrary fixed point and then move the point over the entire parameter domain, where the WLS fitting is calculated and evaluated for each measurement point independently. This method can be regarded in a way as a combination of WLS and piecewise least squares (PLS) [7]. Besides, as a flexible meshless method there is no need to construct meshes in the domain as with finite element method [8]. MLS has been widely used in many engineering fields. For example, it is known that meshless methods have been used to solve mathematical and physical problems where traditional calculation methods are not applicable [9–11], such as the element-free Galerkin method [12], the meshless local Petrov–Galerkin method [13] and the boundary element-free method [14]. In recent years, many scholars have studied and enhanced the MLS method [15, 16].

As an approximation method, the MLS method determines local approximants in the same way as the ordinary least-squares method [17], whereas errors always occur to all variables. To consider the influence of errors of all variables [18], it is more accurate to determine the local approximants using the total least-squares (TLS) method [17, 19]. For practical engineering problems, the measurement data are usually obtained by uniformly measuring curves and surfaces.

However, since the measurement data are not always sufficient to express all the curve and surface information, it is necessary to generate new non-measurement points, which may introduce new errors. Besides, outliers are inevitable and will result in deviation from the measurement data due to the influence of the testing environment and the instrument itself [20]. The moving total least-squares (MTLS) method suffers from the same problem as the MLS method and cannot be properly applied to curve and surface fitting when outliers occur in the measurement data [21–23].

As mentioned above, the MLS and MTLS methods can be greatly influenced by outliers. The fitting results often deviate from the real curve and the performance of fitting is strongly influenced even if only one outlier exists in the measurement data [24, 25]. Therefore, it is critical to avoid or reduce their influence on curve and surface fitting in order to achieve better results in most cases. Some applicable solutions have been proposed [26–28]. One solution is to directly delete the samples which are probably outliers. In this method, a threshold value is set to determine whether the measurement data are outliers, and then the confirmed outliers are deleted from the measurement data before the surface is fitted [29–32]. However, the accuracy of this method is directly related to the threshold value. Therefore, it is vital to choose the threshold appropriately, which is not an easy task. Another method is to assign appropriate small weights to outliers instead of removing them directly, in which case the negative effects of outliers on the curve and surface reconstruction can be reduced indirectly. However, how to add small weights to outliers is actually a challenging problem, especially when there is more than one outlier in the measurement data. Moreover, although it is clear that the negative influences are relatively reduced, it is hard to know the exact impacts that will be generated by these small weights [31].

To avoid setting the threshold or adding small weights subjectively [31, 32], an improved curve and surface fitting approach called the trimmed MTLS (TrMTLS) method is introduced in this paper. In the domain of influence of the TrMTLS method, the TLS method based on singular value decomposition (SVD) [33] with a truncation procedure is adopted to deal with the outliers and the errors of all variables. It has been proved that the impact of outliers is mitigated and the fitting accuracy is improved. Even if there are no outliers, the results of the improved method are still better than those of the MLS and MTLS methods. In section 2, a brief introduction to the MLS method is given. The TrMTLS method is presented in detail in section 3. Examples of curve and surface fitting including numerical simulation and experimental measurements are given in section 4 to verify the performance of the TrMTLS method. Conclusions are shown given section 5.

2. The MLS method

We first give a brief description of the MLS method. To describe the principle of the MLS method, the trial approximation function [34] is defined as

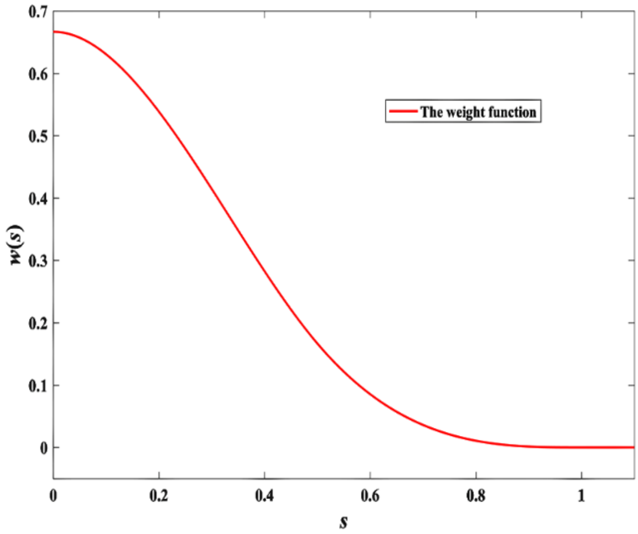

Figure 1. Schematic graph of the cubic spline weight function.

$$u^h(\mathbf{x}) = \sum_{i=1}^{m} p_i(\mathbf{x})\, a_i = \mathbf{p}^T(\mathbf{x})\mathbf{a} \tag{1}$$

where $p_i(\mathbf{x})$, $i = 1, 2, \ldots, m$ are the monomial basis functions, a_i are the coefficients of the basis functions and m is the number of terms in the basis functions.

General polynomial basis functions include the linear basis function, the quadratic basis function and so on, with the linear basis function being widely applied. The two common basis functions can be expressed as follows.

Linear basis function:

$$\begin{cases} \mathbf{p}(\mathbf{x}) = (1, x)^T & (m = 2) \\ \mathbf{p}(\mathbf{x}) = \left(1, x, x^2\right) & (m = 3) \end{cases}.$$

Quadratic basis function:

$$\begin{cases} \mathbf{p}(\mathbf{x}) = \left(1,\ x,\ x^2\right)^T & (m = 3) \\ \mathbf{p}(\mathbf{x}) = \left(1,\ x,\ y,\ x^2,\ xy,\ y^2\right)^T & (m = 6) \end{cases}.$$

At each point of $\mathbf{x}$, an appropriate $\mathbf{a}$ can be chosen so that $u(\mathbf{x})$ is well approximated by $u^h(\mathbf{x})$. To measure the approximation of the function, the approximation function of the discrete weighted L^2 norm can be defined as having the following form:

$$J = \sum_{I=1}^{n} w\left(\|\mathbf{x} - \mathbf{x}_I\| / r\right) \left[\sum_{i=1}^{m} p_i(\mathbf{x}_I)\, a_i - u(\mathbf{x}_I)\right]^2 \tag{2}$$

where r is the radius of the compact influence domain and $w\left(\|\mathbf{x} - \mathbf{x}_I\| / r\right)$ is a weight function, the value of which decreases increasing distance $s = \|\mathbf{x} - \mathbf{x}_I\|$ between $\mathbf{x}$ and $\mathbf{x}_I$. $\mathbf{x}_I$ $(I = 1, 2, \ldots, n)$ is the node in the influence domain of $\mathbf{x}$. Many forms of weight function have been proposed in previous studies. Commonly used weight functions are the exponential weight function and the spline weight function. The cubic spline weight function is applied in this paper; it is expressed as in equation (3) and shown in figure 1:

$$w(s) = \begin{cases} \frac{2}{3} - 4s^2 + 4s^3 & s \leqslant \frac{1}{2} \\ \frac{4}{3} - 4s + 4s^2 - \frac{4}{3}s^3 & \frac{1}{2} < s \leqslant 1 \\ 0 & s > 1 \end{cases}. \tag{3}$$

In the influence domain of $\mathbf{x}$, the coefficients of local approximants are solved by

$$\mathbf{a} = \mathbf{A}^{-1}(\mathbf{x})\,\mathbf{B}(\mathbf{x})\,\mathbf{u} \tag{4}$$

where

$$\mathbf{A}(\mathbf{x}) = \mathbf{P}^T\mathbf{W}(\mathbf{x})\,\mathbf{P}$$

$$\mathbf{B}(\mathbf{x}) = \mathbf{P}^T\mathbf{W}(\mathbf{x})$$

$$\mathbf{P} = \begin{pmatrix} p_1(\mathbf{x}_1) & p_2(\mathbf{x}_1) & \cdots & p_m(\mathbf{x}_1) \\ p_1(\mathbf{x}_2) & p_2(\mathbf{x}_2) & \cdots & p_m(\mathbf{x}_2) \\ \vdots & \vdots & \ddots & \vdots \\ p_1(\mathbf{x}_n) & p_2(\mathbf{x}_n) & \cdots & p_m(\mathbf{x}_n) \end{pmatrix}$$

$$\mathbf{W}(\mathbf{x}) = \mathrm{diag}\left(w_1(s),\, w_2(s),\, \cdots,\, w_n(s)\right)$$

$$\mathbf{u} = \left(u(\mathbf{x}_1),\, u(\mathbf{x}_2),\, \cdots,\, u(\mathbf{x}_n)\right)^T.$$

The approximation function equation (1) can be rewritten as

$$u^h(\mathbf{x}) = \mathbf{p}^T(\mathbf{x})\,\mathbf{a} = \mathbf{p}^T(\mathbf{x})\,\mathbf{A}^{-1}(\mathbf{x})\,\mathbf{B}(\mathbf{x})\,\mathbf{u}. \tag{5}$$

In this paper, we only consider the linear least-squares estimation of the influence domain.

3. The TrMTLS method

3.1. The MTLS method

The TLS method is a method for dealing with the errors-in-variables (EIV) model [33, 35] in which errors of all variables are considered. The function model is defined as

$$\mathbf{AX} = \mathbf{B} \tag{6}$$

where

$$\mathbf{A} = \mathbf{A}_1 + \Delta\mathbf{A}$$

$$\mathbf{B} = \mathbf{B}_1 + \Delta\mathbf{B}.$$

An augmented matrix is constructed by the TLS method based on SVD:

$$\mathbf{C} := \begin{bmatrix}\mathbf{A} & \mathbf{B}\end{bmatrix} = \mathbf{U}\Sigma\mathbf{V}^T \tag{7}$$

where $\mathbf{\Sigma} = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_{n+d})$. Let $\sigma_1 \geqslant \sigma_2, \ldots, \geqslant \sigma_{n+d}$ be the singular values of C; we define the partitionings as follows:

$$\mathbf{V} := \begin{bmatrix}\mathbf{V}_{11} & \mathbf{V}_{12} \\ \mathbf{V}_{21} & \mathbf{V}_{22}\end{bmatrix} \quad \mathbf{\Sigma} := \begin{bmatrix}\mathbf{\Sigma}_1 & 0 \\ 0 & \mathbf{\Sigma}_2\end{bmatrix}. \tag{8}$$

When $\mathbf{V}_{22}$ is non-singular, a solution exists by the TLS method. It is unique only if $\sigma_n \neq \sigma_{n+1}$. On this occasion, the solution by TLS is

$$\hat{X}_{\mathrm{tls}} = -\mathbf{V}_{12}\mathbf{V}_{22}^{-1}. \tag{9}$$

The TLS method based on SVD is applied to the MTLS method to determine the parameters of local approximants. Not only is the calculation faster, but also the order of the basis function is easier to change. The augmented matrix [33, 36] can be expressed as

$$\mathbf{C}_\mathbf{x} := \mathbf{W}_\mathbf{x}\begin{bmatrix}\mathbf{A} & \mathbf{B}\end{bmatrix} = \mathbf{U}_\mathbf{x}\mathbf{\Sigma}_\mathbf{x}\mathbf{V}_\mathbf{x}^T \tag{10}$$

where $\mathbf{W}_\mathbf{x} = \mathrm{diag}(w(\mathbf{x}-\mathbf{x}_1), w(\mathbf{x}-\mathbf{x}_2), \ldots, w(\mathbf{x}-\mathbf{x}_n))$ is the weight matrix. The solution for equation (9) can be rewritten as

$$\mathbf{a} = -\mathbf{V}_{\mathbf{x}12}\mathbf{V}_{\mathbf{x}22}^{-1}. \tag{11}$$

3.2. The TrMTLS method

As mentioned above, the MLS and MTLS methods are sensitive to the outliers in the measurement data. Therefore, the proposed TrMTLS method is a viable alternative that can reduce the influence of outliers. In this method, the residual is defined as

$$r(k) = w\left(y_i - y_{if}\right) \tag{12}$$

where y_i is the real value, y_{if} is the fitting value and w is the weight value.

Let $m < N$, where m is the number of nodes in the influence domain and N is the total number of nodes. Then the truncation procedure [37–39] is

$$\theta_{\mathrm{Tr}} = \arg\max\left\{r^2_{1:m(r)}\right\}. \tag{13}$$

The TLS method with the truncation procedure is applied to determine the local coefficients in the TrMTLS method. In the influence domain of an arbitrary fixed point in the TrMTLS method, the TLS method based on SVD is first adopted to obtain the coefficients of local approximation. Then, the residuals of all nodes can be obtained by the coefficients of local approximation and the appropriate weight function, and the truncation procedure is used to trim the node for which the squared residual is the largest. Finally, the local approximation is recalculated to replace the original value by using the TLS method based on SVD. The arbitrary fixed point is moved over the entire parameter domain, where the truncation procedure is calculated for each point independently.

To further understand the principle of the TrMTLS method, the truncation procedure for the influence domain is shown in figure 2.

As shown in figure 2, $\Delta_e < \Delta_b < \Delta_c < \Delta_d < \Delta_a$ and $w_c < w_a < w_b < w_e < w_d$. In the influence domain of x_i, it can be obtained that $w_d\Delta_d$ of the square is the largest, so node d must be trimmed.

As mentioned above, the MLS method only considers the errors of the dependent variables, with the constraint of local approximation being carried out in the vertical direction. The MTLS method can be considered as an improved MLS method, and it takes into account the errors of all variables, with the constraint of local approximation being carried out in

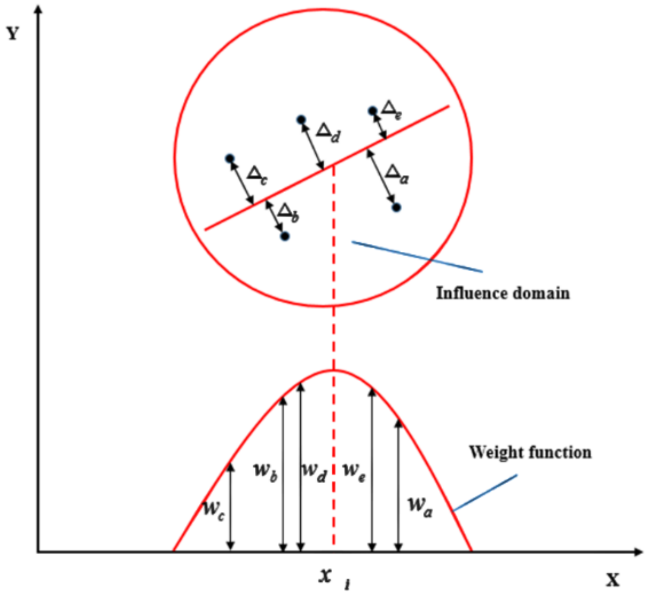

Figure 2. Schematic graph of the TrMTLS method.

the orthogonal direction. However, both methods are sensitive to outliers. Unlike the MTLS method, the TLS method based on SVD with a truncation procedure is adopted for dealing with the outliers and errors of all variables in the influence domain of TrMTLS.

The previous numerical simulation showed that it can be difficult to assign an appropriate weight function to the nodes in the influence domain before the node with the largest squared residual is trimmed. There are two ways to solve this: one is use the weight function whose value decreases with increasing distance between the nodes and the fitting points; the other is to add the same weight value to all nodes in the influence domain. These two methods suit different circumstances. It can be found that the result of the first way is more accurate when there are obvious outliers in the measurement data, and the second way is better when there are only unbiased random errors or outliers with no obvious values. We named the first way 'unweighted TrMTLS' and the second 'weighted TrMTLS'.

The following procedure, as shown in the flowchart in figure 3, is carried out in numerical simulation experiments to verify the performance of the improved method:

Step 1: Add the random errors $(\delta_i, \varepsilon_i)$ and outliers $(0, \Delta y_j)$ to the data (x_i, y_i) for getting tested data (x_{im}, y_{im}).
Step 2: Fit the tested data (x_{im}, y_{im}) by MLS, MTLS and unweighted TrMTLS for getting the fitting value (x_{if}, y_{if}).
Step 3: Calculate the fitting error s of the theoretical value y_i and fitting value y_{if} by

$$s = \sum_{i=1}^{n} |y_i - y_{if}|. \tag{14}$$

Record the values s_1, s_2 and s_3 for MLS, MTLS and TrMTLS, respectively.

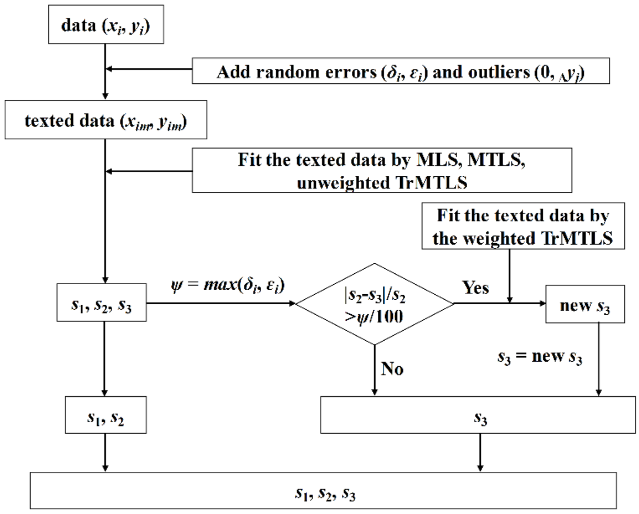

Figure 3. Simulation flowchart of the TrMTLS method.

Step 4: Calculate the value $|s_2 - s_3|/s_2$. If $|s_2 - s_3|/s_2 > \psi/100$, perform Step 5; otherwise, Step 5 is skipped.
Step 5:Fit the tested data (x_{im}, y_{im}) using weighted TrMTLS for getting the fitting value (x_{if}, y_{if}), then recalculate the sum of errors to replace the previous and record the new value of s_3.
Step 6: Repeat Steps 1–5 10 000 times.
Step 7: Average the recorded values of s_1, s_2 and s_3, and take them as the final values of MLS, MTLS and TrMTLS, respectively.

4. Case study

In this section, four examples are given to verify the performance of the TrMTLS method. The MLS method and MTLS method are also applied for comparison.

4.1. Example 1

Consider the aspheric profile function

$$y = \frac{cx^2}{1 + \sqrt{1 - (1+k)\, c^2 x^2}} \tag{15}$$

where $c = 1/1083$ is the reciprocal of the radius of curvature of the base vertex and $k = -1.5$ is the constant of the quadric surface. Select a uniformly distributed set of points (x_i, y_i), $i = 1, 2, \ldots, n$ determined by equation (15). Then, the random errors $(\delta_i, \varepsilon_i)$ and outliers $(0, \Delta y_j)$ are added to the points (x_i, y_i), forming a set of tested data (x_{im}, y_{im}). In this section, normally distributed random errors with a zero mean are added. Outliers are generated by adding Δy_j to some points of the dependent variable. E_j, $j = 1, 2, 3, 4$ are outliers as shown in figure 4. The fitting performance is characterized by equation (14).

In Example 1 let $n = 61$ and $d = (\max(x) - \min(x))/5$. Figure 4 shows the fitting curves obtained using MLS, MTLS

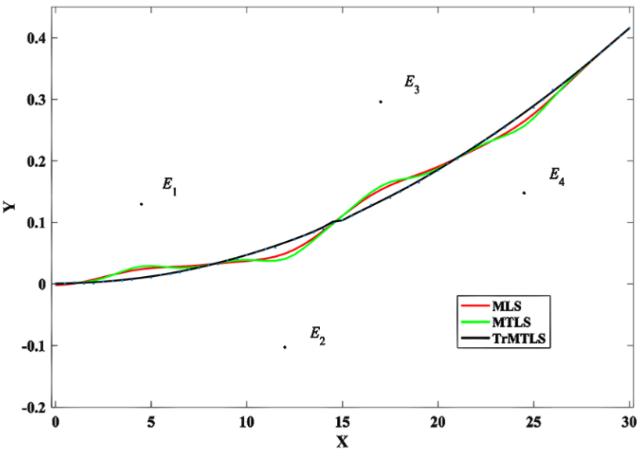

Figure 4. Fitting the aspheric profile curve by the MLS, MTLS and TrMTLS methods.

Table 1. The sum of errors s of three methods for Example 1.

Variance		s		
δ_i	ε_i	s_1	s_2	s_3
0.000 001	0.001	0.508 742	0.570 558	0.048 801
0.000 01	0.001	0.508 678	0.570 478	0.048 792
0.0001	0.001	0.508 798	0.570 637	0.048 815
0.001	0.001	0.508 779	0.570 605	0.048 783
0.001	0.0001	0.508 130	0.568 753	0.046 874
0.001	0.000 01	0.508 135	0.568 739	0.047 012
0.001	0.000 001	0.508 133	0.568 737	0.047 013

and TrMTLS. The sum of errors for these three methods are listed in table 1.

4.2. Example 2

In this example, we consider the oscillation function

$$y = e^{ax}\sin(bx) \tag{16}$$

where $a = 1/30$ and $b = 0.4$. The data are obtained in the same way as in Example 1 and are still fitted by the three methods. In Example 2 let $n = 161$ and $d = (\max(x) - \min(x)) \times 2/25$. The fitting results and curves are shown in table 2 and figure 5, respectively.

4.3. Example 3

In this example, we consider the following function

$$z = \left(x^2 - y^2\right)^2 \tag{17}$$

defined on the region $\Omega = [-1, 1] \times [-1, 1]$. Taking a uniformly distributed set of points, the region is divided into a 33×33 regular node grid. The data are obtained in the same way as in Examples 1 and 2. The fitting results using these three methods are shown in table 3 and the fitted surfaces are shown in figure 6.

In Example 3 let $n = 1089$ and $d = (\max(x) + \max(y))/10$. The fitting performance is characterized by the sum of errors between the theoretical value and the fitting value

Table 2. The sum of errors s of the three methods for Example 2.

Variance		s		
δ_i	ε_i	s_1	s_2	s_3
0.000 001	0.001	2.614 597	2.047 388	0.959 902
0.000 01	0.001	2.614 645	2.047 429	0.959 945
0.0001	0.001	2.614 580	2.047 395	0.959 946
0.001	0.001	2.614 596	2.047 389	0.960 077
0.001	0.0001	2.614 229	2.046 992	0.960 275
0.001	0.000 01	2.614 255	2.047 021	0.960 309
0.001	0.000 001	2.614 258	2.047 030	0.960 295

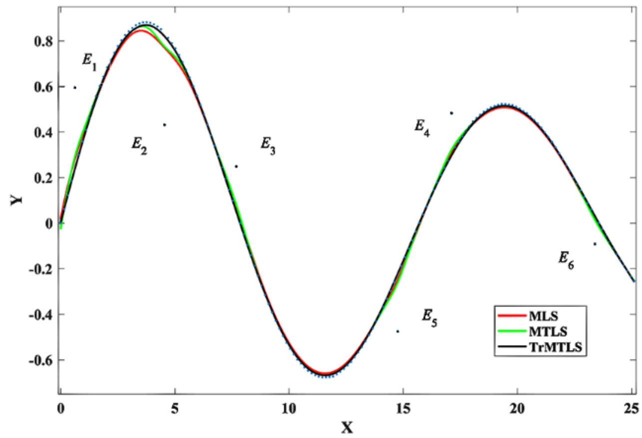

Figure 5. Fitting the oscillation curve with MLS, MTLS and TrMTLS methods.

Table 3. The sum of errors s for the three methods for Example 3.

Variance		s		
σ_x, σ_y	σ_z	s_1	s_2	s_3
0.000 001	0.001	8.267 936	4.924 211	3.702 649
0.000 01	0.001	8.267 709	4.924 068	3.727 679
0.0001	0.001	8.268 510	4.924 925	3.702 433
0.001	0.001	8.293 442	5.010 912	3.971 687
0.001	0.0001	8.283 595	4.994 708	3.708 152
0.001	0.000 01	8.290 440	5.006 132	3.733 076
0.001	0.000 001	8.276 531	4.991 401	3.708 305

$$s = \sum_{i=1}^{n} |z_i - z_{if}| \tag{18}$$

where z_i and z_{if} are the theoretical and fitting values, respectively.

It can be seen from Examples 1–3 that MTLS and MLS methods are sensitive to outliers. Compared with these two methods, the improved algorithm, TrMTLS, obviously gives better results. Even when there are no outliers, the results of the improved method are still better. The function of Example 1 is taken as an example to illustrate the performance of the TrMTLS method when the data only contain random errors. The fitting results and curves are shown in table 4 and figure 7, respectively. As shown in figure 7, all three methods give a nice approximation. From the results of table 4, the

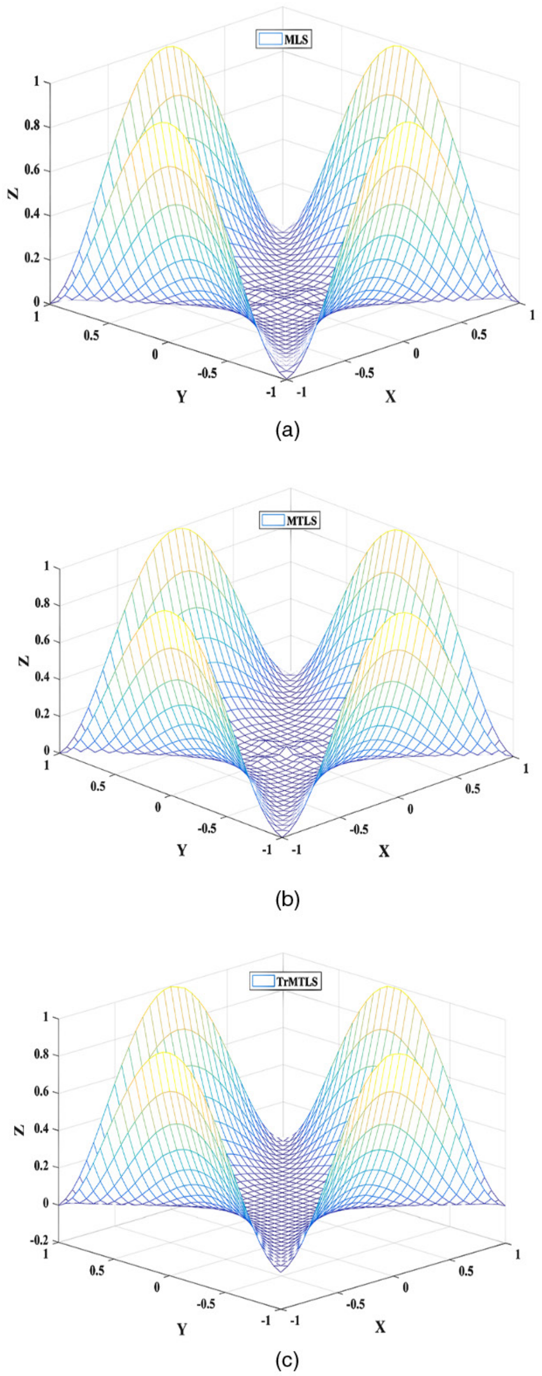

Figure 6. Fitting the surface in Example 3 by three methods: (a) MLS, (b) MTLS and (c) TrMTLS.

improved method (TrMTLS) is more accurate than the other two methods.

4.4. Example 4

An experiment was carried out to further illustrate the performance of the TrMTLS method. As shown in figure 8, the coordinate measuring machine is used to measure the profile of a standard cylinder with a radius of 40.1840 mm. The profile data are obtained by measuring the fixed cylinder surface horizontally with a non-contact displacement KEYENCE LK-G150 sensor. In Example 4 let $n = 950$ and $r = (\max(x) - \min(x)) \times 9/1000$. The repetitive positioning error of the X-axis is about 15 μm and the repetitive measurement error

Table 4. The sum of errors s of the three methods for Example 1.

Variance		s		
δ_i	ε_i	s_1	s_2	s_3
0.000 001	0.001	0.075 53	0.043 5786	0.043 5782
0.000 01	0.001	0.075 39	0.043 4438	0.043 4435
0.0001	0.001	0.075 40	0.043 4348	0.043 4345
0.001	0.001	0.075 56	0.043 6147	0.043 6143
0.001	0.0001	0.074 40	0.041 6549	0.041 6548
0.001	0.000 01	0.074 40	0.041 6153	0.041 6152
0.001	0.000 001	0.074 41	0.041 6154	0.041 6153

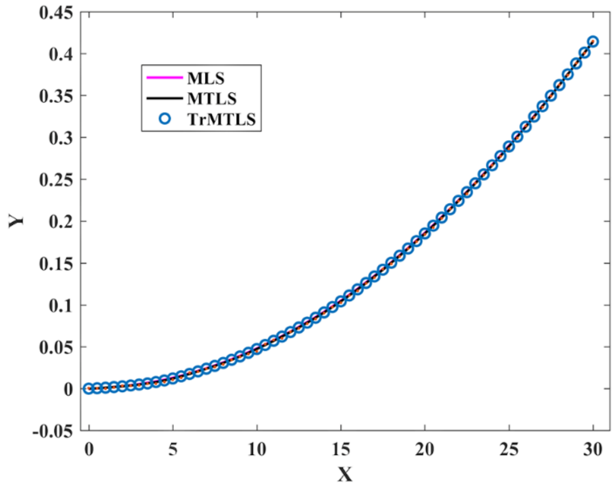

Figure 7. Fitting the curve in Example 1 by the MLS, MTLS and TrMTLS methods.

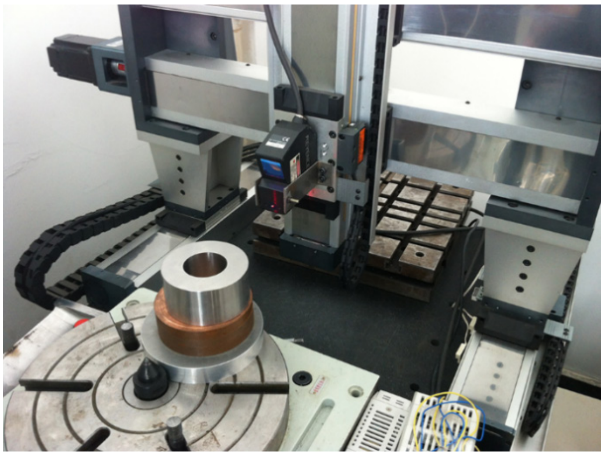

Figure 8. Profile measurement of a standard cylinder by a coordinate measuring machine.

Table 5. The radius obtained with the three methods for Example 4.

Variance		R (mm)		
δ_i	ε_i	MLS	MTLS	TrMTLS
0.015	0.0005	40.1578	40.1600	40.1653

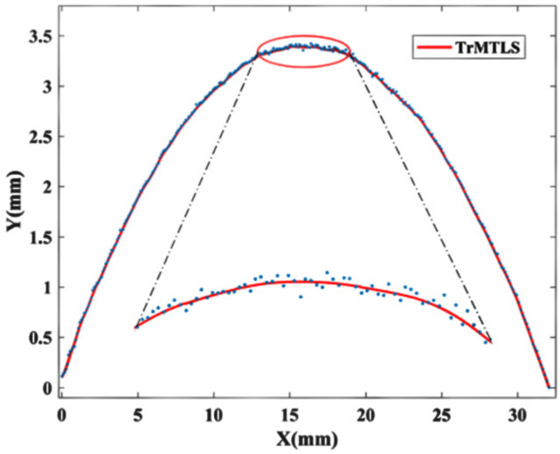

Figure 9. Fitting the profile of the standard cylinder in Example 4 by the TrMTLS method.

of the LK-G150 sensor is about 0.5 µm. All three methods are used to fit the measurement data and the circular regression algorithm based on the least-squares method is applied to obtain the regression radius. The results of MLS, MTLS and TrMTLS are shown in table 5. As shown in figure 9, the fitting curve of the proposed method is shown and the second profile is the local enlargement of a section of the fitting curve. Compared with the other two methods, it can be seen that the result using the TrMTLS method is closest to the standard cylindrical radius under the same conditions. The experimental result verifies the performance of our proposed method.

As mentioned above, the TrMTLS method can deal with the outliers and the random errors of all variables without setting the threshold or adding small weights subjectively. In all the above examples, TrMTLS has better fitting accuracy and robustness than the MLS and MTLS methods. Most importantly, it is noted that the truncation procedure is employed only once, and only one point is trimmed in each influence domain of the TrMTLS method. Although there are multiple outliers in the measurement data, the proposed method can give a better result after application of the truncation procedure for the entire parameter domain independently. Even if there are no outliers, the node with the largest squared residual may be regarded as an outlier. Further research will be carried out to achieve good performance using the TrMTLS method without needing to choose a weight function for trimming the node with the largest squared residual.

5. Conclusions

The advantage of the MLS and MTLS methods is that a shape function with high-order continuity and consistency can be obtained by employing a basis function with low order and choosing a suitable compact support weight function. These are the popular methods for curve fitting because of their good approximation properties. However, due to the way the MLS and MTLS methods are constructed both are sensitive to outliers. To avoid setting the threshold or adding small weights subjectively, an improved curve and surface fitting approach, named TrMTLS, is introduced in this paper. In the influence domain of TrMTLS method, the TLS method based on SVD with a truncation procedure is adopted to deal with the outliers and errors of all variables. To verify the performance of the proposed algorithm, the discrete points generated by numerical simulation and obtained by experimental measurement are fitted by the three methods under the same conditions. From all the fitting results, it can be seen that the TrMTLS method is more robust and accurate than MLS and MTLS, which confirms the validity of the proposed TrMTLS.

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References

[1] Surnam Franke R and Šalkauskas K 1996 Localization of multivariate interpolation and smoothing methods *J. Comput. Appl. Math.* **73** 79–94
[2] Amirfakhrian M and Mafikandi H 2016 Approximation of parametric curves by moving least squares method *Appl. Math. Comput.* **283** 290–8
[3] Wang Q, Zhou W, Chen Y G, Ma G, Chang X L, Miao Y and Chen E 2018 Regularized moving least-square method and regularized improved interpolating moving least-square method with nonsingular moment matrices *Appl. Math. Comput.* **325** 120–45
[4] Shepard D 1968 A two-dimensional interpolation function for irregularly-spaced data *Proc. ACM Natl. Conf.* pp 517–24
[5] Lancaster P and Salkauskas K 1981 Surfaces generated by moving least squares methods *Math. Comput.* **37** 141–58
[6] Lipman Y 2009 Stable moving least-squares *J. Approx. Theory* **161** 371–84
[7] Li W, Song G B and Yao G M 2017 Piece-wise moving least squares approximation *Appl. Numer. Math.* **115** 68–81
[8] Cheng Y, Wang W Q, Peng M J and Zhang Z 2014 Mathematical aspects of meshless methods *Math. Probl. Eng.* **2014** 1–4
[9] Zhang L W and Liew K M 2014 An improved moving least-squares Ritz method for two-dimensional elasticity problems *Appl. Math. Comput.* **246** 268–82
[10] Wei Q and Cheng R J 2014 The improved moving least-square Ritz method for the one-dimensional sine–Gordon equation *Math. Probl. Eng.* **2014** 383219
[11] Zhang Z, Hao S Y, Liew K M and Cheng Y M 2013 The improved element-free Galerkin method for two-dimensional elastodynamics problems *Eng. Anal. Bound. Elem.* **37** 1576–84
[12] Wang J F, Hao S Y and Cheng Y M 2014 The error estimates of the interpolating element-free Galerkin method for two-point boundary value problems *Math. Probl. Eng.* **2014** 641592

[13] Shokri A and Bahmani E 2019 Direct meshless local Petrov–Galerkin (DMLPG) method for 2D complex Ginzburg–Landau equation *Eng. Anal. Bound. Elem.* **100** 195–203
[14] Chen L C, Liu X and Lin X L 2019 The boundary element-free method for 2D interior and exterior Helmholtz problems *Comput. Math. Appl.* **77** 846–64
[15] Maisuradze G G and Thompson D L 2003 Interpolating moving least-squares methods for fitting potential energy surfaces: detailed analysis of one-dimensional applications *J. Chem. Phys.* **119** 10002–14
[16] Breitkopf P, Naceur H, Rassineux A and Villon P 2005 Moving least squares response surface approximation: formulation and metal forming applications *Comput. Struct.* **83** 1411–28
[17] Scitovski R, Ungar Š and Jukic D 1998 Approximating surfaces by moving total least squares method *Appl. Math. Comput.* **93** 219–32
[18] Lei Z, Gu T Q, Zhao J, Ji S J, Sun Q Z and Hu M 2014 An adaptive moving total least squares method for curve fitting *Measurement* **49** 107–12
[19] Markovsky I and Huffel S V 2007 Overview of total least-squares methods *Signal Process.* **87** 2283–302
[20] Ren M J, Cheung C F and Kong L B 2012 A task specific uncertainty analysis method for least-squares-based form characterization of ultra-precision freeform surfaces *Meas. Sci. Technol.* **23** 54005
[21] Mehrabi H and Voosoghi B 2015 Recursive moving least squares *Eng. Anal. Bound. Elem.* **58** 119–28
[22] Cheung C F, Kong L B, To S and Lee W B 2009 Measuring ultra-precision freeform surfaces using a hybrid fitting and matching method *Meas. Sci. Technol.* **20** 105103
[23] Levin D 2015 Between moving least-squares and moving least-ℓ 1 *BIT Numer. Math.* **55** 781–96
[24] Debruyne M, Christmann A, Hubert M and Suykens J A K 2010 Robustness of reweighted least squares kernel based regression *J. Multivar. Anal.* **101** 447–63
[25] Ding J, Liu Q and Sun P P 2019 A robust registration algorithm of point clouds based on adaptive distance function for surface inspection *Meas. Sci. Technol.* **30** 75003
[26] Liu D, Shi Y, Tian Y G and Huang X K 2016 Ramp loss least squares support vector machine *J. Comput. Sci.* **14** 61–8
[27] Suykens J A K, Brabanter J D, Lukas L and Vandewalle J 2002 Weighted least squares support vector machines: robustness and sparse approximation *Neurocomputing* **48** 85–105
[28] Lin Y L, Hsieh J G, Jeng J H and Cheng W C 2015 On least trimmed squares neural networks *Neurocomputing* **161** 107–12
[29] Yang B, Shao Q M, Pan L and Li W B 2018 A study on regularized weighted least square support vector classifier *Pattern Recognit. Lett.* **108** 48–55
[30] Chen C F, Yan C Q and Li Y Y 2015 A robust weighted least squares support vector regression based on least trimmed squares *Neurocomputing* **168** 941–6
[31] Wen W, Hao Z F and Yang X W 2010 Robust least squares support vector machine based on recursive outlier elimination *Soft Comput.* **14** 1241–51
[32] Chuang C C and Lee Z J 2011 Hybrid robust support vector machines for regression with outliers *Appl. Soft Comput.* **11** 64–72
[33] Gu T Q, Ji S J, Lin S W and Luo T Z 2016 Curve and surface reconstruction method for measurement data *Measurement* **78** 278–82
[34] Joldes G R, Chowdhury H A, Witteka A, Doylea B and Miller K 2015 Modified moving least squares with polynomial bases for scattered data approximation *Appl. Math. Comput.* **266** 893–902
[35] Zhang S L and Zhang K 2014 On a basic multivariate EIV model with linear equality constraints *Appl. Math. Comput.* **236** 247–52
[36] Zhang L, Gu T Q, Zhao J, Ji S J, Hu M and Li X B 2013 An improved moving least squares method for curve and surface fitting *Math. Probl. Eng.* **2013** 159694
[37] Avramidis A and Zioutas G 2011 Locally and globally robust penalized trimmed squares regression *Simul. Model. Pract. Theory* **19** 148–60
[38] Wang C, Caja J and Gómez E 2018 Comparison of methods for outlier identification in surface characterization *Measurement* **117** 312–25
[39] Yang L M and Dong H W 2018 Support vector machine with truncated pinball loss and its application in pattern recognition *Chemometrics Intell. Lab. Syst.* **177** 89–99