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Uncertainty evaluation of straightness in coordinate measuring machines based on error ellipse theory integrated with Monte Carlo method

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Abstract

According to the new generation geometrical product specification, it is necessary to provide measurement uncertainty together with measurement results in order to determine the reliability of results. The traditional methods used for the uncertainty evaluation of straightness are laborious and time-consuming owing to a large quantity of repeated measurements or a complicated computational process. Based on the error ellipse theory and the Monte Carlo method, a novel method for uncertainty evaluation is proposed. Through the error ellipse theory, in the measuring space of coordinate measuring machines, the positional uncertainty of sampling points can be more accurately considered to be represented by an ellipse. By integrating the Monte Carlo method, only with limited sets of real measured data in small experimental trials, the uncertainty propagation from a single sampling point to the whole straight line can be demonstrated clearly in the simulation without requiring large amounts of time and labour. The detailed procedures of uncertainty evaluation are given. The straightness uncertainty can then be obtained by statistical analysis of the simulation results. Real straightness measurement experiments were carried out and compared with the results from the proposed method. The difference was no more than 5%, which verified the validity of the method.

Keywords:measurement uncertainty, straightness, error ellipse theory, Monte Carlo method, coordinate measuring machines

(Some figures may appear in colour only in the online journal)

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1. Introduction

Straightness is a crucial part of form and position errors and an essential indicator in geometrical feature measurement of precision shafts and guide rails. The poor performance of mechanical parts is often the result of excessive straightness. According to the new generation geometrical product specification (GPS) [1], the uncertainty of the result must be given to present the reliability of results. Thus, evaluation of the straightness uncertainty is a significant issue.

For simple direct measurement like length, the repeated measurement experiment is one of the most widely used methods in the field, and the uncertainty can be evaluated by the standard deviation of observations according to the Guide to the Expression of Uncertainty in Measurement (GUM) [2]. However, it is quite time-consuming owing to the large number of repetitive laborious tasks. For indirect measurement like volume, GUM gives the expression to determine the combined uncertainty based on the principle of uncertainty propagation. However, it is hard to implement this in some cases due to some additional limitations for GUM. For example, it must be possible to express the output quantity by an analytic function of the input quantities; model linearization could not always satisfy the requirement for high order terms [3], the process of determining the partial derivatives to solve the analytic expression is quite complex, such as the partial derivation of the least square fitted curve.

Hence, many researchers have been dedicated to the exploration of more effective methods of uncertainty evaluation. As the uncertainty is influenced by various factors, including temperature, measurement method, data processing algorithm, sampling strategy, etc, some researchers started with the factors. Cao *et al* [4] described the composition of the uncertainty caused by the measuring process and analysed the uncertainty caused by sample size insufficiency and measuring process. Cui *et al* [5] also pointed out that, with more sample points, the uncertainties from different methods are a little smaller, and it is necessary to focus on the measurement strategies. Okuyama [6] considered cross-axis translational motion and the sensor's random error, and made the uncertainty of the estimated straightness profile least. Feng *et al* [7, 8] emphasized the impact on measurement accuracy of debris attached to the coordinate measuring machine (CMM) stylus tip and proposed methods to reduce such measurement error. Others created novel methods to evaluate the uncertainty. Hüser *et al* [9] proposed a procedure to determine the uncertainty from a single profile using a statistical method. Farooqui *et al* [10] described a method for uncertainty analysis using bootstrap techniques. Calvo *et al* [11] presented a novel method based on vectorial calculus of point coordinates to evaluate straightness uncertainty using CMM. Wiora *et al* [12] devised a weighted combined method and made a comparison with CMM to prove the improvement in the additive measured quantity. Jakubiec *et al* [13] derived an analytical method for evaluating the measurement uncertainty of CMM. Kruth *et al* [14] investigated the influence of feature form deviations and proposed a method that determines uncertainties for feature measurements on CMMs. These achievements contributed a lot to the uncertainty issue. However, considering all the research, the actual distribution characteristics of the sampling points by CMMs has not been investigated thoroughly, and the uncertainty propagation from a single sampling point to the whole feature has not been demonstrated clearly.

However, in recent years there has been increasing interest in using the Monte Carlo method (MCM) to simulate the measurement of different features to obtain the uncertainty [15–18]. Through using MCM, the complicated calculation of sensitivity coefficients in the expression of combined uncertainty can be skipped over. Yet, the traditional MCM mathematically assumes the measurement points in a plane lie within a rectangular area, which contradicts the actual point distribution of CMMs, resulting in an increase in obtained uncertainty.

Hence, a modified method is proposed on the basis of the error ellipse theory integrated with the Monte Carlo method (EE-MCM) to evaluate the straightness uncertainty in the measuring space of CMMs. Based on the error ellipse theory, the measurement process by CMMs is considered to obey the normal distribution. The straightness to be discussed in this paper belongs to planar straightness; a 2D error ellipse is used to represent the uncertainty of sampling points on the CMM. Then the error ellipse is used as the simulation condition in MCM. Finally, an experiment in straightness measurement was carried out to verify this method.

The organization of the paper is as follows. In section 2, the error ellipse theory of sampling points on CMMs is introduced; a method based on EE-MCM is proposed to evaluate the uncertainty of straightness in section 3; an experiment on straightness measurement is conducted to demonstrate the validity of the proposed method in section 4; the uncertainty results by means of the formula method given by GUM, MCM, EE-MCM, and the repeated experiment method, are compared and analysed in section 5; finally, concluding remarks and potential applications of the proposed method are discussed in section 6.

2. Error ellipse theory

In modern industrial production, it is common to evaluate uncertainty using CMMs to measure the objects repeatedly and statistical analysis to acquire an uncertainty interval. The sampling point is the most fundamental component of the coordinate measuring space. When it comes to actually measuring a single point may times, generally, the repeated measurement results are different owing to the random errors. Those sampling points are visibly contained in a closed geometrical zone. When the measurement process satisfies the normal distribution, theoretically, the closed geometrical zone is supposed to be an interval in 1D space, a rectangle in 2D space, or a rectangular solid in 3D space, as illustrated in figure 1. The measurand is supposed to be located in the centre of the zone.

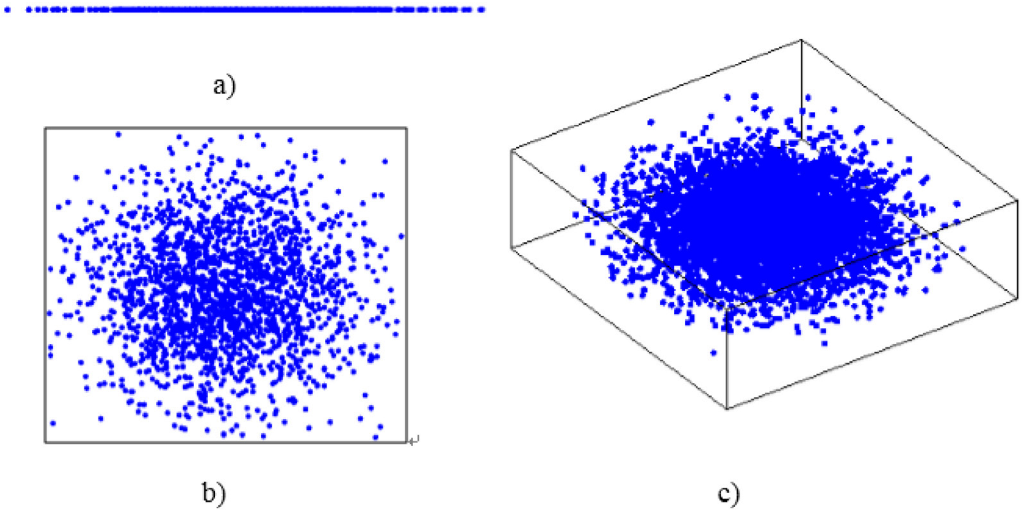

Figure 1. Shapes of the closed geometrical zone in different dimensions, in theory.

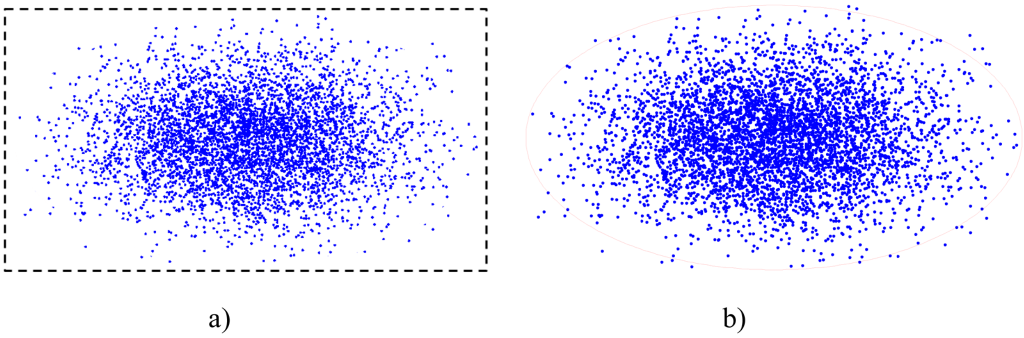

Figure 2. Comparison of the closed geometrical zone in theory and the actual measurement. (a) In theory, (b) in the actual measurement.

However, it was found in the actual measurement by CMMs that the closed geometrical zone in 2D space is not a rectangle but more like an ellipse, which was used by Sladek to determine the area of measurement uncertainty [19]. The difference could reasonably be attributed to the coupling mechanism of the x- and y-axes which are installed in a spatially perpendicular configuration to have a mechanical coupling influence on each other. As a result, the geometrical zone is no longer a rectangle, which is often assumed in the traditional MCM [20, 21]. In this case, an ellipse is more suitable to describe the point's error distribution, as shown in figure 2. In this paper, 'error' in the CMM measurement results refers to the difference between the measurement result and true value as defined in GUM [2].

In 2D space, the Cartesian coordinates can be used to denote the position of a point. Inevitably, there are differences between the sampled coordinate (x_s, y_s) and the real coordinate (x, y) owing to random errors during the measurement process. The uncertainties of sampling points on CMMs can be represented by an error ellipse whose size is determined by the point uncertainty u_x in the x-direction and u_y in the y-direction. In this case, x and y can be replaced by the mean values μ_x and μ_y of the many samples. Hence, the equation of the ellipse can be obtained as

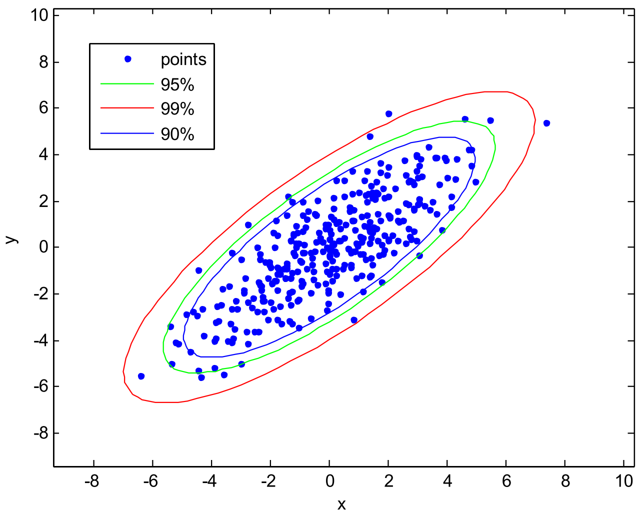

Figure 3. Error ellipse with 95%, 99% and 90% confidence level.

$$P^T M^{-1} P = s \tag{1}$$

where $P = [x_s - \mu_x, y_s - \mu_y]^T$, $M = \begin{bmatrix} {u_x}^2 & u_{xy} \\ u_{xy} & {u_y}^2 \end{bmatrix}$ is the covariance matrix containing the uncertainty information and u_{xy} denotes the covariance between two directions. s is the coefficient which determines the size of the ellipse [22]. Further, s corresponds to the confidence level of a sampling point falling in the error ellipse. Figure 3 indicates the error ellipses with 90%, 95% and 99% confidence level, respectively.

In the case of CMM, u_x and u_y are two independent variables. Thus, equation (1) can be simplified as

$$\frac{(x_s - \mu_x)^2}{{u_x}^2} + \frac{(y_s - \mu_y)^2}{{u_y}^2} = s. \tag{2}$$

Equation (2) shows that s is the quadratic sum of two independent variables. Therefore, s can be considered to obey the chi-squared distribution with two degrees of freedom. Then with a confidence level, s can be obtained by looking up the chi-squared distribution critical value table.

3. Error ellipse theory integrated with Monte Carlo method

GUM is the internationally accepted master document for uncertainty evaluation, and it has already been included in the International Organization for Standardization/International Electrotechnical Commission (ISO/IEC) Guide 98. It provides a combined expression, based on the uncertainty propagation principle, to determine the uncertainty of the measured object which cannot be measured directly through experiments. The precondition is that the relation between the input quantities and output quantities can be expressed as a mathematical analytic function.

The evaluation methods [23] for straightness consist of the minimum zone, two endpoints line and least squares mean line methods. Based on the relatively efficient and widely-used least squares mean line method, the straightness error δ is defined as the difference between the maximum and minimum distances from the point on the line profile to the datum line. Therefore, for a measured straight line with N sampling points, whose datum line function is $y = kx + b$, the expression of straightness error δ can be obtained as:

$$\delta = \frac{y_{\max} - y_{\min} - k(x_{\max} - x_{\min})}{\sqrt{1 + k^2}} \tag{3}$$

where $k$is the slope of the datum line. $(x_{\max}, y_{\max})$is the farthest point and $(x_{\min}, y_{\min})$is the nearest point.

According to the above definition of straightness error, the literature [4] gives the analytical expression of the straightness uncertainty u_δ as the combined uncertainty based on GUM.

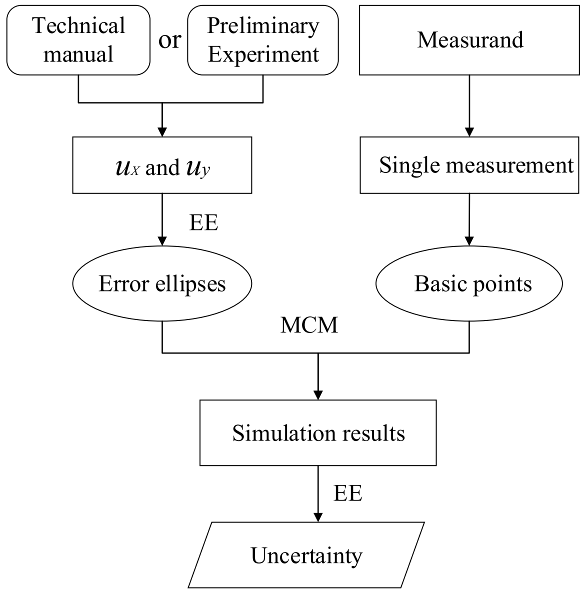

Figure 4. EE-MCM procedure.

$$
\begin{aligned}
{u_\delta}^2 = {} & \left(\frac{\partial\delta}{\partial k} u_k\right)^2 + \left(\frac{\partial\delta}{\partial x_{\max}} u_{x_{\max}}\right)^2 + \left(\frac{\partial\delta}{\partial x_{\min}} u_{x_{\min}}\right)^2 \\
& + \left(\frac{\partial\delta}{\partial y_{\max}} u_{y_{\max}}\right)^2 + \left(\frac{\partial\delta}{\partial y_{\min}} u_{y_{\min}}\right)^2
\end{aligned} \tag{4}
$$

where $k = \frac{\sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i - N \sum_{i=1}^{N} (x_i y_i)}{\left(\sum_{i=1}^{N} x_i\right)^2 - N \sum_{i=1}^{N} x_i^2}$, $u_k = \sqrt{\sum_{i=1}^{N} \left[\left(\frac{\partial k}{\partial x_i} u_{x_i}\right)^2 + \left(\frac{\partial k}{\partial y_i} u_{y_i}\right)^2\right]}$, $\frac{\partial\delta}{\partial k} = \frac{-x_{\max} + x_{\min}}{\sqrt{1+k^2}} - \frac{k[y_{\max} - y_{\min} - k(x_{\max} - x_{\min})]}{(1+k^2)^{3/2}}$, $\frac{\partial\delta}{\partial x_{\max}} = \frac{-k}{\sqrt{1+k^2}}$, $\frac{\partial\delta}{\partial x_{\min}} = \frac{k}{\sqrt{1+k^2}}$, $\frac{\partial\delta}{\partial y_{\max}} = \frac{1}{\sqrt{1+k^2}}$, $\frac{\partial\delta}{\partial y_{\min}} = \frac{-1}{\sqrt{1+k^2}}$,

3.1. Uncertainty evaluation based on EE-MCM

Although the formula for the combined uncertainty yields a clear calculation of the straightness uncertainty, it requires a complex calculation process between the direct measurands and the final straightness uncertainty. In this case, the MCM provides an alternative choice, which can be implemented to estimate the uncertainty through many random virtual sampling experiments on a computer without too much computation. Additionally, it is not required that the measured objects be expressed by an analytic function. Nevertheless, with the application of only MCM, the coupling mechanism of the x- and y-axes structure has not been fully taken into consideration. Therefore, when the error ellipse theory is integrated with the Monte Carlo method, EE-MCM can solve the uncertainty issue more precisely.

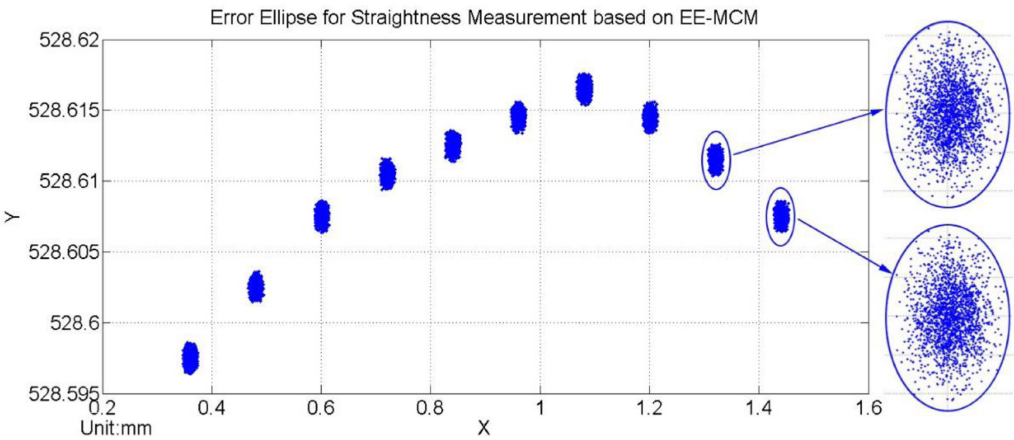

Figure 5. Straightness measurement simulation based on EE-MCM.

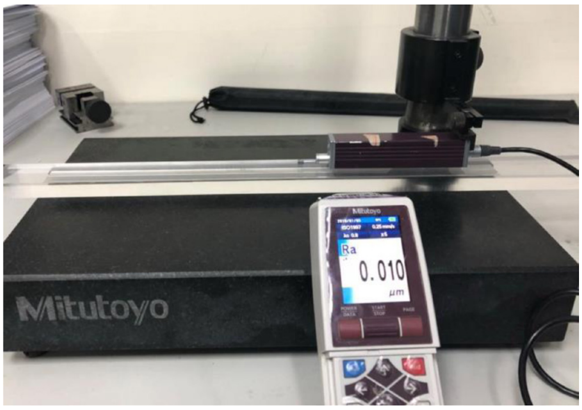

Figure 6. Roughness measurement experiment using the Mitutoyo SJ-210.

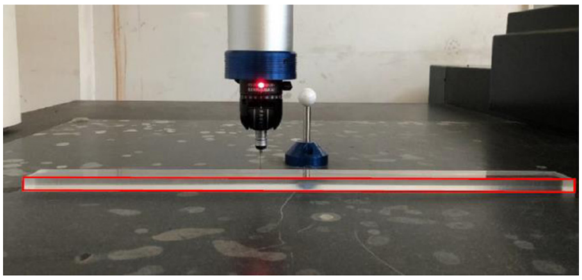

Figure 7. Straightness measurement experiment by the CMM.

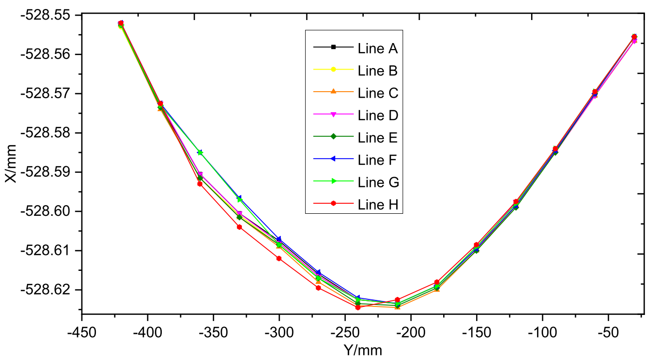

Figure 8. Straightness profile measurement with 14 sampling points in the *x*-direction.

First, the initial condition is the uncertainty of the sampling points in the measuring space, which can be acquired in advance by the technical manual or by statistical analysis through a simple preliminary experiment. One group of original measuring points in the formal experiment can be chosen as the initial basic points in the following simulation. Next, based on the error ellipse theory, in the actual measurement by the CMM, the error distribution of sampling points is considered as the normal distribution. An ellipse can be used to represent the positional uncertainty of sampling points in the measuring space. Then, by integrating the MCM, the simulation of geometrical features measurement was performed. Finally, many groups of simulation results are obtained, and the uncertainty can be determined through statistical analysis of the simulation results. The EE-MCM procedure is outlined in figure 4.

3.2. Simulation case of straightness uncertainty based on EE-MCM

From the perspective of the specific straightness measurement, for example, figure 5 shows the simulation process of straightness measurement with ten sampling points and the simulation was performed T times.

Located at the centre of each cluster of blue dots are the ten original sampling points from the actual measurement of the straightness profile. They are chosen as a group of basic points in the simulation. According to the uncertainty u_x and u_y, which was acquired in advance by the technical manual or preliminary experiment, the error ellipses were generated around each basic point. Next, based on MCM, during each time of simulation, a point was selected randomly from each ellipse according to the normal distribution. Then the ten selected points can be obtained as the measuring points to

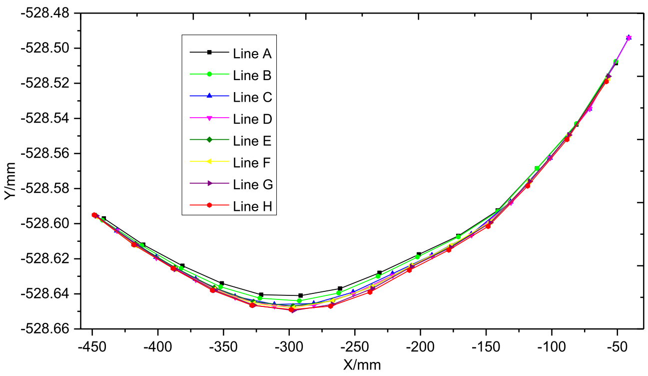

Figure 9. Straightness profile measurement with 14 sampling points in the y-direction.

Table 1. Straightness error in the x-direction with 14 sampling points.

Number	Straightness error in the x-direction (mm)	Number	Straightness error in the x-direction (mm)
1	0.071 36	5	0.072 42
2	0.071 27	6	0.070 71
3	0.072 91	7	0.07088
4	0.071 78	8	0.074 93

Table 2. Straightness error in the y-direction with 14 sampling points.

Number	Straightness error in the y-direction (mm)	Number	Straightness error in the y-direction (mm)
1	0.0832	5	0.0872
2	0.0855	6	0.0857
3	0.0888	7	0.0886
4	0.0897	8	0.0886

form a straightness profile. In this way, the simulation process was repeated T times on the computer. Therefore, T simulated profiles were formed, and T straightness results could be correspondingly calculated. Consequently, the uncertainty of straightness could be determined by analysing the distribution of the straightness results. In figure 5, the ten clusters of blue dots represent ten different sampled positions on the straightness profile. The number of each cluster of blue dots is T, which represents the number of simulation times.

4. Experiment

4.1. Preliminary experiment

A preliminary experiment was conducted on the CMM to obtain the uncertainty of the measurement system to provide the initial condition for the simulation based on EE-MCM and the formula of the combined uncertainty given by GUM.

A polymethyl methacrylate bar with a high-quality surface was chosen for the experiment, so that the influence of surface roughness could be neglected. The roughness of the bar was measured as Ra = 0.010 μm by the Mitutoyo SJ-210, as is shown in figure 6.

The CMM was the VGS Tech V3 696 with the Renishaw MH20i probe system. The ambient temperature was 19.7 °C, and humidity was 41%. A single point was measured 50 times repeatedly in the x- and y-directions, respectively.

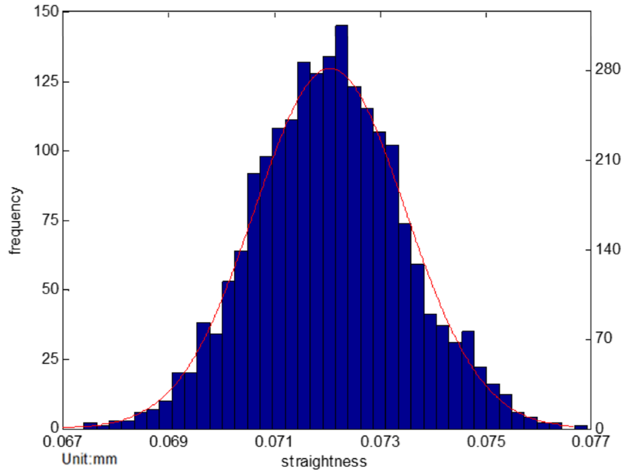

Figure 10. Simulation results in the x-direction based on EE-MCM.

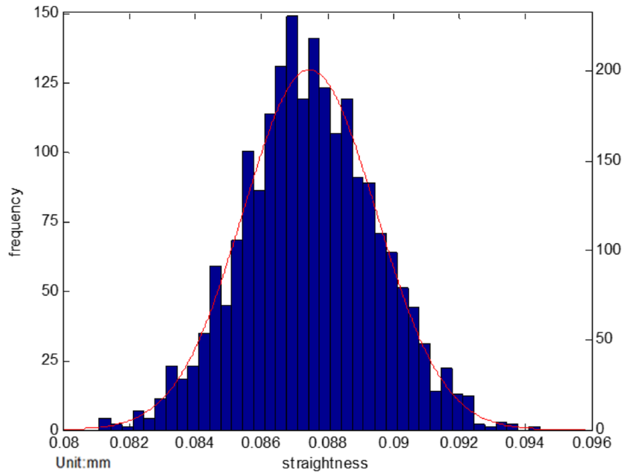

Figure 11. Simulation results in the y-direction based on EE-MCM.

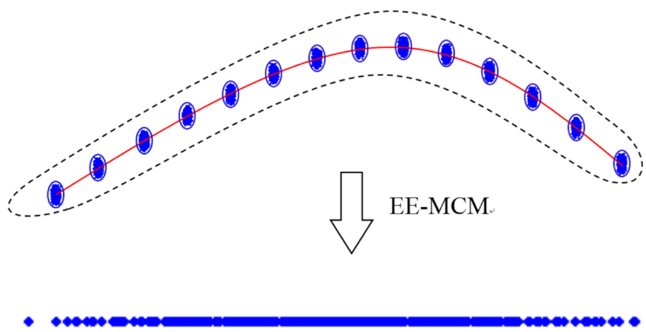

Figure 12. Error interval in the 1D space obtained through EE-MCM.

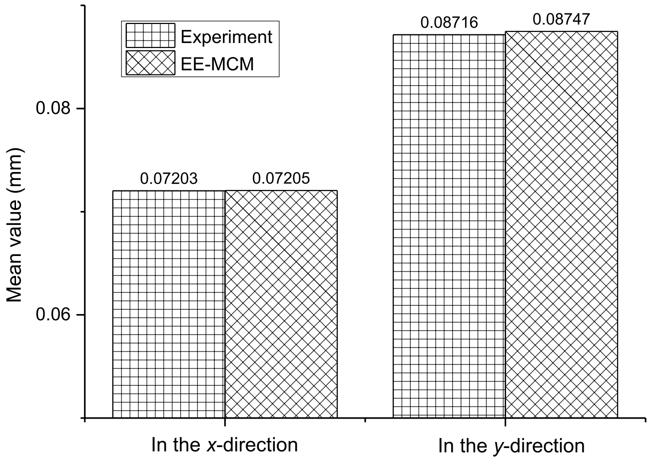

Figure 13. Comparison of the mean values obtained using two different methods.

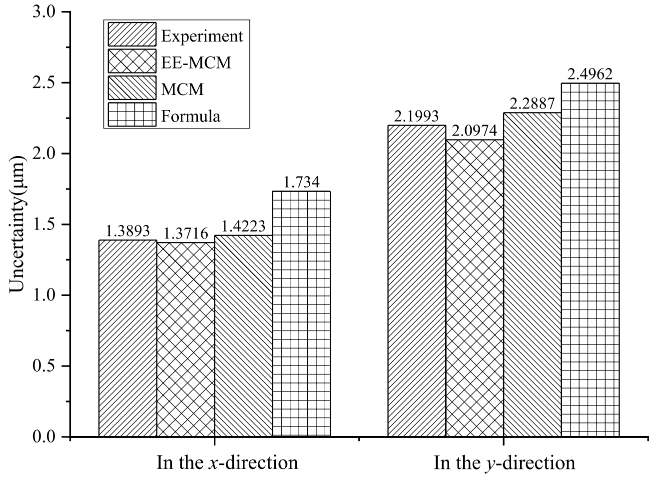

Figure 14. Comparison of the uncertainties using four different methods.

Based on GUM, the standard deviations of the corresponding positional coordinates could be considered as the standard uncertainties of the CMM in the x- and y-directions, which were $u_x = 1.165\,\mu$m and $u_y = 1.677\,\mu$m (original data are listed in appendix A). The results can also be applicable to the uncertainty evaluation of other geometrical features besides straightness. The uncertainty in the z-direction was not required since this paper focuses on the 2D features within the xy-plane.

Based on the formula for the combined uncertainty, $u_{x_{\max}} = u_{x_{\min}} = u_x = 1.165\,\mu$m and $u_{y_{\max}} = u_{y_{\min}} = u_y = 1.677\,\mu$m were substituted into equation (4). Then the straightness uncertainties were calculated as 1.7340 µm in the x-direction and 2.4962 µm in the y-direction.

4.2. Straightness measurement experiment

After the preliminary experiment, measurement experiments on straightness were conducted. In the measurement, the CMM was configured to set in auto mode. The polymethyl methacrylate bar, whose size was $600 \times 40 \times 14$ mm, is horizontally allocated on the worktable of CMM. The edge of the bar with the red mark was evenly sampled at 14 different positions by the CMM probe which moved along a straight line, as shown in figure 7.

The measurements were repeated eight times in the x- and y-directions, respectively. Therefore, 16 groups of sampling point data were acquired, and detailed data are listed in appendices B and C. The straightness results can be read directly from the software of the CMM. Based on the GUM, the standard uncertainty of straightness can be evaluated by the standard deviation.

The line profiles along the x-axis and y-axis are shown separately in figures 8 and 9, respectively. The straightness results in the x- and y-directions, which correspond to the profiles, are listed in tables 1 and 2, respectively.

From table 1, similarly, the standard deviation of the straightness in the x-direction is calculated as $\sigma_{\delta_X} = 1.3893\,\mu$m.

From table 2, the standard deviation of the straightness in the y-direction is calculated as $\sigma_{\delta_Y} = 2.1993\,\mu$m.

Figures 8 and 9 show that the line profiles in the same direction are highly overlapped, illustrating that the straightness was measured precisely.

5. Comparison and discussion

According to the EE-MCM procedure mentioned in section 3.2, one group of original sampling points in the actual experiment was chosen as the basic points for the EE-MCM simulation. Based on the error ellipse theory, with 99% confidence level, the error ellipse of each basic point can be determined by the standard uncertainty u_x and u_y in the preliminary experiment. The simulation was performed 2000 times which satisfies the stabilization criterion [24] that twice the standard deviation (0.0023 mm) is less than the numerical tolerance (0.005 mm). Then 2000 straightness results in the x- and y-directions were obtained. The distributions of the results are exhibited as figures 10 and 11.

To verify that the straightness results obey the normal distribution, Jarque–Bera tests [25] were done with 0.05 significance level on the data of figures 10 and 11. The test results are both zero, which indicates that the sample data obey the normal distribution. Therefore, the error ellipse theory can also be applied to evaluate the uncertainty of the straightness results. While the sampling point with x- and y-coordinates is in the 2D space, the straightness result can be considered to be a point with a single coordinate in the 1D space. In other words, its uncertainty can also be presented by an error ellipse in the 1D space based on the error ellipse theory. As depicted in figure 12, through EE-MCM, the straightness results were obtained as an interval obeying normal distribution, which is mentioned in section 2.

Instead of calculating the standard deviation directly as the standard uncertainty in the traditional MCM, an enveloping ellipse, which could be shown more distinctly in 2D or 3D cases like the centre of a circle, is fitted for the straightness results in EE-MCM. The 1D enveloping ellipse is just the error ellipse of the straightness uncertainty. Thus, the uncertainty in the x-direction can be obtained as 1.3716 μm. Likewise, the uncertainty in the y-direction can be obtained as 2.0974 μm.

The mean values obtained by two different methods, the experiment and EE-MCM, were compared. The results are shown in figure 13.

From figure 13, it can be concluded that the mean values in the two different methods are similar to each other. Therefore, the method based on EE-MCM can also be applied to estimate the straightness measurement result.

More importantly, the uncertainties obtained by the experiment method, EE-MCM, MCM, and formula methods were compared; the uncertainties in the x- and y-directions are shown in figure 14.

From figure 14, the results of the formula method are approximately 19% in the x-direction and 16% in the y-direction greater than the results of EE-MCM. The reason for this is that the computed results based on the formula are under the condition of maximum error propagation. Additionally, it can be easily seen that the results of both the experiment and EE-MCM agree well with each other, which proves the validity of EE-MCM. Specifically, the results of EE-MCM are approximately 1% in the x-direction and 5% in the y-direction smaller than the experimental results, which can reasonably be attributed to the unaccountable influencing factors, such as the dynamic measuring errors during the actual measurement process. Besides, the results of EE-MCM are more accurate than the results of MCM since the distribution of sampling points through the error ellipse theory is more consistent with the actual measurement. Further, the number of samples in the simulation based on EE-MCM is large enough so that the simulation value is closer to the true value.

In EE-MCM, the initial condition was the uncertainty of the sampling points in the coordinate measurement system, which was acquired in the preliminary experiment. Besides, it is also the essential condition in the uncertainty evaluation using the formula method. Moreover, it is the fundamental parameter that can reflect the measuring point accuracy of the whole workspace of the CMM. Therefore, it can also be used in the measurement of other geometrical features by CMM.

Unlike the method based on repeated experiments, EE-MCM requires no repeated labour. Only limited sets of measured data are needed, and the rest of the simulation procedure can be finished by the computer quickly. Furthermore, EE-MCM is more precise than the formula method based on GUM, and does not require complicated formula derivation.

6. Conclusion

A method based on EE-MCM was proposed to evaluate the uncertainties in the measurement of geometric features, like straightness, using CMMs efficiently. The integration of the error ellipse theory and the MCM can achieve more precise results than those obtained using only one method.

Normally, in the measuring space of the coordinate measurement system, the uncertainty of sampling points can be acquired beforehand through the technical manual or a simple preliminary experiment. When evaluating other geometrical features besides straightness, it is not necessary to calculate again the point uncertainty as a reflection on the measurement characteristics of the CMM. Based on the error ellipse theory, it can be represented by an ellipse, which is taken as the initial condition in EE-MCM. The uncertainty propagation from a single sampling point to the whole straight line can be simulated by integrating the MCM. Ultimately, the uncertainty of straightness can be obtained only with a limited set of measured data.

The comparison of the results from different methods demonstrates that EE-MCM is valid and practical, with the following three advantages.

(1) EE-MCM provides a more accurate estimation of measurement uncertainty associated with straightness measurement, compared to the experiment method, MCM and formula methods.
(2) EE-MCM eliminates the need for repeated straightness measurements which make the experiment method time-consuming.
(3) EE-MCM requires no formula derivation, a significant contributor to the complexity of implementing the formula method by GUM.

The method based on EE-MCM was successfully applied to the uncertainty evaluation of straightness. In future research, it could also be extended to other features or to form and position error, such as planes, cylinders, roundness, linear contour, and cylindricity. Additionally, it can also significantly reduce the CMM measurement time since the uncertainty result can be obtained with a single, or very few, CMM measurements. Further advantages of EE-MCM will be shown, especially in the uncertainty evaluation of 3D space measurement.

Acknowledgment

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Appendix A. Measurements repeated 50 times for one certain point to determine the measurement uncertainty of the CMM in the *x*- and *y*-directions

	y-direction			*x*-direction		
	X	*Y*	*Z*	*X*	*Y*	*Z*
1	3.6287	0.0121	−3.0058	0.006	4.8915	−3.0253
2	3.6237	0.0126	−3.0064	0.006	4.8915	−3.0253
3	3.6227	0.0126	−3.0069	0.006	4.8915	−3.0253
4	3.6232	0.0111	−3.0064	0.0055	4.8915	−3.0253
5	3.6227	0.0141	−3.0064	0.0055	4.8915	−3.0253
6	3.6227	0.0126	−3.0064	0.0065	4.8915	−3.0248
7	3.6222	0.0136	−3.0059	0.007	4.8914	−3.0248
8	3.6227	0.0121	−3.0069	0.006	4.8915	−3.0248
9	3.6222	0.0146	−3.0069	0.006	4.8915	−3.0248
10	3.6227	0.0146	−3.0064	0.0065	4.892	−3.0248
11	3.6222	0.0146	−3.0064	0.006	4.892	−3.0248
12	3.6232	0.0146	−3.0064	0.0075	4.8914	−3.0243
13	3.6217	0.0142	−3.0059	0.007	4.8914	−3.0248
14	3.6222	0.0136	−3.0054	0.0065	4.8915	−3.0248
15	3.6217	0.0137	−3.0059	0.006	4.8915	−3.0243
16	3.6222	0.0136	−3.0059	0.0075	4.8914	−3.0243
17	3.6217	0.0132	−3.0059	0.0065	4.8915	−3.0243
18	3.6222	0.0136	−3.0049	0.0065	4.8915	−3.0243
19	3.6222	0.0141	−3.0054	0.009	4.8914	−3.0253
20	3.6217	0.0142	−3.0059	0.0065	4.8915	−3.0243
21	3.6222	0.0156	−3.0054	0.0065	4.8915	−3.0248
22	3.6222	0.0161	−3.0054	0.007	4.8919	−3.0238
23	3.6222	0.0156	−3.0054	0.0059	4.8915	−3.0238
24	3.6222	0.0156	−3.0054	0.0074	4.8914	−3.0233
25	3.6217	0.0167	−3.0059	0.0075	4.8914	−3.0248
26	3.6222	0.0156	−3.0064	0.007	4.8914	−3.0238
27	3.6212	0.0142	−3.0059	0.007	4.8919	−3.0243
28	3.6222	0.0126	−3.0049	0.0075	4.8914	−3.0248
29	3.6212	0.0112	−3.0054	0.008	4.8914	−3.0248
30	3.6222	0.0151	−3.0059	0.0085	4.8914	−3.0243
31	3.6217	0.0132	−3.0054	0.0075	4.8914	−3.0238
32	3.6212	0.0147	−3.0059	0.0085	4.8914	−3.0248
33	3.6217	0.0142	−3.0069	0.0085	4.8919	−3.0238
34	3.6217	0.0142	−3.0054	0.0085	4.8914	−3.0238
35	3.6217	0.0127	−3.0054	0.008	4.8914	−3.0248
36	3.6217	0.0102	−3.0049	0.0085	4.8914	−3.0243
37	3.6217	0.0097	−3.0044	0.008	4.8919	−3.0248
38	3.6207	0.0097	−3.0044	0.0085	4.8914	−3.0243
39	3.6212	0.0102	−3.0054	0.0085	4.8914	−3.0243
40	3.6212	0.0102	−3.0054	0.0085	4.8914	−3.0243
41	3.6217	0.0122	−3.0054	0.0085	4.8919	−3.0243
42	3.6202	0.0117	−3.0049	0.0085	4.8919	−3.0243
43	3.6207	0.0117	−3.0054	0.0085	4.8914	−3.0243
44	3.6212	0.0127	−3.0054	0.0085	4.8919	−3.0248
45	3.6217	0.0132	−3.0049	0.009	4.8914	−3.0243
46	3.6217	0.0137	−3.0049	0.009	4.8914	−3.0243
47	3.6222	0.0136	−3.0054	0.009	4.8914	−3.0243
48	3.6217	0.0142	−3.0049	0.009	4.8919	−3.0243
49	3.6207	0.0142	−3.0049	0.0095	4.8914	−3.0248
50	3.6212	0.0147	−3.0049	0.0095	4.8914	−3.0243

Appendix B. Coordinates of the evenly sampled points and the straightness value in the *y*-direction acquired by CMM

	1		2		3		4	
	X	*Y*	*X*	*Y*	*X*	*Y*	*X*	*Y*
1	−441.1131	−528.597	−51.1423	−528.508	−41.3785	−528.494	−431.174	−528.604
2	−411.103	−528.612	−81.1292	−528.543	−71.3639	−528.535	−401.163	−528.62
3	−381.2888	−528.624	−111.116	−528.569	−101.35	−528.563	−371.152	−528.632
4	−351.2804	−528.634	−141.104	−528.593	−131.339	−528.588	−341.142	−528.642
5	−321.271	−528.6405	−171.092	−528.608	−161.325	−528.606	−311.132	−528.647
6	−291.2611	−528.641	−202.311	−528.619	−191.312	−528.618	−281.12	−528.647
7	−261.2507	−528.637	−232.301	−528.63	−221.302	−528.629	−251.107	−528.64
8	−231.2386	−528.628	−262.292	−528.64	−251.292	−528.639	−221.094	−528.63
9	−201.2261	−528.6175	−292.283	−528.644	−281.28	−528.646	−191.081	−528.619
10	−171.2169	−528.607	−322.271	−528.643	−311.267	−528.646	−161.071	−528.607
11	−141.2075	−528.5925	−352.259	−528.636	−341.254	−528.642	−131.061	−528.588
12	−111.1972	−528.5685	−382.247	−528.626	−371.242	−528.632	−101.05	−528.563
13	−81.1888	−528.5435	−412.237	−528.613	−401.23	−528.619	−71.0404	−528.535
14	−51.1804	−528.5085	−442.23	−528.598	−431.218	−528.604	−41.2445	−528.494
δ	0.0832		0.0855		0.0888		0.0897	

	5		6		7		8	
	X	*Y*	*X*	*Y*	*X*	*Y*	*X*	*Y*
1	−56.9214	−528.516	−446.994	−528.596	−56.7983	−528.516	−448.516	−528.595
2	−86.9073	−528.55	−416.981	−528.613	−86.7837	−528.55	−418.504	−528.612
3	−116.893	−528.576	−386.97	−528.626	−116.768	−528.576	−388.49	−528.626
4	−146.879	−528.599	−356.959	−528.638	−146.754	−528.6	−358.477	−528.638
5	−176.869	−528.613	−326.948	−528.646	−176.74	−528.614	−328.492	−528.647
6	−206.858	−528.624	−296.936	−528.648	−206.727	−528.625	−298.49	−528.649
7	−236.846	−528.636	−266.924	−528.644	−236.715	−528.637	−268.477	−528.647
8	−267.145	−528.644	−236.911	−528.636	−266.704	−528.646	−238.463	−528.639
9	−297.135	−528.647	−206.897	−528.624	−296.691	−528.65	−208.449	−528.627
10	−327.121	−528.645	−176.885	−528.613	−326.676	−528.647	−178.436	−528.615
11	−357.105	−528.637	−146.875	−528.599	−356.662	−528.638	−148.426	−528.602
12	−387.091	−528.625	−116.864	−528.576	−386.647	−528.626	−118.413	−528.579
13	−417.078	−528.612	−86.8517	−528.55	−416.634	−528.612	−88.4018	−528.552
14	−447.066	−528.596	−56.8418	−528.517	−446.775	−528.596	−58.3909	−528.519
δ	0.0872		0.0857		0.0886		0.0886	

Appendix C. Coordinates of the evenly sampled points and the straightness value in the *x*-direction acquired by CMM

	1		2		3		4	
	Y	*X*	*Y*	*X*	*Y*	*X*	*Y*	*X*
1	−420.239	−528.5525	−30.2338	−528.557	−420.237	−528.553	−30.2338	−528.5565
2	−390.238	−528.573	−60.2344	−528.571	−390.237	−528.574	−60.2354	−528.5705
3	−360.239	−528.5905	−90.2351	−528.585	−360.237	−528.592	−90.2361	−528.585
4	−330.24	−528.6005	−120.235	−528.599	−330.238	−528.602	−120.237	−528.5975
5	−300.239	−528.6075	−150.236	−528.61	−300.237	−528.609	−150.237	−528.6095
6	−270.239	−528.616	−180.235	−528.619	−270.237	−528.618	−180.238	−528.619
7	−240.238	−528.6225	−210.236	−528.624	−240.237	−528.624	−210.239	−528.6235
8	−210.237	−528.6235	−240.236	−528.623	−210.236	−528.625	−240.238	−528.6225
9	−180.236	−528.619	−270.238	−528.617	−180.236	−528.62	−270.24	−528.6165
10	−150.236	−528.6095	−300.238	−528.609	−150.236	−528.61	−300.242	−528.608
11	−120.236	−528.5985	−330.239	−528.601	−120.237	−528.599	−330.242	−528.6005
12	−90.2351	−528.5845	−360.239	−528.592	−90.2361	−528.585	−360.242	−528.5905
13	−60.2349	−528.5705	−390.237	−528.574	−60.2354	−528.571	−390.24	−528.5725
14	−30.2348	−528.5565	−420.238	−528.553	−30.2358	−528.557	−420.242	−528.552
δ	0.071 361 665		0.071 269 305		0.072 907 737		0.071 777 058	

	5		6		7		8	
	Y	*X*	*Y*	*X*	*Y*	*X*	*Y*	*X*
1	−420.242	−528.553	−30.2393	−528.556	−420.245	−528.553	−30.2463	−528.556
2	−390.242	−528.574	−60.2399	−528.57	−390.245	−528.573	−60.2464	−528.57
3	−360.243	−528.592	−90.24	−528.585	−360.247	−528.585	−90.2465	−528.584
4	−330.243	−528.602	−120.241	−528.599	−330.247	−528.597	−120.248	−528.598
5	−300.243	−528.609	−150.241	−528.61	−300.247	−528.609	−150.247	−528.609
6	−270.242	−528.617	−180.242	−528.619	−270.246	−528.617	−180.248	−528.618
7	−240.242	−528.624	−210.243	−528.624	−240.246	−528.623	−210.249	−528.623
8	−210.241	−528.624	−240.243	−528.622	−210.245	−528.624	−240.25	−528.625
9	−180.24	−528.62	−270.244	−528.616	−180.245	−528.619	−270.252	−528.621
10	−150.239	−528.61	−300.245	−528.607	−150.246	−528.609	−300.252	−528.615
11	−120.24	−528.599	−330.245	−528.597	−120.245	−528.598	−330.253	−528.606
12	−90.239	−528.585	−360.244	−528.585	−90.2455	−528.584	−360.253	−528.595
13	−60.2389	−528.57	−390.244	−528.573	−60.2449	−528.57	−390.252	−528.573
14	−30.2388	−528.556	−420.245	−528.552	−30.2453	−528.556	−420.253	−528.552
δ	0.072 423 201		0.070 710 921		0.0708 769 93		0.074 933 031	

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References

[1] ISO/TS 14253-3 2011 *Geometrical Product Specifications (GPS)—Inspection by Measurement of Workpieces and Measuring Equipment—Part 3: Guidelines for Achieving Agreements on Measurement Uncertainty Statements* (International Standard Organization)
[2] BIPM, IEC, IFCC, ISO, IUPAC, IUPAP and OIML 2008 *Evaluation of Measurement Data—Guide to the Expression of Uncertainty in Measurement* (JCGM 100:2008) 1st edn (Sèvres: BIPM Joint Committee for Guides in Metrology) (GUM 1995 with minor corrections)
[3] Herrador M A and González A G 2004 Evaluation of measurement uncertainty in analytical assays by means of Monte-Carlo simulation *Talanta* **64** 415–22
[4] Cao Y *et al* 2013 A study on mutative scale straightness measurement based on uncertainty analysis *Measurement* **46** 145–53
[5] Cui C, Fu S and Huang F 2009 Research on the uncertainties from different form error evaluation methods by CMM sampling *Int. J. Adv. Manuf. Technol.* **43** 136–45
[6] Okuyama E 2010 Two-point method considering cross-axis translational motion and sensor's random error *Precis. Eng.* **34** 49–54
[7] Feng X, Kinnell P K and Lawes S 2017 Development of CO_2 snow cleaning for *in situ* cleaning of μCMM stylus tips *Meas. Sci. Technol.* **28** 015007
[8] Feng X, Pascal J and Lawes S 2017 A microscopy approach for *in situ* inspection of micro-coordinate measurement machine styli for contamination *Meas. Sci. Technol.* **28** 095010
[9] Hüser D *et al* 2016 Procedure to approximately estimate the uncertainty of material ratio parameters due to inhomogeneity of surface roughness *Meas. Sci. Technol.* **27** 085005
[10] Farooqui S A, Doiron T and Sahay C 2009 Uncertainty analysis of cylindricity measurements using bootstrap method *Measurement* **42** 524–31
[11] Calvo R, Gómez E and Domingo R 2014 Vectorial method of minimum zone tolerance for flatness, straightness, and their uncertainty estimation *Int. J. Precis. Eng. Manuf.* **15** 31–44
[12] Wiora J, Kozyra A and Wiora A 2016 A weighted method for reducing measurement uncertainty below that which results from maximum permissible error *Meas. Sci. Technol.* **27** 035007
[13] Jakubiec W, Płowucha W and Starczak M 2012 Analytical estimation of coordinate measurement uncertainty *Measurement* **45** 2299–308
[14] Kruth J *et al* 2009 Uncertainty determination for CMMs by Monte Carlo simulation integrating feature form deviations *CIRP Annals* **58** 463–6
[15] BIPM, IEC, IFCC, ISO, IUPAC, IUPAP and OIML 2008 *Evaluation of Measurement Data—Supplement 1 to the 'Guide to the Expression of Uncertainty in Measurement'—Propagation of Distributions using a Monte Carlo method* (JCGM 101:2008) (Sèvres: BIPM Joint Committee for Guides in Metrology)
[16] Cox M G and Siebert B R L 2006 The use of a Monte Carlo method for evaluating uncertainty and expanded uncertainty *Metrologia* **43** S178–8
[17] Wübbeler G, Krystek M and Elster C 2008 Evaluation of measurement uncertainty and its numerical calculation by a Monte Carlo method *Meas. Sci. Technol.* **19** 084009
[18] Shahanaghi K and Nakhjiri P 2010 A new optimized uncertainty evaluation applied to the Monte-Carlo simulation in platinum resistance thermometer calibration *Measurement* **43** 901–11
[19] Sładek J 2011 *Modeling and Evaluation of Coordinate Measuring Machines Accuracy* (Cracow: Cracow University of Technology)
[20] Lira I 2011 Monte Carlo evaluation of the uncertainty associated with the construction and use of a fitted curve *Measurement* **44** 2156–64
[21] Chen A and Chen C 2016 Comparison of GUM and Monte Carlo methods for evaluating measurement uncertainty of perspiration measurement systems *Measurement* **87** 27–37
[22] Du Z, Wu Z and Yang J 2016 Error ellipsoid analysis for the diameter measurement of cylindroid components using a laser radar measurement system *Sensors* **16** 714
[23] GB/T11336 2004 *Measurement of Departures from Straightness* (Beijing: AQSIQ and SAC)
[24] Solaguren-Beascoa Fernández M, Alegre Calderón J M and Bravo Díez P M 2009 Implementation in MATLAB of the adaptive Monte Carlo method for the evaluation of measurement uncertainties *Accr. Qual. Assur.* **14** 95–106
[25] Jarque C M 2011 Jarque–Bera test *International Encyclopedia of Statistical Science* (Berlin: Springer)