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Rearrangement term in the folding model of the nucleon optical potential

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Abstract

Based on the mean-field determination of the single-particle energy in nuclear matter that contains naturally a rearrangement term (RT) implied by the Hugenholtz–van Hove theorem, the folding model of the nucleon optical potential (OP) is extended to take into account the RT using the effective, density dependent CDM3Yn interaction. With the exchange part of the nucleon folded OP treated exactly in the Hartree–Fock manner, a compact nonlocal version of the folding model is suggested in the present work to determine explicitly the isospin-dependent, nonlocal central term of the nucleon OP. To solve the optical model (OM) equation with a complex nonlocal OP, the calculable *R*-matrix method is used to analyse the elastic neutron and proton scattering on ^{40,48}Ca, ^{90}Zr, and ^{208}Pb targets at low energies. The inclusion of the RT into the folding model calculation of the nonlocal nucleon OP was shown to be essential for the overall good OM description of elastic nucleon scattering. To validate the nonlocal version of the folding model, the OM results given by the nonlocal folded nucleon OP are also compared with those given by the global parametrization of the nonlocal OP using the analytical nonlocal form factor suggested by Perey and Buck.

Keywords: nucleon optical potential, folding model, rearrangement term

(Some figures may appear in colour only in the online journal)

1. Introduction

Over a wide range of the single-particle (SP) energies, the nucleon motion in medium is overwhelmingly governed by the nuclear mean field, known as the shell-model potential for bound states and the optical potential (OP) for scattering states. The mean-field, SP potential is also the key quantity in the many-body studies of the equation of state of nuclear matter

(NM) as well as the structure of finite nuclei [1, 2]. The nucleon OP in the NM limit has been well studied in the Brueckner–Hartree–Fock (BHF) calculations of NM using the free nucleon–nucleon (NN) interaction [2–5], or the mean-field calculation of NM on the Hartree–Fock (HF) level using different choices of the effective NN interaction [6–10]. The mean-field prediction for the nucleon OP in the NM limit provides a vital input for the microscopic models of the nucleon OP of finite nuclei. In particular, the widely-used folding model of the nucleon OP (see, e.g. [10–13]).

The microscopic many-body studies of NM have shown the important role by the Pauli blocking effects, and the increasing strength of the three-body interaction as well as other higher-order NN correlations at high densities of NM [2]. These in-medium effects are effectively taken into account by the density dependence explicitly embedded in different versions of the effective NN interaction used in the nuclear structure and nuclear reactions studies. In the present work, we focus on the CDM3Yn density dependent versions [14] of the M3Y-Paris interaction [15] which have been successfully used in the HF studies of NM [6, 7, 16–18] as well as in the folding model calculation of the nucleon and nucleus–nucleus OP [11, 12, 14, 19–21]. In general, the folding model calculation of the nucleon OP is done on the HF level, and the folded OP lacks, therefore, the higher-order *rearrangement* term that arises naturally in the Landau theory of infinite Fermi systems [22]. Such a rearrangement term (RT) also presents in the SP potential when it is determined from the total energy of NM using the Hugenholtz–van Hove (HvH) theorem [23, 24], which is exact for all the interacting Fermi systems, independent of the interaction between fermions. In our recent HF study of NM [10], the density dependence of the CDM3Yn interaction (with $n = 3, 6$) was modified to reproduce on the HF level the SP potential obtained from the total NM energy using the HvH theorem. A strong impact of the RT on the strength and shape of the folded nucleon OP was found [10] essential for a good optical model (OM) description of the elastic $n+^{208}$Pb scattering at energies of 30.4 and 40 MeV.

Because the standard local approximation [11] was used in [10] to localize the (Fock-type) exchange term of the folded OP, it remains uncertain how the RT affects the OM results of elastic nucleon scattering when the antisymmetrization of the nucleon–nucleus system is exactly taken into account, and the folded nucleon OP becomes *nonlocal*. Although some versions of the nonlocal folding model with the exact treatment of the exchange term are already available in the literature (see, e.g. [13]), none of them has included the RT into the HF-type folding model calculation. Therefore, the main goal of the present study is to explore the impact of the rearrangement contribution to the OM description of nucleon elastic scattering by the nonlocal folded nucleon OP that treats the exchange kernel exactly. For this purpose, a compact nonlocal version of the folding model of the nucleon OP is suggested, where the RT is taken into account using the modified density dependence of the CDM3Yn interaction suggested in [10]. As an important mean-field aspect of the SP potential, the RT is always explicitly taken into account in numerous variational HF calculations of nuclear structure using the effective density-dependent NN interaction. However, the RT has been so far neglected in most of the HF-type folding model calculations of the nucleon OP, i.e. the single-nucleon potential at positive energies. Our present research is expected to shed light on the important role of the RT in the OM description of elastic nucleon scattering using the nonlocal folded OP. Furthermore, a consistent folding model for the nonlocal nucleon mean-field potential including the RT should also be of interest for the studies of nuclear reactions at low energies, in particular, those relevant for the nuclear astrophysics, where the effect of the nonlocality of the nucleon–nucleus potential has been shown to be quite significant [25, 26].

In general, solving the Schrödinger equation with a nonlocal potential readily leads to an integro-differential equation that is more complicated to solve than a standard differential equation with a local potential. For the elastic nucleon scattering, the use of the nonlocal OP leads to an explicit angular-momentum dependence of the integral equation. At variance with the traditional methods for the solution of the integro-differential equation, we have chosen in the present work the calculable *R*-matrix method [27] to solve the OM equation with the nonlocal folded nucleon OP. This *R*-matrix method was recently extended [27, 28] to include the Lagrange mesh and Gauss–Legendre quadrature integration that significantly simplify the numerical calculation. This method was tested in our recent OM analysis [29] of elastic nucleon scattering on different targets at energies up to 40 MeV, using the phenomenological nonlocal nucleon OP [30–33]. To validate the present nonlocal folding model of the nucleon OP, the OM results given by the nonlocal folded OP are also compared with those given by the global parametrization of the nonlocal nucleon OP suggested recently [31–33] using the analytical form of the Perey–Buck nonlocal form factor [30].

2. Single-particle potential in nuclear matter

Because an effective, density dependent NN interaction is the key input for the folding model calculation of the nucleon OP, we discuss first the density dependent CDM3Yn interaction which is based on the mean-field description of the SP potential in NM. Originally, parameters of the density dependence of the CDM3Yn interaction were parametrized [14] to reproduce the saturation properties of symmetric NM in the HF calculation. Later on, these parameters were updated to include a realistic isovector part [10, 12]. On the HF level, the CDM3Yn interaction is proven to give a good description of the EOS of NM [18]. As an illustration, we show in figure 1 the HF results for the NM energy per nucleon obtained with the CDM3Yn interaction in comparison with the results of the *ab initio* variational calculation of NM using the Argonne V18 interaction [34]. One can see a nice agreement of the HF results with those of the *ab initio* calculation, especially, at low densities up to the saturation density $\rho_0 \approx 0.16\ \text{fm}^{-3}$ which is known to be accessible by elastic nucleon–nucleus scattering. In the present work, we focus particularly on the impact of the RT in the folding model description of the nonlocal nucleon OP, given a significant contribution by the RT to the *local* folded nucleon OP shown in [10].

In general, according to Landau theory for an infinite system of interacting fermions [22], the single-nucleon energy is determined [10] as the derivative of the energy per nucleon $\varepsilon \equiv E/A$ of NM with respect to the nucleon momentum distribution $n_\tau(k)$ as

$$E_\tau(\rho, k) = \frac{\partial \varepsilon}{\partial n_\tau(k)} = \frac{\hbar^2 k^2}{2m_\tau} + U_\tau(\rho, k), \text{ where } \tau = n \text{ or } p. \tag{1}$$

$E_\tau(\rho, k)$ is, thus, the change of the energy of NM at the nucleon density ρ caused by the removal or addition of a nucleon with the momentum k. The SP potential $U_\tau(\rho, k)$ consists of both the HF and rearrangement terms

$$U_\tau(\rho, k) = U_\tau^{(\text{HF})}(\rho, k) + U_\tau^{(\text{RT})}(\rho, k). \tag{2}$$

The explicit expressions of $U_\tau(k)$ obtained in the HF calculation of NM using a density dependent NN interaction have been given in [10]. At the Fermi momentum ($k \to k_F$), $E_\tau(k_F)$ determined from equations (1) and (2) is exactly the Fermi energy given by the Hugenholtz–van Hove (HvH) theorem [23]. We note that the HvH theorem is satisfied on the HF level only when the effective NN interaction is density independent, with the RT equal zero [10, 35]. In the mean-field calculation (1)–(2) of the SP potential, the RT originates naturally from the density dependence of the effective NN interaction that presumably accounts for the

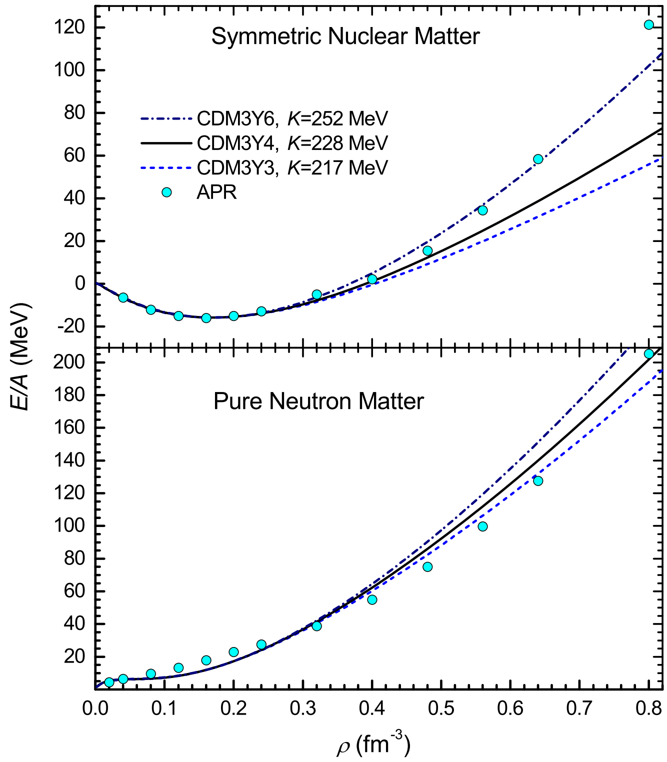

Figure 1. Energy per nucleon of the symmetric NM and pure neutron matter given by the HF calculation using the density dependent CDM3Yn interaction. K is the nuclear incompressibility obtained at the saturation density $\rho_0 \approx 0.16$ fm^{-3}. The circles are results of the *ab initio* variational calculation by Akmal, Pandharipande and Ravenhall (APR) [34].

higher-order NN correlations as well as the three-body force. Indeed, the RT was shown in the BHF studies of the SP potential in NM [1, 4, 5] to be due to the higher-order terms like the second-order diagram in the perturbative expansion of the mass operator or contribution from the three-body forces.

For the spin-saturated NM, the direct (D) and exchange (EX) terms of the central part of the CDM3Yn interaction [12, 14] are used explicitly in the HF calculation of the SP potential in NM

$$v_{\mathrm{D(EX)}}(\rho, s) = F_0(\rho) v_{00}^{\mathrm{D(EX)}}(s) + F_1(\rho) v_{01}^{\mathrm{D(EX)}}(s) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2. \tag{3}$$

The radial parts of the isoscalar (IS) and isovector (IV) two-body force $v_{00(01)}^{\mathrm{D(EX)}}(s)$ are kept unchanged as determined from the original M3Y-Paris interaction [15], in terms of three Yukawas. The parameters of the IS density dependence $F_0(\rho)$ were determined in the HF calculation [14] to reproduce the empirical saturation point of symmetric NM, with the nuclear incompressibility K around 230 MeV (see figure 1). The parameters of the IV density dependence $F_1(\rho)$ were determined and fine tuned [10] by the isospin dependence of nucleon OP in asymmetric NM given by the BHF calculation by Jeukenne, Lejeune and Mahaux (JLM) [36] and folding model description of the charge exchange (p, n) scattering to the isobar analog states in medium-mass nuclei [21]. Based on the exact expression of the RT of the SP potential given by the HvH theorem at different densities of NM, a compact method was suggested [10] to account for the RT on the HF level. Namely, the density dependence of the CDM3Yn interaction is added by a correction term originated from the RT, $F_{0(1)}(\rho) \to F_{0(1)}(\rho) + \Delta F_{0(1)}(\rho)$, so that the total SP potential can be calculated on the standard HF level

$$U_\tau(\rho, k) = \sum_{k'\sigma'\tau'} \langle \boldsymbol{k}\sigma\tau, \boldsymbol{k}'\sigma'\tau'|v_{\mathrm{D}}|\boldsymbol{k}\sigma\tau, \boldsymbol{k}'\sigma'\tau'\rangle + \langle \boldsymbol{k}\sigma\tau, \boldsymbol{k}'\sigma'\tau'|v_{\mathrm{EX}}|\boldsymbol{k}'\sigma\tau, \boldsymbol{k}\sigma'\tau'\rangle, \quad (4)$$

where $|\boldsymbol{k}\sigma\tau\rangle$ are the ordinary plane waves. Treating explicitly the isospin dependence, the SP potential (4) can be expressed [10] in terms of the IS and IV components as

$$U_\tau(\rho, k) = [F_0(\rho) + \Delta F_0(\rho)]U_0^{(\mathrm{M3Y})}(\rho, k) \pm [F_1(\rho) \pm \Delta F_1(\rho)]U_1^{(\mathrm{M3Y})}(\rho, k), \quad (5)$$

where $(-)$ sign pertains to $\tau = p$ and $(+)$ sign to $\tau = n$. $U_0^{(\mathrm{M3Y})}$ and $U_1^{(\mathrm{M3Y})}$ are the IS and IV parts of the SP potential, respectively, given by the HF calculation of NM using the original density independent M3Y interaction. More details on the density dependent functions $F_{0(1)}(\rho)$, $\Delta F_{0(1)}(\rho)$, and $U_{0(1)}^{(\mathrm{M3Y})}(\rho, k)$ are given in [10]. Because the original M3Y interaction is momentum independent, the momentum- or energy dependence of the SP potential (5) is entirely determined by the exchange terms of $U_{0(1)}^{(\mathrm{M3Y})}$. It is obvious from equation (1) that the in-medium nucleon momentum k is determined self-consistently from the SP energy E_τ as

$$k = \sqrt{\frac{2m_\tau}{\hbar^2}[E_\tau(\rho, k) - U_\tau(\rho, k)]}. \quad (6)$$

With the density dependence of the CDM3Yn interaction fine tuned to reproduce the saturation properties of NM as shown in figure 1, the present HF approach provides a continuous description of both the SP potential for nucleons bound in NM, $U_\tau(\rho, k)$ with $k < k_F$, and the nucleon optical potential, $U_\tau(\rho, k)$ with $k > k_F$. This is the well-known *continuous approximation* for the SP potential [1, 37], where the nucleon OP in NM is determined as the mean-field potential felt by a nucleon incident on NM at the energy $E > 0$. Complying with the Landau theory for a system of interacting fermions [22], the nucleon OP (or the SP potential at positive energies) is also determined by the relations (1) and (2), so that the nucleon OP consists again of both the HF term and RT. The IS part of the nucleon OP, i.e. the nucleon OP in symmetric NM is determined as

$$\begin{aligned} U_0(E, \rho) &= [F_0(\rho) + \Delta F_0(\rho)]U_0^{(\mathrm{M3Y})}(\rho, k(E, \rho) \\ &= [F_0(\rho) + \Delta F_0(\rho)]\left[J_0^D + \int \hat{j}_1(k_F r) j_0(k(E, \rho)r) v_{00}^{\mathrm{EX}}(r) \mathrm{d}^3 r\right], \end{aligned} \quad (7)$$

$$\text{where } J_0^{\mathrm{D}} = \int v_{00}^{\mathrm{D}}(r)\mathrm{d}^3 r, \;\; \hat{j}_1(x) = 3j_1(x)/x = 3(\sin x - x\cos x)/x^3. \quad (8)$$

$k(E, \rho)$ is the in-medium momentum of the incident nucleon propagating in the mean field of bound nucleons in NM, and is determined by the same relation (6) but with E_τ replaced by the incident energy E. Within the time-independent HF formalism, the energy- and momentum dependences of the nucleon OP are treated on the same footing via the relation (6) as illustrated in figure 2. Therefore, an important constraint for the present study is that at $E > 0$ the energy dependence of the potential (7) should agree reasonably with the observed energy dependence of the nucleon OP. The total SP potential in symmetric NM (7) evaluated at ρ_0 using the CDM3Y6 interaction is compared with the empirical data [38–40] in figure 2. One can see that the inclusion of the RT significantly improves the agreement with the empirical data at low energies ($E < 50$ MeV). The HF results shown in figure 1 also confirms that at low energies the energy dependence of the nucleon OP is mainly determined by the Fock-type

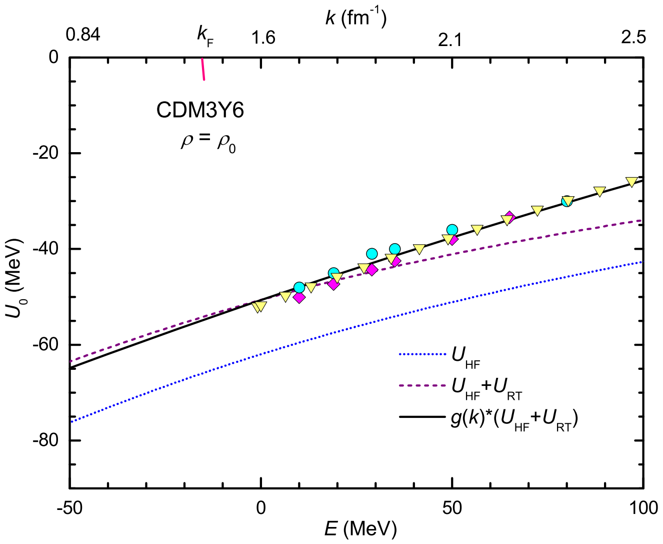

Figure 2. Single-nucleon potential in symmetric NM (7) determined at $\rho \approx \rho_0$ with and without the RT using the CDM3Y6 interaction, in comparison with the empirical data for the nucleon OP taken from [38] (circles), [39] (squares), and [40] (triangles). The momentum dependent factor $g(k)$ was obtained [10] by the χ^2 fit of the calculated potential (7) at $E > 0$ to the empirical data (solid line).

exchange term of the nucleon OP, i.e. by the antisymmetrization effect. At higher energies, the agreement worsens, and this is a well expected effect, because the energy dependence of the nucleon OP in NM was shown in the microscopic BHF calculation [37] to originate not only from the exchange part, but also from the direct part of the Brueckner G-matrix. To have a realistic momentum dependence of the nucleon OP at higher energies or momenta, the nucleon OP given by the HF calculation was scaled [10] by a momentum dependent function $g(k)$ determined from the χ^2 fit to the empirical data, $U(\rho, E) = g(k(E, \rho))U_0(\rho, E)$ (see figure 2). In the present work, we focus on the folding model analysis of elastic nucleon scattering at low energies ($E \leqslant 45$ MeV), and assume $g(k) \approx 1$ in the folding calculation of the nucleon OP.

3. Folding model of the nucleon optical potential

3.1. Nucleon folded OP with a nonlocal exchange kernel

The folding model of the nucleon OP is known to generate the first-order term of the microscopic nucleon OP within the Feshbach's formalism of nuclear reactions [41]. The success of the folding approach in the description of elastic nucleon–nucleus scattering at low and medium energies confirms that the first-order term of the Feshbach's microscopic OP is indeed the dominant part of the nucleon OP. Applying the *local density approximation* (LDA), commonly adopted in the HF calculations of nuclear structure, the plane waves $|\boldsymbol{k}'\sigma'\tau'\rangle$ in the SP potential (4) can be replaced by the SP wave functions $|j\rangle$ of the target nucleons. Then, the central OP of the elastic nucleon scattering on the target A can be evaluated as

$$U(k) = \sum_{j\in A}[\langle \boldsymbol{k}, j|v_{\rm D}|\boldsymbol{k}, j\rangle + \langle \boldsymbol{k}, j|v_{\rm EX}|j, \boldsymbol{k}\rangle]. \tag{9}$$

The antisymmetrization of the nucleon–nucleus system is done in the HF manner, taking into account explicitly the knock-on exchange. As a result, the exchange term of the nucleon–nucleus potential (9) becomes *nonlocal*, and the OM equation for the elastic nucleon scattering at the energy E is an integro-differential equation

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + U_{\rm D}(R) + V_{\rm C}(R) + V_{\rm s.o.}(R)(\boldsymbol{L}.\boldsymbol{\sigma})\right]\chi(\boldsymbol{R}) + \int K(\rho, \boldsymbol{R}, \boldsymbol{r})\chi(\boldsymbol{r}){\rm d}^3r = E\chi(\boldsymbol{R}), \tag{10}$$

where $V_{\rm s.o.}(R)$ is the spin–orbit potential, and the Coulomb potential $V_{\rm C}(R)$ is included only for elastic proton scattering. The scattering wave function $\chi(\boldsymbol{R})$ is obtained from the solution of the OM equation (10) at each nucleon–nucleus distance R. The energy dependent mean-field part consists of the local direct potential $U_{\rm D}(R)$ and the exchange integral with a nonlocal, density dependent kernel $K(\rho, \boldsymbol{R}, \boldsymbol{r})$. The mean-field part of the nucleon OP can be expressed, in a manner consistent with the Lane representation, in terms of the isoscalar and isovector parts as

$$\begin{aligned} U_{\rm D}(R) &= U^{\rm D}_{\rm IS}(R) \pm U^{\rm D}_{\rm IV}(R),\\ K(\rho, \boldsymbol{R}, \boldsymbol{r}) &= K_{\rm IS}(\rho, \boldsymbol{R}, \boldsymbol{r}) \pm K_{\rm IV}(\rho, \boldsymbol{R}, \boldsymbol{r}), \end{aligned} \tag{11}$$

where $(-)$ sign pertains to proton OP and $(+)$ sign to neutron OP. The IS and IV terms in equation (11) are determined using, respectively, the IS and IV parts of the nucleon density matrices as

$$\begin{aligned} U^{\rm D}_{\rm IS(IV)}(R) &= \int [\rho_n(\boldsymbol{r}) \pm \rho_p(\boldsymbol{r})]v^{\rm D}_{00(01)}(\rho, s){\rm d}^3r,\\ K_{\rm IS(IV)}(\rho, \boldsymbol{R}, \boldsymbol{r}) &= [\rho_n(\boldsymbol{R}, \boldsymbol{r}) \pm \rho_p(\boldsymbol{R}, \boldsymbol{r})]v^{\rm EX}_{00(01)}(\rho, s), \end{aligned} \tag{12}$$

where $s = |\boldsymbol{R} - \boldsymbol{r}|$. The nucleon density matrix is determined from the SP wave functions of target nucleons as

$$\rho_\tau(\boldsymbol{r}, \boldsymbol{r}') = \sum_{j\in A}\varphi_j^{(\tau)*}(\boldsymbol{r})\varphi_j^{(\tau)}(\boldsymbol{r}'), \text{ with } \rho_\tau(\boldsymbol{r}) \equiv \rho_\tau(\boldsymbol{r}, \boldsymbol{r}), \text{ and } \tau = n, p. \tag{13}$$

Within the adopted LDA, the parameters of the density dependence of the CDM3Yn interaction determined in the HF calculation of NM are readily used in the HF-type folding model calculation of the nucleon OP of finite nuclei (9), where the density dependent functional $F_{0(1)}(\rho) + \Delta F_{0(1)}(\rho)$ is given consistently by the local target density $\rho(\boldsymbol{r})$ appearing in equations (11) and (12). The direct potential $U_{\rm D}(R)$ is obtained simply by folding the local nucleon density matrices with the direct part $v^{\rm D}_{00(01)}(\rho, s)$ of the density dependent CDM3Yn interaction (see more details in [11]), including the contribution of the rearrangement term. We show here the explicit expression of the IV part of the direct folded potential

$$U^{\rm D}_{\rm IV}(R) = \int [\rho_n(\boldsymbol{r}) - \rho_p(\boldsymbol{r})][F_1(\rho(\boldsymbol{r})) \pm \Delta F_1(\rho(\boldsymbol{r}))]v^{\rm D}_{01}(s){\rm d}^3r, \tag{14}$$

where the $(\pm)$ signs are used in the same way as in equation (11). One can see that the contribution of the RT to the IV part of the direct potential $U^{\rm D}_{\rm IV}$ via $\Delta F_1(\rho)$ is the same for both the proton and neutron OP.

Note that the energy dependence of the folded nucleon OP is implicitly embedded in the exchange integral of equation (10) when the nonlocal exchange term is treated exactly. This procedure is cumbersome and involves the explicit angular-momentum dependence of the exchange kernel. Using the multipole decomposition of the radial Yukawa function in the exchange component of the CDM3Yn interaction (3)

$$v_{00(01)}^{\mathrm{EX}}(s) = \sum_{\lambda\mu} \frac{4\pi}{2\lambda + 1} X_{00(01)}^{(\lambda)}(R, r) Y_{\lambda\mu}^{*}(\hat{\boldsymbol{R}}) Y_{\lambda\mu}(\hat{\boldsymbol{r}}), \tag{15}$$

we obtain, after integrating out the angular dependence, the following radial OM equation for each partial wave L

$$\begin{aligned} -\frac{\hbar^2}{2\mu}\left[\frac{\mathrm{d}^2}{\mathrm{d}R^2} - \frac{L(L+1)}{R^2}\right]\chi_{LJ}(R) + [U_{\mathrm{D}}(R) + V_{\mathrm{C}}(R) \\ + A_{LJ} V_{\mathrm{s.o.}}(R)]\chi_{LJ}(R) + \int_0^\infty K_{LJ}(\rho, R, r)\chi_{LJ}(r)\mathrm{d}r = E\chi_{LJ}(R), \end{aligned} \tag{16}$$

where the s.o. coupling coefficient $A_{LJ} = L$ if $J = L + 1/2$, and $A_{LJ} = -L-1$ if $J = L - 1/2$. One can see that the use of the nonlocal OP leads to an explicit angular-momentum dependence of the integral equation for the scattering wave function. The nonlocal exchange kernel is determined explicitly as

$$K_{LJ}(\rho, R, r) = [K_{LJ}^{\mathrm{IS}}(\rho, R, r) \pm K_{LJ}^{\mathrm{IV}}(\rho, R, r)], \tag{17}$$

$$\begin{aligned} K_{LJ}^{\mathrm{IS}}(\rho, R, r) = [F_0(\rho(r)) + \Delta F_0(\rho(r))]\sum\nolimits_{nlj,\lambda} \frac{u_{nlj}^{(n)}(R)u_{nlj}^{(n)}(r) + u_{nlj}^{(p)}(R)u_{nlj}^{(p)}(r)}{R\ r} \\ \times (2j+1)X_{00}^{(\lambda)}(R, r)\begin{pmatrix} L & l & \lambda \\ 0 & 0 & 0 \end{pmatrix}^2, \end{aligned} \tag{18}$$

$$\begin{aligned} K_{LJ}^{\mathrm{IV}}(\rho, R, r) = [F_1(\rho(r)) \pm \Delta F_1(\rho(r))]\sum\nolimits_{nlj,\lambda} \frac{u_{nlj}^{(n)}(R)u_{nlj}^{(n)}(r) - u_{nlj}^{(p)}(R)u_{nlj}^{(p)}(r)}{R\ r} \\ \times (2j+1)X_{01}^{(\lambda)}(R, r)\begin{pmatrix} L & l & \lambda \\ 0 & 0 & 0 \end{pmatrix}^2. \end{aligned} \tag{19}$$

Here $u_{nlj}^{(\tau)}(r)$ is the radial part of the SP wave function $\varphi_{nlj}^{(\tau)}(\boldsymbol{r})$ of the target nucleon. Note that in equations (17) and (19), the $(-)$ sign is used with the proton OP and $(+)$ sign with the neutron OP. Thus, the contribution of the RT to the IV part of the exchange kernel is also the same for both the proton and neutron OP as found for the IV part of the direct potential (14). The explicit representation of the nucleon OP in terms of the IS and IV parts is helpful for the investigation of the contribution of valence neutrons to the OP. Furthermore, the form factor (FF) of the charge exchange (p, n) reaction to the isobar analog state (IAS) is determined, in the Lane isospin coupling scheme, entirely by the IV part of the nucleon OP [21, 42]. Therefore, the present nonlocal folding model can be used to calculate the nonlocal charge exchange FF in the future folding model studies of the (p, n) reaction to IAS.

3.2. Local approximation for the folded nucleon OP

Although it is established that the nucleon OP is nonlocal in the coordinate space due to the Pauli blocking (as shown above) and the multichannel coupling, over the years the nucleon OP is mainly assumed in the local form for the OM analysis of elastic nucleon scattering. The local OP that describes properly the nucleon elastic scattering is the key input for the distorted wave Born approximation (DWBA) or coupled channel (CC) analyses of different direct reaction processes induced by the incident nucleon. Here the (bare) real OP accounts for the purely elastic scattering and imaginary OP accounts for the absorption of flux by those nonelastic reaction channels that are not explicitly taken into account in the CC calculation. It is of interest, therefore, to assess the accuracy of the local version [11, 12] of the nonlocal folding model suggested in the present work.

Applying a local WKB approximation [43, 44] for the change in the scattering wave function in the OM equation (10) induced by the exchange of spatial coordinates of the incident nucleon and that bound in the target

$$\chi(\boldsymbol{r}) = \chi(\boldsymbol{R} + \boldsymbol{s}) \simeq \chi(\boldsymbol{R})\exp(\mathrm{i}\boldsymbol{k}(E, \boldsymbol{R}).\boldsymbol{s}), \tag{20}$$

the exchange integral in equation (10) can be evaluated independently using the nonlocal nucleon density matrix. This gives rise to a *local* exchange term of the folded nucleon–nucleus potential (9) that depends explicitly on energy via the local momentum of the incident nucleon $k(E, R)$

$$\begin{aligned} U_{\mathrm{EX}}(E, R) &= U_{\mathrm{IS}}^{\mathrm{EX}}(E, R) \pm U_{\mathrm{IV}}^{\mathrm{EX}}(E, R), \\ U_{\mathrm{IS(IV)}}^{\mathrm{EX}}(E, R) &= \int [\rho_n(\boldsymbol{R}, \boldsymbol{r}) \pm \rho_p(\boldsymbol{R}, \boldsymbol{r})] j_0(k(E, R)s) v_{00(01)}^{\mathrm{EX}}(\rho, s)\mathrm{d}^3r, \end{aligned} \tag{21}$$

where the ($\pm$) signs are used in the same way as in equation (11). The local momentum $k(E, R)$ of the incident nucleon propagating in the target mean field is determined from the real part of the total folded potential $U(E, R) = U_{\mathrm{D}}(R) + U_{\mathrm{EX}}(E, R)$ as

$$k^2(E, R) = \frac{2\mu}{\hbar^2}[E - \mathrm{Re}\, U(E, R) - V_{\mathrm{C}}(R)]. \tag{22}$$

The method used to evaluate the direct (12) and exchange (21) folded potentials has been discussed earlier (see, e.g. [11]). Using a realistic local approximation for the nonlocal density matrix in the exchange potential (21), the nuclear density $\rho(\boldsymbol{r})$ obtained in any structure model or directly deduced from the electron scattering data can be used in the folding calculation of the nucleon OP. A preliminary folding model study of elastic $n+^{208}$Pb scattering using the neutron OP obtained with the local approximation (21) for the exchange term has shown a significant contribution of the RT to the nucleon folded OP [10]. It should be noted that the RT is commonly taken into account in the variational HF calculation of nuclear structure using some density dependent NN interaction. However, the RT has not been included so far in most of the HF-type folding model calculations of the nucleon OP, and the main goal of the present work is to show the impact of the RT on both the local and nonlocal folded nucleon OP.

4. Elastic nucleon scattering on ^{40,48}Ca, ^{90}Zr, and ^{208}Pb targets

The new version of the folding model for the nonlocal and local nucleon OP discussed above in sections 3.1 and 3.2 has been used in the present work to calculate the nucleon OP for the OM study of the elastic neutron and proton scattering on ^{40,48}Ca, ^{90}Zr, and ^{208}Pb targets. One can see from equations (12)–(13) that the folding calculation of the *nonlocal* OP requires explicitly the single-particle wave functions of all nucleons bound in the target. We have used here the SP wave functions given by the HF calculation of finite nuclei, using a complete basis of spherical Bessel functions [45] and the finite-range D1S Gogny interaction [46].

In the context of a *complex* folded OP, it is necessary to have a realistic complex parametrization of the density dependent CDM3Yn interaction. For this purpose, the imaginary density dependence of the CDM3Yn interaction was determined using the same density dependent functionals $F_{0(1)}(\rho)$ as those used in equation (3) for the real interaction, with the parameters determined at each energy to reproduce on the HF level [12] the energy dependent imaginary nucleon OP given by the JLM parametrization of the BHF results for NM [36]. The folded complex nonlocal nucleon OP as well as its local version were further used as the input for the OM calculation of elastic nucleon scattering using the extended *R*-matrix method [27]. In the present OM calculation, the nonlocal mean-field part of the nucleon OP is supplemented by the local Coulomb and spin–orbit potentials taken from the global systematics CH89 of the nucleon OP [39].

The reliability of the folded OP in the OM study of elastic nucleon scattering is best to be probed in the analysis of elastic neutron scattering from a heavy target at low energies, where the Coulomb interaction is absent and the mean-field dynamics is well established. The elastic $n+^{208}$Pb scattering data accurately measured over a wide angular range at energies of 26, 30.4, and 40 MeV [47–49] turned out to be a very good test ground for this purpose. Because the parameters of the (real) CDM3Yn interaction were adjusted by the realistic HF description of NM as shown in figure 1, *no renormalization* of the strength of the *real* folded potential (12) was allowed in the present OM analysis to test its proximity to the real nucleon OP as well as the impact of the RT to the real folded OP. While the imaginary folded nucleon OP based on the JLM parametrization of the G-matrix interaction delivers a good OM description of elastic proton scattering, it gives consistently a stronger absorption in the neutron OP, and an overall renormalization of the *imaginary* neutron folded potential by a factor $\sim$0.8 is needed for a good OM description of elastic neutron scattering data at the considered energies. From the OM results obtained with the CDM3Y6 interaction shown in figure 3 for elastic $n+^{208}$Pb scattering one can see that the inclusion of the RT into the folding model calculation is essential for a good OM description of the data over the whole angular range. We note that the results obtained with the complex CDM3Y3 and CDM3Y4 interactions are almost the same as those shown in figure 3, with a minor difference that is hardly noticeable on the logarithmic scale. In the absence of the Coulomb interaction, the oscillation pattern of the elastic neutron cross section over the whole angular range can be reproduced only with the inclusion of the RT. It can also be seen in figure 3 that the local approximation (21)–(22) for the exchange term of the folded nucleon OP is quite reasonable, and the local folded OP with the RT included also gives a reasonable OM description of the data. About the same impact by the RT and good accuracy of the local folding approach can be seen in the OM results for elastic neutron scattering on the medium-mass ^{40,48}Ca and ^{90}Zr targets (see figure 4).

The elastic $p+^{208}$Pb scattering data measured at 30.4, 35, and 45 MeV [53, 54] are compared in figure 5 with the OM results given by the same three versions of the folded OP as those discussed in figure 3. The inclusion of the RT into the folding model calculation was found to be also vital for a good OM description of the elastic proton scattering data over the

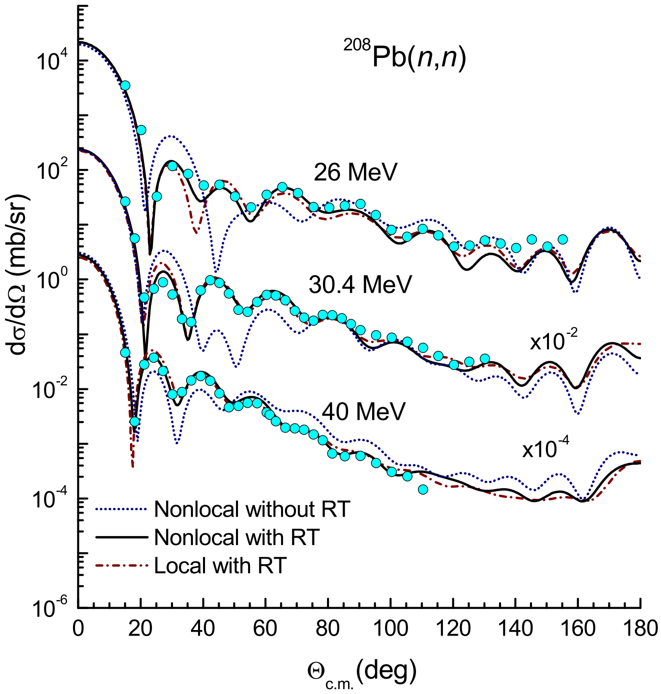

Figure 3. OM description of the elastic $n+^{208}$Pb scattering data measured at 26, 30, and 40 MeV [47–49] given by the complex *nonlocal* folded OP obtained with the CDM3Y6 interaction with or without the inclusion of the RT, in comparison with that given by the *local* folded OP with the RT included.

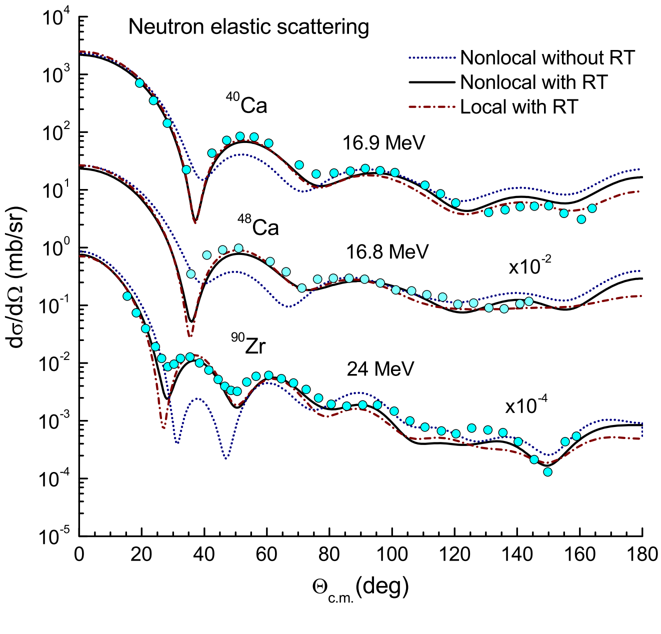

Figure 4. The same as figure 3 but for the data of the elastic neutron scattering on ^{40,48}Ca and ^{90}Zr target measured [50–52] at 17 and 24 MeV, respectively.

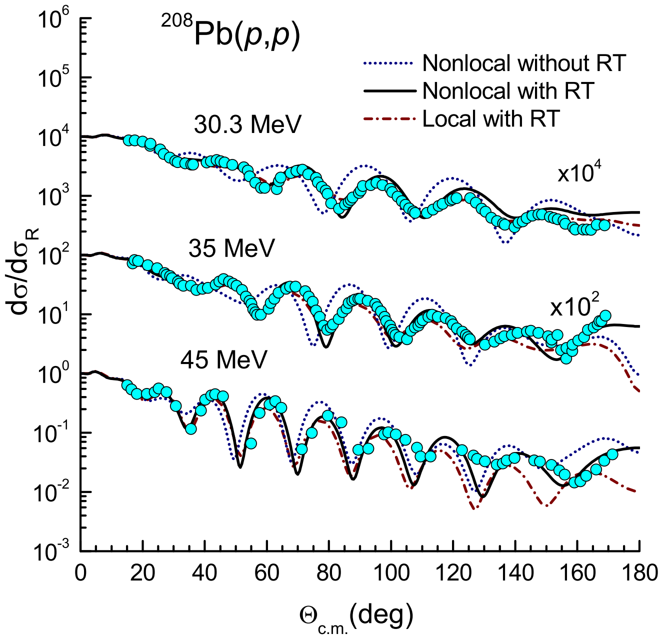

Figure 5. The same as figure 3 but for the elastic $p+^{208}$Pb scattering data measured at 30, 35, and 45 MeV [53, 54]. The calculated elastic scattering cross sections and data points are plotted in ratio to the Rutherford cross section at the corresponding angles.

whole angular range as is seen in figure 5. At the forward angles, the effect of the RT in the elastic $p+^{208}$Pb scattering is slightly weaker than that found in the elastic $n+^{208}$Pb scattering because the elastic cross section there has a significant contribution from the Coulomb scattering which is not affected by the inclusion of the RT in the folding calculation. The same impact by the RT, but with a more pronounced difference between the results given by three versions of the folded OP can be seen in the OM results for the elastic $p+^{40}$Ca scattering shown in figure 6. For this double magic target, the results of our folding model analysis show unambiguously the importance of taking into account both the RT and nonlocality of the folded nucleon OP. Although, the nonlocal and local folded OP's give about the same OM results for the elastic proton scattering at the forward- and medium angles, the data at the backward angles can be properly reproduced only by the nonlocal folded OP, especially at the proton energy of 45 MeV. The effect of the RT found here in the folding model description of elastic nucleon-nucleus scattering at low energies should be complementary to the rearrangement of the SP configurations established in the single-nucleon removal reactions [55].

5. Microscopic nonlocal folded OP versus the global parametrization

Although the practical OM calculations of elastic nucleon scattering are usually done using some global parameters of the local nucleon OP (see, e.g. [39, 57]), some OM studies have been aimed to explore the use of an explicit nonlocal nucleon OP and deduce the global parameters for that purpose. We note here the early work by Perey and Buck (PB) [30] and the recent revision of the PB parametrization by Tian, Pang, and Ma (TPM) [31], where the

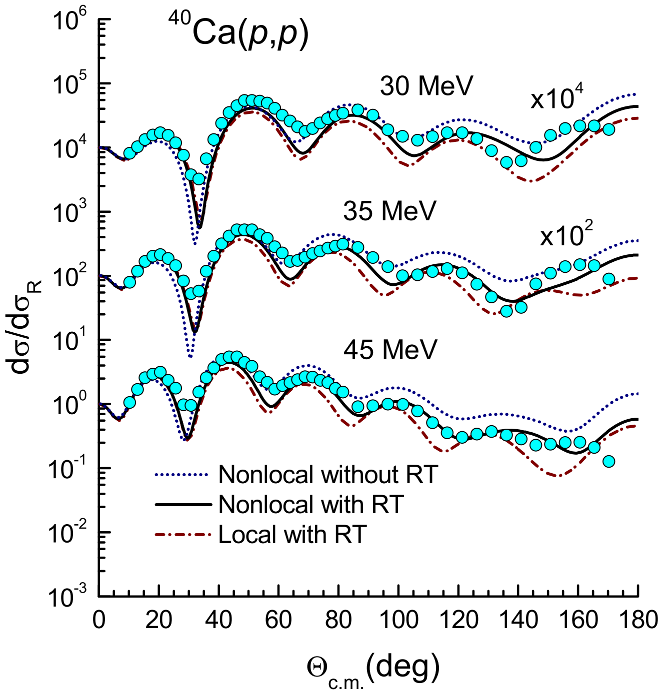

Figure 6. The same as figure 5 but for the elastic $p+^{40}$Ca scattering data measured at 30, 35, and 45 MeV [56].

nonlocal nucleon OP is built up from a Woods–Saxon form factor multiplied by a nonlocal Gaussian. While the PB parameters were adjusted to the best OM fit of the two data sets (elastic $n+^{208}$Pb scattering at 7.0 and 14.5 MeV), those of the TPM potential were fitted to reproduce the data of elastic nucleon scattering on ^{32}S, ^{56}Fe, ^{120}Sn, and ^{208}Pb targets at energies of 8 to 30 MeV. More recently, an energy dependence has been introduced explicitly into the imaginary parts of the PB and TPM potentials, dubbed as PBE and TPME potentials, with the parameters adjusted to achieve the overall good OM description of nucleon elastic scattering on ^{40}Ca, ^{90}Zr, and ^{208}Pb targets at energies E of 5–45 MeV [32, 33]. Given the microscopic nucleon folded OP constructed from the realistic SP wave functions of the target nucleons using the mean-field based density dependent CDM3Yn interaction, it is of interest to compare its predicting power with that of the global parametrization. The OM results for the elastic $n+^{208}$Pb scattering at 26, 30.4, and 40 MeV given by the nonlocal folded OP obtained with the CDM3Y6 interaction are compared with the OM results given by the PB parametrization of the nonlocal neutron OP [30] and the recent energy dependent version PBE [32] in figure 7. One can see that the nonlocal folded OP performs quite well, with the predicted elastic cross section agreeing closely with the data like that given by the global PBE potential. As expected, the original (energy independent) PB parametrization fails to account for these data that were measured at energies higher than those considered by Perey and Buck [30]. It can be seen in figure 8 that the elastic $n+^{208}$Pb cross section predicted by the nonlocal folded OP agrees with the data slightly better than that predicted by the global TPM and TPME parametrizations of the nonlocal OP. Thus, these results confirm nicely the reliability of the present nonlocal version of the folding model in the calculation of the nucleon OP for medium and heavy targets. The model can be used, therefore, to predict the complex nucleon

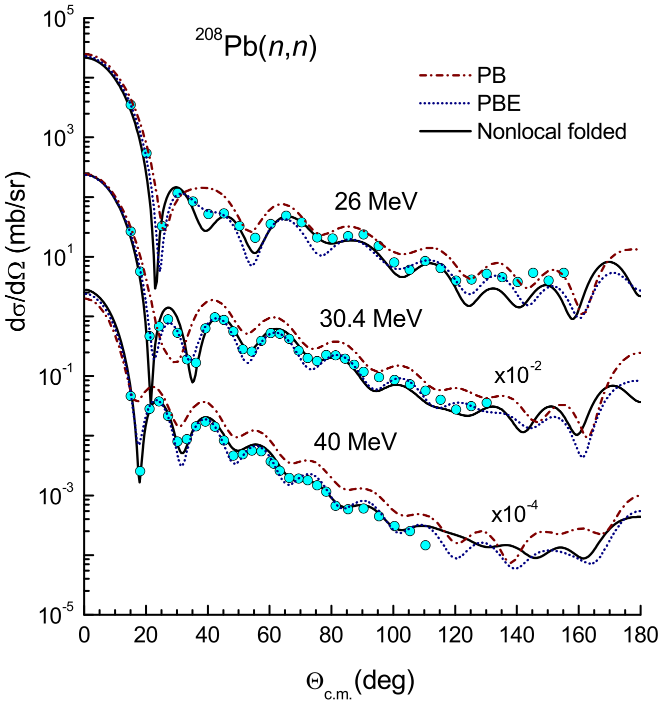

Figure 7. OM description of the elastic $n+^{208}$Pb scattering data measured at 26, 30, and 40 MeV [47–49] given by the nonlocal folded OP obtained with the CDM3Y6 interaction including the RT, in comparison with the OM results given by the original PB parametrization of the nonlocal neutron OP [30] and its recent energy dependent version PBE [32].

OP of the short-lived, unstable nuclei (when no elastic scattering data are available) for further use in the direct reaction analysis, provided the realistic SP wave functions are available for these radioactive nuclei, which are not an easy task for the nuclear structure models.

6. Summary

The folding model of the nonlocal nucleon OP, with the exchange potential calculated exactly in the HF manner, is generalized to include the rearrangement term using the CDM3Yn interaction with a complex density dependence. The obtained OM results for the elastic neutron and proton scattering on ^{40,48}Ca, ^{90}Zr, and ^{208}Pb targets at different energies have shown that the inclusion of the RT into the folding model calculation of the nucleon OP is essential for a good OM description of the considered elastic data. Although the RT is widely taken into account in numerous variational HF calculations of nuclear structure using the effective density-dependent NN interaction, the results of the present work have confirmed, for the first time, the important role of the RT in the (local and nonlocal) HF-type folding model of the nucleon OP.

The OM results given by the complex nonlocal folded OP with the RT included are further compared with those given by the global parametrization of the nonlocal nucleon OP suggested recently [31, 32], based on the analytical nonlocal form factor suggested many years ago by Perey and Buck [30]. This comparison has confirmed the reliability of the

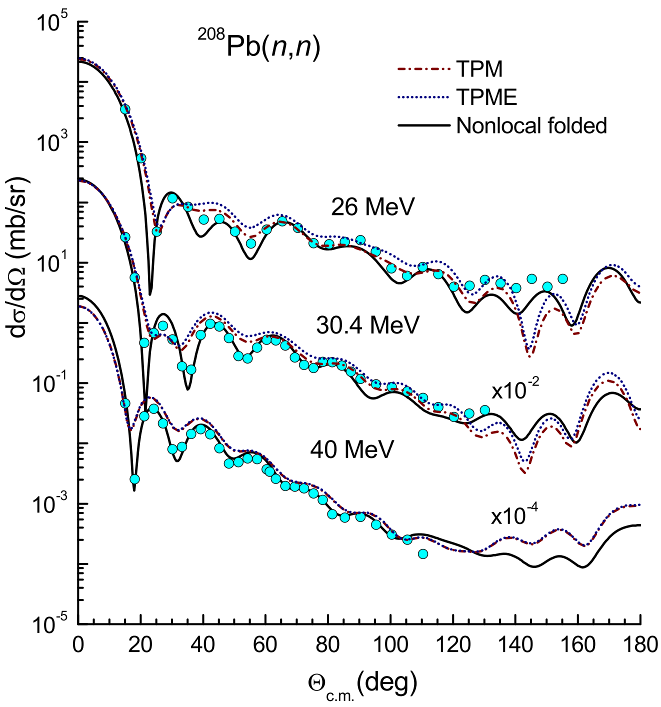

Figure 8. The same as figure 7 but in comparison with the OM results given by the TPM parametrization of the nonlocal neutron OP [31] and its energy dependent version TPME [32].

present nonlocal folding model for the medium and heavy targets, and the model can be used, therefore, to predict the nucleon OP of the short-lived unstable nuclei (for which no elastic scattering data are available) using the realistic SP wave functions given by the nuclear structure studies.

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We thank Pierre Descouvemont for his helpful discussions and comments on the calculable *R*-matrix method [27, 28]. The present research has been supported, in part, by the National Foundation for Scientific and Technological Development (NAFOSTED Project No. 103.04-2 017.317).

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