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Optical solitons with M-truncated derivative and conservation laws for NLSE equation which describe pseudospherical surfaces

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Abstract

The present work analyzes the optical solitons with M-truncated and conservation laws (Cls) of the nonlinear Schrodinger equation that explain pseudospherical surfaces. The well-known integration scheme which is generalized Bernoulli sub-ODE method is utilized to construct such optical soliton solutions. For the successful existence of the solutions, the constraints conditions have been presented. The discussion for the physical features of the obtained solutions is reported. Moreover, we establish Cls for the equation under consideration by means of multiplier approach.

Keywords: generalized Bernoulli sub-ODE method, M-truncated derivative, optical solitons

(Some figures may appear in colour only in the online journal)

1. Introduction

In both theoretical and applied sciences, the research of physical events through mathematical models is an important component. Such models often lead to nonlinear systems and some prototypical equations in amazingly big numbers of instances. Solitons are an intrinsically nonlinear phenomenon and their history is closely linked to the growth of nonlinear wave equations theory. In latest decades there has been a steady increase in interest in optical solitons. The field has significant potential for technological applications and poses many interesting study issues from both a basic and an applied perspective. Many fields of science and engineering employ a nonlinear partial differential equations (PDEs) to determine the characteristic of these phenomena. In the scrutiny of PDEs, a lot of authors give a priority in constructing soliton and exact solutions.

NLSEs are a PDEs with exciting performances for many years. This is because of its expansive spectrum of applicability. There are a types of NLSEs that can be employed to interpret real phenomena in various domains among which is Bose–Einstein, deliquescence [1, 2], nonlinear optics [3–5], fluid dynamics [6] to mention a few. In the present time, many studies can be found in this regard [7–13]. Optical solitons are localized electromagnetic waves that spread in nonlinear dispersive media with the unchanged intensity as a result of the effects of the stability for the dispersion and nonlinearity [14–23].

Furthermore, Cls generate numerous ideas on the systems of equations that are modelled by PDEs. The Cls have been very important aspects in the investigation of system of PDEs, because the integrability, internal properties, existence and uniqueness of equations can be reached through their Cls. Several schemes have been suggested in the literature for the constructions of Cls of a system of a particular equations [24–29].

Pseudospherical surfaces is described by a differential system for a 2-vector-valued function $U(x, t)$ if it possesses some certain conditions with regard to the existence of smooth real functions F_{ij}, $i \in [1, 3]$, $j \in [1, 2]$, depending on U only and a finite number of derivatives, thus we have

$$\sigma_i = F_{i1}\mathrm{d}x + F_{i2}\mathrm{d}t,\ i \in [1, 3], \tag{1}$$

hold for the following

$$\mathrm{d}\sigma_1 = \sigma_3 \wedge \sigma_2,\ \mathrm{d}\sigma_2 = \sigma_1 \wedge \sigma_3,\ \mathrm{d}\sigma_3 = \sigma_1 \wedge \sigma_2. \tag{2}$$

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A differential equation for a real valued function $U(x, t)$ is said to be integrable kinematically if it has the integrability property of a family of one-parameter for linear situations [30–36]

$$V_x = P(\gamma)V,\ V_t = Q(\gamma)V, \tag{3}$$

with $P(\gamma)$ and $Q(\gamma)$ depicting the $SL(2, R)$-valued functions for $x, t,$. Therefore, an equation is said to be integrable kinematically if it has the same property with the zero curvature property

$$\frac{\partial P(\gamma)}{\partial t} - \frac{\partial Q(\gamma)}{\partial x} + [P(\gamma), Q(\gamma)] = 0, \tag{4}$$

in which $trP(\gamma) = trQ(\gamma) = 0$, for every γ. Additionally, a differential equation is said to be strictly integrable kinematically if its integrable kinematically and diagonal entries of the matrix $P(\gamma)$ presented above are γ and $-\gamma$.

Many renowned physicists and applied mathematicians have been contributing to some or all of the aforementioned fields over the past decades. The involved-engineers, physicists, and mathematicians-are still operating within their own limits to a big extent. One of the aims of this article is to establish some optical soliton solutions and Cls, hoping to stimulate future communication with distinct backgrounds among scientists. The optical solitons are computed via a well-known analytical technique with m-truncated derivative, while the Cls are constructed by means of multiplier technque

The rest of the paper is organized in the following direction: In section two we state the description of m-trucated derivative and the form of the governing equation, in section three we establish some optical solitons solutions for the governing equation. Section four provides the Cls for the governing equation by means of multiplier approach and finally the paper is concluded in section five.

2. Truncated M-fractional derivative

We define the truncated Mittag–Leffler function of one parameter by

$$_iE_\beta(z) = \sum_{k=0}^{i} \frac{z^k}{\Gamma(\beta k + 1)}. \tag{5}$$

Truncated M-fractional derivative (TMD) is a fractional derivative that has been introduced in [37]. This derivative has expunged the obstacles with the existing derivatives. It is defined in the following definition.

Definition 2.1. Assume that f: $(0, \infty) \to \mathbb{R}$, the TMD of f with order γ exhibited $_iT_M^{\gamma,\beta}$ is given by

$$_iT_M^{\gamma,\beta} f(\tau) = \lim_{\epsilon\to 0} \frac{f(\tau +_i E_\beta(\epsilon\tau^{-\gamma})) - f(\tau)}{\epsilon}, \tag{6}$$

for $\tau > 0$, and $_iE_\beta\gamma \in (0, 1)$, $\beta > 0$ is a truncated Mittag–Leffler function of one parameter, as defined in (5). Note that, if f is γ-differentiable in some open interval $(0, a)$, $a > 0$, and $\lim_{\tau\to 0^+}(_iT_M^{\gamma,\beta} f(\tau))$. Then we attain

$$_iT_M^{\gamma,\beta} f(0) = \lim_{\tau\to 0^+}(_iT_M^{\gamma,\beta} f(\tau)). \tag{7}$$

Theorem 2.2. *Surmise that* f: $(0, \infty) \to \mathbb{R}$ *is* $\gamma-$ *differentiable for* $\tau_0 > 0$, *with* $\gamma \in (0, 1]$, $\beta > 0$, *then* f *is continuous at* τ_0.

Theorem 2.3. *Let* $0 < \gamma \leqslant 1$, $\beta > 0$, $a, b \in \mathbb{R}$, f, g, *γ-differentiable, at a point* $\tau > 0$. *Then*

- $_iT_M^{\gamma,\beta}(af + bg) = a_iT_M^{\gamma,\beta}(f) + b_iT_M^{\gamma,\beta}(f)$, $a, b \in \mathbb{R}$
- $_iT_M^{\gamma,\beta}(t^\mu) = \mu\tau^{\mu-\gamma}$, $\mu \in \mathbb{R}$
- $_iT_M^{\gamma,\beta}(fg) = f_i T_M^{\gamma,\beta}(g) + g_i T_M^{\gamma,\beta}(f)$,
- $_iT_M^{\gamma,\beta}\left(\frac{f}{g}\right) = \frac{g_i T_M^{\gamma,\beta}(f) - f_i T_M^{\gamma,\beta}(g)}{g^2}$,
- If f is differentiable, then $_iT_M^{\gamma,\beta}(f)(\tau) = \frac{\tau^{1-\gamma}}{\Gamma(\beta+1)}\frac{df}{d\tau}$,
- $_iT_M^{\gamma,\beta}(fog)(\tau) = f'(g(\tau))_iT_M^{\gamma,\beta} g(\tau)$, for f differentiable at g.

2.1. Governing equation

Assume that M^2 is a differentiable surface [38], parameterized by x, t. Let also, the following

$$\begin{aligned}\sigma_1 &= 2\sigma dx + (-4\gamma\sigma + 2V_x)dt,\\ \sigma_2 &= 2\gamma dx + (2(V^2 + \sigma^2) - 4\gamma^2)dt,\\ \sigma_3 &= -2V dx + (4\gamma V + 2\sigma_x)dt.\end{aligned} \tag{8}$$

Then M^2 is a PSS if and only if U satisfies the NLSE

$$i_0^E\mathcal{D}_{M,t}^{\gamma,\beta}U +_0^E\ \mathcal{D}_{M,x}^{2\gamma,\beta} + 2|U|^2U = 0, \tag{9}$$

where $\mathcal{D}_{M,.}^{2\gamma,\beta}$ denotes the m-truncated derivative. To obtain a travelling wave solutions for equation (9), we substitute $U(x, t) = V + i\sigma$ in equation (9) to obtain

$$\begin{aligned}{}_0^E\mathcal{D}_{M,t}^{\gamma,\beta}V +_0^E\ \mathcal{D}_{M,x}^{2\gamma,\beta}\sigma + 2(V^2 + \sigma^2)\sigma = 0,\\ -_0^E\mathcal{D}_{M,t}^{\gamma,\beta}\sigma +_0^E\ \mathcal{D}_{M,x}^{2\gamma,\beta}V + 2(V^2 + \sigma^2)V = 0,\end{aligned} \tag{10}$$

where $\mathcal{D}_{M,.}^{2\gamma,\beta}$ denotes the m-truncated derivative. Plugging the transformation $V(x, t) = u(\xi)$, $\sigma(x, t) = v(\xi)$, $\xi = \frac{\lambda\Gamma(\beta+1)}{\gamma}(x^\gamma - kt^\gamma + c)$, where λ, k, c are real constants and will be found later, into equation (10), we have

$$\begin{aligned}-k\lambda u'(\xi) + 2v(\xi)(u(\xi)^2 + v(\xi)^2) + \lambda^2 v''(\xi) = 0,\\ k\lambda v'(\xi) + \lambda^2 u''(\xi) + 2u(\xi)(u(\xi)^2 + v(\xi)^2) = 0.\end{aligned} \tag{11}$$

3. Application

According to Bernoulli sub-ODE method [39], equation (11) has possessed the solution as comes next

$$u = a_0 + a_1\Phi(\xi),\ v = b_0 + b_1\Phi(\xi), \tag{12}$$

where $\Phi(\xi)$ satisfies

$$\Phi' = \mu\Phi^2 - \lambda\Phi. \tag{13}$$

where μ is a nonzero constant. Plugging equation (13) along with equation (12) into equation (11), we get

$$\begin{aligned}&2(b_1\Phi(\xi)+b_0)((a_1\Phi(\xi)+a_0)^2+(b_1\Phi(\xi)+b_0)^2)\\&\quad+a_1k\lambda\Phi(\xi)(\lambda-\mu\Phi(\xi))+b_1\lambda^2\Phi(\xi)(\lambda-\mu\Phi(\xi))\\&\quad\times(\lambda-2\mu\Phi(\xi))=0,\\&2(a_1\Phi(\xi)+a_0)((a_1\Phi(\xi)+a_0)^2+(b_1\Phi(\xi)+b_0)^2)\\&\quad+a_1\lambda^2\Phi(\xi)(\lambda-\mu\Phi(\xi))(\lambda-2\mu\Phi(\xi))\\&\quad+b_1k\lambda\Phi(\xi)(\mu\Phi(\xi)-\lambda)=0.\end{aligned}\tag{14}$$

Collecting the terms in $\Phi^i(i=0,1,2,3,4,5,6,7)$, one reaches

$$\begin{aligned}&2b_0(a_0^2+b_0^2)=0,\\&b_1(2a_0^2+6b_0^2+\lambda^4)+a_1(4a_0b_0+k\lambda^2)=0,\\&2b_1(a_1^2+b_1^2+\lambda^2\mu^2)=0,\\&a_1(4a_0b_1-k\lambda\mu)\\&\quad+2a_1^2b_0+3b_1(2b_0b_1-\lambda^3\mu)=0,\\&2a_0(a_0^2+b_0^2)=0,\\&a_1(6a_0^2+2b_0^2+\lambda^4)+b_1(4a_0b_0-k\lambda^2)=0,\\&2a_1(a_1^2+b_1^2+\lambda^2\mu^2)=0,\\&a_1(4b_0b_1-3\lambda^3\mu)\\&\quad+b_1(2a_0b_1+k\lambda\mu)+6a_0a_1^2=0.\end{aligned}\tag{15}$$

Solving (15), we obtain $a_0=a_0,\quad k=k,\quad \lambda=\lambda,\quad \mu=\mu,\quad a_1=\frac{-4a_0^2\mu-k^2\mu}{4a_0\lambda}$, $b_0=\frac{a_0k}{\lambda^2}$ $b_1=\frac{\lambda^2(a_1k+2b_0\lambda\mu)}{k^2}$. Using the obtained results, we reach the following dark and singular optical solitons

$$\begin{aligned}V(x,t)&=a_0-\frac{4a_0^2+k^2}{8a_0}\\&\quad\times\left(\tanh\left(\frac{\lambda}{2}\left(\frac{\lambda\Gamma(\beta+1)}{\gamma}(x^\gamma-kt^\gamma+c)\right)\right)-1\right),\\\sigma(x,t)&=\frac{a_0k}{\lambda^2}+\frac{\lambda^3(a_1k+2b_0\lambda\mu)}{2\mu k^2}\\&\quad\times\left(\tanh\left(\frac{\lambda}{2}\left(\frac{\lambda\Gamma(\beta+1)}{\gamma}(x^\gamma-kt^\gamma+c)\right)\right)-1\right),\end{aligned}\tag{16}$$

or

$$\begin{aligned}V(x,t)&=a_0-\frac{4a_0^2+k^2}{8a_0}\\&\quad\times\left(\coth\left(\frac{\lambda}{2}\left(\frac{\lambda\Gamma(\beta+1)}{\gamma}(x^\gamma-kt^\gamma+c)\right)\right)-1\right),\\\sigma(x,t)&=\frac{a_0k}{\lambda^2}+\frac{\lambda^3(a_1k+2b_0\lambda\mu)}{2\mu k^2}\\&\quad\times\left(\coth\left(\frac{\lambda}{2}\left(\frac{\lambda\Gamma(\beta+1)}{\gamma}(x^\gamma-kt^\gamma+c)\right)\right)-1\right).\end{aligned}\tag{17}$$

4. Results and discussion

The novel GBM and multiplier scheme have been employed to establish soliton solutions and Cls for the equation under consideration, respectively. We computed Cls because of its importance in determining the integrability of differential equation and in providing partial information on the existence of a particular movement, even if the equations are too complicated to require a complete solution. The governing equation has possessed conserved vectors and internal properties which are presented in the section 5. These results has affirmed its integrability. We established soliton solutions because such solutions tend to play a significant role in explaining various phenomena and processes throughout the natural sciences. This is due to their capability to check the accuracy or estimate errors in statistical, asymptotic and estimated analytical solutions. To this aim, we get motivated and computed these results which will be of great benefit to the literature.

The physical and outlook views of the solutions obtained by GBM are portrayed by means of graphical representations. The types of solutions attained are the dark optical soliton reported in (16), the singular optical soliton reported in (17).

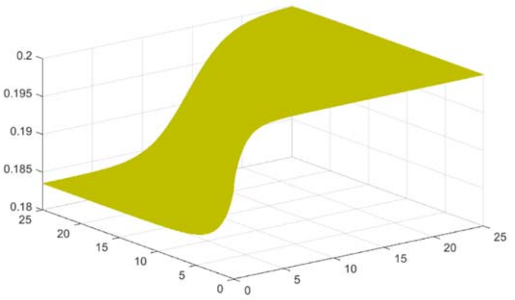
Dark optical soliton describes the solitary waves with less strength than the context, the different soliton solutions represent a solitary wave with discontinuous derivatives; an example of such solitary waves are compactions with finite (compact) support, and peaks with discontinuous first derivative peaks. Due to their efficiency and, of course, versatility in long distance optical communication, these types of solitary waves are extremely important.

It should be remembered that optical fibers are thin long strings of ultra-pure glass or plastic so that light can be transmitted from one end to the next without any attenuation or loss. In order to have a clear vision on the effect of parameters to the transmission of solitons, suitable values of parameters have been considered in the following graphs: In figure 1, we choose $a_0=0.2$, $\lambda=1.2$, $k=0.7$, $c=0.1$, $a_1=0.5$, $b_0=0.9$, $\mu=0.55$. And in figure 2, we choose $a_0=0.2$, $\lambda=1.2$, $k=0.9$, $c=0.1$, $a_1=0.55$, $b_0=0.19$, $\mu=0.5$.

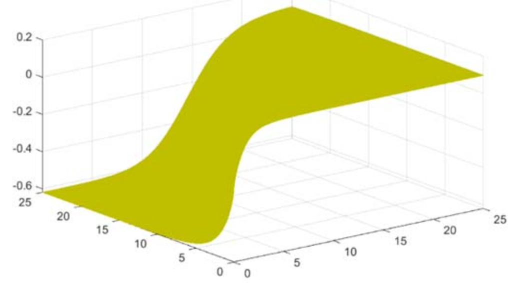
5. Conservation laws via multiplier approach

Following the multipliers approach as depicted in [40], we construct the first-order multipliers $\Lambda^1(x,t,\sigma,V,\sigma_x,V_x,\sigma_t,V_t,)$, $\Lambda^2(x,t,\sigma,V,\sigma_x,V_x,\sigma_t,V_t)$ for the equation under consideration and they are given by

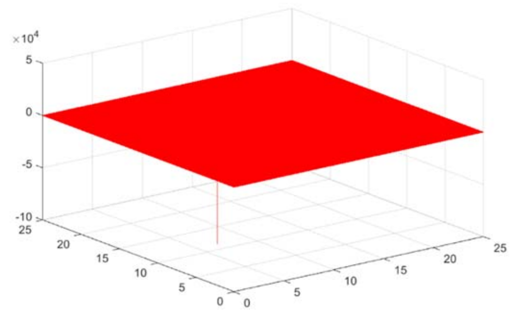
$$\begin{aligned}\Lambda^1&=\frac{1}{2}(c_1x+2c_2)V-\frac{1}{2}(2c_1t+2c_3)\sigma_x-c_4\sigma_t,\\\Lambda^2&=\frac{1}{2}(c_1x+2c_2)\sigma+\frac{1}{2}(2c_1t+2c_3)V_x-c_4V_t,\end{aligned}\tag{18}$$



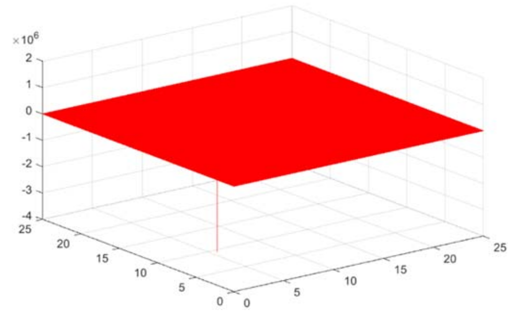
(a) V(x,t) with $\gamma = 0.5$, $\beta = 0.9$ for (16).



(b) $\sigma(x, t)$ with $\gamma = 0.5$ for (16).

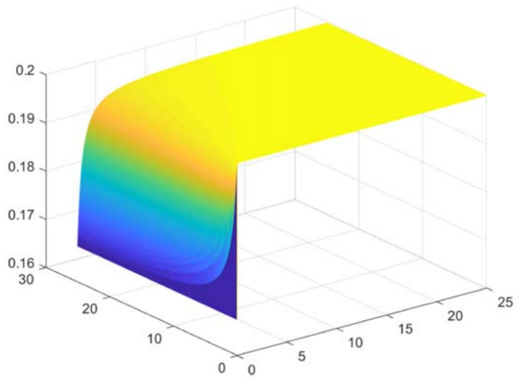


(c) V(x,t) with $\gamma = 0.5$, $\beta = 0.9$ for (17).

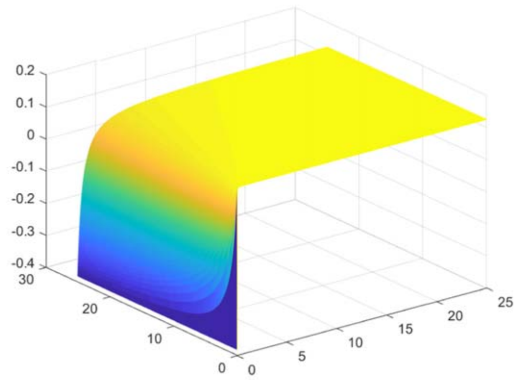


(d) $\sigma(x, t)$ with $\gamma = 0.5$, $\beta = 0.9$ for (17).

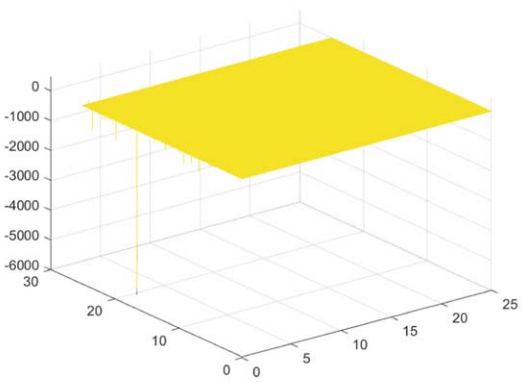
Figure 1. Physical features with suitable values of the parameters.



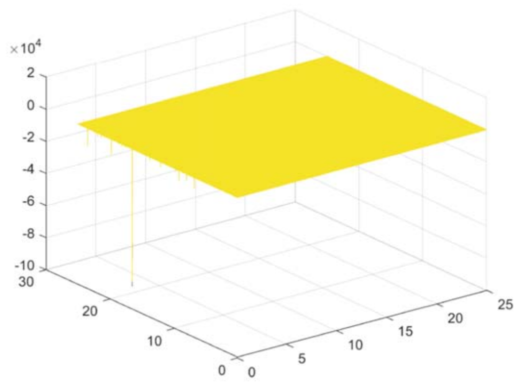
(a) V(x,t) with $\gamma = 0.55$, $\beta = 0.199$ for (16).



(b) $\sigma(x, t)$ with $\gamma = 0.5$ for (16).



(c) V(x,t) with $\gamma = 0.55$, $\beta = 0.99$ for (17).



(d) $\sigma(x, t)$ with $\gamma = 0.55$, $\beta = 0.99$ for (17).

Figure 2. Physical features with suitable values of the parameters.

where c_1, c_2 are some arbitrary constants. Subsequently, one attains the following fluxes:

$$T^t = -\frac{1}{2}V^4c_4 - V^2\sigma^2c_4 + \frac{1}{4}V^2c_1x + \frac{1}{2}V^2c_2\sigma_x\sigma^3c_1t^2 + \frac{1}{2}c_4\sigma_x^2 - 2\sigma^3\sigma_xc_3t - V\sigma_xc_1t - V\sigma_xc_3 + \frac{1}{2}c_4V_x^2,$$
$$T^x = \frac{1}{2}Vc_1\sigma + Vc_3\sigma_t + Vc_3\sigma_t - \frac{1}{2}c_3V^4 + 2\sigma^3\sigma_tc_3t - \frac{1}{2}V_xc_1\sigma_x + Vc_1\sigma_{tt} - \sigma_xc_4\sigma_t + \sigma_xVc_2 - \frac{1}{2}\sigma_x^2c_1t + \frac{1}{2}\sigma_xVc_1x - \frac{1}{2}\sigma_x^2c_3 - t\sigma^2c_1V^2 + \sigma_t\sigma^3c_1t^2 - \frac{1}{2}c_1tV^4 - \sigma^2c_3V^2 - \frac{1}{2}V_x^2c_3 - V_xc_4V_t - V_xc_2\sigma - \frac{1}{2}V_x^2c_1t. \tag{19}$$

By situations disconnected without a constants, one reaches the following fluxes:

Case 1.

$$\Lambda^1 = \frac{1}{2}V_x - \sigma_{xt},\ \Lambda^2 = V_{xt} + \frac{1}{2}\sigma_x. \tag{20}$$

Subsequently, we obtain the following fluxes

$$\mathbf{T^t} = \frac{1}{4}V^2x - \sigma_x\sigma^3t^2 - V\sigma_{xt},$$
$$\mathbf{T^x} = \frac{1}{2}V\sigma - \frac{1}{2}V_x\sigma_x + V\sigma_{tt} - \frac{1}{2}\sigma_x^2t + \frac{1}{2}V\sigma_{xx} - tV^2\sigma^2 + \sigma_t\sigma^3t^2 - \frac{1}{2}tV^4 - \frac{1}{2}V_x^2t. \tag{21}$$

Case 2.

$$\Lambda^1 = \sigma,\ \Lambda^2 = V. \tag{22}$$

Subsequently, we obtain the following fluxes

$$\mathbf{T^t} = \frac{1}{2}V^2,$$
$$\mathbf{T^x} = V\sigma_x - \sigma V_x. \tag{23}$$

Case 3.

$$\Lambda^1 = -\sigma_x,\ \Lambda^2 = V_x. \tag{24}$$

Subsequently, we obtain the following fluxes

$$\mathbf{T^t} = -2\sigma^3\sigma_{xt} - V\sigma_x,$$
$$\mathbf{T^x} = V\sigma_t - \frac{1}{2}V^4 + 2\sigma^3\sigma_{tt} - \frac{1}{2}\sigma_x^2 - \sigma^2V^2 - \frac{1}{2}V_x^2. \tag{25}$$

Case 4.

$$\Lambda^1 = -\sigma_t,\ \Lambda^2 = V_t. \tag{26}$$

Subsequently, we obtain the following fluxes

$$\mathbf{T^t} = -\frac{1}{2}V^4 - \sigma^2V^2 + \frac{1}{2}\sigma_x^2 + \frac{1}{2}V_x^2,$$
$$\mathbf{T^x} = -V_tV_x - \sigma_t\sigma_x. \tag{27}$$

6. Conclusion

In this work we have studied the optical solitons with M-truncated derivative and Cls for the NLSE which describe Pseudospherical Surfaces. The novel GBM have been utilized to extract such novel solutions. For the successful existence of the solutions, the constraints conditions have been reported. The discussion for the physical features of the obtained solutions have been presented. Moreover, the construction for the conservation laws have been done by means of multiplier approach.

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