IOP Publishing Physica Scripta
Phys. Scr. 95 (2020) 035109 (10pp) https://doi.org/10.1088/1402-4896/ab4411

Digital adiabatic passage in multi-state systems

N Irani[1], **M Saadati-Niari**[2] **and M Amniat-Talab**[1]

[1] Physics Department, Faculty of Sciences, Urmia University, P.B. 165, Urmia, Iran
[2] Department of Physics, Faculty of Sciences, University of Mohaghegh Ardabili, PO Box 179, Ardabil, Iran

E-mail: m.saadati@uma.ac.ir

Received 17 April 2019, revised 17 August 2019
Accepted for publication 12 September 2019
Published 6 February 2020

Abstract

Digital adiabatic passage in multi-pod and multi-lambda systems is investigated. We show that using digital variation of control fields, one can create a coherent superposition of ground states in multi-state systems. In order to design the propagator at every step, the multi-state systems are simplified to a three-state system using Morris-Shore transformation. In this method, the number of steps is arbitrary and by increasing the number of steps the population of excited states decreases to negligible small values. Sensitivity of the final fidelity with respect to exact value of $t_{\max}$ decreases by increasing the number of steps.

Keywords: STIRAP, digital adiabatic passage, multi-pod, multi-lambda

(Some figures may appear in colour only in the online journal)

1. Introduction

In recent years, creation of coherent superposition of states due to its application in quantum processes such as quantum information processing [1] and nonlinear optics [2], has been considered extensively. Multi-pod and multi-Lambda linkages are two important quantum systems in which coherent superposition of states can be generated. In a multi-pod linkage, an arbitrary number of ground states are coupled to an excited state and the multi-lambda linkage pattern is composed of an initial ground state, an arbitrary number of excited states and the target ground states such that the number of excited states is equal to the number of target states. In resent years, STIRAP (stimulated Raman adiabatic passage) [3–9] and π-pulse [10] techniques have been developed and are used to create a coherent superposition of ground states in multi-pod and multi-lambda systems [11–15].

Shapiro *et al.* [16] proposed a technique, named piecewise adiabatic passage (PAP), Which can be used to transfer the population in three-state Λ-like systems by a train of femtosecond pulses. Rangelov and Vitanov [17] proposed another technique for producing complete population transfer in three-state Λ-like systems by a train of coincident pulse pairs, in which for large number of pulse pairs the maximum population in the middle state is reduced to a negligible small value. Recently, the proposed technique in [17] has been extended to multi-state systems [18–20], nuclear state population transfer [21, 22] and hyperbolic-tangent pulses [23]. Vatikus and Greentree [24] proposed another scheme of adiabatic passage, with a stepwise change of the fields, named digital adiabatic passage (DAP). In this scheme the tunnel matrix elements (Rabi frequencies) are forced to vary in discrete steps, rather than smoothly. Recently, it has been shown that DAP technique can be used in optical waveguides in the case of waveguide separation that is varied digitally [25, 26].

Here we show that DAP technique can be implemented in multi-pod and multi-lambda systems to generate an arbitrary coherent superposition of ground states. For this purpose, by using Morris-Shore (MS) transformation [27–29] we first reduce the multi-pod and multi-lambda systems to a three-state Λ-like system . Then we introduce fractional digital adiabatic passage (F-DAP) in MS basis and extend it to original basis. The tunnel matrix elements are piecewise constant with time and are designed to create an arbitrary coherent superposition of all ground states from an initial state. In this method, the Rabi frequencies are changed abruptly in each step and by increasing the number of steps,

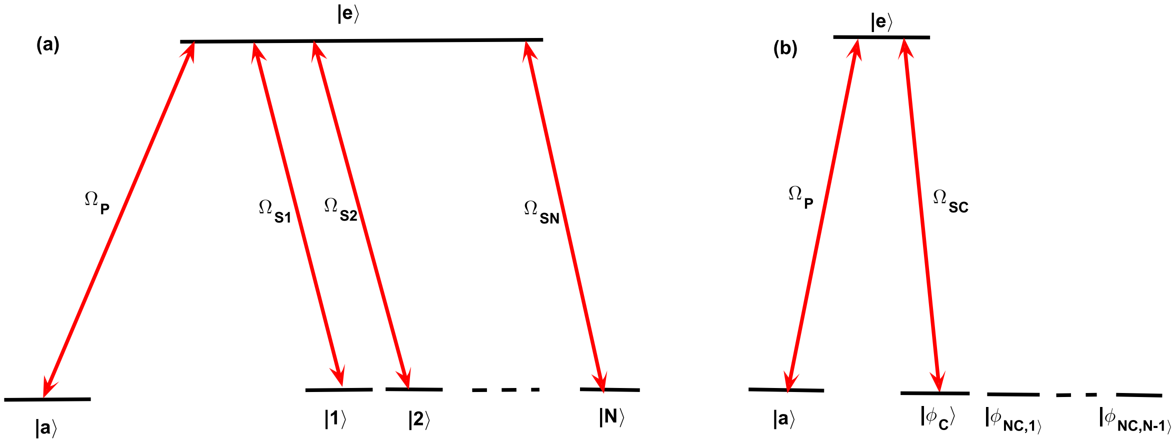

Figure 1. Multi-state system with multi-pod linkage pattern (a) in original basis, (b) in MS basis.

maximum population of excited states reduces to a negligible small values.

This paper is organized as follows. In section 2 we introduce the Hamiltonian of a multi-pod system. Using a suitable MS transformation we reduce the Hamiltonian in original basis to a three state Hamiltonian in MS basis. By designing stepwise sinusoidal Rabi frequencies we generate a coherent superposition of desired states. In section 3, we show how DAP technique can be applied in muti-lambda systems. Finally, in section 4 we provide a summary of the results.

2. Superposition of states in *N*-pod systems using digital pulses

2.1. Reduction to Λ-system using MS transformation

We consider a system consists of $N+1$ ground states and an excited state $|e\rangle$ [see figure 1(a)]. The initial state $|a\rangle$ is coupled to the excited state with a pump pulse $\Omega_P(t)$. Other ground states $|1\rangle$, $|2\rangle,\cdots,|N\rangle$ are also coupled to the excited state with Stokes pulses $\Omega_{S1}(t)$, $\Omega_{S2}(t),\cdots,\Omega_{\rm SN}(t)$. We impose that the Stokes pulses have the same shape. The initial Hamiltonian in original basis $\{|a\rangle, |e\rangle, |1\rangle, |2\rangle,\cdots,|N\rangle\}$ can be written as ($\hbar=1$):

$$\hat{H}(t)=\begin{pmatrix}0 & \Omega_P(t) & 0 & 0 & \cdots & 0\\ \Omega_P^*(t) & \Delta & \Omega_{S1}^*(t) & \Omega_{S2}^*(t) & \cdots & \Omega_{\rm SN}^*(t)\\ {\rm O} & \Omega_{S1}(t) & 0 & 0 & \cdots & 0\\ 0 & \Omega_{S2}(t) & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & \Omega_{\rm SN}(t) & 0 & 0 & \cdots & 0\end{pmatrix}, \quad (1)$$

where the single-photon detuning measures the frequency offset from resonance. The Rabi frequencies of the couplings between the ground states and the excited state respectively denoted as

$$\Omega_P(t)=|\Omega_P(t)|{\rm e}^{{\rm i}\phi_P}, \quad (2a)$$

$$\Omega_{Sj}(t)=|\Omega_{Sj}(t)|{\rm e}^{{\rm i}\phi_{Sj}},\ j=1,\cdots,N. \quad (2b)$$

We assume that all of the laser pulses are in exact resonance with their transitions ($\Delta=0$) and the phases have constant values. Following MS transformation technique [27–29], the initial Hamiltonian (1) can be transformed into an effective Λ-system reading in the basis, $\{|a\rangle, |e\rangle, |\phi_C\rangle\}$ [see figure 1(b)] as has been shown in [13, 14, 19, 30]. The Hamiltonian in the new basis reads:

$$\hat{H}_{\rm MS}(t)=\begin{pmatrix}0 & \Omega_P(t) & 0\\ \Omega_P^*(t) & 0 & \Omega_{\rm SC}^*(t)\\ 0 & \Omega_{\rm SC}(t) & 0\end{pmatrix}, \quad (3)$$

where

$$|\phi_C\rangle=\chi_1|1\rangle+\chi_2|2\rangle+\cdots+\chi_N|N\rangle, \quad (4)$$

and $\Omega_{\rm SC}(t)=\Omega_{S1}(t)/\chi_1$. In the above equation χ_i, ($i=1, 2,\cdots, N$) is defined as follows:

$$\chi_1=x_1x_2\cdots x_{N-1}, \quad (5a)$$

$$\chi_2={\rm e}^{{\rm i}(\phi_{S2}-\phi_{S1})}x_1'x_2\cdots x_{N-1}, \quad (5b)$$

$$\vdots$$

$$\chi_N={\rm e}^{{\rm i}(\phi_{\rm SN}-\phi_{S1})}x_{N-1}'. \quad (5c)$$

Here $x_j'=\sin\theta_j$ and $x_j=\cos\theta_j$, ($j=1, 2, \ldots, N-1$), where $0\leqslant\theta_j<\pi/2$ is an angle parametrizing the Stokes amplitudes as follows:

$$\frac{|\Omega_{S2}(t)|}{|\Omega_{S1}(t)|}=\tan\theta_1, \quad (6a)$$

$$\frac{|\Omega_{S3}|}{\sqrt{\sum_{i=1}^{2}|\Omega_{Si}(t)|^2}}=\tan\theta_2, \quad (6b)$$

$$\vdots$$

$$\frac{|\Omega_{\rm SN}(t)|}{\sqrt{\sum_{i=1}^{N-1}|\Omega_{Si}(t)|^2}}=\tan\theta_{N-1}. \quad (6c)$$

2.2. Digital fractional adiabatic passage in MS basis

In the following we impose that $\phi_P = \phi_{S1} = 0$. The eigenvectors of the reduced Hamiltonian (3) in MS basis are

$$|D_\pm\rangle = \frac{\Omega_P|a\rangle \pm \sqrt{\Omega_P^2 + \Omega_{SC}^2}|e\rangle + \Omega_{SC}|\phi_C\rangle}{\sqrt{2(\Omega_P^2 + \Omega_{SC}^2)}}, \tag{7a}$$

$$|D_0\rangle = \frac{\Omega_{SC}|a\rangle - \Omega_p|\phi_C\rangle}{\sqrt{\Omega_P^2 + \Omega_{SC}^2}}, \tag{7b}$$

with corresponding eigenenergies

$$E_\pm = \pm\sqrt{\Omega_P^2 + \Omega_{SC}^2}, \quad E_0 = 0. \tag{8}$$

Here $|D_0\rangle$ is referred to the dark state in the context of STIRAP. In F-STIRAP our aim is to transfer the population from the initial state $|a\rangle$ to the final state $|\phi_C\rangle$ such that we can create a coherent superposition of these states as:

$$|\Psi_T(t_f)\rangle = \cos\varphi|a\rangle - \sin\varphi|\phi_C\rangle. \tag{9}$$

where φ is an angle parmetrizing the Ω_P and Ω_{SC} pulses in MS basis. Using smoothly varying control function for $\Omega_P(t)$ and $\Omega_{SC}(t)$, with the constraints that $\Omega_{SC}(t=0) \gg \Omega_P(t=0)$ and $\frac{\Omega_{SC}(t=t_{max})}{\Omega_P(t=t_{max})} \neq 0$, the population is transferred fractionally from $|a\rangle$ to $|\phi_C\rangle$. Our strategy is to digital control of the time evolution of the system in order to fractional population transfer between the states $|a\rangle$ and $|\phi_C\rangle$. For digital control we use sinusoidal functions in S steps as the pulses in the kth step are

$$\Omega_P^{(k)} = \Omega_M \sin\left[\frac{\xi\varphi}{S-1}\right], \quad \Omega_{SC}^{(k)} = \Omega_M \cos\left[\frac{\xi\varphi}{S-1}\right], \tag{10}$$

where

$$\xi = \left\lfloor \frac{St}{t_{max}} \right\rfloor, \quad t \in [t_{max}^{(k-1)}, t_{max}^{(k)}). \tag{11}$$

In order to calculate the unitary evolution operator for kth step in MS basis we define the second MS basis so called bright and dark states [31] as follows:

$$|v_1\rangle = \sin\vartheta_k|a\rangle + \cos\vartheta_k|\phi_C\rangle, \tag{15a}$$

$$|v_2\rangle = |e\rangle, \tag{15b}$$

$$|v_3\rangle = \cos\vartheta_k|a\rangle - \sin\vartheta_k|\phi_C\rangle, \tag{15c}$$

where

$$\tan\vartheta_k = \frac{\Omega_P^{(k)}}{\Omega_{SC}^{(k)}}. \tag{16}$$

Due to piecewise constant nature of the digital Hamiltonian, the mixing angle ϑ_k is constant. The MS Hamiltonian in the dark-bright basis is $H_{db} = T^\dagger H_{MS} T$, where

$$\hat{T} = \begin{pmatrix} \sin\vartheta_k & 0 & \cos\vartheta_k \\ 0 & 1 & 0 \\ \cos\vartheta_k & 0 & -\sin\vartheta_k \end{pmatrix}, \tag{17}$$

and

$$\hat{H}_{db} = \begin{pmatrix} 0 & \Omega_M & 0 \\ \Omega_M & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{18}$$

The dark-bright propagator at the end of kth step can be calculated as follows:

$$\hat{U}_{db}(\tau_k) = \exp(-iH\tau_k) = \begin{pmatrix} \cos(\Omega_M\tau_k) & -i\sin(\Omega_M\tau_k) & 0 \\ -i\sin(\Omega_M\tau_k) & \cos(\Omega_M\tau_k) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{19}$$

where $\tau_k = t_{max}^{(k)}$. The transition propagator in the kth step of MS basis $\hat{U}_{MS}(\tau_k) = T\hat{U}_{db}T^\dagger$ is (20)

$$\hat{U}_{MS}(\tau_k) = \begin{pmatrix} \cos^2\vartheta_k + \sin^2\vartheta_k\cos(\Omega_M\tau_k) & -i\sin\vartheta_k\sin(\Omega_M\tau_k) & \sin\vartheta_k\cos\vartheta_k[\cos(\Omega_M\tau_k) - 1] \\ -i\sin\vartheta_k\sin(\Omega_M\tau_k) & \cos(\Omega_M\tau_k) & -i\cos\vartheta_k\sin(\Omega_M\tau_k) \\ \sin\vartheta_k\cos\vartheta_k[\cos(\Omega_M\tau_k) - 1] & -i\cos\vartheta_k\sin(\Omega_M\tau_k) & \sin^2\vartheta_k + \cos^2\vartheta_k\cos(\Omega_M\tau_k) \end{pmatrix}. \tag{20}$$

Using digital pulses, the difference between kth and $(k+1)$th dark state is

$$\eta(k) = 1 - |\langle D_0(k)|D_0(k+1)\rangle|^2 = \sin^2\left[\frac{\varphi}{(S-1)}\right]. \tag{12}$$

In the limit of $\eta \ll 1$, the total error in the F-DAP is the sum over all the individual errors,

$$\eta_T = \sum_{k=1}^{S}\eta(k) = S\sin^2\left[\frac{\varphi}{(S-1)}\right]. \tag{13}$$

which becomes in the limit of large S as

$$\eta_T = \frac{\varphi^2}{S}. \tag{14}$$

The total propagator during the F-DAP evolution is

$$\hat{U}_{MS}(t_f) = \hat{U}_{MS}(\tau_S)\hat{U}_{MS} \times (\tau_{S-1}) \cdots \hat{U}_{MS}(\tau_k) \cdots \hat{U}_{MS}(\tau_2)\hat{U}_{MS}(\tau_1). \tag{21}$$

In this method we take $t_{max} = t_{max}^{(S)} = S\pi/\Omega_M$ and $t_{max}^{(k)} = kt_{max}/S$. Figure 2 shows the evolution of Rabi frequencies and population, for $S = 5,15$ which leads to find a state with equal superposition of $|a\rangle$ and $|\phi_C\rangle$. The results show that by increasing the number of steps, S, the maximum population of $|e\rangle$ is negligible which resembles STIRAP in three-state systems.

It is seen from equation (20) that when $t_{max} = 2nS\pi/\Omega_M$, with $n \in \mathbb{N}$ in each case, then $\tau_k = 2nk\pi/\Omega_M$ and $\hat{U}(\tau_k)$ will be an identity matrix. Hence the overall evolution must also be identity. Figure 3 shows the population of state $|\phi_C\rangle$ at the end

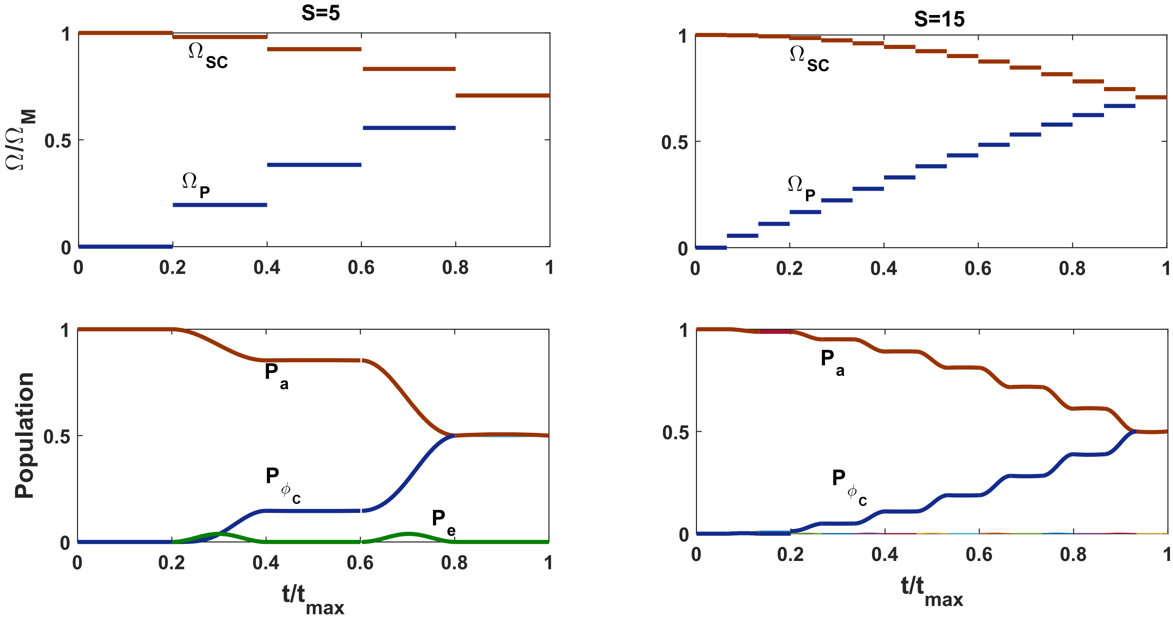

Figure 2. Fractional digital adiabatic passage evolution of Rabi frequencies and population as a function of fractional time $t/t_{\max}$ in MS basis for $S = 5$, 15 with $\varphi = \pi/4$ and $t_{\max} = S\pi/\Omega_M$. Half population transfer is achieved in both cases in stepwise manner. However, the population $P_e(t\ )$ of the intermediate state is different depending on the number of steps. The Rabi frequencies are designed using (10).

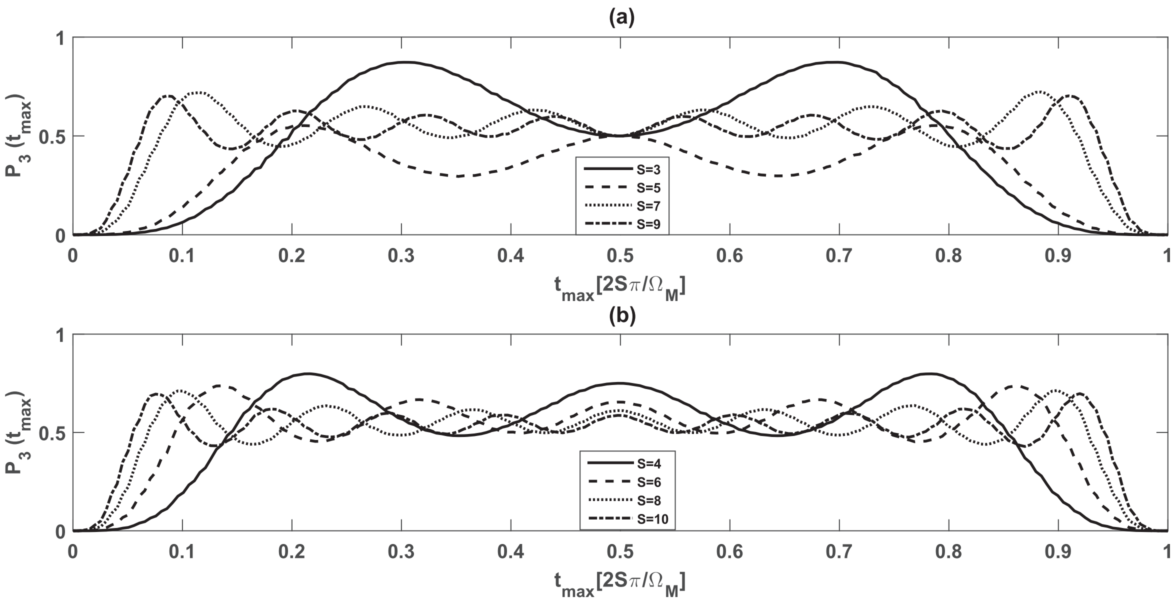

Figure 3. Final population of $|\phi_C\rangle$ after Half-DAP in MS basis using sin and cos protocol as a function of $t_{\max}$ for the first four values of S ((a) odd valus ($S = 3$, 5, 7, and 9) and (b) even values ($S = 4$, 6, 8, and 10). When $\cos(\tau_k\Omega_M) = 1$, then the propagator becomes identity, which is seen by two nulls in the population.

of the Half-DAP protocol as a function of $t_{\max}$ for different values of S. The time axis is considered between two resonance $t_{\max} = 0$ and $t_{\max} = 2S\pi/\Omega_M$ corresponding to $U_{\rm MS}(\tau_k) = I$. It is observed that sensitivity of the system with respect to the exact value of $t_{\max}$ decreases with increasing the number of steps which means the robustness of the population transfer.

2.3. Digital pulses in original basis

The purpose of this section is to create an arbitrary coherent superposition of all ground states in a $(N + 1)$-pod system by F-DAP technique. Using (4) and (9), F-DAP in original basis leads to

$$|\Psi(t_f)\rangle = \cos\varphi\ |a\rangle - \sin\varphi\ (\chi_1|1\rangle + \chi_2|2\rangle + \ldots + \chi_N|N\rangle). \tag{22}$$

For creation of a desired superposition of all ground states

$$|\Psi(t_f)\rangle = c_a|a\rangle + \sum_{j=1}^{N} c_j|j\rangle,\quad |c_a|^2 + \sum_{j=1}^{N}|c_j|^2 = 1, \tag{23}$$

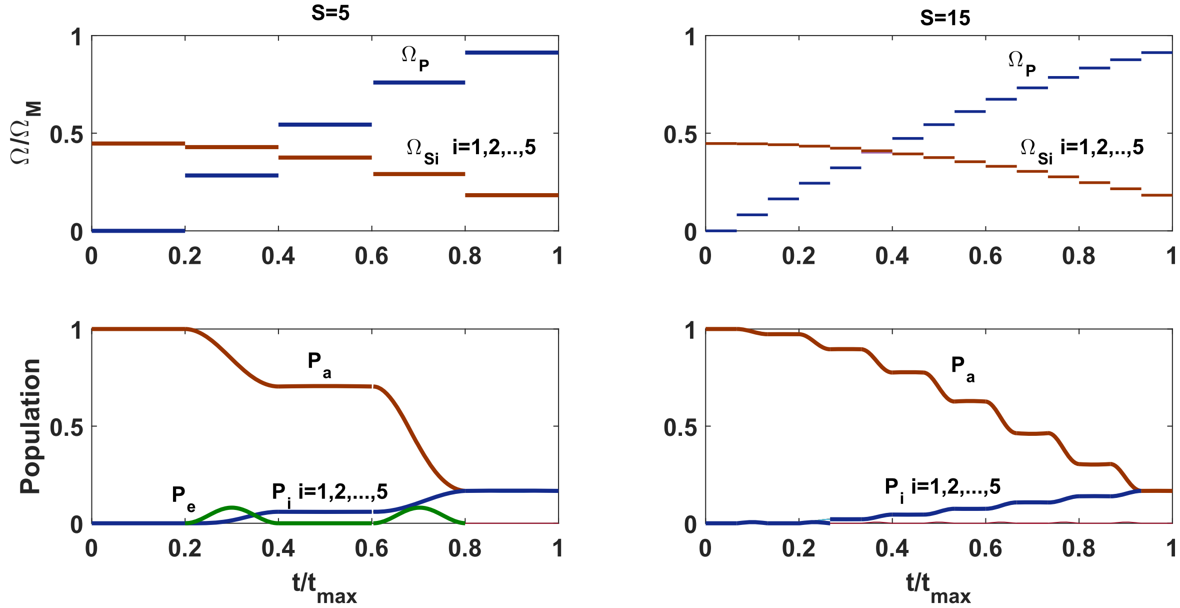

Figure 4. Time evolution of Rabi frequencies and population in a $(N+1)$-pod system with $N=5$ using $S=5{,}15$ steps that leads to a superposition of ground states with equal amplitudes at the end of the evolution. The Rabi frequencies are designed using (24).

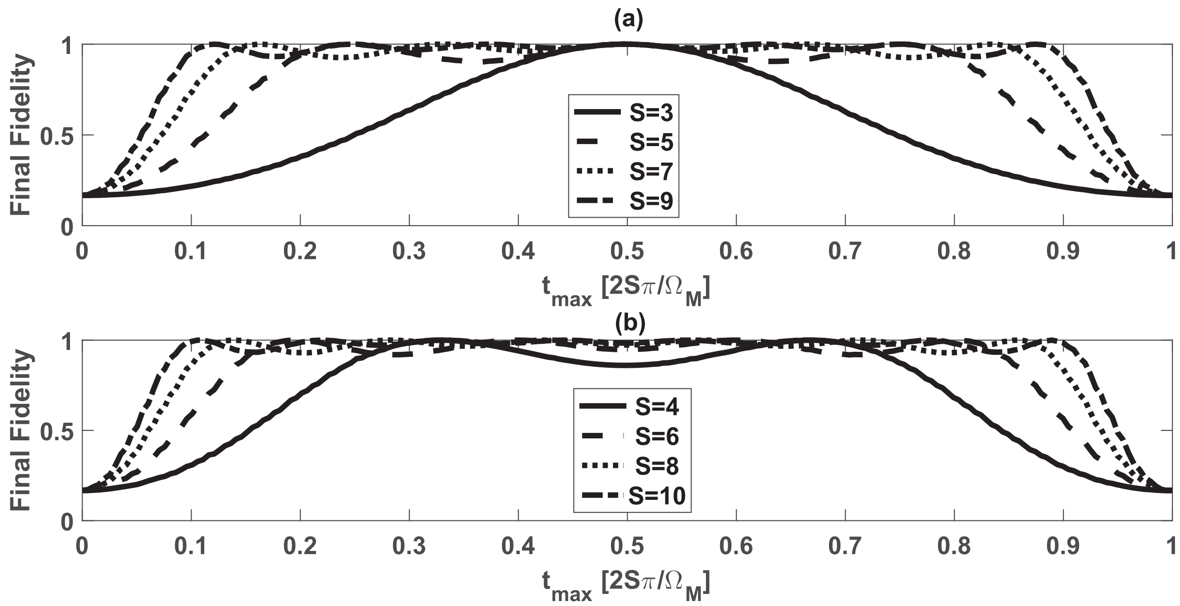

Figure 5. Final fidelity of the system with respect to the desired state $|\Psi_{\text{desired}}\rangle = \frac{1}{\sqrt{6}}(|a\rangle - |1\rangle - |2\rangle - |3\rangle - |4\rangle - |5\rangle)$ in a $(N+1)$-pod system with $N=5$ using the sinusoidal protocol as a function of $t_{\max}$ for the first four values S ((a) odd values ($S=3$, 5, 7, and 9) and (b) even values ($S=4$, 6, 8, and 10)). When $\cos(\tau_k\Omega_M)=1$, the propagator becomes identity, as it is seen the population starts from 0.168 and ends at the same value.

at the end of evolution, we use sinusoidal functions for Rabi frequencies in S steps such that the pulses in the kth step are

$$\Omega_P^{(k)} = \Omega_M \sin\left[\frac{\xi\beta}{S-1}\right], \tag{24a}$$

$$\Omega_{S1}^{(k)} = \zeta_1\Omega_M \cos\left[\frac{\xi\beta}{S-1}\right], \tag{24b}$$

$$\Omega_{S2}^{(k)} = \zeta_2\Omega_M \cos\left[\frac{\xi\beta}{S-1}\right], \quad \vdots \tag{24c}$$

$$\Omega_{SN}^{(k)} = \zeta_N\Omega_M \cos\left[\frac{\xi\beta}{S-1}\right], \tag{24d}$$

where

$$\beta = \arccos(c_a), \tag{25a}$$

$$\zeta_i = \frac{c_i}{\sqrt{1-|c_a|^2}}. \tag{25b}$$

By adjusting β and ζ_i appropriately, we can create a coherent superposition of states with desired amplitudes. Figure 4 shows an example for a $(N+1)$-pod system, $N=5$, using

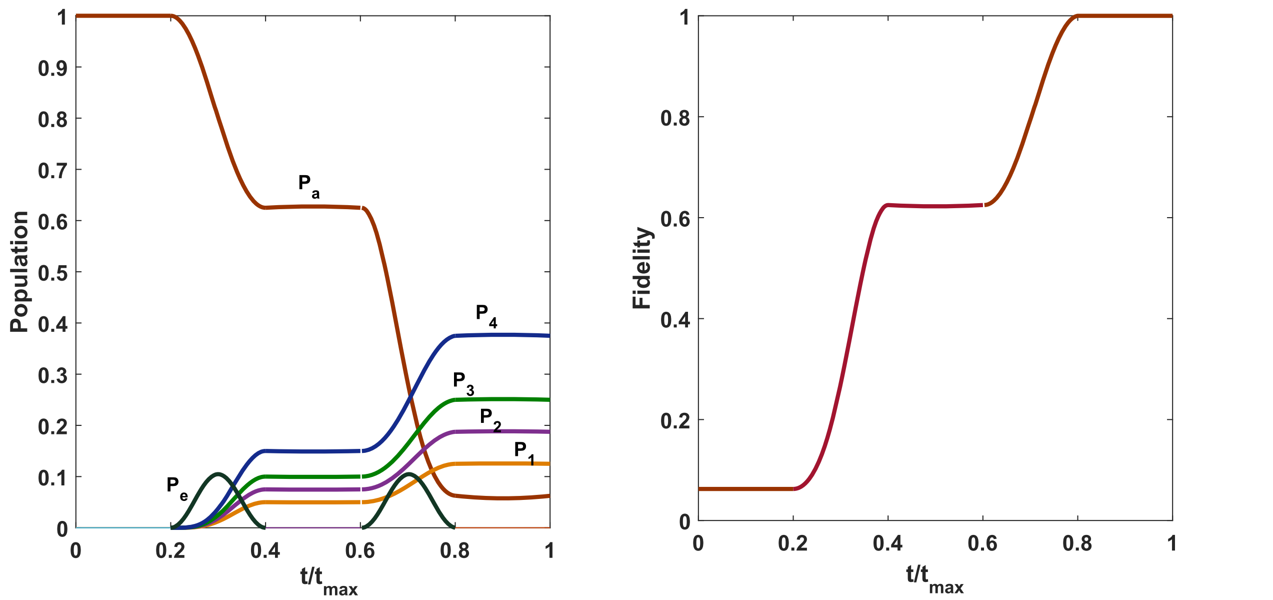

Figure 6. Time evolution of populations (left frame) and fidelity (right frame) in a $N+1$-pod system with $N=4$ using $S=5$ steps that lead to (26). The Rabi frequencies are designed using (24).

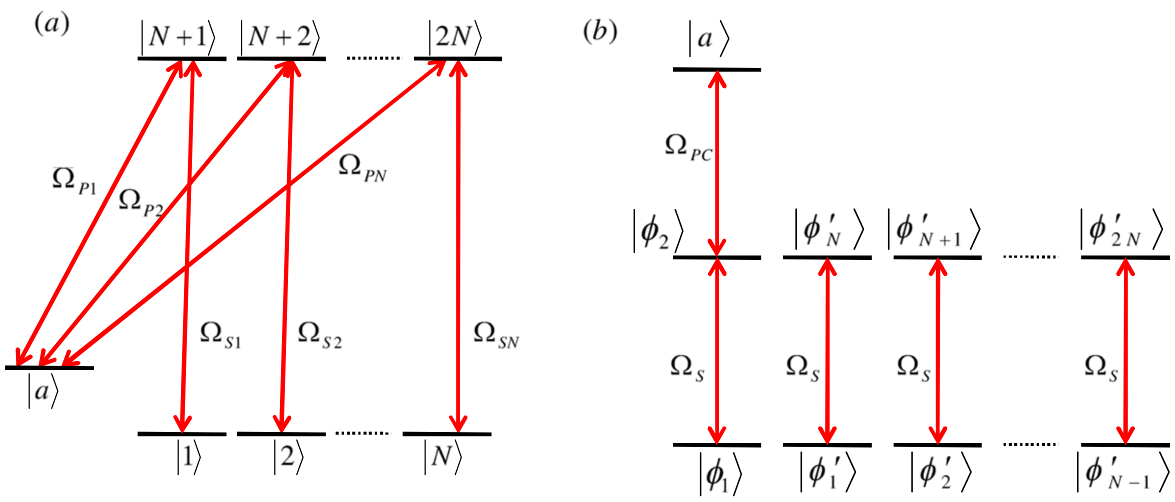

Figure 7. Multi-state system with multi-lambda linkage pattern (a)- in original basis, (b)- in MS basis.

$S=5$, 15 steps. In both cases, the population is transferred from the initial state $|a\rangle$ to a superposition of all ground states with equal amplitudes at the end of the dynamics. The transient population of the state $|e\rangle$ is damped to a negligible value for $S=15$. In figure 5 we have plotted the fidelity of final state, $F=|\langle\Psi_{\text{desired}}|\Psi(t_f)\rangle|^2$ with respect to the desired state $|\Psi_{\text{desired}}\rangle=\frac{1}{\sqrt{6}}(|a\rangle-|1\rangle-|2\rangle-|3\rangle-|4\rangle-|5\rangle)$, as a function of $t_{\max}$ for different values of S in a $(N+1)$-pod system with $N=5$. We observe that for different values of $t_{\max}$, as the number of steps increases, the final fidelity remains close to 1. Figure 6 shows time evolution of populations and fidelity in $(N+1)$-pod system with $N=4$ using $S=5$ steps that leads to a superposition of ground states with unequal amplitudes and different phases as follows:

$$|\Psi(t_f)\rangle=\sqrt{\frac{1}{16}}|a\rangle-\sqrt{\frac{2}{16}}|1\rangle-\sqrt{\frac{3}{16}}e^{i\pi/3}|2\rangle-\sqrt{\frac{4}{16}}e^{i\pi/6}|3\rangle-\sqrt{\frac{6}{16}}e^{i\pi/2}|4\rangle. \tag{26}$$

3. Superposition of states in multi-lambda systems using digital pulses

3.1. Reduction to Λ-system using MS transformation

In this section we consider a multi-lambda system [11, 18] as shown in figure 7(a). The resonant real pump pulses couple the

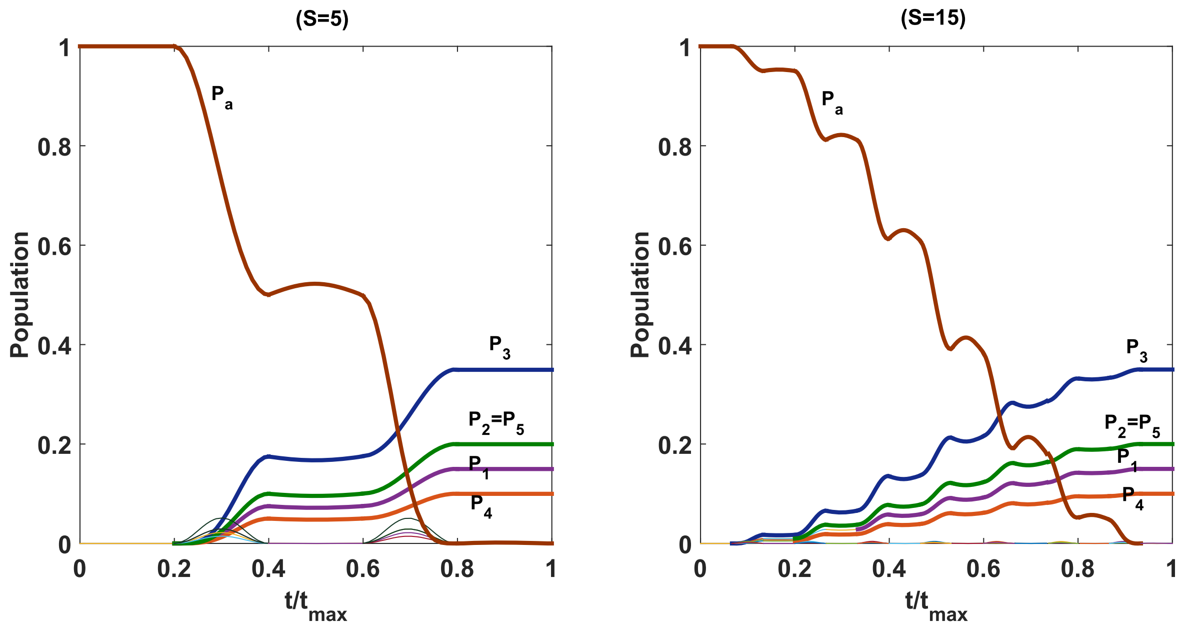

Figure 8. Time evolution of populations in a five-lambda system using $S = 5$, 15 steps of digital pulses that leads to (34). The Rabi frequencies are designed using (32) and (33).

initial state $|a\rangle$ to a set of excited states $|m\rangle (m = N + 1, N + 2, \ldots, 2N)$ via Rabi frequencies $\Omega_{Pn}(t) (n = 1, 2, \ldots, N)$. The resonant real Stokes pulses $\Omega_{Sn}(t)$ also couple the excited states $|m\rangle = |N + n\rangle (n = 1, 2 \ \ldots, N)$ to the ground states $|n\rangle$. The effective Hamiltonian of such a system in the space $\{|1\rangle, |2\rangle, \cdots, |2N\rangle, |a\rangle\}$ is given by:

$$\hat{H}(t) = \begin{pmatrix} 0 & 0 & \cdots & 0 & \Omega_{S1}(t) & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & \Omega_{S2}(t) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \vdots & \vdots & \vdots & \Omega_{\mathrm{SN}}(t) & 0 \\ \Omega_{S1}(t) & 0 & \cdots & 0 & \vdots & \vdots & \vdots & 0 & \Omega_{\mathrm{P1}}(t) \\ 0 & \Omega_{S2}(t) & \cdots & 0 & \vdots & \vdots & \vdots & \vdots & \Omega_{\mathrm{P2}}(t) \\ \vdots & \vdots & \ddots & 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \Omega_{\mathrm{SN}}(t) & 0 & 0 & 0 & 0 & \Omega_{\mathrm{PN}}(t) \\ 0 & 0 & \cdots & 0 & \Omega_{\mathrm{P1}} & \Omega_{\mathrm{P2}} & \cdots & \Omega_{\mathrm{PN}} & 0 \end{pmatrix}. \quad (27)$$

Following MS transformation technique , the initial Hamiltonian can be transformed into an effective Λ-system reading in the basis $\{|a\rangle, |\phi_2\rangle, |\phi_1\rangle\}$ and $N - 1$ two state systems which are separated from each other [see figure 7(b)] [18], in which $|\phi_1\rangle$ and $|\phi_2\rangle$ are superposition of ground states and excited states of original basis, respectively. In order to satisfy MS condition in this system we require that the pump Rabi frequencies have the same shape and also all of the Stokes Rabi frequencies have the same temporal pule shape:

$$\Omega_{S1}(t) = \Omega_{S2}(t) = \cdots = \Omega_{\mathrm{SN}}(t) = \Omega_S(t). \quad (28)$$

The Hamiltonian in the subspace $\{|a\rangle, |\phi_2\rangle, |\phi_1\rangle\}$ can be read as follows:

$$\hat{H}_{\mathrm{MS}}(t) = \begin{pmatrix} 0 & \Omega_{\mathrm{PC}}(t) & 0 \\ \Omega_{\mathrm{PC}}(t) & 0 & \Omega_S(t) \\ 0 & \Omega_S(t) & 0 \end{pmatrix}, \quad (29)$$

where

$$|\phi_1\rangle = \chi_1|1\rangle + \chi_2|2\rangle + \cdots + \chi_N|N\rangle, \quad (30a)$$

$$|\phi_2\rangle = \chi_1|N + 1\rangle + \chi_2|N + 2\rangle + \cdots + \chi_N|2N\rangle, \quad (30b)$$

$$\Omega_{\mathrm{PC}}(t) = \frac{\Omega_{P1}(t)}{\chi_1}. \quad (30c)$$

In equation (30), χ_i can be calculated using (5) and (6) by substituting Ω_{Pi} instead of Ω_{Si}.

3.2. Digital pulses in original basis

The purpose of this subsection is to create an arbitrary coherent superposition of all ground states in a multi-lambda system. For creation of a desired superposition of all ground states at the end of the evolution as

$$|\Psi(t_f)\rangle = \sum_{j=1}^{N} c_j|j\rangle, \quad \sum_{j=1}^{N}|c_j|^2 = 1, \quad (31)$$

we use sinusoidal functions in S steps as the pulses in the kth step are follows:

$$\Omega_{P1}^{(k)} = c_1\Omega_M \sin\left[\frac{\xi\pi}{2(S-1)}\right], \quad (32a)$$

$$\Omega_{P2}^{(k)} = c_2\Omega_M \sin\left[\frac{\xi\pi}{2(S-1)}\right],$$

$$\vdots \quad (32b)$$

$$\Omega_{\mathrm{PN}}^{(k)} = c_N\Omega_M \sin\left[\frac{\xi\pi}{2(S-1)}\right], \quad (32c)$$

and

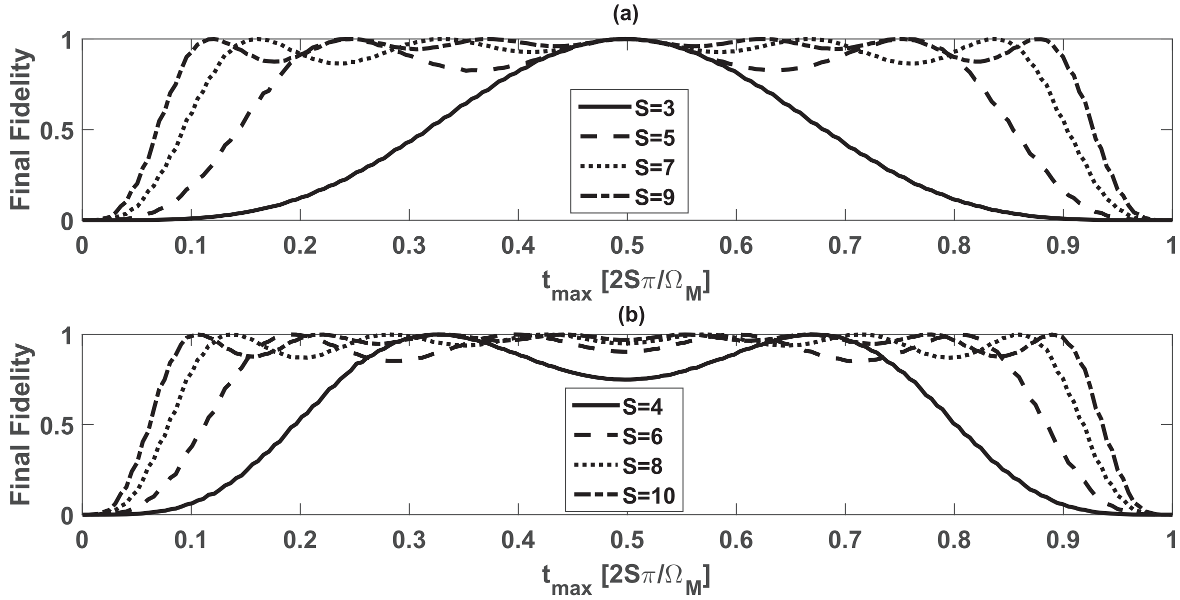

Figure 9. Final fidelity of five-lambda system for desired state (34) as a function of $t_{\max}$ using the sinusoidal protocol for the first four values S (a) odd values ($S = 3, 5, 7$, and 9) and (b) even values ($S = 4, 6, 8$, and 10). When $\cos(\tau_k\Omega_M) = 1$, the propagator becomes identity, as it is seen the population starts from zero and ends at the same value.

$$\Omega_{S1}^{(k)} = \Omega_{S2}^{(k)} = \cdots = \Omega_{\mathrm{SN}}^{(k)} = \Omega_S^{(k)} = \Omega_M \cos\left[\frac{\xi\pi}{2(S-1)}\right], \tag{33}$$

where ξ is given by (11). Figure 8 shows the evolution of population in a five-lambda system, $N = 5$, for $S = 5$, 15 steps of digital pulses that leads to

$$|\Psi(t_f)\rangle = \sqrt{\frac{3}{20}}|1\rangle + \sqrt{\frac{4}{20}}|2\rangle + \sqrt{\frac{7}{20}}|3\rangle + \sqrt{\frac{2}{20}}|4\rangle + \sqrt{\frac{4}{20}}|5\rangle. \tag{34}$$

In both cases the population is transferred from state $|a\rangle$ to an equal superposition of ground states. The transient population of the intermediate states is damped as S increases, where for $S = 15$ it gives negligibly small values. Figure 9 also shows the fidelity of the final state with respect to the desired state (34) as a function of $t_{\max}$ for different values of S in a five-lambda system. It is seen, similar to multi-pod systems as the number of steps increases the robustness of final fidelity of the system also increases with respect to $t_{\max}$.

4. Conclusion and discussion

In this paper, we studied generation of coherent superposition of states in multi-pod and multi-lambda systems, with resonant condition, using DAP technique. We showed that this technique can be extended to fractional population transfer in a three-state system. By applying MS transformation in both multi-pod and multi-lambda systems, we found special values of pulse parameters to designe appropiate digital Rabi frequencies for creatation a desired coherent superposition of ground states. The results of numerical studies show that DAP technique in multi-state systems is robust against digitization of the control parameters and so it resembles simple adiabatic passage. Moreover, we showed that if the step number of digital sinusoidal pulses increases, the robustness of final fidelity of generated superposition state, with respect to $t_{\max}$, increases. Recently, the technique of three state DAP implemented experimentally in three optical waveguides [26]. In [32] an optical beam splitter with one input and N output waveguide channels ($N + 1$-pod linkage) is introduced. The DAP technique in multi-pod systems could be implemented experimentally in such optical beam splitter. Also the DAP technique in multi-state systems can be used to creation of fast qudit gates.

We note that while the analytic solution for digital adiabatic passage is applicable to exact resonance only, the technique can be modified to include nonzero single-photon detuning. In order to numerical study we consider $(N + 1)$-pod system with $N = 1$ as our aim is to transfer the population from state $|a\rangle$ to $\frac{1}{\sqrt{2}}(|a\rangle - |1\rangle)$. In figure 10 we plot the final fidelity of desired state $|\Psi(t_f)\rangle = \frac{1}{\sqrt{2}}(|a\rangle - |1\rangle)$ versus the single-photon detuning for ($S = 5$, 15) steps of digital pulses. The figure demonstrates that by increasing the number of steps, this technique is applicable for some range of nonzero single-photon detuning.

At the end, we study how the final fidelity is affected by increasing the number of levels with the same steps of digital pulses. In order to study the robustness of the protocol for more complicated level systems, we choice three $(N + 1)$-pod systems with $N = 5$, 10, 15. Our aim is to create coherent superposition of all ground states with equal amplitudes in these systems using $S = 3$ steps of digital pulses. In figure 11 we have plotted the fidelity of final state as a function of $t_{\max}$ using $S = 3$ steps of digital pulses in these systems. It can be seen that on all systems, $F = 1$ is achieved for the same value

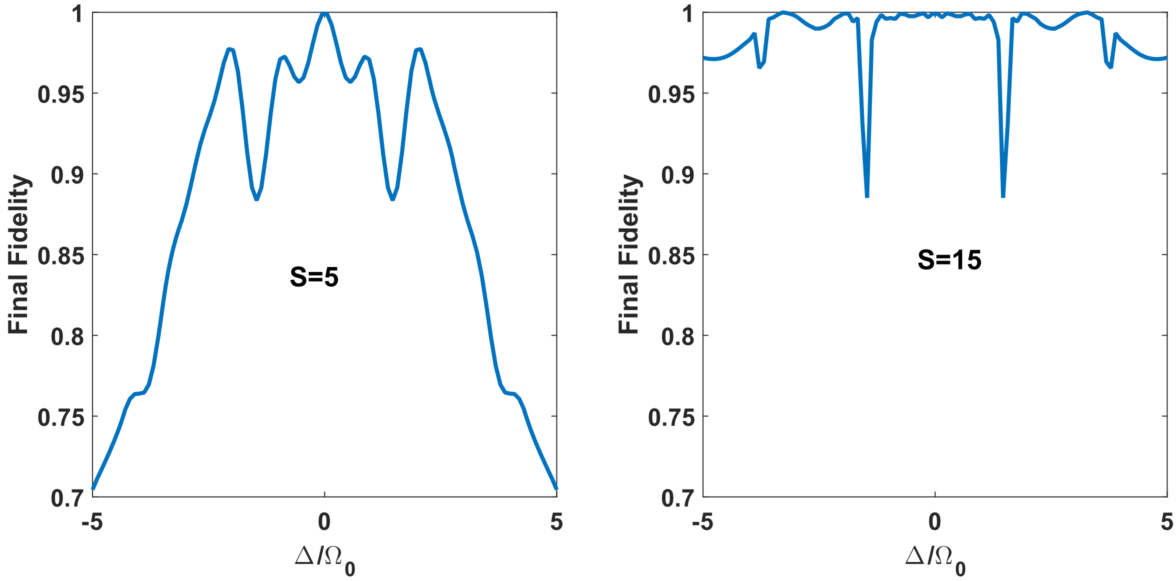

Figure 10. Final fidelity of desired state $|\Psi(t_f)\rangle = \frac{1}{\sqrt{2}}(|a\rangle + |1\rangle)$ in a two-pod system vs detuning Δ for $S = 5$ (left frame) and $S = 15$ (right frame) number of digital pulses.

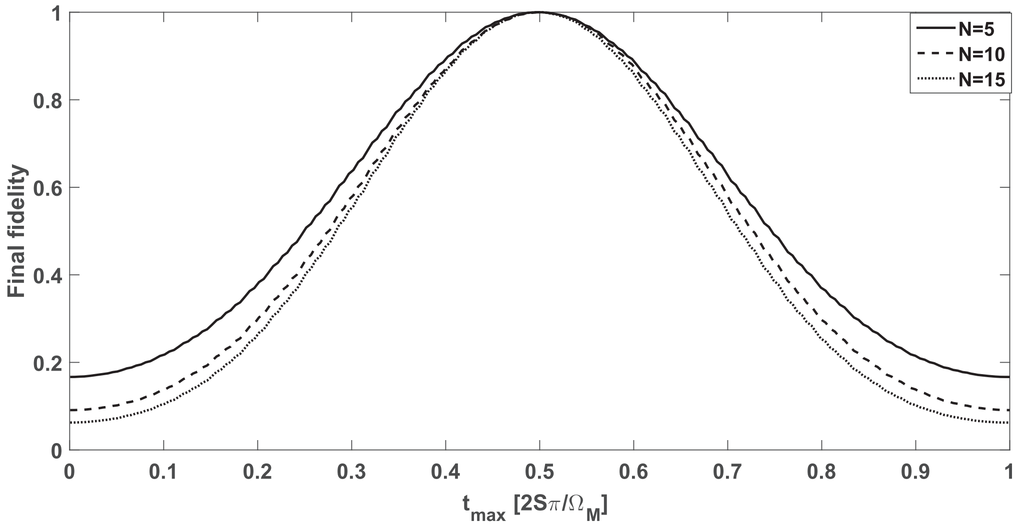

Figure 11. Final fidelity of desired states (coherent superposition of all ground states) for three $(N + 1)$-pod systems ($N = 5$, 10, 15) as a function of $t_{\max}$ using $S = 3$ steps of digital pulses.

of $t_{\max}$. However, with deviation from this value of $t_{\max}$, it is harder and harder to achieve higher fidelities for larger systems.

References

[1] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press) 0–521–63503–9

[2] král P, Thanopulos I and Shapiro M 2007 *Rev. Mod. Phys.* **79** 53

[3] Gaubatz U, Rudecki P, Schiemann S and Bergmann K 1990 *J. Chem. Phys.* **92** 5363

[4] Bergmann K, Theuer H and Shore B W 1998 *Rev. Mod. Phys.* **70** 1003

[5] Vitanov N V, Fleischhauer M, Shore B W and Bergmann K 2001 *Adv. At. Mol. Opt. Phys.* **46** 55

[6] Vitanov N V, Halfmann T, Shore B W and Bergmann K 2001 *Annu. Rev. Phys. Chem.* **52** 763

[7] Bergmann K, Vitanov N V and Shore B W 2015 *J. Chem. Phys.* **142** 170901

[8] Vitanov N V, Rangelov A A, Shore B W and Bergmann K 2017 *Rev. Mod. Phys.* **89** 015006

[9] Shore B W 2017 *Adv. Opt. Photonics* **9** 563

[10] Shore B W 1990 *The Theory of Coherent Atomic Excitation* (New York: Wiley)

[11] Unanyan R G, Shore B W and Bergmann K 2001 *Phys. Rev.* A **63** 043401

[12] Thanopulos I, Král P and Shapiro M 2004 *Phys. Rev. Lett* **92** 113003

[13] Kyoseva E S and Vitanov N V 2006 *Phys. Rev.* A **73** 023420

[14] Amniat-Talab M, Saadati-Niari M, Guérin and Nader-Ali R 2011 *Phys. Rev.* A **83** 013817

[15] Bevilacqua G, Schaller G, Brandes T and Renzoni F 2013 *Phys. Rev.* A **88** 013404
[16] Shapiro E A, Milner V, Menzel-Jones C and Shapiro M 2007 *Phys. Rev. Lett.* **99** 033002
[17] Rangelov A A and Vitanov N V 2012 *Phys. Rev.* A **85** 043407
[18] Saadati-Niari M and Amniat-Talab M 2014 *J. Mod. Opt.* **61** 877
[19] Saadati-Niari M and Amniat-Talab M 2014 *J. Mod. Opt.* **61** 1492
[20] Saadati-Niari M 2016 *Ann. Phys.* **372** 138
[21] Nedaee-Shakarab B, Saadati-Niari M and Zolfagharpour F 2016 *Phys. Rev.* C **94** 045601
[22] Nedaee-Shakarab B, Saadati-Niari M and Zolfagharpour F 2017 *Phys. Rev.* C **96** 044619
[23] Mirza-Zadeh S, Saadati-Niari M and Amniat-Talab M 2018 *Laser. Phys. Lett* **15** 095105
[24] Vatikus J A and Greentree A D 2013 *Phys. Rev.* A **87** 063820
[25] Vatikus J A, Steel M J and Greentree A D 2017 *Opt. Express* **25** 5466
[26] Ng V, Vatikus J A, Chaboyer Z J, Nguyen T, Dawes J M, Withford M J, Greentree A D and Steel M J 2017 *Opt. Express* **25** 2552
[27] Morris J R and Shore B W 1983 *Phys. Rev.* A **27** 906
[28] Rangelov A A, Vitanov N V and Shore B W 2006 *Phys. Rev.* A **74** 053402
[29] Shore B W 2014 *J. Mod. Opt.* **61** 787
[30] Rousseaux B, Guérin S and Vitanov N V 2013 *Phys. Rev.* A **87** 032328
[31] Vitanov N V 1998 *J. Phys.* B **31** 709
[32] Rangelov A A and Vitanov N V 2012 *Phys. Rev.* A **85** 055803