

Trajectory Tracking Control for Double-steering Automated Guided Vehicle Based on Model Predictive Control

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Abstract. In order to solve the trajectory tracking control problem of double-steering automated guided vehicle (AGV), considering various constraints in practical work, a trajectory tracking controller based on model predictive control is designed. Firstly, the kinematic model of double-steering AGV is established. Then, a trajectory tracking model based on model predictive control is designed to achieve fast and accurate tracking performance. Finally, the influence of the predicted time-domain length on the system performance is analyzed by simulation, and the simulation experiment is compared with that of the AGV based on the proportional integral differential trajectory tracking controller. The experiment result shows that the model predictive controller can meet all kinds of constraints in actual work, and complete the tracking of the continuous high curvature target path efficiently and accurately in real-time with high robustness.

1. Introduction

AGV has become an important equipment in intelligent logistics, intelligent manufacturing and flexible production. According to the different number of driving wheels, the AGV is usually divided into single-steering AGV, double-steering AGV and multi-steering AGV. Among them, AGV with double steering wheels is worth focusing on because of its flexible body motion and rich application scenarios.

At home and abroad, there are many researches on AGV trajectory tracking, and the horizontal and vertical correction control are often used to achieve AGV trajectory tracking. According to the single steering wheel AGV, Ibari[1] designed an inverse controller based on Lyapunov stability theory to track the single steering wheel AGV, which ensured the stability of tracking error; Xiong Zhonggang[2] proposed an intelligent path control method based on immune fuzzy PID, which realized the path tracking of straight line and curve of small agricultural machinery in complex environment. For the double steering wheel AGV, Hemami[3] analysed the relationship between the front and rear steering wheel angles, and proposed a vehicle trajectory tracking scheme under low-speed driving; Yuan[4] designed a trajectory tracking controller for the double steering wheel semi-trailer tractor based on the inversion method, which controlled the front and rear steering wheels to travel along the same trajectory, and improved the vehicle mobility. The above models contain many parameters, and the parameter value selection method is only for specific scenarios, so it is difficult to achieve generalization. In fast path tracking, AGV may not adjust in time and deviate greatly. Aiming at the four-wheel omni-directional AGV, Zhang Ye and others[5] put forward an integrated control system of four-wheel steering combined with active suspension, using the model predictive control algorithm, taking various performance indexes of the vehicle body as the control goal, to meet the



stability requirements of the four-wheel steering of the vehicle body under the limit condition. Pandu S P[6] proposes a trajectory tracking controller based on backstepping to track a given trajectory. Nguyen hung[7] designed a sliding mode dynamic controller to make the speed of the AGV converge to the speed control input and realize the AGV uniform speed tracking track.

In the aspect of tracking control, the common controller models are PID controller, feedforward feedback controller, linear quadratic regulator[8] and model predictive controller[9]. Among them, model predictive control (MPC) is mainly used in the control of nonlinear and highly coupled complex systems. In order to solve the problem of track tracking in a longer time span, Felipe Kuhne[10] and others proposed an AGV control scheme with nonholonomic constraints, which was directly dealt with by model predictive control. After the error model is linearized continuously, the quadratic programming is used to calculate the model predictive control. Pacheco[11] compared MPC based tracking controller with PID based tracking controller, and the results showed that MPC controller has better tracking effect on the basis of ensuring response speed.

Up to now, the research on the track tracking of AGV with double steering wheel is not sufficient enough. In this paper, the research object is AGV with two steering wheels and four submissive wheels arranged at the four corners of the base. On the basis of its kinematic model, a dual steering wheel AGV rectification controller based on model prediction is designed, and the process of AGV rectification control is simulated by MATLAB software to verify the real-time and robustness of the rectification algorithm.

2. Kinematic model of AGV

This paper focuses on double-steering AGV with the structure shown in Figure 1. Since the vehicle movement is realized by dynamic adjustment of front and rear steering wheels, the AGV kinematic model is simplified as the movement of front and rear driving wheels. When the kinematic model is established, the AGV body structure is assumed to be rigid body, that is to say, it conforms to the principle of rigid body translation. At the same time, the influence of AGV suspension parts on the body is ignored. Each steering subsystem has one driving wheel. By adjusting the angle of the front and rear wheel, AGV can track the straight and curve path.

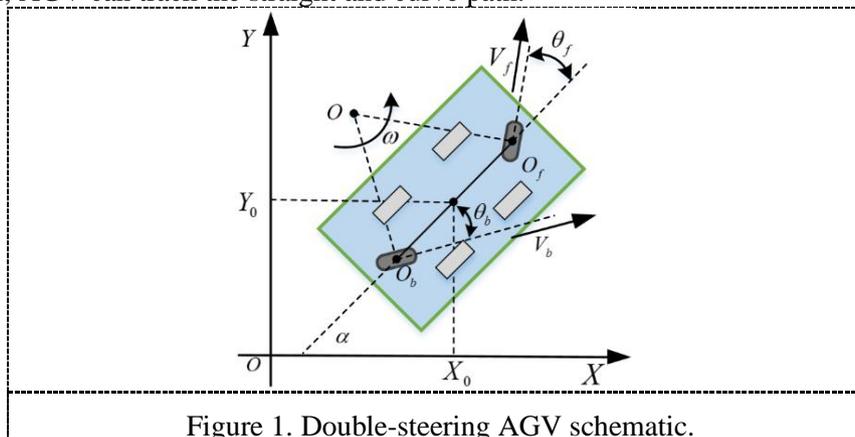


Figure 1. Double-steering AGV schematic.

Establish the plane rectangular coordinate system $\{XOY\}$, O_f , O_b are the contact points of the front and rear driving wheels with ground, respectively. The wheelbase is L . When the AGV body is tracking an arc path, the angles of the front and rear driving wheels are the same because of the same curvature of the arc path. Set the angle between the travel direction of front and rear drive wheel and the X axis of $\{X'CY'\}$ AGV coordinate system as, θ_f , θ_b , take v_f , v_b as the travel speed of the front and rear wheels, and the turning radius as R .

As shown in Figure 1, the geometric center of the vehicle body (X_0, Y_0) is the midpoint of the line between the front and rear steering wheel. Under the condition of uniform load, the geometric center is considered as the center of mass of the AGV body, which represent the location of the AGV body.

The linear velocity state quantity of the center of mass in the global coordinate system is,

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \cos \theta_f & \cos \theta_b \\ \sin \theta_f & -\sin \theta_b \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} v_f \\ v_b \end{bmatrix} \quad (1)$$

Considering the kinematic characteristics of rigid body, according to the sine theorem, the linear and angular velocities can be expressed as

$$v_c = \sqrt{v_x^2 + v_y^2} \quad (2)$$

$$\omega_c = \frac{v_f \sin(\theta_f + \theta_b)}{L \cos \theta_b} \quad (3)$$

When model predictive control is applied to correct the trajectory of double-steering AGV, the position and orientation state of AGV system is recorded as

$$\mathbf{X} = [x_0 \ y_0 \ \theta]^T = [x \ y \ \theta]^T \quad (4)$$

Where x and y represent the position of the body center of mass in the global coordinate system $\{XOY\}$. The linear velocity and angular velocity of the double-steering AGV are taken as the control inputs, which are expressed by the body centroid velocity and angular velocity of the vehicle and denoted as

$$\mathbf{u} = [v_c \ \omega_c]^T = [v \ \omega]^T \quad (5)$$

Assuming that the wheels are rolling without sliding, the kinematic model of the double-steering AGV can be given as follows

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(x, u) = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_c \\ w_c \end{bmatrix} \quad (6)$$

3. Trajectory tracking model of double-steering AGV based on model predictive control

3.1. Tracking error model of double-steering AGV

In order to set AGV tracking path and set up the operation reference trajectory, any point of the reference trajectory meets the state space expression (7)

$$\dot{\mathbf{x}}_r = f(x_r, u_r) \quad (7)$$

Where the reference trajectory state quantity is $\mathbf{X}_r = [x_r \ y_r \ \theta_r]^T$, its reference input control quantity is $\mathbf{u}_r = [v_r, w_r]^T$. (x_r, u_r) is the reference trajectory vehicle coordinate, θ_r is the reference heading angle. In order to linearize the predictive control model, Taylor expansion for equation (7) at the reference trajectory point are taken, ignoring the higher-order term and keeping the first-order term, and the linearization error model of the double-steering AGV can be obtained as follows

$$\dot{\mathbf{x}} = f(x_r, u_r) + \frac{\partial f(x, u)}{\partial x} \Big|_{\substack{x=x_r \\ u=u_r}} (x - x_r) + \frac{\partial f(x, u)}{\partial u} \Big|_{\substack{x=x_r \\ u=u_r}} (u - u_r) \quad (8)$$

Make equation (8) to minus (7) can get

$$\dot{\mathbf{X}}' = \begin{bmatrix} \dot{x} - \dot{x}_r \\ \dot{y} - \dot{y}_r \\ \dot{\theta} - \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} 0 & 0 & -v_r \sin \theta_r \\ 0 & 0 & v_r \cos \theta_r \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x - x_r \\ y - y_r \\ \theta - \theta_r \end{bmatrix} + \begin{bmatrix} \cos \theta_r & 0 \\ \sin \theta_r & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v - v_r \\ w - w_r \end{bmatrix} \quad (9)$$

Where $x' = x - x_r$ represents the state error between the actual and reference movement position of the vehicle body. $u' = u - u_r$ indicate the control input error. Equation (9) is a continuous model. By using forward difference approximation of x' , the following AGV linear time-varying discrete error model is obtained

$$\begin{cases} x(k+1) = \mathbf{A}_d x(k) + \mathbf{B}_d u(k) \\ y(k) = \mathbf{C}_d x(k) \end{cases} \quad (10)$$

In which the linear time-varying matrices are as follows:

$$\mathbf{A}_d = \begin{bmatrix} 1 & 0 & -v_r \sin \theta_r T \\ 0 & 1 & v_r \cos \theta_r T \\ 0 & 0 & 1 \end{bmatrix} \quad (11)$$

$$\mathbf{B}_d = \begin{bmatrix} \cos \theta_r T & 0 \\ \sin \theta_r T & 0 \\ 0 & T \end{bmatrix} \quad (12)$$

$$\mathbf{C}_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (13)$$

Where, T is the sampling period, k is the sampling time.

3.2. Trajectory tracking controller model

The incremental expression of the discrete error model at system time k is as follows:

$$\boldsymbol{\psi}(k) = \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix} \quad (14)$$

$$\Delta u(k) = u(k) - u(k-1) \quad (15)$$

The state space expression of the kinematic model is:

$$\begin{cases} \boldsymbol{\psi}(k+1) = \tilde{\mathbf{A}}_d \boldsymbol{\psi}(k) + \tilde{\mathbf{B}}_d \Delta u(k) \\ \boldsymbol{\eta}(k) = \tilde{\mathbf{C}}_d \boldsymbol{\psi}(k) \end{cases} \quad (16)$$

Where,

$$\tilde{\mathbf{A}}_d = \begin{bmatrix} \mathbf{A}_d & \mathbf{B}_d \\ \mathbf{0}_{m \times n} & \mathbf{I}_m \end{bmatrix} \quad (17)$$

$$\tilde{\mathbf{B}}_d = \begin{bmatrix} \mathbf{B}_d \\ \mathbf{I}_m \end{bmatrix} \quad (18)$$

$$\tilde{\mathbf{C}}_d = [\mathbf{C}_d \quad \mathbf{0}] \quad (19)$$

The finite prediction time domain is assumed to be p , and the state of the system is predicted in the time domain $[\tau, \tau + p]$. The control time domain is c , and the system control sequence is generated in the time domain $[\tau, \tau + c]$. So the prediction equation of the system in the prediction time domain is

$$Y(k) = \phi_p \boldsymbol{\psi}(k) + \mathbf{Q}_c \Delta U(k) \quad (20)$$

Where, $Y(k)$ is the system output quantity; ϕ_p is the state quantity parameter; Q_c is the control quantity parameter; $\Delta U(k)$ is the control increment sequence, expressed by the formula as

$$Y(k) = [\eta(k), \dots, \eta(k+p)]^T \quad (21)$$

$$\phi_p = [\tilde{C}_d, \tilde{C}_d \tilde{A}_d, \dots, \tilde{C}_d \tilde{A}_d^{c-1}, \dots, \tilde{C}_d \tilde{A}_d^{p-1}]^T \quad (22)$$

$$\Delta U(k) = [\Delta u(k), \dots, \Delta u(k+c)]^T \quad (23)$$

$$Q_c = \begin{bmatrix} \tilde{C}_d \tilde{B}_d & 0 & 0 & 0 \\ \tilde{C}_d \tilde{A}_d \tilde{B}_d & \tilde{C}_d \tilde{B}_d & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{C}_d \tilde{A}_d^{c-1} \tilde{B}_d & \tilde{C}_d \tilde{A}_d^{c-2} \tilde{B}_d & \dots & \tilde{C}_d \tilde{B}_d \\ \tilde{C}_d \tilde{A}_d^c \tilde{B}_d & \tilde{C}_d \tilde{A}_d^{c-1} \tilde{B}_d & \dots & \tilde{C}_d \tilde{A}_d \tilde{B}_d \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{C}_d \tilde{A}_d^{p-1} \tilde{B}_d & \tilde{C}_d \tilde{A}_d^{p-2} \tilde{B}_d & \dots & \tilde{C}_d \tilde{A}_d^{p-c-1} \tilde{B}_d \end{bmatrix} \quad (24)$$

In the system prediction equation, it is necessary to establish an accurate objective function $\Delta U(k)$ to solve the control increment, so as to obtain the appropriate control quantity sequence in the control time domain $[\tau, \tau+c]$. In the actual operation of AGV, considering the practical constraints such as the feasible region of AGV, the steering angle and the travel speed, the trajectory tracking controller model in this paper can be described as:

$$\begin{cases} x_{k+1} = A_d x_k + B_d \Delta u_k \\ y_{t,k} = C_{t,d} x_k \end{cases} \quad (25)$$

$$\min_{\Delta U} J_k = \sum_{i=1}^p \|\eta(k+i) - \eta_r(k+i)\|_Q^2 + \sum_{i=1}^{c-1} \|\Delta u(k+i)\|_R^2 + \rho \gamma^2 \quad (26)$$

s.t.

$$u_{\min} \leq u(k) \leq u_{\max}, k = 0, 1, 2, \dots$$

$$\Delta u_{\min} \leq \Delta u(k) \leq \Delta u_{\max}, k = 0, 1, 2, \dots$$

$$y_{t,\min} \leq y_t(k) \leq y_{t,\max}, k = 0, 1, 2, \dots$$

The objective function not only ensures that the model can track the target trajectory accurately, but also ensures that the control quantity conforms to the actual performance limit of AGV. Q is the prediction time domain weight factor; R is the control time domain weight factor; ρ is the weight coefficient; γ is the relaxation variable. In order to solve the problem in a larger feasible region, relaxation variables are introduced into the objective function. The selection of relaxation variables should not be too large, otherwise the model can not guarantee high accuracy even if it has better real-time performance.

3.3. Model algorithm of trajectory tracking controller

In order to obtain the optimal control sequence of the system, the optimal trajectory tracking problem is transformed into a constrained linear quadratic programming problem (QP) for solution. By substituting the prediction equation (20) into the objective function equation (26), we can get

$$J_k = [\Delta U^T(k), \gamma]^T H [\Delta U^T(k), \gamma] + G [\Delta U^T(k), \gamma] \quad (27)$$

s.t.

$$H = \begin{bmatrix} S_k^T Q S_k + R & 0 \\ 0 & \rho \end{bmatrix}$$

$$G = \begin{bmatrix} 2E_k^T Q S_k & 0 \end{bmatrix}$$

Where G is the control increment coefficient matrix, E_k is the tracking error in predictive time domain $[\tau, \tau + p]$. For the quadratic real function with linear constraints, this paper uses the general interior point method to solve the problem, and obtains the internal control input increment in $[\tau, \tau + c]$ as follows:

$$\Delta U_t = [\Delta u_t, \Delta u_{t+1}, \dots, \Delta u_{t+c-1}]^T \quad (28)$$

Taking the first element of the control sequence as the increment of the actual input control quantity, it is calculated that the control quantity acts on the actual system, namely:

$$u(t) = u(t-1) + \Delta u_t \quad (29)$$

When the system enters the next sampling period, the system recalculates the control input increment, and realizes the double-steering AGV tracking control by iteration.

4. Simulation and result analysis

In order to verify the validity of the above mentioned AGV predictive controller model, this paper designs a trajectory tracking simulation experiment based on MATLAB platform. In the experiment, the default parameters of the robot motion model and MPC controller are shown in Table 1, and the control quantity constraints and control increment constraints are shown in Table 2.

Table 1. Default parameter values of motion model and MPC controller.

| | Value | Parameter meaning |
|----------|---------------------|-------------------------------|
| L | 0.87 M | Wheelbase |
| W | 0.66 M | Tread |
| T | 0.1 s | sampling period |
| N | 5 | Prediction time domain |
| γ | 10 | Relaxation variable |
| Q | diag(3,3,0.5) | Prediction time domain weight |
| R | $0.1I_{2 \times 2}$ | Control time domain weight |
| X_0 | $[0 \ 0.5 \ 0.1]^T$ | Initial position |

4.1. Analysis of the influence of prediction time domain on the system

The prediction time domain reflects predictive ability of response system to the future output, and the prediction time-domain length affects the track tracking significantly. When the prediction time domain is long, the solution time of the model is long, which affects the fast dynamic performance of the system. Therefore, it is necessary to optimize the prediction time domain separately for MPC trajectory tracking algorithm, and select the appropriate parameters through system simulation to achieve the balance of real-time and stability.

Table 2. Model constraint range.

| | Constraint scope | Parameter meaning |
|------------|-------------------------|----------------------------|
| v | [-1.5 m/s, 1.5 m/s] | Line speed |
| w | [-1.2 rad/s, 1.2 rad/s] | angular velocity |
| Δv | [-0.35 m/s, 0.35 m/s] | Linear speed increment |
| Δw | [-0.2 rad/s, 0.2 rad/s] | Angular velocity increment |

In the experiment, the speed of the AGV is 0.4 m/s; the angular speed is 0.074 rad/s, and the predictive time domain satisfies $N \in \{5,6,7,8\}$. The results are shown in Figure 2. By increasing the prediction time domain in a reasonable range, the AGV achieves better steady-state tracking effect. Figure 2 (b) reflects the change of linear speed input and steering angle input in the initial stage of the steering wheel. The increase of N will significantly reduce the steady-state error, but the longer prediction time domain increases the system overshoot. When the sampling period is less than 7, the angular velocity of the steering wheel angle of the robot meets the performance limit of the robot. Otherwise, the variation of the steering angle exceeds the maximum bearing angular velocity of the system (0.2 rad/s).

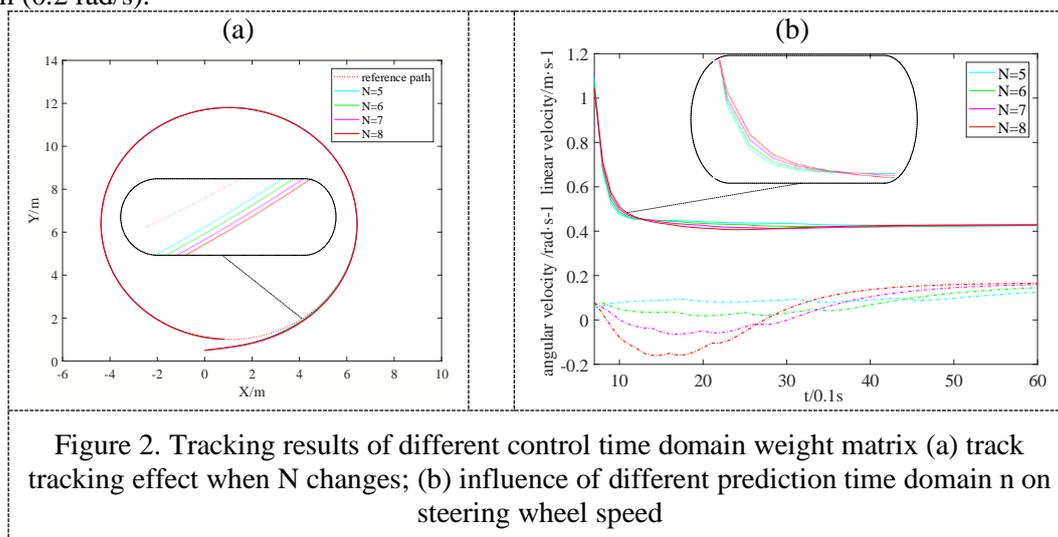


Figure 2. Tracking results of different control time domain weight matrix (a) track tracking effect when N changes; (b) influence of different prediction time domain n on steering wheel speed

4.2. Comparative experiment analysis of MPC controller and PID controller

In order to verify the trajectory tracking algorithm based on model predictive control, a comparative simulation experiment is designed to compare the tracking performance of PID controller and MPC controller. The influence of the MPC model parameters on the tracking effect is analysed through the experiments mentioned above, in which the sampling period is selected as $N=7$ and the control time-domain weight matrix $R=0.1I_{2 \times 2}$.

Using the steering ability to evaluate the stability of trajectory tracking algorithm. For this purpose, the reference path is set as a straight line combining with an S-shaped curve high curvature sine curve. Where the sine curve track meets the following requirements

$$y_r = 3.7 \sin\left(\frac{\pi}{8} * x_r\right) \quad (30)$$

The comparison of simulation results are shown in Figure 3.

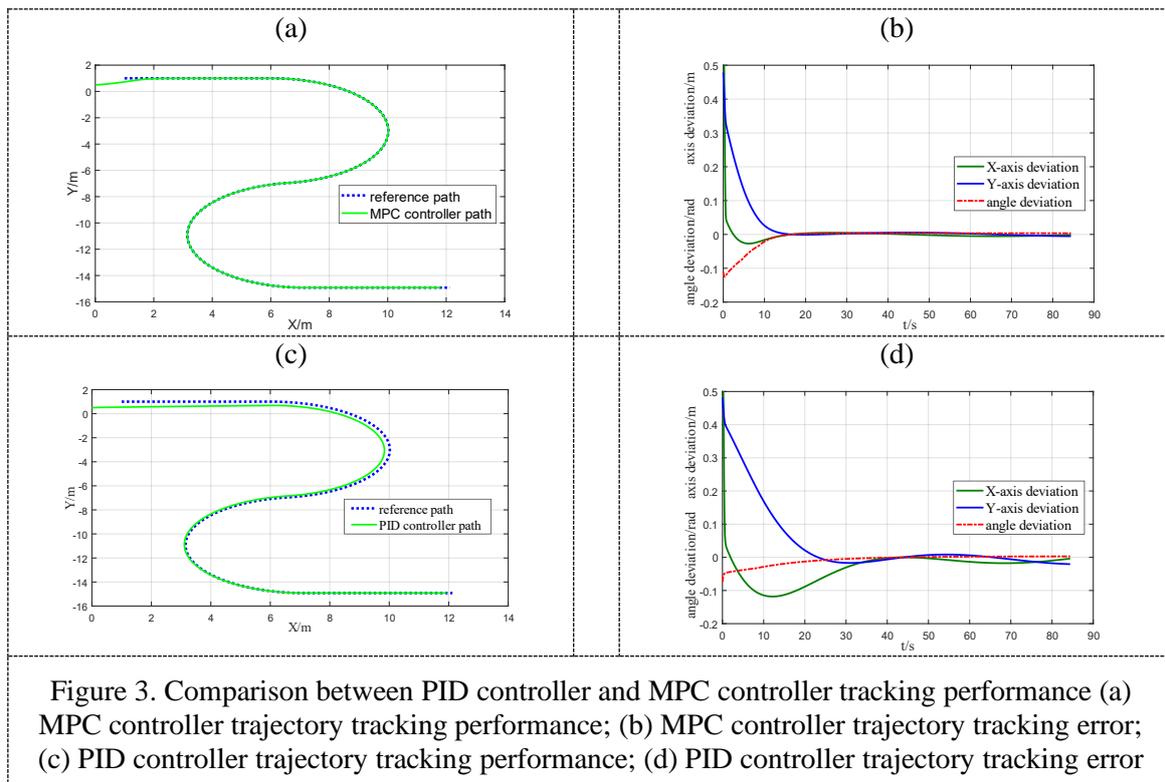


Figure 3. Comparison between PID controller and MPC controller tracking performance (a) MPC controller trajectory tracking performance; (b) MPC controller trajectory tracking error; (c) PID controller trajectory tracking performance; (d) PID controller trajectory tracking error

According to figure3(a) and 3(c), for the arc with larger curvature, the tracking path of the AGV based on PID controller will have a large overshoot, while MPC controller shows better control characteristics. When the running time is at 33.7 s, the AGV reaches the maximum curvature for the first time. At this point, the trajectory of PID controller with stable running trend shows a large overshoot of 19 cm in one direction again, while the trajectory error of MPC controller keeps within the range of $[-1, 1.1]$ after 18 s.

It can be seen from Fig. 3 (b) and (d) that when the AGV moves for a period of time, it will quickly and stably meet the tracking index requirements within a certain range. The AGV with PID controller takes a longer time to reach steady state, and the steady state error is larger.

Through above experimental results, double-steering AGV based on MPC track controller shows better rapidity and stability in circular and sinusoidal path with large curvature. After choosing reasonable parameters, the tracking performance of MPC is better than that of PID algorithm.

5. Conclusion

In this paper, the double-steering AGV is taken as research object. In order to study the kinematic performance of double-steering AGV, the kinematic model is established. At the same time, a tracking algorithm based on model predictive control is designed, which makes the AGV complete the tracking task quickly and stably. Finally, the feasibility and effectiveness of the model predictive control trajectory tracking algorithm is verified using MATLAB platform simulation experiment. Through the actual tracking effect of the controller under different model parameters, the optimal controller parameters are obtained. At the same time, through the simulation experiment, the trajectory tracking path based on the PID controller is compared with that of the MPC after parameter optimization, which proves that the MPC algorithm has higher stability. In this paper, the AGV controller only considers the case of uniform load, the next step will consider the influence of load and inertia.

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