The cluster-delay consensus of nonlinear multi-agent systems via impulsive control

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Abstract. Based on the impulsive control strategy, the cluster-delay consensus of nonlinear multi-agent systems is studied in this paper for the first time. Different from the traditional continuous control method, impulsive control only acts on the systems at discrete impulsive moments, so it has the advantages of low control costs, fast response speed and strong adaptability. In addition, by the impulsive protocol, the state information of all neighboring agents is used to update their own state at impulse instants. Based on the graph theory and Lyapunov stability theory, some sufficient consensus criteria are given. Finally, the correctness of theoretical results is illustrated by numerical simulation.

1. Introduction

In current society, the research of multi-agent systems (MASs) based on the distributed cooperative control technology has been widely used in many fields of social life, such as sociology [1], economics [2], formation control of unmanned aerial vehicles or robots [3], etc. As a significant topic in the field of the technology, the consensus means that all agents in the same MASs or cluster eventually tend to a common state. The study about consensus has already made abundant achievements [4-6] and attracted more and more attention in academic circles.

In practical applications, all agents within MASs can be divided into multiple clusters (i.e. subgroups) by the degree of correlation among agents. In the same cluster, the degree of association or coupling strength between agents is higher. Meanwhile, the coupling strength between agents in different clusters is generally smaller. If each cluster within MASs can have different common state in the end, then it said that the cluster consensus of the systems can be reached. Up to now, the cluster consensus (also known as group consensus) has been studied in [7-9], etc.

In particular, if the common state of one cluster is chosen as the leading state, the remaining clusters' common states will eventually be consistent with the corresponding delay states of the leading state. In this case, as a special case of group consensus, the concept of cluster-delay consensus was developed in [10]. The authors have been studied the cluster-delay consensus of first-order nonlinear MASs via continuity control. Based on graph theory and Lyapunov stability theory, some sufficient criteria for consensus are given. Consider a first-order nonlinear MASs with delays, the cluster consensus has been studied via pinning control with periodic intermittent effect in [11]. Similarly, through designing a specific intermittent control protocol, the cluster-delay synchronization of a directed network has been investigated in [12]. On account of the transmission ways of information in different clusters, the new notion of layered intermittence was developed in [13]. Based on that, the cluster-delay consensus of MASs with aperiodic intermittent communication has been researched. Compared with the traditional way of continuous control in [7-13], the impulsive control has some advantages of low cost and high efficiency, which is widely used in the research of consensus for MASs [14-17], etc.

Motivated by the above discussions, this paper studies the cluster-delay consensus for a class of first-order nonlinear MASs via impulsive control. Compared with [10-13], this paper has the following innovations: firstly, impulsive control is used to make sure that the MASs can achieve the cluster-delay consensus while reducing the control costs and improving control efficiency; secondly, in the construction of the controller, agents receive not only the state information of adjacent nodes in their own cluster, but also the state information of adjacent nodes in other clusters.

The rest of this paper is as follows: In section 2, the related concepts in graph theory, the construction of models and protocols are briefly introduced. In section 3, sufficient consensus criteria are derived by mathematical theoretical derivation. In section 4, the correctness of theoretical results is illustrated by some simulations. In section 5, a brief summary of this paper is given.

2. Problem formulation and preliminaries

2.1. Graph theory

For convenience, the structure of MASs is represented by a topology graph. If the connection between any two agents in the systems is bidirectional, then the graph is undirected. Otherwise, it is a directed graph. For a MASs composed by N agents, let $G=(\Theta,\bar{\Omega},A)$ denotes its directed topology graph without self-cycling, where $\Theta=\{\Theta_1,\ldots,\Theta_N\}$ is the node set, $\bar{\Omega}=\{(\Theta_j,\Theta_i): i,j=1,\ldots,N\}\subset\Theta\times\Theta$ is the edge set, $A=\left[a_{ij}\right]$ is the weighted adjacency matrix. Let (Θ_j,Θ_i) denotes an edge from Θ_j to Θ_i, and its weight is a_{ij}. For convenience, let $a_{ij}=1$ denotes Θ_i can receive the state information of Θ_j. Conversely, $a_{ij}=0$ means that there is no connection between the two nodes. Let $d_i=d_{in}(\Theta_i)=\sum_{j=1,j\neq i}^{N}a_{ij}$ be the in-degree of node, then the degree matrix can be expressed as $D=diag(d_i,i=1,\ldots,N)$. Furthermore, let $L=D-A=\left[l_{ij}\right]$ be the Laplacian matrix, where $l_{ij}=\begin{cases}-a_{ij}, i\neq j,\\ -\sum_{j=1,j\neq i}^{N}l_{ij}, i=j.\end{cases}$

Let $B=diag(b_1,\ldots,b_N)$ denotes the connection matrix between follower agents and leader, where the connection weight $b_i=1$ means that follower agents can receive the state information of leader, vice versa.

2.2. Problem description and protocol construction

Consider a first-order nonlinear MASs composed by N follower agents, and its dynamics is described by

$$\dot{x}_i(t)=f(x_i(t),t)+Ax_i(t)-P_{\hat{i}}(s_{\hat{i}}(t)-S_1(t-\tau_{\hat{i}}))+u_i(t),\quad i=1,2,\cdots,N, \tag{1}$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\cdots,x_{in}(t))^T\in R^n$ is the position state of agent i, $u_i(t)\in R^n$ is the control input, $f(x_i(t),t)\in R^n$ is a nonlinear vector-valued function, the matrix $A\in R^{n\times n}$. The agents in MASs are divided into F clusters, and the index sets of these clusters are such as $V_1=\{1,2,\cdots,\tilde{m}_i\}$,…, $V_i=\{\tilde{m}_1+\cdots+\tilde{m}_{i-1}+1,\cdots,\tilde{m}_1+\cdots+\tilde{m}_{i-1}+\tilde{m}_i\}$,…, $V_F=\{\tilde{m}_1+\cdots+\tilde{m}_{F-1}+1,\cdots,N\}$, where $N=\tilde{m}_1+\tilde{m}_2+\cdots\tilde{m}_F$. Let $\hat{i}=1,2,\cdots,F$ is the subscript of cluster which the ith agent belongs.

In [10], the dynamics of each follower agents was modeled by

$$\dot{x}_i(t)=f(x_i(t))-\sum_{j\in V_{\hat{i}}}l_{ij}x_j(t)-\sum_{j\notin V_{\hat{i}}}l_{ij}x_j(t)+\bar{u}_i(t), \tag{2}$$

where $\bar{u}_i(t) = -k_{\hat{i}}(s_{\hat{i}}(t) - s_1(t-\tau_{\hat{i}})) - \sigma_i(x_i(t) - s_{\hat{i}}(t)) + \sum_{j \notin V_{\hat{i}}} l_{ij}(x_j(t) - x_i(t))$ is the control input, j denotes the neighboring agents of agent i, $K_{\hat{i}}$ and σ_i are connection weights, $i = 1, 2, \cdots, N$.

Furthermore, we can get

$$\dot{x}_i(t) = f(x_i(t)) - \sum_{j \in V_{\hat{i}}} l_{ij} x_j(t) - \sum_{j \notin V_{\hat{i}}} l_{ij} x_i(t) - k_{\hat{i}}(s_{\hat{i}}(t) - s_1(t-\tau_{\hat{i}})) - \sigma_i(x_i(t) - s_{\hat{i}}(t)). \tag{3}$$

Obviously, on the one hand, the continuous protocol $\bar{u}_i(t)$ will not go into effect when the state information of neighboring agents can not be received continuously. On the other hand, by systems (3), the update of state $x_i(t)$ is only affected by the neighboring agents in the same cluster, and the state information of neighboring agents in other clusters is not fully utilized.

Therefore, motivated by systems (3), the novel protocol (4) is designed in this paper. Two improvements are as follows: firstly, the impulsive control strategy is used to make the systems to reach the cluster-delay consensus while reducing control costs and improving control efficiency; secondly, each agent receives the state information of all neighboring agents to update their current states.

$$u_i(t) = \sum_{k=1}^{+\infty} \delta(t-t_k)(\alpha B_k \sum_{j=1}^{N} a_{ij}(x_j(t) - S_{\hat{j}}(t) - (x_i(t) - S_{\hat{i}}(t))) + (B_k - I_n)\beta(x_i(t) - S_{\hat{i}}(t))), \tag{4}$$

where $\delta(t)$ is the Dirac function, α and β are coupling strengths, $B_k \in R^{n\times n}$ is impulsive gain matrix, I_n represents the identity matrix of order n, $\hat{i} = 1, 2, \cdots, F$, $\hat{j} = 1, 2, \cdots, F$ and $S_1(t) \neq S_2(t) \neq \cdots \neq S_F(t)$.

Virtual leaders are introduced into MASs, and its dynamics is

$$\dot{S}_h(t) = f(S_h(t), t) + AS_h(t) - P_h(S_h(t) - S_1(t-\tau_h)), \quad h = 1, 2, \cdots, F, \tag{5}$$

where $S_h(t) \in R^n$, let $S_1(t)$ (the position state of the first virtual leader) be the leading state, and the final common states of other clusters are the delay states of $S_1(t)$, τ is time delay and $\tau_1 = 0$, P_h is the connection weight.

As the basic premise of the follow-up content, several definitions and assumptions are given.

Definition 1: The cluster consensus of MASs with (1) and (5) is said to be reached, if the solutions of systems (1) and systems (5) satisfy $\lim_{t\to+\infty} \|\tilde{x}_i(t)\| = 0$, where $\tilde{x}_i(t) = x_i(t) - S_{\hat{i}}(t)$.

Definition 2: The delay consensus of MASs with (5) is said to be reached, if the solutions of systems (5) satisfy $\lim_{t\to+\infty} \|e_h(t)\| = 0$, where $e_h(t) = S_h(t) - S_1(t-\tau_h)$.

Definition 3: The MASs with (1) and (5) is said to reach cluster-delay consensus, if the solutions of systems (1) and systems (5) satisfy $\lim_{t\to+\infty} \|\tilde{x}_i(t)\| = 0$ and $\lim_{t\to+\infty} \|e_h(t)\| = 0$, respectively, where $\tilde{x}_i(t) = x_i(t) - S_{\hat{i}}(t)$ and $e_h(t) = S_h(t) - S_1(t-\tau_h)$.

Assumption 1: For any vector $\{x_i(t), x_0(t)\} \in R^n$, there exists a non-negative constant ϕ such that $\| f(x_i(t), t) - f(x_0(t), t)\| \le \varphi \|x_i(t) - x_0(t)\|$.

Assumption 2: In the topology of MASs, the virtual leaders in their own cluster are global reachable nodes, and the first leader is also the global reachable node of other leaders.

By using the Kronecker product, systems (5) can be rewritten as

$$\dot{e}(t) = (I_F \otimes I_n)(F(S(t), t) - F(S_1(t-\tau), t-\tau)) + (I_F \otimes A)e(t) - (\Lambda \otimes I_n)e(t), \tag{6}$$

where $F(S_1(t-\tau), t-\tau) = (f^T(S_1(t-\tau_1), t-\tau_1), \cdots, f^T(S_1(t-\tau_F), t-\tau_F))^T \in R^{F\times n}$, $e(t) \in R^{F\times n}$ and $F(S(t), t) = (f^T(S_1(t), t), \cdots, f^T(S_F(t), t))^T \in R^{F\times n}$, $\Lambda = diag(P_1, P_2, \cdots, P_F)$ is a diagonal matrix.

Based on (1) and (4), the systems' model can be obtained as

$$\begin{cases} \dot{x}_i(t) = f(x_i(t),t) + Ax_i(t) - P_{\hat{i}}(S_{\hat{i}}(t) - S_1(t-\tau_{\hat{i}})), \quad t \neq t_k, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) \\ \qquad = \alpha B_k \sum_{j=1}^{N} a_{ij}(x_j(t_k^-) - S_{\hat{j}}(t_k^-) - (x_i(t_k^-) - S_{\hat{i}}(t_k^-))) + (B_k - I_n)\beta(x_i(t_k^-) - S_{\hat{i}}(t_k^-)), \quad t = t_k, \end{cases} \tag{7}$$

where $\{t_k\}$ is the impulse sequence, which satisfies $0 \le t_0 < \cdots < t_k$ and $\lim_{k\to+\infty} t_k = +\infty$. Let the state of agent i is left-hand continuous at impulse instants, that is $x_i(t_k) = x_i(t_k^-)$. Let $\Delta t_k = t_{k+1} - t_k$ denotes the impulsive interval. There exists a real number R^* such that $\sup\{t_{k+1} - t_k\} = R^* < +\infty$.

Systems (7) can be rewritten as

$$\begin{cases} \dot{\tilde{x}}(t) = (I_N \otimes A)\tilde{x}(t) + (I_N \otimes I_n)(F(x(t),t) - F(S(t),t)), \quad t \neq t_k, \\ \tilde{x}(t_k^+) = (I_{Nn} - \alpha L \otimes B_k + (\beta I_n) \otimes (B_k - I_n))\tilde{x}(t_k^-), \quad t = t_k, \end{cases} \tag{8}$$

where $\tilde{x}^T(t) \in R^{N\times n}$ and $F(x(t),t) = (f^T(x_1,t),\cdots,f^T(x_N,t))^T \in R^{N\times n}$.

3. Main Results

By the impulsive protocol (4), several sufficient criteria for cluster-delay consensus of first-order nonlinear MASs with (1) and (5) are given.

Theorem 1: Based on the above preparations, if the following conditions (1) and (2) are satisfied, and there exists the solutions of systems (1) and systems (5) such that $\lim_{t\to+\infty}\|\tilde{x}_i(t)\| = 0$ and $\lim_{t\to+\infty}\|e_h(t)\| = 0$, then the cluster-delay consensus of MASs with (1) and (5) is said to be reached via impulsive control.

Condition (1): There exists a constant ϑ such that $1 < \vartheta \le 1\big/\left\{\lambda_{k2}\exp\left[2\lambda_{k1}\left(t_k - t_{k-1}\right)\right]\right\}$, where $r > 0$, $E = I_{Nn} - \alpha L \otimes B_k + (\beta I_N) \otimes (B_k - I_n)$, $\lambda_{k1} = \lambda_{\max}(I_N \otimes A + rI_{Nn})$ and $\lambda_{k2} = \lambda_{\max}(E^T E) \in (0,1)$ are the maximum eigenvalue of the corresponding matrix, respectively.

Condition (2): There exist appropriate parameters such that matrix $Q = (\varepsilon I_F) \otimes I_n + I_F \otimes A - \Lambda \otimes I_n$ is negative definite , where $\varepsilon > 0$ and $\Lambda = diag(P_1, P_2, \cdots, P_F)$.

Proof ① Construct the Lyapunov function $V(t) = \tilde{x}^T(t)\tilde{x}(t)/2$.

When $t \in (t_{k-1}, t_k], k \in N_+$, the derivative of the function along the solution trajectory of systems (8) is expressed as $\dot{V}(t) = \tilde{x}^T(t)\dot{\tilde{x}}(t)$.

From Assumption 1, if there exists a constant $r > 0$ such that $F(x(t),t) - F(S(t),t) \le r\tilde{x}(t)$, then we have $\dot{V}(t) \le \tilde{x}^T(t)(I_N \otimes A + rI_{Nn})\tilde{x}(t)$. Due to $(I_N \otimes A + rI_{Nn})^T = (I_N \otimes A + rI_{Nn})$, we can get

$$\dot{V}(t) \le \lambda_{k1}\tilde{x}^T(t)\tilde{x}(t) = 2\lambda_{k1}V(t). \tag{9}$$

The integral of equation (9) is obtained easily, that is

$$V(t) \le V(t_{k-1}^+)\exp\left[2\lambda_{k1}\left(t - t_{k-1}\right)\right]. \tag{10}$$

When $t = t_k, k \in N_+$, it can be derived as follows.

$$V(t_k^+) \le \lambda_{k2}\tilde{x}^T(t_k^-)\tilde{x}(t_k^-)/2 = \lambda_{k2}V(t_k^-). \tag{11}$$

For $t = (t_k, t_{k+1}], k \in N_+$, the expression of $V(t)$ is computed. When $t \in (t_0, t_1]$, we have $V(t_1) \le V(t_0^+)\exp\left[2\lambda_{k1}\left(t_1 - t_0\right)\right]$. According to equation (11), one has $V(t_1^+) \le \lambda_{12}V(t_0^+)\exp\left[2\lambda_{k1}\left(t_1 - t_0\right)\right]$. Similarly, when $t \in (t_1, t_2]$, we can get $V(t) \le \lambda_{12}V(t_0^+)\exp\left[2\lambda_{k1}\left(t - t_0\right)\right]$. If $k = 2$, we have $V(t_2^+) \le \lambda_{12}\lambda_{22}V(t_0^+)\exp\left[2\lambda_{k1}\left(t_2 - t_0\right)\right]$. By analogy, for $t = (t_k, t_{k+1}]$, equation (12) can be obtained by mathematical induction.

$$\begin{aligned} V(t) &\le \lambda_{12}\lambda_{22}\cdots\lambda_{k2}V(t_0^+)\exp\left[2\lambda_{k1}\left(t-t_0\right)\right] \\ &=\left(1/\vartheta^k\right)V(t_0^+)\exp\left[2\lambda_{k1}\left(t-t_k\right)\right]\vartheta\lambda_{k2}\exp\left[2\lambda_{k1}\left(t_k-t_{k-1}\right)\right]\cdots\vartheta\lambda_{12}\exp\left[2\lambda_{k1}\left(t_1-t_0\right)\right]. \end{aligned} \tag{12}$$

If there exist $\vartheta > 1$ and $\lambda_{k2} \in (0,1)$ such that $\vartheta \le 1\Big/\left\{\lambda_{k2}\exp\left[2\lambda_{k1}\left(t_k-t_{k-1}\right)\right]\right\}$, then equation (12) can be rewritten as

$$V(t) \le V(t_0^+)\exp\left[2\lambda_{k1}\left(t-t_k\right)\right]\Big/\vartheta^k. \tag{13}$$

By equation (13), there exists $\sup\left\{t_{k+1}-t_k\right\} = R^* < +\infty$ such that $V(t) \to 0$ and $\lim_{t\to+\infty}\left\|\tilde{x}_i(t)\right\| = 0$, where $t \to +\infty$. Thus, the cluster consensus of MASs (1) and (5) can be reached via impulsive control.

②Construct the Lyapunov function $W(t) = e^T(t)e(t)/2$. The derivative of the function along the solution trajectory of systems (7) is expressed as $\dot{W}(t) = e^T(t)\dot{e}(t)$.

By Assumption 1, if there exists a constant $\varepsilon > 0$ such that $F(S(t),t) - F(S_1(t-\tau),t-\tau) \le \varepsilon e(t)$, then we have

$$\dot{W}(t) \le e^T(t)Qe(t). \tag{14}$$

Equation (14) shows that if $\dot{W}(\mathrm{t}) \le 0$ always hold, matrix Q must and will be negative definite. Therefore, suppose that there exist $\varepsilon > 0$ and $\Lambda = diag(P_1, P_2, \cdots, P_F)$ such that $\lambda_{\max}(Q) < 0$, then equation (14) can be rewritten as $\dot{W}(\mathrm{t}) \le 2\lambda_{\max}(Q)W(t)$. Obviously, it is easy to get that

$$W(t) \le W(0)\exp\left[2\lambda_{\max}\left(Q\right)t\right]. \tag{15}$$

According to equation (15), when $t \to +\infty$, we can get $W(t) \to 0$ and $\lim_{t\to+\infty}\left\|e_h(t)\right\| = 0$. That is to say, the delay consensus of MASs (5) can be reached.

In summary, the cluster-delay consensus of first-order nonlinear MASs (1) and (5) can be reached via impulsive control. The proof is completed.

4. Numerical Simulation

In this section, we illustrate the feasibility of the above theoretical results by numerical simulation.

The MASs composed by eight follower agents are considered as shown in figure 1, where the virtual leaders from the first cluster to the third cluster are S_1, S_2 and S_3, respectively. For convenient, let $n = 1$, the initial position states of each agent are chosen as $S_1(0) = 1$, $S_2(0) = 5$, $S_3(0) = 10$, $x_1(0) = -4$, $x_2(0) = 5$, $x_3(0) = 12$, $x_4(0) = -10$, $x_5(0) = 15$, $x_6(0) = 7$, $x_7(0) = -0.5$ and $x_8(0) = 10$. Let $f(x_i(t),t) = x_i(t)\sin(t^2)$ and $f(S_h(t),t) = S_h(t)\sin(t^2)$. Obviously, we have $\left|f(x_i(t),t) - f(S_{\hat{i}}t),t)\right| \le \left|x_i(t) - S_{\hat{i}}(t)\right|$ and $\left|f(S_h(t),t) - f(S_1(t-\tau_h),t)\right| \le \left|S_h(t) - S_1(t-\tau_h)\right|$, then $r = \varepsilon = 1$. Meanwhile, the other parameters are chosen as $\alpha = 0.1$, $\beta = 0.2$, $t_{k+1} - t_k = 0.01$, $B_k = diag(0.7, \cdots, 0.7) \in R^{8\times 8}$, $\Lambda = diag(12,12,12)$ and $A = I_n$.

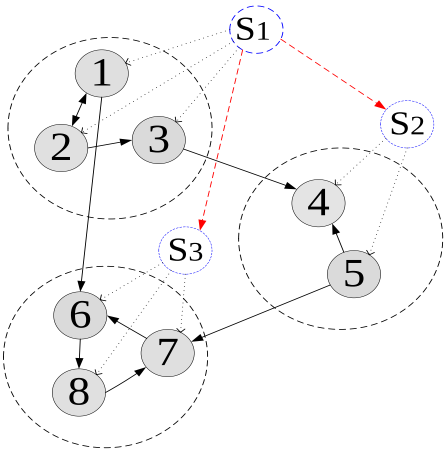

Figure.1 The topology graph of MASs.

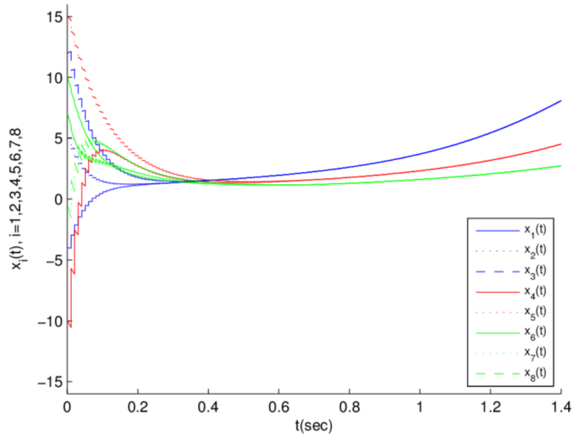

Figure.2 The position states of agents.

Based on the above parameters, a series of results can be obtained by calculation, such as $\lambda_{k1}=2$, $\lambda_{k2}\approx 0.925\in(0,1)$, $1<\vartheta\le 1.039$ and $\lambda_1=\lambda_2=\lambda_3=-10<0$. That is to say, the conditions (1) and (2) in Theorem 1 are satisfied. Suppose that $\tau_2=0.3$s and $\tau_3=0.6s$, then the following figures are obtained by simulations.

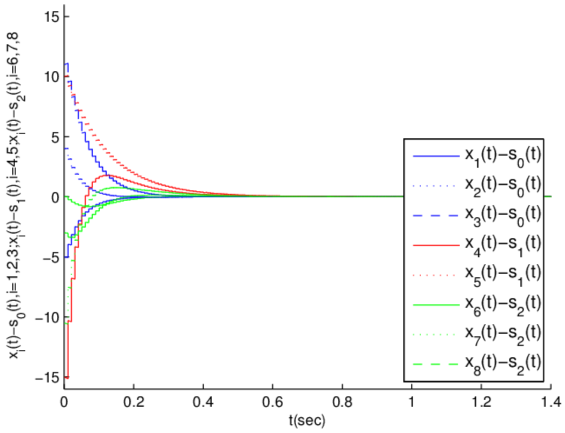

Figure.3 The state errors between each follower agent and their corresponding virtual leaders.

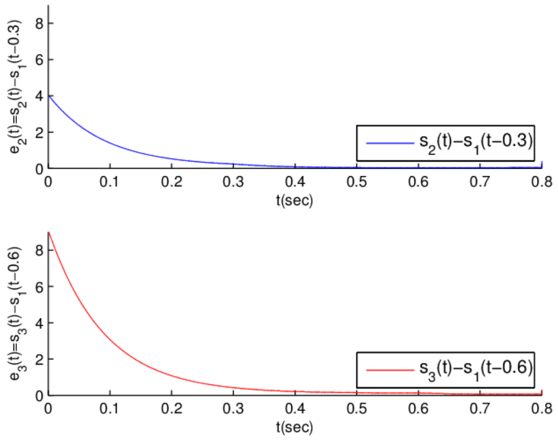

Figure.4 The trajectories of state errors $e_2(t)$ and $e_3(t)$.

According to figure 2 and figure 3, the state of agents in each cluster converges to the corresponding virtual leader's state, and equation $\lim_{t\to+\infty}\|\tilde{x}_i(t)\|=0$ is ensured to be hold. Therefore, the cluster consensus of MASs (1) and (5) can be achieved via impulsive control. According to figure 4, it can be seen that $\lim_{t\to+\infty}\|e_h(t)\|=0$. Therefore, the delay consensus of MASs (5) can be achieved via impulsive control. In summary, the correctness of protocol (4) is illustrated by simulation. Therefore, the cluster-delay consensus of MASs (1) and (5) can be achieved via impulsive control.

5. Conclusions

For a first-order nonlinear MASs, the impulsive protocol is designed to make the systems to reach the cluster-delay consensus. According to the impulsive protocol, the update of controller of *ith* agent is related to the state information of all neighboring agents. Based on the graph theory and Lyapunov stability theory, several sufficient criteria for the cluster-delay consensus are given. Finally, the feasibility of the theoretical results is illustrated through simulations.

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