

# Analysis of resistance characteristics of liquid flow in microfluidic channels

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**Abstract.** In order to investigate the resistance characteristics of liquid flow in microfluidic channels, the mathematical models of liquid flow resistance in microfluidic channels are established in this paper. The microfluidic channel samples with different parameters are designed and fabricated by using soft lithography technology. The effects of channel parameters of channel aspect ratio  $\alpha$ , channel equivalent diameter  $d_l$  and channel length/diameter ratio  $l/d_l$  on liquid flow resistance are analysed and tested through experiments. This paper reveals the flow resistance performances in microfluidic channels and provides research basis for improving the flow characteristics and optimizing the microfluidic channel structures.

## 1. Introduction

Nowadays, microfluidic technology has been developed widely, various microfluidic devices have been applied for medical diagnose<sup>1,2</sup>, chemical detecting<sup>3,4</sup> and optical analysis<sup>5</sup>. Kim et al. presented a new approach that integrates all components into a single device<sup>6</sup>, which controlled exposure of isolated single cells to a high pH buffer. Shaegh et al. reported a multi-analyte optical sensing module for dynamic measurements of pH and dissolved oxygen levels in the culture medium<sup>7</sup>, which can be adapted for applications in various microfluidic cell culture and organ-on-chip devices.

In recent studies, we fabricated a microfluidic liquid colour-changing lens<sup>8,9</sup> based on the PDMS material. PDMS optical membranes with microfluidic channel structures were fabricated by soft lithography technology<sup>10</sup>. Some applications of the microfluidic lens were realized, including vision protection and camouflage<sup>11</sup>. However, the liquid flow resistance characteristics in the microfluidic channels have not been in-depth investigated until now.

In this paper, the mathematical models of liquid flow resistance in the channels are established, the deviation between liquid flow resistance in microfluidic channels and that in conventional pipelines is provided and discussed. Different microfluidic channel samples are designed and fabricated for flow resistance tests. The effects of channel parameters of channel aspect ratio, channel equivalent diameter and channel length/diameter ratio on flow resistance are investigated.



## 2. Mathematical models of liquid flow resistance in microfluidic channels

In this paper, soft lithography technology is employed to make the microfluidic channels, the channel cross section is rectangular. The liquid flow in microfluidic channels is regarded as laminar flow. According to fluid mechanics, the Reynolds number  $Re$  of liquid flow can be expressed as:

$$Re = \frac{\bar{v} d_l}{\mu} \quad (1)$$

where  $\bar{v}$  is the average flow velocity,  $\rho$  is liquid density,  $d_l$  is channel equivalent diameter,  $\mu$  is liquid hydrodynamic viscosity.

The channel equivalent diameter  $d_l$  can be expressed as:

$$d_l = \frac{2wh}{w+h} \quad (2)$$

where  $w$  is the channel width and  $h$  is the channel depth.

The flow characteristics in microfluidic channels with rectangular cross section can be expressed as<sup>12</sup>:

$$Q = \frac{wh^3 \Delta p}{12\mu l} \left[ 1 - \sum_{n, \text{odd}} \frac{1}{n^5} \frac{192h}{\pi^5 w} \tanh(n\pi \frac{w}{2h}) \right] \quad (3)$$

where  $Q$  is the volume flow rate in channels,  $\Delta p$  is the differential pressure at both ends of channels,  $l$  is the channel length.

In this study, all the microfluidic channels are flat with rectangular cross section, that is, the channel depth  $h$  is less than the channel width  $w$ , therefore, equation (3) can be simplified as follows<sup>13</sup>:

$$Q = \frac{wh^3 (1 - 0.63 \frac{h}{w}) \Delta p}{12\mu l}, \quad h < w \quad (4)$$

According to equation (1) to (4) and Bernoulli's equation, the loss of liquid laminar flow in the channels can be expressed as:

$$\Delta p = f \cdot \frac{3w\rho l \bar{v}^2}{8h(w+h)(1 - 0.63 \frac{h}{w})} \quad (5)$$

where  $f$  is the flow resistance coefficient in channels.

The average flow velocity  $\bar{v}$  can be expressed as:

$$\bar{v} = \frac{Q_m}{\rho wh} \quad (6)$$

where  $Q_m$  is the mass flow rate in channels ( $kg/s$ ).

According to equation (1), (2) and (6), it can be concluded that the relationship between Reynolds number, the average liquid velocity and channel parameters can be expressed as follows:

$$Re = \frac{2Q_m}{\mu(w+h)} \quad (7)$$

Inserting equation (6) into equation (5) leads to:

$$\Delta p = f \cdot \frac{3lQ_m^2}{8\rho wh^3(w+h)(1 - 0.63 \frac{h}{w})} \quad (8)$$

According to equation (7) and (8), the Poisson number of liquid laminar flow in channels can be expressed as:

$$Po = f \cdot Re = \frac{16\rho wh^3(1-0.63\frac{h}{w})\Delta p}{3\mu l Q_m} \quad (9)$$

The resistance coefficient  $f_0$  of liquid laminar flow in fully developed conventional circular pipelines can be expressed as:

$$f_0 = \frac{64}{Re} \quad (10)$$

Compared to conventional circular pipelines, the flow characteristics in microfluidic channels with rectangular cross section are greatly affected by channel aspect ratio  $\alpha$  ( $\alpha = w/h$ ). The relationship between the resistance coefficient in microfluidic channels and that in conventional circular pipelines can be expressed as:

$$f = f_0 \cdot F \quad (11)$$

where  $F$  is the resistance coefficient adjustment factor.

In equation (11), the resistance coefficient adjustment factor  $F$  can be expressed as:

$$F = \frac{3}{2}(1-1.3553/\alpha+1.9467/\alpha^2-1.7012/\alpha^3+0.9564/\alpha^4-0.2537/\alpha^5) \quad (12)$$

where  $\alpha$  is the channel aspect ratio,  $\alpha = w/h$ ,  $0 < 1/\alpha < 1$ .

According to equation (10) to (12), the following expression can be obtained:

$$f = \frac{96}{Re}(1-1.3553/\alpha+1.9467/\alpha^2-1.7012/\alpha^3+0.9564/\alpha^4-0.2537/\alpha^5) \quad (13)$$

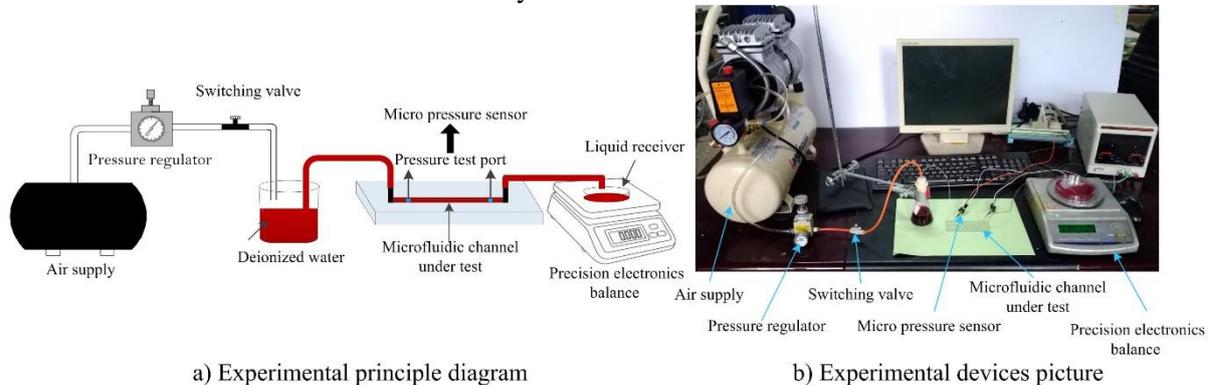
### 3. Experimental tests on flow resistance characteristics in microfluidic channels

In order to investigate the relationship between channel parameters and flow resistance characteristics, microfluidic channel samples with different structures are designed and made, as shown in table 1, the thickness of channel top film is 5mm, ignoring the influence of channel deformation. The key channel parameters are studied in this paper, including channel aspect ratio  $\alpha$ , channel equivalent diameter  $d_l$  and channel length/diameter ratio  $l/d_l$ .

Table 1. Microfluidic channel samples with rectangular cross section for flow resistance tests

Number	Channel parameters					
	$w$ ( $\mu\text{m}$ )	$h$ ( $\mu\text{m}$ )	$\alpha$	$d_l$ ( $\mu\text{m}$ )	$l$ (mm)	$l/d_l$
1	400	100	4	160.0	50	312
2	600	100	6	171.43	55	320
3	800	100	8	177.78	60	337
4	1000	100	10	181.82	60	330
5	1200	100	12	184.62	60	325
6	600	150	4	240.0	75	312
7	800	200	4	320.86	100	312
8	400	100	4	160.0	40	256
9	400	100	4	160.0	30	187
10	400	100	4	160.0	20	125
11	400	100	4	160.0	15	94
12	400	100	4	160.0	10	63
13	400	100	4	160.0	5	31

Figure 1 shows the experimental system for liquid flow resistance tests. The liquid pressure at inlet and outlet of the channel is measured by micro pressure sensor (Honeywell Corp.), and the mass flow rate of liquid flow is measured by precision electronic balance. During the experiments, only the flow resistance characteristics are tested when Reynolds number  $Re$  is less than 1000.



a) Experimental principle diagram  
b) Experimental devices picture  
Figure 1. Experimental system for flow resistance tests of microfluidic channels.

Figure 2 shows the test results of flow resistance coefficient  $f$  under different  $\alpha$  with a certain  $d_l$  and  $l/d_l$  (number 1-5 in table 1). It can be seen that, the resistance coefficient calculated according to equation (10) and (13) are 0.183 and 0.209 when  $\alpha$  is 4 and  $Re$  is 349, the test result is 0.225. When  $\alpha$  is 12 and  $Re$  is 491, the calculated results according to equations are 0.130 and 0.175 respectively, and the test result is 0.205. Therefore, it can be concluded that the experimental results are basically consistent with the calculated results of equation (13), the flow characteristics in the rectangular microchannel also accord with the prediction of traditional theory. By comparing of the results of equation (10) and (13), the deviation between them gradually increases as  $\alpha$  increases. Therefore, the calculation method for conventional pipelines is not precise enough for microfluidic channels with rectangular cross section.

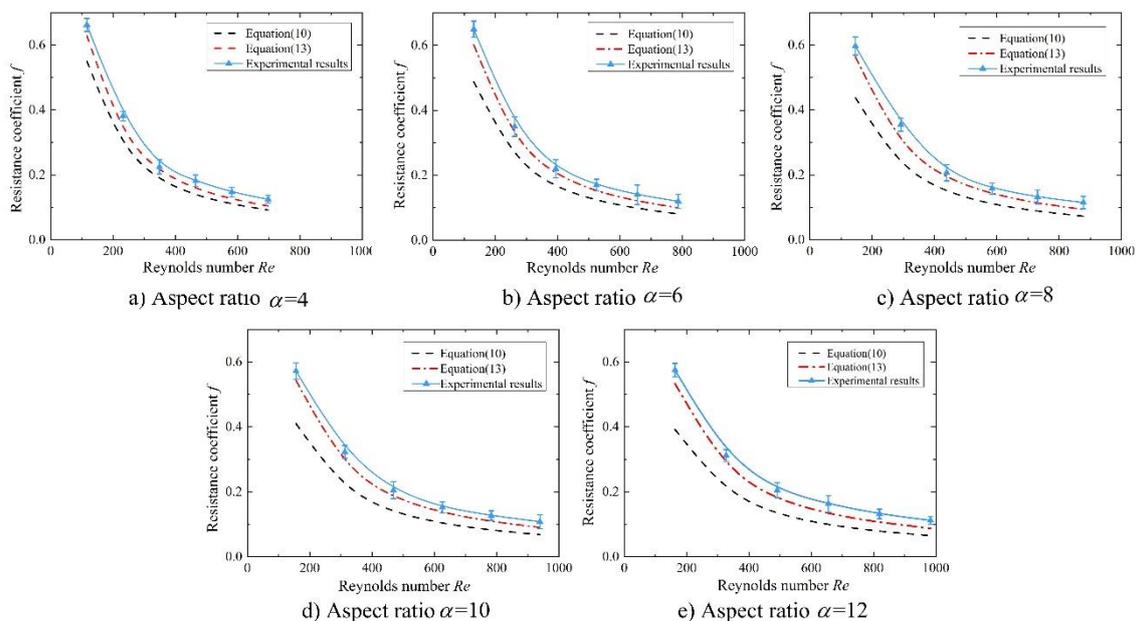


Figure 2. Flow resistance coefficient of microfluidic channels with different aspect ratios.

In order to visually analyse the influence of channel aspect ratio  $\alpha$  on flow resistance characteristics, the test results in figure 2 are drawn in the same coordinate system, as shown in figure

3. Figure 3(a) shows that the smaller the aspect ratio  $\alpha$  is, the smaller the resistance coefficient  $f$  becomes when the channel depth  $h$  is  $100\ \mu\text{m}$ . Meanwhile, the resistance coefficient  $f$  gradually decreases as the Reynolds number  $Re$  increases. Figure 3(b) shows that the Poisson number  $Po$  increases as the increase of Reynolds number  $Re$  and channel aspect ratio  $\alpha$ .

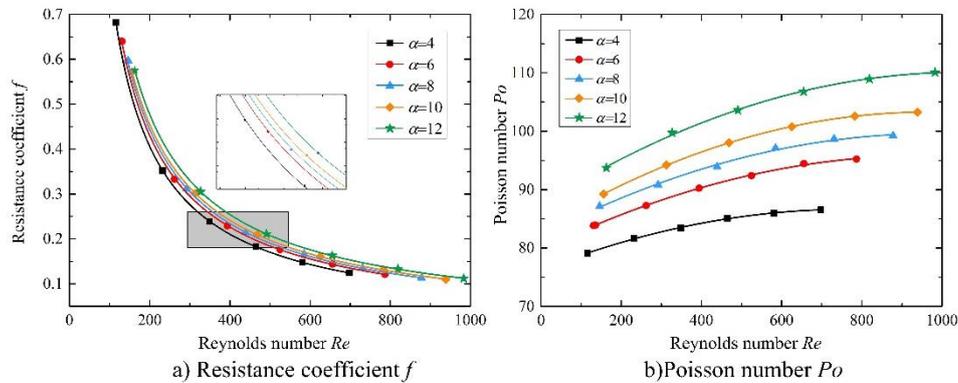


Figure 3. Flow resistance characteristics of microfluidic channels with different aspect ratios.

Figure 4 presents the test results of liquid resistance coefficient with different equivalent diameters  $d_l$  (number 1, 6, 7 in table 1). It can be concluded that the equivalent diameter  $d_l$  decreases as the resistance coefficient  $f$  increase with a certain  $\alpha$ , and the Poisson number  $Po$  gradually decreases as the equivalent diameter  $d_l$  increases. Compared with the constant Poisson number ( $Po = 72.9$ ) of conventional pipelines, the variation range of Poisson number in microfluidic channels is about 6%~40%. Therefore, the flow characteristics in microfluidic channels are greatly affected with the change of equivalent diameter  $d_l$ .

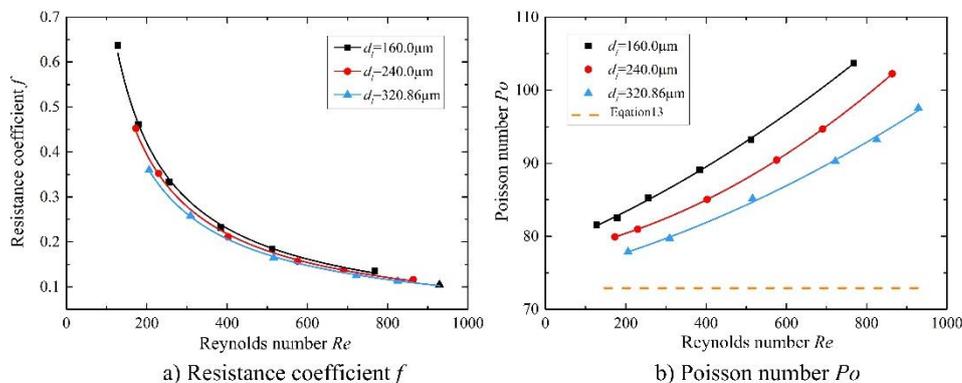


Figure 4. Flow resistance characteristics of microfluidic channels with different equivalent diameters.

Figure 5 shows the relationship between liquid resistance coefficient  $f$  and length-diameter ratio  $l/d_l$  when  $\alpha$  is 4,  $d_l$  is  $160\ \mu\text{m}$  and  $Re$  is 512 (number 8-13 in table 1). It can be concluded that when  $l/d_l$  is smaller than 63, the resistance coefficient  $f$  changes significantly, and when  $l/d_l$  is larger than 63, it gradually approaches the constant value of equation (13), and the inlet effect of liquid flow in channels can be ignored in this region. Therefore, in order to eliminate the inlet effect, the length-diameter ratio  $l/d_l$  of all channel is larger than 63 in this paper.

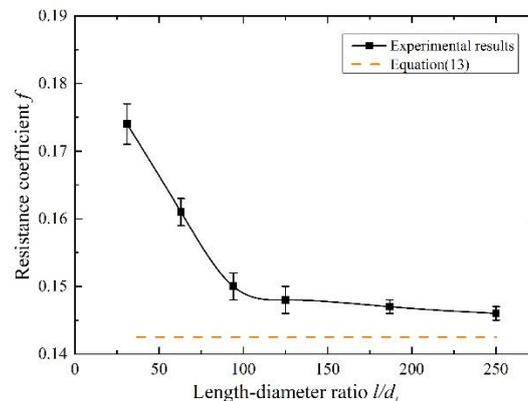


Figure 5. Flow resistance coefficient of microfluidic channels with different length-diameter ratios.

#### 4. Conclusions

From the theoretical analysis and experimental investigations in this study, it can be seen that there is a certain deviation between the flow resistance characteristics of liquid flow in microfluidic channels and that in traditional macroscopic pipelines. The test results show that the liquid flow in microfluidic channels is greatly affected by the channel parameters, including channel aspect ratio  $\alpha$ , channel equivalent diameter  $d_l$  and channel length-diameter ratio  $l/d_l$ . This study reveals the flow resistance performances and provides the research basis for channel structure optimization in further applications.

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