

# Adaptive Dynamic Sliding Mode Control Laws for Attitude Stabilization of Flexible Spacecraft

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**Abstract.** In this paper, an adaptive dynamic sliding mode control technique is adopted to stabilize the attitude of flexible spacecraft with disturbance. The mathematical model of flexible spacecraft is constructed by the modified Rodrigues parameters. The adaptive control technique is applied to guarantee stabilization of flexible spacecraft with disturbance. By introducing a dynamic switching function, the chattering caused by sign function of the traditional sliding mode control law can be eliminated. Finally, by comparing with the adaptive control law proposed in [12], a numerical example is employed to illustrate the effectiveness of the designed control law.

## 1. Introduction

Attitude stabilization control has been a vital part of spacecraft control design. The performance of spacecraft is greatly influenced by internal uncertainties and external disturbances, etc. Nowadays, the space missions become more and more complex. Thus, high accuracy and high efficiency are required in spacecraft attitude stabilization. Not only spacecraft attitude stabilization but also anti-interference performance should be guaranteed at same time [1]. Many attitude control methods have been proposed to ensure the spacecraft attitude stable, such as PID control [2], sliding mode control [3], adaptive control [4], and optimal control [5], etc.

Sliding mode control is insensitive to internal uncertainty and external disturbance in nonlinear systems. In [6], a sliding mode controller was designed to deal with the spacecraft tracking problems, and the stability of the closed-loop system is analyzed. Sliding mode control methods consist of equivalent control and switching control. The sign function of switching control will cause chattering problem when the frequency is high. Many modified sliding mode control methods were used to solving this problem. In [7], by transferring the discontinuous control variables to high order derivative of sliding mode switching function, the high order sliding mode control law was designed to track the spacecraft attitude and eliminate chattering. Also, by substituting saturation function for sign function, a variable-structure sliding mode control method is proposed to process spacecraft attitude maneuvers [8]. In [9], by introducing discontinuity term of sign function into the first or higher order derivatives of the control law, a new switching function was designed by the derivative of previous switching function. A dynamic sliding mode control method was constructed to stabilize attitude and eliminate chattering of flexible spacecraft.

Adaptive control methods were universally applied in spacecraft attitude stabilization area because of its strong adaptive ability to disturbance. In [10], a robust adaptive controller with parameter update laws was designed to solve the dead-zone nonlinear problem in spacecraft attitude tracking. In [11], an adaptive finite-time backstepping control law was constructed to achieve attitude tracking in finite-time. In [12], adaptive control law was complemented to stabilize attitude of flexible spacecraft with



disturbance. But over-adaptive problem is easy to occur in adaptive control technique. Thus, the adaptive sliding mode control law was proposed to stabilize the flexible spacecraft attitude. In [13], a backstepping based adaptive sliding mode control law was designed to solve the attitude problem of rigid spacecraft with parameter variation and external disturbance.

In this paper, adaptive dynamic sliding mode control method was proposed to ensure stabilization and eliminate chattering in flexible spacecraft attitude control. The attitude maneuver of flexible spacecraft was established by the modified Rodrigues parameters. Adaptive control was used to reduce the effect caused by external disturbance, while dynamic sliding mode control was constructed to stabilize the spacecraft attitude and eliminate chattering. The Lyapunov law was used to analyze that the controller designed by adaptive dynamic sliding mode control technique can guarantee the stability of the flexible spacecraft with disturbance. Finally, the simulation results showed that the adaptive dynamic sliding mode control law can stabilize attitude of the flexible spacecraft with disturbance and eliminate chattering.

## 2. Problem formulation

In this section, the attitude kinematics of flexible spacecraft can be described by the modified Rodrigues parameters as follows

$$\dot{p} = \frac{1}{4} \{ (1 - p^T p) \mathbf{I}_{3 \times 3} + 2p^\times + 2pp^T \} \omega = F(p)\omega, \quad (1)$$

where  $\omega = [\omega_1 \ \omega_2 \ \omega_3]^T \in \mathbf{R}^3$  represents the angular velocity of the rigid body of flexible spacecrafts, and  $p = [p_1 \ p_2 \ p_3]^T \in \mathbf{R}^3$  represents the attitude description relative to inertial system, there exists

$$p = \mathbf{n} \tan(\phi / 4),$$

where  $\mathbf{n}$  is unit Euler main axis vector,  $\phi \in (0, 360^\circ)$  is Euler axis rotation angle. Define  $x^\times$  as a skew-symmetric matrix

$$x^\times = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix},$$

Supposing that the elastic deformations is small, by applying Euler's theorem, the attitude kinematics of flexible spacecraft with disturbance can be built as

$$\begin{cases} J\dot{\omega} + \delta^T \dot{\eta} = -\omega^\times (J\omega + \delta^T \dot{\eta}) + u + d, \\ \ddot{\eta} + C\dot{\eta} + K\eta = -\delta\ddot{\omega}, \end{cases} \quad (2)$$

where symmetric matrix  $J$  is the inertia matrix of the flexible spacecraft,  $J_{mb}$  is the inertia matrix of rigid body of flexible spacecraft, and  $\sigma$  is the coupling matrix of rigid body and flexible appendages,  $\eta$  is the flexible modal displacement of flexible appendages.  $u$  is the external torque acts on rigid body,  $d$  is the boundary external disturbance torque acts on flexible spacecraft.  $C, K$  is the damping matrix and stiffness matrix of flexible spacecraft respectively. Considering  $N$ -order flexible modal, define

$$C = \text{diag} \{ 2\zeta_i \omega_{ni}, i = 1, \dots, N \},$$

$$K = \text{diag} \{ \omega_{ni}^2, i = 1, \dots, N \}.$$

where  $\zeta_i, \omega_{ni}, i = 1, 2, \dots, N$  are the vibration frequency and damping ratio of the flexible modal displacements.

Based on the modified Rodrigues parameters, the mathematical model of flexible spacecraft can be described as

$$\begin{cases} \dot{p} = F(p)\omega, \\ \begin{bmatrix} \dot{\eta} \\ \dot{\psi} \end{bmatrix} = B_1 \begin{bmatrix} \eta \\ \psi \end{bmatrix} + B_2 \delta\omega, \\ \dot{\omega} = J_{mb}^{-1} \left[ -\omega^\times (J_{mb} \omega + \delta^T \psi) + \delta^T (C\psi + K\eta - C\delta\omega) + u + d \right]. \end{cases} \quad (3)$$

Where

$$B_1 = \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix}, B_2 = \begin{bmatrix} -I \\ C \end{bmatrix}.$$

In this paper, the control law should be designed to stabilize the attitude  $(p, \omega)$ , which means  $\lim_{t \rightarrow \infty} p(t) = 0, \lim_{t \rightarrow \infty} \omega(t) = 0$ . At the same time, the chattering of flexible appendages should be reduced.

### 3. Design of Adaptive Dynamic Sliding Mode Control Law

In this section, the adaptive dynamic sliding mode control method is proposed to stabilize attitude of flexible spacecraft with disturbance and eliminate chattering. Firstly, a switching function was designed as follow

$$s = \omega + \lambda p, \quad (4)$$

where  $\lambda$  is a strictly positive constant. When the state variables arrive to the sliding mode surface  $s = 0$ , the results that  $\lim_{t \rightarrow \infty} p(t) = 0, \lim_{t \rightarrow \infty} \omega(t) = 0$  can be concluded, which means the flexible spacecraft system can be stabilized on the sliding model surface  $s = 0$ .

Considering the following Lyapunov function with  $k_1$  is strictly positive constant,

$$V_1 = 2k_1 p^T p, \quad (5)$$

when  $s = 0$ , its derivative of (5) is given by

$$\begin{aligned} \dot{V}_1 &= 4k_1 p^T \dot{p} = k_1 p^T \left[ (1 - p^T p) I_{3 \times 3} + 2p^\times + 2pp^T \right] \omega \\ &= -\lambda k_1 p^T p + \lambda k_1 p^T p^T p p - 2\lambda k_1 p^T p^\times p - 2\lambda k_1 p^T p p^T p \\ &= -\lambda k_1 p^T p - \lambda k_1 p^T p p^T p - 2\lambda k_1 p^T p^\times p, \end{aligned} \quad (6)$$

where  $p^T p^\times p$  is given by

$$p^T p^\times p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix} \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = 0,$$

It can be obtained that

$$\dot{V}_1 = -\lambda k_1 p^T p - \lambda k_1 (p^T p)^2 \leq 0. \quad (7)$$

which implies that  $\lim_{t \rightarrow \infty} p(t) = 0$ . From the equation (4), we can conclude that  $\lim_{t \rightarrow \infty} \omega(t) = 0$ , so the flexible spacecraft model is asymmetric stable on the sliding mode surface.

Once the state variables arrive at the sliding mode surface, the system can be stabilized. Its derivative of (4) is given by

$$\begin{aligned}
\dot{s} &= \dot{\omega} + \lambda \dot{p} \\
&= J_{mb}^{-1} \left[ -\omega^\times (J_{mb} \omega + \delta^T \psi) + \delta^T (C\psi + K\eta - C\delta\omega) + u + d \right] + \lambda F(p)\omega \\
&= A(p, \omega, \eta, \psi) + J_{mb}^{-1} u + J_{mb}^{-1} d,
\end{aligned} \tag{8}$$

where

$$A(p, \omega, \eta, \psi) = J_{mb}^{-1} \left[ -\omega^\times (J_{mb} \omega + \delta^T \psi) \right] + J_{mb}^{-1} \left[ \delta^T (C\psi + K\eta - C\delta\omega) \right] + \lambda F(p)\omega.$$

To construct a first-order dynamic system, the first derivative of sliding mode  $s = 0$  was applied to achieve a new switching function as follow

$$\sigma = \dot{s} + \alpha s = A(p, \omega, \eta, \psi) + J_{mb}^{-1} u + J_{mb}^{-1} d + \alpha s, \tag{9}$$

where  $\alpha$  is a strictly positive constant.

It is known that  $\dot{s} + \alpha s = 0$  is a first-order asymmetric stable dynamic system when  $\alpha \neq 0$ . It can be concluded that  $\lim_{t \rightarrow \infty} s = 0, \lim_{t \rightarrow \infty} \dot{s} = 0$ . The derivative of (9) is obtained as

$$\begin{aligned}
\dot{\sigma} &= \dot{A}(p, \omega, \eta, \psi) + J_{mb}^{-1} \dot{u} + J_{mb}^{-1} \dot{d} + \alpha \dot{s} \\
&= \dot{A}(p, \omega, \eta, \psi) + J_{mb}^{-1} \dot{u} + J_{mb}^{-1} \dot{d} + \alpha \left( A(p, \omega, \eta, \psi) + J_{mb}^{-1} u + J_{mb}^{-1} d \right) \\
&= \dot{A}(p, \omega, \eta, \psi) + \alpha A(p, \omega, \eta, \psi) + \alpha J_{mb}^{-1} u + \alpha J_{mb}^{-1} d + J_{mb}^{-1} \dot{u} + J_{mb}^{-1} \dot{d},
\end{aligned} \tag{10}$$

Define  $v = \dot{u}, \bar{A}(p, \omega, \eta, \psi) = \dot{A}(p, \omega, \eta, \psi) + \alpha A(p, \omega, \eta, \psi) + \alpha J_{mb}^{-1} u$ , which can be concluded that the real input of the first-order dynamic system  $\dot{\sigma} = \dot{s} + \alpha s = 0$  is  $v$ . A new model can be obtained as follow:

$$\dot{\sigma} = \bar{A}(p, \omega, \eta, \psi) + J_{mb}^{-1} v + \alpha J_{mb}^{-1} d + J_{mb}^{-1} \dot{d}, \tag{11}$$

$u$  is continuous even  $v$  is not continuous, so it can be concluded that the discontinuous input  $v$  does not cause chattering problem of sliding mode surface  $s = 0$ .

Remark 1: For any vector  $x = [x_1 \ x_2 \ x_3]^T \in \mathbb{R}^3$ ,  $\text{sign}(x) = \begin{bmatrix} \frac{x_1}{|x_1|} & \frac{x_2}{|x_2|} & \frac{x_3}{|x_3|} \end{bmatrix}$  holds.

When  $d \neq 0$  and supposing that  $\|d\|, \|\dot{d}\|$  are boundary, design the adaptive control law as follows:

$$\begin{cases} v = J_{mb} \left( -\bar{A}(p, \omega, \eta, \psi) + \theta \right), \\ \theta = -k_1 \sigma - k_2 \text{sign}(\sigma) - \hat{d}(t). \end{cases} \tag{12}$$

Define  $d^* = \alpha J_{mb}^{-1} d + J_{mb}^{-1} \dot{d}, \tilde{d}(t) = \hat{d}(t) - d^*$ . The dynamic system (11) became

$$\dot{\sigma} = -k_1 \sigma - k_2 \text{sign}(\sigma) - \left[ \hat{d}(t) - d^* \right],$$

Consider the following Lyapunov function candidate

$$V_3(t) = \frac{1}{2} \sigma^T \sigma + \frac{1}{2\gamma} \tilde{d}^T(t) \tilde{d}(t), \tag{13}$$

The time derivative of (13) can be written as

$$\begin{aligned}
\dot{V}_3(t) &= \sigma^T \dot{\sigma} + \frac{1}{\gamma} \tilde{d}^T(t) \dot{\hat{d}}(t) \\
&= \sigma^T \left[ -k_1 \sigma - k_2 \text{sign}(\sigma) - \tilde{d}(t) \right] + \frac{1}{\gamma} \tilde{d}^T(t) \dot{\hat{d}}(t) \\
&= -k_1 \sigma^T \sigma - k_2 \sigma^T \text{sign}(\sigma) - \sigma^T \tilde{d}(t) + \frac{1}{\gamma} \tilde{d}^T(t) \dot{\hat{d}}(t) \\
&= -k_1 \sigma^T \sigma - k_2 \sigma^T \text{sign}(\sigma) + \frac{1}{\gamma} \tilde{d}^T(t) \left[ \dot{\hat{d}}(t) - \gamma \sigma \right],
\end{aligned} \tag{14}$$

Choose  $\dot{\hat{d}}(t) = \gamma \sigma$ , the derivative of  $V_3(t)$  became

$$\dot{V}_3(t) = -k_1 \sigma^T \sigma - k_2 \sigma^T \text{sign}(\sigma) < 0. \tag{15}$$

Thus, the control law is designed as

$$\begin{cases} v = -J_{\text{mb}} \bar{A}(p, \omega, \eta, \psi) - J_{\text{mb}} (k_1 \sigma - k_2 \text{sign}(\sigma) - \hat{d}(t)), \\ \dot{\hat{d}}(t) = \gamma \sigma. \end{cases} \tag{16}$$

which implies  $\lim_{t \rightarrow \infty} \sigma = 0, \lim_{t \rightarrow \infty} \tilde{d}(t) = 0$ . Combining (13) and (4), the conclusion  $\lim_{t \rightarrow \infty} p(t) = 0$ ,  $\lim_{t \rightarrow \infty} \omega(t) = 0$  holds.

It is difficult to get the value of  $\dot{A}(p, \omega, \eta, \psi)$  in actual system. A robust differentiator which is convergent in finite time is designed to overcome this drawback.  $z_0, z_1$  is the real-time estimations of  $A(p, \omega, \eta, \psi)$  and  $\dot{A}(p, \omega, \eta, \psi)$  respectively, and  $\gamma_0 > \gamma_1 > 0$ .

$$\begin{aligned}
\dot{z}_0 &= -\gamma_0 \|z_0 - A(p, \omega, \eta, \psi)\|^{1/2} \times \text{sign}(z_0 - A(p, \omega, \eta, \psi)) + z_1, \\
\dot{z}_1 &= -\gamma_1 \text{sign}(z_0 - A(p, \omega, \eta, \psi)).
\end{aligned} \tag{17}$$

In conclusion, the adaptive sliding mode control law for flexible spacecraft is established as follows:

$$\begin{cases} s = \omega + \lambda p, \\ \sigma = \dot{s} + \alpha s, \\ \dot{u} = v = J_{\text{mb}} \left( -\bar{A}(p, \omega, \eta, \psi) - k_1 \sigma - k_2 \text{sign}(\sigma) - \hat{d}(t) \right), \\ \dot{\hat{d}}(t) = \gamma \sigma, \\ \dot{z}_0 = -\gamma_0 \|z_0 - A(p, \omega, \eta, \psi)\|^{1/2} \times \text{sign}(z_0 - A(p, \omega, \eta, \psi)) + z_1, \\ \bar{A}(\omega, \eta, \psi) = \dot{A}(p, \omega, \eta, \psi) + \alpha A(p, \omega, \eta, \psi) + \alpha J_{\text{mb}}^{-1} u, \\ A(p, \omega, \eta, \psi) = J_{\text{mb}}^{-1} \left[ -\omega^\times (J_{\text{mb}} \omega + \delta^T \psi) + \delta^T (C\psi + K\eta - C\delta\omega) \right] + \lambda F(p)\omega \end{cases} \tag{18}$$

where  $\lambda, k_1, k_2, \alpha$  are strictly positive, and  $\gamma_0 > \gamma_1 > 0$ . For this system, even a sign function is included in the input  $v$ , the control torque  $u$  is still a continuous vector. It is obvious to know that this adaptive dynamic sliding mode control method can stabilize attitude and eliminate chattering of flexible spacecraft with disturbance.

#### 4. Example

In this section, compared with the adaptive control law in [12], the simulation results showed the effectiveness of the proposed adaptive dynamic sliding mode control law. The moment of inertia  $J$  is chosen as

$$J = \begin{bmatrix} 1543.9 & -2.3 & -2.8 \\ -2.3 & 417.6 & -35 \\ -2.8 & -35 & 1713.3 \end{bmatrix} (\text{kg} \cdot \text{m}^2),$$

The coupling matrix of the rigid body and flexible appendages of flexible spacecraft is

$$\delta = \begin{bmatrix} -9.4733 & -15.5877 & 0.0052 \\ -0.5331 & 0.4855 & 18.0140 \\ 0.5519 & 4.5503 & 16.9974 \\ -12.1530 & 11.7138 & -0.0002 \end{bmatrix} (\text{kg}^{1/2} \cdot \text{m}),$$

Considering the flexible spacecraft has four-order flexible modal, the natural frequency is

$$\omega_{n1} = 0.7681, \omega_{n2} = 1.1038, \omega_{n3} = 1.8733, \omega_{n4} = 2.5496.$$

The corresponding damping is

$$\xi_1 = 0.005607, \xi_2 = 0.00862, \xi_3 = 0.01283, \xi_4 = 0.02516$$

the initial angular velocity is

$$\omega(0) = [0 \ 0 \ 0]^T,$$

and the initial flexible model displacement values are

$$p(0) = \begin{bmatrix} -0.2243 \\ 0.6728 \\ -0.4485 \end{bmatrix},$$

Choosing the parameters as following

$$\eta_i(0) = 0.001, \psi_i(0) = 0.001, i = 1, 2, 3, 4$$

Supposing that the disturbance is sine error function as

$$d = \begin{bmatrix} 0.01 \sin(0.1t) \\ 0.01 \sin(0.2t) \\ 0.01 \sin(0.3t) \end{bmatrix}.$$

and take both initial values of adaptive control laws are zero vectors in [12] and the proposed method in this paper. The red solid line represents the proposed adaptive dynamic sliding mode control law (ADSMC), while the blue dotted line represents the adaptive control law (Adaptive) in [12], the simulation results are as follows.

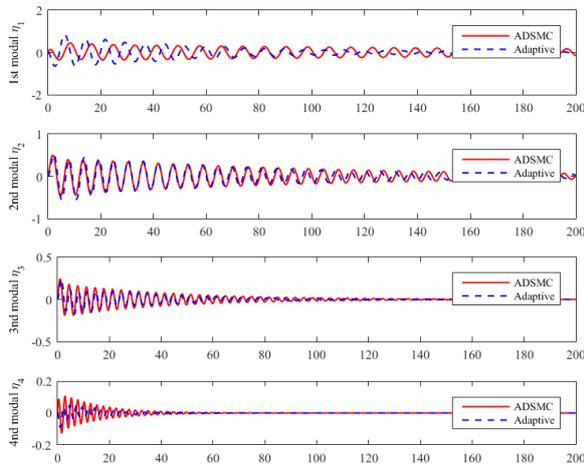


Fig. 1. The response of the flexible modal displacement  $\eta$

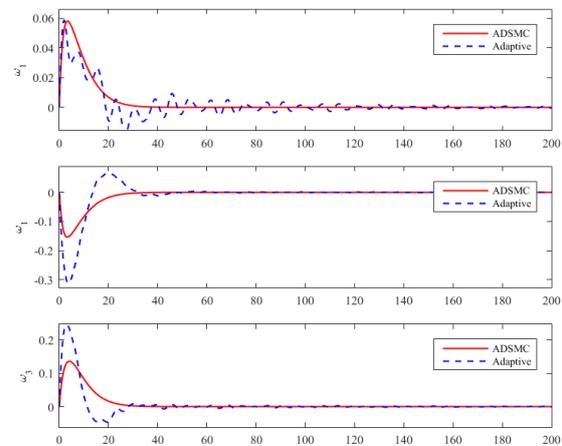


Fig. 2. The response of the control torque  $u$

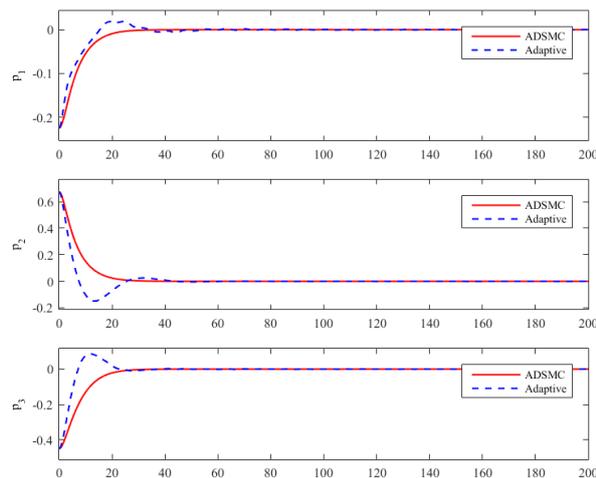


Fig. 3. The response of the attitude angular velocity  $\omega$

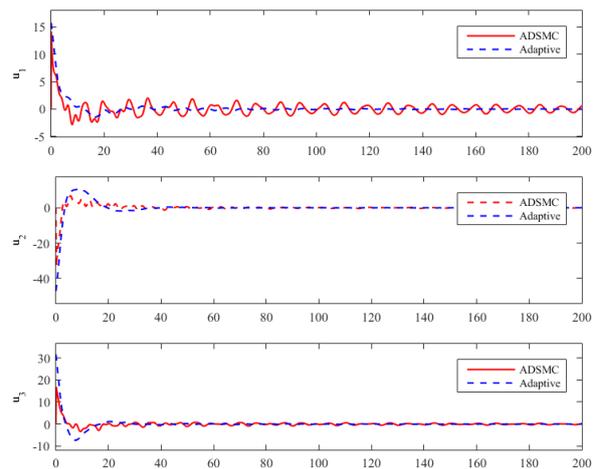


Fig. 4. The response of the modified Rodrigues Parameter  $p$

Fig.1 shows the flexible modal displacements of two kinds of control law. It can be concluded that both of their flexible modal displacements decrease to zero as time increases. It can be drawn that both control torque  $u$  tend to zero when time increases, but the control torque  $u$  in [12] is greater than the proposed method(ADSMC), which means the proposed method can complete target by using smaller control torque. Fig.3 represents the response of attitude angular velocity  $\omega$ , it can be obtained that the attitude angular velocity  $\omega$  goes to zero which means the flexible spacecraft arrived stabilization under the effect of disturbance. But we can know that the attitude angular velocity converges smoother under the proposed ADSMC law than the adaptive control law in [12]. Fig.4 proves the convergence of the modified Rodrigues parameter  $p$ , from where we can draw a conclusion that both of control law can stabilize the attitude of flexible spacecraft with disturbance, but the proposed method can arrive stable state faster and smoother. The adaptive dynamic control law proposed in this paper can stabilize the attitude around 30s, also the chattering is eliminated even on the sliding mode surface.

## 5. Conclusions

In this paper, an adaptive dynamic sliding mode control law was proposed to stabilize the attitude and eliminate the chattering of flexible spacecraft with disturbance. By introducing the adaptive technique, the attitude can be stabilized faster even under disturbance. And the chattering problem can be solved by constructing a one-order dynamic system. The simulation results proved the efficiency of the proposed method.

## References

- [1] K. Tsuchiya, "Dynamics of a spacecraft during extension of flexible appendages," *Journal of Guidance Control and Dynamics*, vol. 6, no. 2, pp. 100--103, 2015.
- [2] L. I. Guangxing, B. Shaohua, "Nonlinear pid attitude tracking control research of flexible spacecraft," *Aerospace Control*, 2009.
- [3] V. I. Utkin, "Variable structure systems with sliding modes," *IEEE Transactions on Automatic Control*, vol.22, no. 2, pp. 212--222, 2003.
- [4] S. N. Singh, "Nonlinear adaptive attitude control of spacecraft," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-23, no.3, pp.371--379, 2007.
- [5] Y. Park, "Robust and optimal attitude stabilization of spacecraft with external disturbances," *Aerospace Science and Technology*, vol.9, no.3, pp. 253--259, 2005.
- [6] Y. P. Chen and S. C. Lo, "Sliding-mode controller design for spacecraft attitude tracking maneuvers," *IEEE Transactions on Aerospace and Electronic Systems*, vol.29, no.4, pp.1328--1333, 1993.
- [7] P.M. Tiwari, S.Janardhanan, and M.U. Nabi, "Attitude control using higher order sliding mode," *Aerospace Science and Technology*, vol.54, pp.108--113, 2016.
- [8] Dwyer, A.W. Thomas, III, and H.Sira-Ramirez, "Variable-structure control of spacecraft attitude maneuvers," *Journal of Guidance Control and Dynamics*, vol.11, no.3, pp. 262--270, 1988.
- [9] R.D. QinghuaZhu and G.Ma, "attitude control of flexible spacecraft based on dynamic sliding mode control," *Control Theory and Application*, vol.35, no.10, pp. 1430--1435, 2018.
- [10] B.Wu, X.Cao, and X.Lei, "Robust adaptive control for attitude tracking of spacecraft with unknown dead-zone," *Aerospace Science and Technology*, vol.45, pp. 196--202, 2015.
- [11] G.Yong and S.Shenmin, "Adaptive finite-time backstepping control for attitude tracking of spacecraft based on rotation matrix," *Chinese Journal of Aeronautics*, vol.27, no.2, 2014.
- [12] Y.Zhang, Y.Wu, L.He, and A.Wu, "Adaptive control for disturbance attenuation of flexible spacecraft," *The 37th China control conference* 2018.
- [13] B. Cong, X.Liu, and C.Zhen, "Backstepping based adaptive sliding mode control for spacecraft attitude maneuvers," *Aerospace Science and Technology*, vol. 30, no.1, pp. 1--7, 2013.