A Correction of Lorentz Transformation

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Abstract. As a correction of the Lorentz transformation, the Magic transformation eliminates a hidden logical danger of the Lorentz transformation. With the second order square matrix, a new derivation of the Magic transformation is given. The Magic transformation may be a reasonable modification of the Lorentz transformation in theory, and it is hoped that it will arouse the attention of physicists.

1. Introduction

In order to explain the experimental results of Michelson-Morey, as early as 1900, J. J. Larmor set up the formulae now generally known as the Lorentz transformation [1] (P.2). It is as follows.

Assuming that S system and S' system coincide with the coordinate origin at the time of t=0 and the coordinate axis of S' system always keeps parallel with the coordinate axis of S system. Compared with S system, S' system moves in a straight line with a uniform speed v and $\hat{\boldsymbol{v}}$ is the unit vector of the speed $\boldsymbol{v}$. The coordinate of the event P in S system is $(\boldsymbol{x}, t)$ and the coordinate in S' system is $(\boldsymbol{x}', t')$. The speed of light is c and the Lorentz transformation gives the relation between $(\boldsymbol{x}, t)$ and $(\boldsymbol{x}', t')$[2](P.69). That is

$$\boldsymbol{x}' = \boldsymbol{x} - (\hat{\boldsymbol{v}}\cdot\boldsymbol{x})\hat{\boldsymbol{v}} + \frac{(\hat{\boldsymbol{v}}\cdot\boldsymbol{x})\hat{\boldsymbol{v}} - \boldsymbol{v}t}{\sqrt{1-\left(v^2/\mathrm{c}^2\right)}},$$
$$t' = \frac{t - \boldsymbol{v}\cdot\boldsymbol{x}/\mathrm{c}^2}{\sqrt{1-\left(v^2/\mathrm{c}^2\right)}}. \tag{1}$$

Assuming that $\boldsymbol{u}'$ is the velocity of a particle in S' system and $\boldsymbol{u}$ is the velocity of it in S system, Lorentz transformation also gives the relation between $\boldsymbol{u}'$ and $\boldsymbol{u}$. That is [2] (P.69)

$$\boldsymbol{u}' = \frac{\sqrt{1-\left(v^2/\mathrm{c}^2\right)}}{1-\left(\boldsymbol{u}\cdot\boldsymbol{v}/\mathrm{c}^2\right)}\left(\boldsymbol{u} - (\boldsymbol{u}\cdot\hat{\boldsymbol{v}})\hat{\boldsymbol{v}}\right) + \frac{(\boldsymbol{u}\cdot\hat{\boldsymbol{v}})\hat{\boldsymbol{v}} - \boldsymbol{v}}{1-\left(\boldsymbol{u}\cdot\boldsymbol{v}/\mathrm{c}^2\right)}. \tag{2}$$

Where $\hat{\boldsymbol{v}}$ is a unit vector in the direction $\boldsymbol{v}$ (Printing errors in the original formula were corrected when quoted).

In four dimensional space-time and 4-velocity notation, assuming that $\boldsymbol{x}=(x_1, x_2, x_3)^{\mathrm{T}}$, $\boldsymbol{X}=(x_1, x_2, x_3, \mathrm{i}ct)^{\mathrm{T}}$, $\boldsymbol{X}'=(x_1', x_2', x_3', \mathrm{i}ct')^{\mathrm{T}}$, $\boldsymbol{v}=(v_1,v_2,v_3)^{\mathrm{T}}$, $\boldsymbol{V}=(V_1,V_2,V_3,V_4)^{\mathrm{T}}=(\gamma v_1, \gamma v_2, \gamma v_3, \mathrm{i}c\gamma)^{\mathrm{T}}$, $V^2=V_1^2+V_2^2+V_3^2$, $\gamma = 1\big/\sqrt{1-\left(v^2/\mathrm{c}^2\right)}$, Eq. (1) can be written conveniently as[3]

$$\boldsymbol{X}'=\boldsymbol{L}(\boldsymbol{V})\boldsymbol{X},$$

$$\boldsymbol{L}(\boldsymbol{V})=\begin{pmatrix} 1+\frac{V_1^2(\gamma-1)}{V^2} & \frac{V_1V_2(\gamma-1)}{V^2} & \frac{V_1V_3(\gamma-1)}{V^2} & -\frac{V_1}{\mathrm{ic}} \\ \frac{V_1V_2(\gamma-1)}{V^2} & 1+\frac{V_2^2(\gamma-1)}{V^2} & \frac{V_2V_3(\gamma-1)}{V^2} & -\frac{V_2}{\mathrm{ic}} \\ \frac{V_1V_3(\gamma-1)}{V^2} & \frac{V_2V_3(\gamma-1)}{V^2} & 1+\frac{V_3^2(\gamma-1)}{V^2} & -\frac{V_3}{\mathrm{ic}} \\ \frac{V_1}{\mathrm{ic}} & \frac{V_2}{\mathrm{ic}} & \frac{V_3}{\mathrm{ic}} & \gamma \end{pmatrix}.$$

In 2003, Wu and Li found a logical contradiction in Lorentz transformation [4]. It is as follows.

Suppose another frame S'' is in a rectilinear motion with constant velocity $\boldsymbol{u}$ relative to the frame S, then S'' is in a rectilinear motion with constant velocity $\boldsymbol{u}'$ given by Eq. (2) relative to the frame S'. In frame S″, the space-time coordinates of the event P can be obtained in two ways:

$$\boldsymbol{x}''=\boldsymbol{x}'\boldsymbol{L}(\boldsymbol{u}')=\boldsymbol{x}\boldsymbol{L}(\boldsymbol{v})\,\boldsymbol{L}(\boldsymbol{u}') \tag{3}$$

and

$$\boldsymbol{x}''=\boldsymbol{x}\boldsymbol{L}(\boldsymbol{u}). \tag{4}$$

However, if $\boldsymbol{u}$ and $\boldsymbol{v}$ are not parallel, then the results of the two ways are not the same!

Therefore, Wu and Li proposed a new transformation called the Magic transformation [4]:

$$\boldsymbol{M}(\boldsymbol{V})=\frac{1}{\mathrm{ic}}\begin{pmatrix} V_4 & \pm V_3 & \mp V_2 & -V_1 \\ \mp V_3 & V_4 & \pm V_1 & -V_2 \\ \pm V_2 & \mp V_1 & V_4 & -V_3 \\ V_1 & V_2 & V_3 & V_4 \end{pmatrix}, \tag{5}$$

It is believed that $\boldsymbol{X}'=\boldsymbol{L}(\boldsymbol{V})\boldsymbol{X}$ should be revised to $\boldsymbol{X}'=\boldsymbol{M}(\boldsymbol{V})\boldsymbol{X}$. In 2011, Wu, Zhang, et al gave the velocity transformation formula between the old and the new coordinate systems under the magic transformation [5]. That is

$$\boldsymbol{u}'=\frac{1}{1-\left(\boldsymbol{u}\cdot\boldsymbol{v}/\mathrm{c}^2\right)}\left(\boldsymbol{u}-\boldsymbol{v}\pm\frac{\boldsymbol{u}\times\boldsymbol{v}}{\mathrm{c}}\mathrm{i}\right).$$

This is obviously different from Eq. (2) under the Lorentz transformation given.

In this paper, a new derivation of the Magic transformation is proposed, and a new form of the Magic transformation is also given.

2. Square matrix of the event coordinates in relativity theory

The special theory of relativity negates the simultaneous absoluteness in Newtonian mechanics, and holds that the space-time interval (rather than the time difference) of two events is invariant in the coordinate transformation of different inertial systems. According to this interval invariance, the second order square matrix can be used to represent the coordinates of the events [6-7]:

$$\boldsymbol{X}=\begin{pmatrix} \mathrm{c}t+x & y+\mathrm{i}z \\ y-\mathrm{i}z & \mathrm{c}t-x \end{pmatrix}, \tag{6}$$

It is noted that the determinant $|\boldsymbol{X}|$ is equal to the space-time interval s^2 between the event P at the coordinate (ct, x, y, z) and the event O at the origin of time and space (0,0,0,0). Its determinant is invariant when $\boldsymbol{X}$ is multiplied on the left or the right by a second order square matrix whose determinant equals 1. Therefore, the transformation coefficient Matrix of coordinates between different Inertial Systems shall be a second order square matrix whose determinant is equal to 1.

Eq. (6) is used to derive the inherent time τ to obtain another representation of the 4-velocity:

$$\boldsymbol{U}=\begin{pmatrix} \gamma\mathrm{c}+u_x & u_y+\mathrm{i}u_z \\ u_y-\mathrm{i}u_z & \gamma\mathrm{c}-u_x \end{pmatrix}. \tag{7}$$

Here

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$$\gamma = 1\Big/\sqrt{1-\left(v^2/\mathrm{c}^2\right)} = \sqrt{1+\left(u^2/\mathrm{c}^2\right)},$$
$$v^2 = v_x^2 + v_y^2 + v_z^2,$$
$$u_x = \gamma v_x, \quad u_y = \gamma v_y, \quad u_z = \gamma v_z.$$

For a stationary particle relative to a coordinate system, the 4-velocity is:

$$\boldsymbol{U}(0) = \begin{pmatrix} \mathrm{c} & 0 \\ 0 & \mathrm{c} \end{pmatrix} = \mathrm{c}\boldsymbol{E}.$$

Similarly, this form can be used to write the four dimensional momentum, the four dimensional acceleration, the four dimensional force as well as other four dimensional vectors of particles.

3. New derivation of the Magic transformation

Assuming that S' system moves at a constant velocity of 4-velocity

$$\boldsymbol{V} = \begin{pmatrix} V_0 + V_1 & V_2 + \mathrm{i}V_3 \\ V_2 - \mathrm{i}V_3 & V_0 - V_1 \end{pmatrix}$$

(here $V_0=\gamma_0\mathrm{c}=\mathrm{c}\sqrt{1+\dfrac{V_1^2+V_2^2+V_3^2}{\mathrm{c}^2}} = \sqrt{\mathrm{c}^2+V_1^2+V_2^2+V_3^2}$) with respect to S system, and the rest of the assumptions is the same as before. The coordinate of the event P occurring at X coordinate in S system is X' in S' system. Assuming that

$$\boldsymbol{X}'=\boldsymbol{M}\boldsymbol{X}, \tag{8}$$

Therefore

$$\boldsymbol{U}' = \left(\frac{d\boldsymbol{M}}{d\tau}\right)\boldsymbol{X} + \boldsymbol{M}\boldsymbol{U}.$$

Obviously, $\boldsymbol{M}$ should only depend on $\boldsymbol{V}$. Because S' system moves in a uniform linear motion with respect to S system, $\boldsymbol{V}$ does not change with time, and thus $\boldsymbol{M}$ does not change with time. That is $\dfrac{d\boldsymbol{M}}{d\tau} = 0$. Therefore

$$\boldsymbol{U}' = \boldsymbol{M}\boldsymbol{U}. \tag{9}$$

That is, under the inertial system transformation, the 4-velocity should have the same transformation form as the coordinate.

When $\boldsymbol{U}=\boldsymbol{V}$, it is obvious that $\boldsymbol{U}'=\boldsymbol{U}(0)=\mathrm{c}\boldsymbol{E}$, then

$$\boldsymbol{M}(\boldsymbol{V})\boldsymbol{V} = \mathrm{c}\boldsymbol{E},$$
$$\boldsymbol{M}(\boldsymbol{V}) = \mathrm{c}\boldsymbol{V}^{-1}. \tag{10}$$

The elementary line transformation is used to obtain the inverse matrix $\boldsymbol{V}^{-1}$ of the 4-velocity $\boldsymbol{V}$. As

$$\left(\begin{array}{cc|cc} V_0+V_1 & V_2+\mathrm{i}V_3 & 1 & 0 \\ V_2-\mathrm{i}V_3 & V_0-V_1 & 0 & 1 \end{array}\right) \xrightarrow{\mathrm{r}_1\times 1/(V_0+V_1)} \left(\begin{array}{cc|cc} 1 & (V_2+\mathrm{i}V_3)/(V_0+V_1) & 1/(V_0+V_1) & 0 \\ V_2-\mathrm{i}V_3 & V_0-V_1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{\mathrm{r}_2-(V_2-\mathrm{i}V_3)\mathrm{r}_1} \left(\begin{array}{cc|cc} 1 & (V_2+\mathrm{i}V_3)/(V_0+V_1) & 1/(V_0+V_1) & 0 \\ 0 & V_0-V_1-\left(V_2^2+V_3^2\right)/(V_0+V_1) & (-V_2+\mathrm{i}V_3)/(V_0+V_1) & 1 \end{array}\right) =$$

$$\left(\begin{array}{cc|cc} 1 & (V_2+\mathrm{i}V_3)/(V_0+V_1) & 1/(V_0+V_1) & 0 \\ 0 & \mathrm{c}^2/(V_0+V_1) & (-V_2+\mathrm{i}V_3)/(V_0+V_1) & 1 \end{array}\right) \xrightarrow{\mathrm{r}_2\times(V_0+V)/\mathrm{c}^2}$$

$$\left(\begin{array}{cc|cc} 1 & (V_2+\mathrm{i}V_3)/(V_0+V_1) & 1/(V_0+V_1) & 0 \\ 0 & 1 & (-V_2+\mathrm{i}V_3)/\mathrm{c}^2 & (V_0+V_1)/\mathrm{c}^2 \end{array}\right) \xrightarrow{\mathrm{r}_1-\mathrm{r}_2(V_2+\mathrm{i}V_3)/(V_0+V_1)}$$

$$\left(\begin{array}{cc|cc} 1 & 0 & \dfrac{1+(V_2^2+V_3^2)/\mathrm{c}^2}{V_0+V_1} & \dfrac{-V_2-\mathrm{i}V_3}{\mathrm{c}^2} \\ 0 & 1 & (-V_2+\mathrm{i}V_3)/\mathrm{c}^2 & (V_0+V_1)/\mathrm{c}^2 \end{array}\right) = \left(\begin{array}{cc|cc} 1 & 0 & (V_0-V_1)/\mathrm{c}^2 & (-V_2-\mathrm{i}V_3)/\mathrm{c}^2 \\ 0 & 1 & (-V_2+\mathrm{i}V_3)/\mathrm{c}^2 & (V_0+V_1)/\mathrm{c}^2 \end{array}\right).$$

Therefore,

$$\boldsymbol{V}^{-1} = \frac{1}{\mathrm{c}^2}\begin{pmatrix} V_0-V_1 & -V_2-\mathrm{i}V_3 \\ -V_2+\mathrm{i}V_3 & V_0+V_1 \end{pmatrix},$$

$$\boldsymbol{M}(\boldsymbol{V}) = \frac{1}{\mathrm{c}}\begin{pmatrix} V_0-V_1 & -V_2-\mathrm{i}V_3 \\ -V_2+\mathrm{i}V_3 & V_0+V_1 \end{pmatrix}. \tag{11}$$

To all appearances $|\boldsymbol{M}|=1$.
Eq. (8) is rewritten by Eq. (11) as follows

$$\boldsymbol{X}' = \boldsymbol{M}\boldsymbol{X} = \frac{1}{\mathrm{c}}\begin{pmatrix} V_0-V_1 & -V_2-\mathrm{i}V_3 \\ -V_2+\mathrm{i}V_3 & V_0+V_1 \end{pmatrix}\begin{pmatrix} \mathrm{c}t+x & y+\mathrm{i}z \\ y-\mathrm{i}z & \mathrm{c}t-x \end{pmatrix} = \begin{pmatrix} \mathrm{c}t'+x' & y'+\mathrm{i}z' \\ y'-\mathrm{i}z' & \mathrm{c}t'-x' \end{pmatrix},$$

Among them,

$$t' = \gamma_0 t - \frac{V_1}{\mathrm{c}}x - \frac{V_2}{\mathrm{c}}y - \frac{V_3}{\mathrm{c}}z, \quad x' = -V_1 t + \gamma_0 x + \mathrm{i}\left(-\frac{V_3}{\mathrm{c}}y + \frac{V_2}{\mathrm{c}}z\right),$$

$$y' = -V_2 t + \gamma_0 y + \mathrm{i}\left(\frac{V_3}{\mathrm{c}}x - \frac{V_1}{\mathrm{c}}z\right), \; z' = -V_3 t + \gamma_0 z + \mathrm{i}\left(-\frac{V_2}{\mathrm{c}}x + \frac{V_1}{\mathrm{c}}y\right),$$

$$\gamma_0 = \sqrt{1+\frac{V_1^2+V_2^2+V_3^2}{\mathrm{c}^2}}.$$

This is consistent with the conclusion of [4-5].

4. More concise derivation

In Eq. (11), it is found that the magic transformation coefficient matrix $\boldsymbol{M}(\boldsymbol{V})$ can be obtained by dividing the four dimensional velocity $\boldsymbol{V}$ by c and then replacing the three spatial components with their opposite numbers respectively. This conclusion can be obtained in a more concise way.

The inverse matrix is obtained by both sides of (10), then

$$\boldsymbol{M}^{-1}(\boldsymbol{V}) = \frac{1}{\mathrm{c}}\boldsymbol{V},$$

Both sides of (9) are multiplied by $\boldsymbol{M}^{-1}(\boldsymbol{V})$ on the left to obtain

$$\boldsymbol{U} = \boldsymbol{M}^{-1}\boldsymbol{U}'.$$

That is

$$\boldsymbol{U} = \frac{1}{\mathrm{c}}\boldsymbol{V}\boldsymbol{U}'.$$

On the other hand, due to the symmetry, S system moves in a uniform straight line at 4-velocity

$$\boldsymbol{V}^* = \begin{pmatrix} \gamma_0\mathrm{c}-V_1 & -V_2-\mathrm{i}V_3 \\ -V_2+\mathrm{i}V_3 & \gamma_0\mathrm{c}+V_1 \end{pmatrix} = -(\boldsymbol{V}-\gamma_0\mathrm{c}\boldsymbol{E}) + \gamma_0\mathrm{c}\boldsymbol{E}$$

with respect to S system, there is a transformation

$$\boldsymbol{U} = \boldsymbol{M}(\boldsymbol{V}^*)\boldsymbol{U}'.$$

Therefore,

$$\boldsymbol{M}(\boldsymbol{V}^*) = \frac{1}{\mathrm{c}}\boldsymbol{V} = \frac{1}{\mathrm{c}}\left(\boldsymbol{V}^*\right)^*,$$

$$\boldsymbol{M}(\boldsymbol{V}) = \frac{1}{\mathrm{c}}\boldsymbol{V}^* = \frac{1}{\mathrm{c}}\begin{pmatrix} \gamma_0\mathrm{c} - V_1 & -V_2 - \mathrm{i}V_3 \\ -V_2 + \mathrm{i}V_3 & \gamma_0\mathrm{c} + V_1 \end{pmatrix}.$$

This is just the formula (11).

5. Discussion

According to the Magic transformation, imaginary numbers will inevitably appear at the components such as the coordinate and the velocity of the particles under the general inertia system transformation, so the second order Hermitian matrix used to describe these vectors will not surely become the Hermitian matrix under the magic transformation. This is why the transformation is named the Magic transformation. On the other hand, under the special inertial system transformation, the component of the spatial coordinate of the particle is not equal to 0 in the direction parallel to the relative velocity of the inertial system, but the component perpendicular to this relative velocity is 0. Then the Magic transformation is equivalent to the Lorentz transformation. In this case, the second order Hermitian matrix which describes the coordinates and the velocities of particles becomes a diagonal matrix, and both kinds of transformations transform the diagonal matrix into a diagonal matrix. Therefore, the Magic transformation should be the reasonable modification of Lorentz transformation in theory. The test of its conformity with the objective reality is expected to arouse the attention of theoretical and experimental physicists.

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