Fiber optic gyro signal denoising based on variational mode decomposition and adaptive noise cancellation

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Abstract. The output signal of the fiber optic gyro (FOG) often contains a large amount of noise, which makes it difficult to model and calibrate. Therefore, it must be denoised. In this paper, an adaptive noise cancellation method based on variational mode decomposition (VMD) is proposed. The FOG signal was decomposed into a series of mode components with different center frequencies, and the correlation coefficient method was used to determine the number of decomposition layer. The low-frequency signal and the high-frequency noise components were distinguished by the frequency distribution of each component. They were respectively accumulated to use as the ready denoising signal and the reference noise, then obtaining the denoised signal by adaptive noise cancellation. Using FOG measured data, the proposed method was taken into experiment compared with traditional methods. Allan variance analysis verified that the proposed method effectively suppresses the noise in the FOG signal.

1. Introduction

Fiber Optic Gyro (FOG) has the advantages of simple structure, fast start-up and small size. It has broad application prospects in inertial navigation systems [1]. However, due to factors such as internal structure and processing technology, FOG output often contains a large amount of noise and abnormal sampling values, which makes it difficult to obtain high accuracy for calibration and error modeling. Therefore, the output signal needs to be denoised [2][3].

Threshold filtering effectively extracts useful signals from noise on the perspective of frequency domain, and solves the real-time problem through sliding data window [2]. It is a hotspot in recent years. The most widely used method is wavelet transform (WT) threshold denoising. The full-frequency wavelet multi-scale denoising method proposed in [4] has achieved better results than the traditional filtering method. However, it does not solve the basis function selection and decomposition layer number determination problem. Ensemble Empirical mode decomposition (EEMD) solves the shortcomings of WT and is successfully applied to FOG signal denoising [3][5]. However, since it is based on a recursive algorithm, modal aliasing is prone to occur. There are still some deficiencies in the theoretical basis and the computational efficiency.

Variational mode decomposition (VMD) [6], as a new non-recursive decomposition method, overcomes the shortcomings of modal aliasing in EEMD. It has strict mathematical derivation and high decomposition precision. It obtains the center frequency and bandwidth of each component by iteratively searching for the optimal solution of the variational model. Since the high and low frequencies are still partially doped after decomposition, it is easily to discard the useful signals or retain the noise in part of the modes. The adaptive noise cancellation technique uses the reference noise to cancel the noise in the noisy signal through the filter, thereby realizing the extraction of the useful signal. It can be

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applied to the stationary or non-stationary signal, and has good suppression performance for noise [7].

This paper proposes a method for FOG output signal denoising based on VMD and adaptive noise cancellation. FOG output signal is decomposed into a set of mode components with different center frequencies, and the useful signals and noise components are identified according to the frequency distribution. The high-frequency noise components are accumulated as the reference noise and the remaining low-frequency components are accumulated as the ready denoising signal, then obtaining the denoised signal by adaptive noise cancellation. The denoising effect of the method is verified by the measured data of FOG compared with different methods.

2. Variational mode decomposition

Under the constraint that the sum of the mode components is equal to the signal, VMD obtains a variational model satisfies that the sum of the estimated bandwidths of the mode components to be the smallest by continuously searching within the frequency range. The intrinsic mode function (IMF) of the VMD is defined as:

$$u_k(t) = A_k(t)\cos(\phi_k(t)) \tag{1}$$

Where $u_k(t)$ is the kth mode component, $A_k(t)$ and $\phi_k(t)$ are instantaneous amplitude and phase. Mixed estimated center frequency of $u_k(t)$ is ω_k. Using the H Gaussian smoothing of the demodulated signal to estimate the bandwidth of $u_k(t)$, the above constraints can be described as:

$$\begin{gathered}\min_{\boldsymbol{u}_k\boldsymbol{\omega}_k}\left\{\sum_k \| \partial_t[(\delta(t)+\frac{j}{\pi t})*u_k(t)]e^{-j\omega_k t}\|_2^2\right\}\\ s.t.\sum_k \boldsymbol{u}_k = \boldsymbol{x}\end{gathered} \tag{2}$$

Where $\delta(t)$ is the impact function, * represents convolution operation, $\boldsymbol{u}_k = \{u_1, u_2, \ldots, u_k\}$, $\boldsymbol{\omega}_k = \{\omega_1, \omega_2, \ldots, \omega_k\}$, $\boldsymbol{x}$ is the ready decomposing signal. Using the penalty factor α and the Lagrange multiplication operator $\lambda(t)$, introducing the alternating direction method of the multiplication operator, $\hat{u}_k^{n+1}, \omega_k^{n+1}$ and $\lambda(t)$ are constantly updated, and the binding variational problems is transformed into non-binding one:

$$L(\boldsymbol{u}_k, \boldsymbol{\omega}_k, \boldsymbol{\lambda}) = \alpha\sum_k \| \partial_t[(\delta(t)+\frac{j}{\pi t})u_k(t)]\times e^{-j\omega_k t}\|_2^2 + \| x(t) - \sum_k u_k(t)\|_2^2 + \left\langle \lambda(t), x(t) - \sum_k u_k(t)\right\rangle \tag{3}$$

Calculate the extreme value of the above Lagrange function, then the frequency domain expressions of the mode component $\boldsymbol{u}_k$ and the center frequency $\boldsymbol{\omega}_k$ are respectively:

$$\hat{u}_k^{n+1}(\omega) = \frac{\hat{f}(\omega) - \sum_{i<k}\hat{u}_i^{n+1}(\omega) - \sum_{i>k}\hat{u}_i^{n}(\omega) + \frac{\hat{\lambda}^n(\omega)}{2}}{1 + 2\alpha(\omega - \omega_k^n)^2} \tag{4}$$

$$\omega_k^{n+1} = \frac{\int_0^\infty \omega |\hat{u}_i^{n+1}(\omega)|^2 \, d\omega}{\int_0^\infty |\hat{u}_i^{n+1}(\omega)|^2 \, d\omega} \tag{5}$$

Performing an inverse Fourier transform on $\hat{u}_k(\omega)$, the actual part is $u_k(t)$. Iterate continuously until the accuracy is satisfied [6][8].

3. Adaptive noise cancellation

As shown in figure 1, the adaptive filter has two inputs, reference signal $z(n)$ and desired signal $d(n)$, and output signal is $y(n)$. Here, the signal source containing noise is used as the desired signal, and the noise source is used as the reference signal. After multiple iterations, the difference between the desired signal and the output signal $e(n)$ is the denoised signal. This process is called noise cancellation, which means removing the portion of the signal source that is strongly correlated with the noise source.

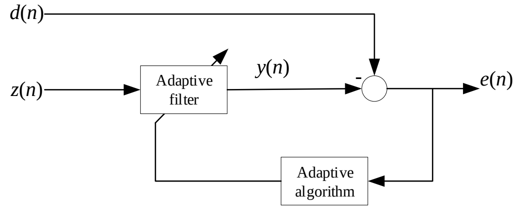

Figure 1. Adaptive filter structure

The most important part of adaptive noise cancellation is the design of adaptive algorithms. For non-stationary signals, recursive least squares (RLS) has better adaptability. RLS algorithm is based on the least squares criterion, which minimizes the sum of squares of all errors from the initial to current time:

$$\min \boldsymbol{J}(n)=\sum_{i=0}^{n}\boldsymbol{e}^2(i)=\sum_{i=0}^{n}\lambda^{n-i}[\boldsymbol{d}(i)-\boldsymbol{y}(i)]^2 \tag{6}$$

Where λ is exponential weighting factor, $0<\lambda<1$, usually chosen near 1, $y(i)=\boldsymbol{w}^H(n)\boldsymbol{z}(i)$, $\boldsymbol{w}$ is adaptive filter weight vector. RLS can be described as adjusting weight vector $\boldsymbol{w}$ so that the sum of squared of the filter output errors is minimal for all input signals at each moment. Solving equation (6) and finding partial derivatives for $\boldsymbol{w}$:

$$\frac{\partial \boldsymbol{J}(\boldsymbol{w})}{\partial \boldsymbol{w}^*}=\frac{\partial}{\partial \boldsymbol{w}^*}\sum_{i=0}^{n}\lambda^{n-i}[\boldsymbol{d}(i)-\boldsymbol{w}^H(n)\boldsymbol{z}(i)]\times[\boldsymbol{d}(i)-\boldsymbol{w}^H(n)\boldsymbol{z}(i)]^* \tag{7}$$

Define the auto-correlation and cross-correlation matrix of the signal as

$$\boldsymbol{R}(n)=\sum_{i=0}^{n}\lambda^{n-i}\boldsymbol{z}(i)\boldsymbol{z}^{\mathrm{H}}(i) \tag{8}$$

$$\boldsymbol{r}(n)=\sum_{i=0}^{n}\lambda^{n-i}\boldsymbol{z}(i)d^*(i) \tag{9}$$

Then the optimal solution is:

$$\boldsymbol{w}_{opt}(n)=\boldsymbol{R}^{-1}(n)\boldsymbol{r}(n) \tag{10}$$

The iterations of $\boldsymbol{R}(n)$ and $\boldsymbol{r}(n)$ are:

$$\boldsymbol{R}(n)=\lambda\boldsymbol{R}(n-1)+\boldsymbol{z}(n)\boldsymbol{z}^{\mathrm{H}}(n) \tag{11}$$

$$\boldsymbol{r}(n)=\lambda\boldsymbol{r}(n-1)+\boldsymbol{z}(n)\boldsymbol{d}^*(n) \tag{12}$$

The update formula for weights $\boldsymbol{w}(n)$ can be obtained by combining (10)-(12):

$$\boldsymbol{w}(n)=\boldsymbol{w}(n-1)+\boldsymbol{k}(n)[\boldsymbol{d}(n)-\boldsymbol{w}^{\mathrm{H}}(n-1)\boldsymbol{z}(n)] \tag{13}$$

Where gain vector $\boldsymbol{k}(n)=\dfrac{\boldsymbol{P}(n-1)\boldsymbol{x}(n)}{\lambda+\boldsymbol{z}^{\mathrm{H}}(n)\boldsymbol{P}(n-1)\boldsymbol{x}(n)}$, $\boldsymbol{P}(n)=\boldsymbol{R}^{-1}(n)$, estimation error $\boldsymbol{e}(n)=\boldsymbol{d}(n)-\boldsymbol{w}^{\mathrm{H}}(n-1)\boldsymbol{z}(n)$ is the required denoised signal [7][9].

4. Adaptive noise cancellation based on variational mode decomposition

4.1. Determination of the number of VMD layers

VMD can customize the number of decomposition layer *K*. However, different *K* has a great influence on decomposition effect. Too large to cause over-decomposition, appear false components, too small to cause under-decomposition, some frequency components in the signal are not decomposed. The correlation coefficient method is used to determine the number of layers. The original signal is decomposed into 2,3, …, *K* layers in turn, the correlation coefficients between each mode component and the original signal is calculated. Set a threshold, when the correlation coefficient of a certain mode in a certain time of decomposition is less than the threshold, the decomposition is terminated. The number of layers of the previous decomposition is taken as the final decomposition layer. The threshold can be set as 1/10 of the maximum correlation coefficient that occurs in each time of VMD [10].

4.2. High and low frequency mode component judgment

Generally, the demarcation point of the high and the low frequency is defined as $f_s / 2^4$ according to different output frequencies f_s [11]. However, in order to maximize the retention of useful signals in the higher frequency bands, this paper defines the demarcation point as $f_s / 2^3$. Since VMD decomposes the signal into mode components that are dominant at different frequencies, each component has a center frequency and bandwidth. Calculate the proportion of the frequency distribution in the high and low frequency bands of the component. If the ratio of high frequency (or low frequency) exceeds 80%, it is defined as the high frequency (or low frequency) component. If the ratios of the two parts are both less than 80%, then the point is further set as $f_s / 2^2$.

4.3. Adaptive noise cancellation based on variational mode decomposition

The main steps of adaptive noise cancellation based on VMD (VMD-RLS) are as follows: (1) Determine the number of decomposition layer by using the correlation coefficient method, and decompose the noisy signal into K layers. (2) Calculate the distribution proportion of each component frequency to determine that the component is high or low frequency, and respectively accumulating as the reference signal and the desired signal for adaptive noise cancellation. (3) Using adaptive noise cancellation to filter out noise in the ready denoising signal. (4) Output the denoised signal.

5. FOG measured data verification and results analysis

In order to verify the validity and denoising accuracy of the VMD-RLS algorithm, a certain FOG is placed on the laboratory leveling turntable statically. The sensitive axis points to the zenith. The data output frequency is 200 Hz, and the acquisition time is 1h. Take 50,000 data points after stable output as the ready denoising signal, and obtain the time domain waveform and spectrum shown in figure 2. It can be seen that the signal is basically covered by noise, especially near 30Hz and 90Hz.

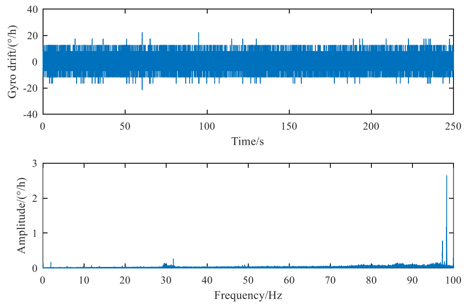

Figure 2. Time domain waveform and spectrum of FOG signal

The signal is decomposed by VMD, and the number of layers in each time of decomposition is one more than the previous. The correlation coefficient between each mode component and the original signal for each time of decomposition is calculated in table 1:

Table 1. Correlation coefficients between mode components and original signal under different decomposition layers

Decomposition layer	Correlation coefficient					Threshold
	IMF1	IMF2	IMF3	IMF4	IMF5	
2	0.9017	0.1132	—	—	—	0.0901
3	0.9004	0.0966	0.3965	—	—	0.0900
4	0.8958	0.0910	0.1575	0.3879	—	0.0896
5	0.8955	0.0865	0.1026	0.1420	0.3819	0.0896

Table 1 shows that at the fifth time decomposition, the correlation coefficient between IMF2 and the original signal is 0.0865, which is less than the threshold value of 0.0896. This decomposition is judged as over-decomposition of the original signal. So, take 4 as the number of decomposition layer of VMD. The time and frequency domain of each mode component after decomposition are shown in figure 3.

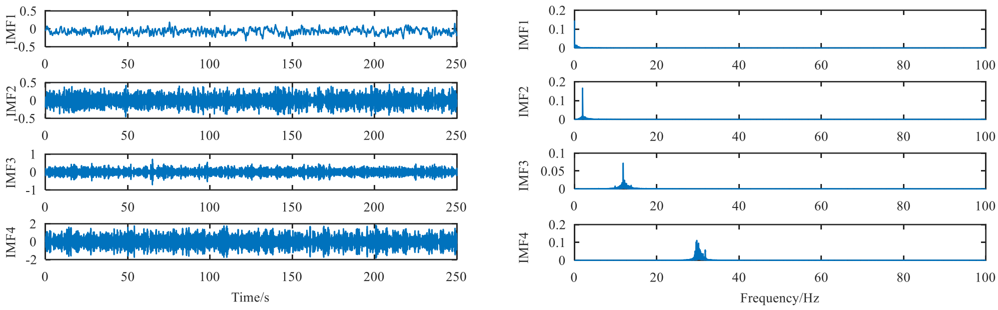

Figure 3. Waveform and spectrum of VMD mode components

It can be seen that the mode spectrum distribution of the VMD is relatively concentrated. Calculating the high and low frequency distribution of each mode is shown in table 2. It can be seen that IMF1~IMF3 are low frequency signal components, and IMF4 is noise component. The two parts are separately accumulated as ready denoising signal and reference noise.

Table 2. Proportion of high and low frequency bands of mode components

Mode component	Proportion (%)	
	High frequency (>25Hz)	Low frequency (<25Hz)
IMF1	0.43	99.57
IMF2	0.65	99.35
IMF3	0.32	99.68
IMF4	99.53	0.47

The ready denoising signal and the reference noise are used as inputs to the cancellation system to obtain denoised signal. In order to show the denoising effect, Kalman filter (KF), WT, EEMD-based noise cancellation (EEMD-RLS) and VMD threshold denoising (VMD) are used as comparisons. The denoising effect of different methods are shown in figure 4.

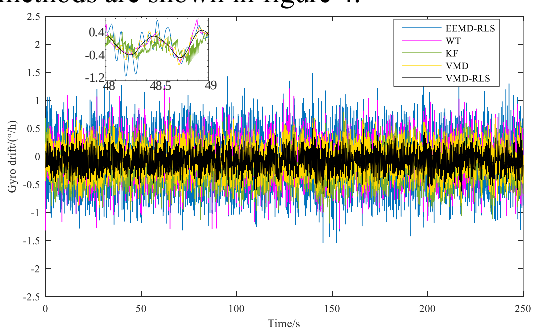

Figure 4. Comparison results of different methods

It can be seen from the figure that the VMD-RLS method has achieved good denoising effect. In order to quantify the denoising results of each method, the Allan variance is used as evaluation criteria. Figure 5 shows the Allan double logarithmic curve of the original gyro signal and various methods denoised signals. From the original signal, the Allan variance curve of FOG is mainly represented by quantization noise and angular random walk in a short correlation time. The effect of bias instability is small. The coefficients of quantization noise Q and angular random walk N obtained by fitting are shown in table 3.

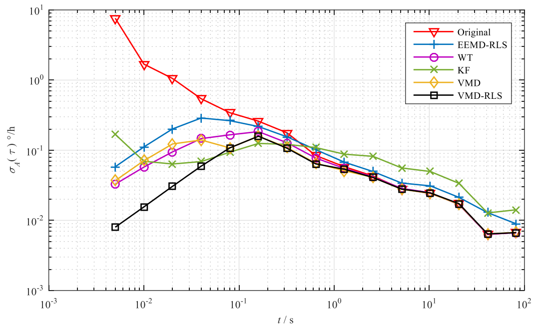

Figure 5. Allan variance double logarithmic curve of different denoising methods

Table 3. Comparison of Allan variance for denoising results

Methods	Q / μrad	N / (°)/h$^{1/2}$
Original	4.75e-2	5.01e-4
EEMD-RLS	8.14e-4	6.85e-5
WT	4.59e-4	3.87e-5
KF	1.95e-3	1.16e-4
VMD	5.15e-4	4.34e-5
VMD-RLS	1.12e-4	9.42e-6

It can be seen from figure 5 and table 3 that the various methods have a significant order of magnitude reduction for the quantization noise and the angular random walk coefficient. VMD-RLS is better than WT, VMD and other denoising method. It is verified that the proposed denoising algorithm is better than the traditional direct filtering high frequency method. KF performs poorly. From the partial amplification of the denoising result (figure 4), the denoised signal after KF is more jitter than other methods. The high-frequency characteristics are more obvious, so the noise coefficient is greater. The performance of EEMD-RLS is not ideal. It can be seen from the partial mode spectrum of EEMD (figure 6) that the frequency span is large and there is modal aliasing, which fails to achieve effective separation of noise and signal, resulting in the desired signal and reference noise are of poor quality, and thus the denoising effect is poor.

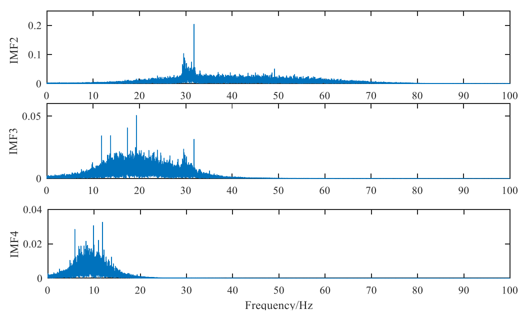

Figure 6. Spectrum of partial mode components of EEMD

6. Conclusion

This paper analyzes the importance of FOG output signal denoising. Based on this, VMD-RLS is proposed. The correlation coefficient method is used to determine the number of decomposition layer. The low frequency signal and the high frequency noise are determined by the mode component frequency distribution. The high and low frequency demarcation points are adjusted, the range of the low frequency band is appropriately expanded to retain the useful signal to the greatest extent. The signal and noise components are separately added for adaptive noise cancellation to obtain denoised signal.

Using FOG measured data for verification experiments, the results show that VMD-RLS has achieved the best denoising effect, the coefficients of quantization noise and angular random walk are both reduced by orders of magnitude.

Compared with the traditional algorithms, VMD-RLS also shows its advantages. Compared with KF, the local jitter of the signal after denoising is smaller, and the high-frequency noise attenuation is more obvious. Compared with WT, it is not necessary to select the basis function, and the applicability is better. Compared with EEMD-RLS, it can better separate the noise and useful signals. The spectrum is relatively concentrated, the aliasing phenomenon is not obvious. It provides more accurate signals for adaptive noise cancellation. Compared with VMD by directly filtering out high frequency components, it retains the useful signal to the greatest extent while removing noise.

Acknowledgments

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References

[1] Passaro, M.N., Cuccovillo, A., Vaiani, L. (2017) Gyroscope technology and applications: A Review in the industrial perspective. Sensor,17:1-22.

[2] Gao, W., Zu, Y., Wang, W. (2012) Research on real-time de-noising of FOG based on second generation wavelet transform. Chinese Journal of Scientific Instrument, 33:774-780.

[3] Cui, B.B., Chen, X.Y., Song, R. (2015) Application of EMD threshold filtering for fiber optical gyro drift signal de-noising. Acat Opica Sinica,35:61-66..

[4] Wang, W., Xiang, Z., Wang, G.D. (2012) Denoising for optic gyroscope based on wavelet full frequency-domain threshold processing. Infrared and Laser Engineering, 41:3401-3405.

[5] Zhang, Q.H., Gu, S.S., Liu, J.Y. (2016) A gyro signal characteristics analysis method based on empirical mode decomposition. Journal of Sensors, https://doi.org/10.1155/2016/3613878

[6] Konstantin, D., Dominique, Z.(2014) Variational Mode Decomposition. IEEE Transactions on Signal Processing, 62:531-544.

[7] Zhang, L.Y., Zhou, J.C., Li, B. (2015) Research on automatic noise spectral estimation suppression technique and its application. Journal of Harbin Institute of Technology, 47:31-35.

[8] Huang, W.X., Liu, D.W. (2019) Mine microseismic signal denosing based on variational mode decomposition and independent component analysis. Journal of Vibration and Shock, 38:56-63.

[9] Cioffi, J., Kailath, T. (1984) Fast, recursive-least-squares transversal filters for adaptive filtering. IEEE Transactions on Acoustics, Speech, and Signal Processing,32:304-337.

[10] Ping Z.K., Tse, P.W., Chu, F.L. (2005) A comparison study of improved Hilbert-Huang transform and wavelet transform: Application to fault diagnosis for rolling bearing. Mechanical Systems and Signal Processing, 19:974-988.

[11] Yin, D.L., Duan, Z.S., Guo, B.L. (2016) Method for extraction of weak signal based on EEMD and SVD. Coal Mine Machinery, 37:154-156.