Research on missile overload control technology with prescribed performance

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Abstract. Considering that the present control methods for missile overload don't take the transient and steady state with prescribed performance into consideration. On the basis of backstepping idea, a controller design method with prescribed performance is proposed for longitudinal channel overload motion model of missile in order to improve the performance of missile control systems. It is proved that the system is stable in the sense of Lyapunov, the tracking error of the system satisfy the prescribed transient and steady performance. The given simulation results demonstrate the effectiveness of the proposed control method.

1. Introduction

In recent years, the missile defense system has been continuously upgraded, and the requirement of missile maneuverability is increasingly high, which makes the missile gradually develop from the traditional subsonic to supersonic and low-altitude penetration to large-space maneuverability penetration. It makes the missile overload control technology with stronger mobility go into people's viewpoint.

The control method based on sliding mode variable structure is often adopted for missile overload control system. Gu W.J. et al. proposed of linear / adaptive variable structure control technology to improve robust performance of composite system to adapt large airspace maneuvering of missile. Reference [1] adopted the method of PID control or adaptive control according to the size of overload signal. Reference [2] and [3] introduced the virtual target proportional guidance method in overload control technology. Reference [3] adopted the SINS overload control scheme to keep the missile stable in high/low and flat flight trajectory. Reference [4-7] adopted integral sliding mode variable structure control method. In order to improve the dynamic performance of the system, Reference [4] used the integral of overload to construct the sliding mode surface. In Reference [5], a sliding mode surface with error integral terms was designed for the output redefinition system to solve the robustness problem of the output redefinition technology. In Reference [6], different sliding mode surfaces were adopted according to the error size to ensure the tracking error response speed. Reference [8] and [9] adopted the control method of second-order sliding mode variable structure. Reference [8] transformed overload output tracking into a state tracking problem, and constructed a second-order dynamic sliding mode to make the system zero dynamic stability. Reference [9] designed an angle of attack observer based on the design of a second-order sliding mode controller.

The overload control method based on backstepping method is suitable for large airspace maneuvering missile. The method ensures stability of the overall system by constructing Lyapunov function. Reference [10] used backstepping method to design the control system and proved the global asymptotic stability of the system through Lyapunov stability theory. Reference [11] adopted the inversion design idea and introduced the intelligent fuzzy term to enable the system to have strong robustness property.

In addition to these two control methods, some researchers adopted other overload control method. Reference [12] adopted pole assignment method to design overload control system. Reference [13] adopted the inverse system design method of nonlinear control to control the longitudinal and normal overload of the missile and achieved a good control effect.

Most of the above control methods can improve the stability or flexible performance of the missile control system, but these methods do not clearly consider the steady state and transient performance. In reality, the missile needs high maneuverability to penetrate enemy defense systems and better transient performance to attack moving targets. A control system with good stable performance can't meet the practical requirements of missiles. Considering that the control method with prescribed performance can meet the requirements of both transient and steady-state performance, the defect of the existing control method in the design of missile overload controller can be overcome.

Prescribed performance problem originated from Miller and Davison creative work [14]. They designed a controller for a linear system, and guaranteed that the system error was less than the prescribed value in an appointed time, and this method adopted the non-decreasing dynamic gains and piecewise constant switching method. References [15-18] studied further the prescribed performance controller design method for nonlinear systems. Llchmann et al. investigated a prescribed performance control method without dynamic and non-monotonic gain which guaranteed the agility during the choice of transient procedure [16]. Bechlioulis proposed a robust adaptive controller for MIMO nonlinear systems which could be linearized via feedback. They transformed the constrained system into an equivalent unconstrained one, and realized the maximum overshoot was less than the prescribed small constant [17]. Zhang et al. proposed a robust control method with prescribed performance by using dynamic surface technique under backstepping design framework, but it only could realize the prescribed performance of outputs.

In order to improve the performance of the missile system overload control, we adopted the backstepping method with prescribed performance to design controller, so that the control system can meet the requirements of prescribed transient and steady performance at the same time.

The rest of the paper is arranged as follows. In Sect. 2, we introduce the system description and preparation knowledge. In Sect3, according to the missile longitudinal overload model, the backstepping controller based on prescribed performance is designed. We give the simulation results and the conclusions respectively in Sects. 4 and 5.

2. System Description and Preparation Knowledge

2.1. Missile overload motion model

The missile longitudinal channel overload motion mode is as follow referring to the modeling method in Reference [19].

$$\begin{cases} \dot{\alpha}=\omega_z-\dfrac{1}{mV}\left(P\sin\alpha+C_y^{\alpha}\,\alpha+C_y^{\delta_z}\delta_z\right)+\dfrac{g}{V}\cos\theta \\ \dot{\omega}_z=\dfrac{1}{J_z}\left(M_z^{\alpha}\alpha+M_z^{\omega_z}\omega_z+M_z^{\delta_z}\delta_z+M_z^{\dot{\alpha}}\dot{\alpha}\right) \\ n_y=\dfrac{1}{mg}\left(P\sin\alpha+C_y^{\alpha}\alpha+C_y^{\delta_z}\delta_z\right) \end{cases} \tag{1}$$

where, α is attack angle, ω_z is pitching angular velocity, m is missile's weight, V is missile's flight speed, P is missile gravity, δ_z is elevator deflection angle, C_y^{α} , $C_y^{\delta_z}$ are respectively the partial derivatives of lift coefficients to the attack angle and elevator deflection angle, J_z is missile's moment of Inertia, M_z^{α}, $M_z^{\omega_z}$, $M_z^{\delta_z}$ and $M_z^{\dot{\alpha}}$ are Missile's torque coefficients, n_y is normal overload.

According to the actual situation of missile flight, the following treatment is carried out to simplify the overload model:

- Considering that $\frac{g\cos\theta}{V}$ is relatively small, especially in the case of large missile velocity inclination, its influence can be ignored.
- Considering that the influence of pitch rudder angle on lift is small, $C_y^{\delta_z}\delta_z$ can be ignored.
- $\frac{M_z^{\alpha}}{J_z}$ is small and the change rate of attack is small, so $\frac{M_z^{\alpha}}{J_z}$ can be ignored.

The simplified model can be expressed as follow:

$$\begin{cases} \dot{\alpha}=\omega_z-\frac{1}{mV}\left(P\sin\alpha+\mathrm{C}_y^{\alpha}\alpha\right) \\ \dot{\omega}_z=\frac{1}{J_z}\left(M_z^{\alpha}\alpha+M_z^{\omega_z}\omega_z+M_z^{\delta_z}\delta_z\right) \\ n_y=\frac{1}{mg}\left(P\sin\alpha+C_y^{\alpha}\alpha\right) \end{cases} \tag{2}$$

2.2. Prescribed performance and error transformation function [18]

Suppose e is the tracking error. The prescribed performance $\varpi(t):\mathbb{R}_+\to\mathbb{R}_+-\{0\}$ is a positive descending function, and meets equation (3) as $t\geq 0$

$$\begin{cases} -\sigma\varpi(t)<e(t)<\varpi(t),\text{if } e(0)>0 \\ -\varpi(t)<e(t)<\sigma\varpi(t),\text{if } e(0)<0 \end{cases} \tag{3}$$

where, $0<\sigma\leq 1$, $\lim_{t\to\infty}\varpi(t)=\varpi_\infty>0$,and ϖ_∞ is the maximum allowable value of steady state error.

Transform the constraint form as equation (3) to unconstraint form by error transformation function

$$e(t)=\varpi(t)\Phi(S) \tag{4}$$

where, S is the error after transformation, $\Phi(S)$ is a smooth, strictly increasing and reversible function and it meets the characters as equation(5)

$$\begin{cases} -\sigma<\Phi(S)<1,\text{if } e(0)>0 \\ -1<\Phi(S)<\sigma,\text{if } e(0)<0 \end{cases} \tag{5}$$

$$\begin{cases} \lim_{S\to-\infty}\Phi(S)=-\sigma,\ \lim_{S\to\infty}\Phi(S)=1,\text{if } e(0)>0 \\ \lim_{S\to-\infty}\Phi(S)=-1,\ \lim_{S\to\infty}\Phi(S)=\sigma,\text{if } e(0)<0 \end{cases} \tag{6}$$

Remark 1. We can learn from equation (6) that the equation (5) holds if S is bounded. From $\varpi(t)>0$ and equation(4),we also know that $-\sigma\varpi(t)<\varpi(t)\Phi(S)=e(t)<\varpi(t)$ as $e(0)>0$, and $-\sigma\varpi(t)<\varpi(t)\Phi(S)=e(t)<\varpi(t)$ as $e(0)<0$, so Eq(3) holds. Therefore, we can realize the prescribed performance as long as $S\in L_\infty$.

3. Design of missile overload controller based on backstepping method

Without considering the disturbance of missile during flight, according to the established missile pitch overload model (2), the overload controller is designed by backstepping method with prescribed performance.

In order to derive the control law and analyze stability analysis of the model easily, the model can be expressed as follow:

$$\begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1)x_2 \\ \dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)u \\ y = f(x_1) \end{cases} \tag{7}$$

where, $x_1 = \alpha$, $x_2 = \omega_z$, $u = \delta_z$, $f_1(x_1) = -\frac{1}{mV}\left(P\sin\alpha + C_y^\alpha \alpha\right)$, $g_1(x_1) = 1$, $f_2(x_1, x_2) = \frac{1}{J_z}\left(M_z^\alpha \alpha + M_z^{\omega_z}\omega_z\right), g_2(x_1, x_2) = \frac{M_z^{\delta_z}}{J_z}$, $f(x_1) = \frac{1}{mg}\left(P\sin\alpha + C_y^\alpha \alpha\right)$.

For this system, the backstepping controller with prescribed performance is designed.

Step1. Consider the first subsystem in system (7): $\dot{x}_1 = f_1(x_1) + g_1(x_1)x_2$.

Define the error state quantity e_1,we have $e_1 = y - y_d$,where, y_d is expected overload. Define prescribed function $\varpi_1(t) = (2 - 0.01)e^{-t} + 0.01$ and performance error transformation function $\Phi_1(S_1) = \frac{e^{S_1} - e^{-S_1}}{e^{S_1} + e^{-S_1}}$ .Let $e_1 = \varpi_1 \Phi_1(S_1)$,we have $S_1 = \Phi_1^{-1}\left(\frac{e_1}{\varpi_1}\right) = \varphi_1(b)$,where, $\varphi_1(b_1) = \frac{1}{2}\ln\frac{1+b_1}{1-b_1}$, $b_1 = \frac{e_1(t)}{\varpi_1(t)}$.

The derivatives of e_1 and $\Phi_1(S_1)$ are

$$\dot{e}_1 = \dot{y} - \dot{y}_d = f'(x_1)\dot{x}_1 = f'(f_1 + g_1 x_2) \tag{8}$$

$$\dot{S}_1 = \frac{\dot{\varphi}_1}{\varpi_1}\left(\dot{e}_1 - \frac{\dot{\varpi}_1}{\varpi_1}e_1\right) \tag{9}$$

Substituting e_1 into equation (9), we have

$$\dot{S}_1 = \frac{\dot{\varphi}_1}{\varpi_1}\left(f' \cdot f_1 + f' \cdot g_1 x_2 - \dot{y}_d - \frac{\dot{w}_1}{w_1}f + \frac{\dot{w}_1}{w_1}y_d\right) \tag{10}$$

Define Lyapunov function for the first subsystem $V_1 = \frac{1}{2}S_1^2$. Taking the derivative of V_1 gets

$$\dot{V}_1 = S_1\dot{S}_1 = S_1\frac{\dot{\varphi}_1}{\varpi_1}\left(f' \cdot f_1 + f' \cdot g_1 x_2 - \dot{y}_d - \frac{\dot{w}_1}{w_1}f + \frac{\dot{w}_1}{w_1}y_d\right) \tag{11}$$

Choose virtual control law as

$$x_{2,d} = -\frac{1}{f' \cdot g_1}\left(-f' \cdot f_1 - \dot{y}_d - \frac{\dot{\varpi}_1}{\varpi_1}f + \frac{\dot{\varpi}_1}{\varpi_1}y_d + k_1\frac{\dot{\varphi}_1}{\varpi_1}S_1\right) \tag{12}$$

where, $k_1 > 0$ is a constant.

Substituting equation (12) into equation (11), we have $\dot{V}_1 = k_1 S_1^2 \leq 0$.

Step2. Consider the second subsystem in system (13) $\dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)u$

Define the error state quantity e_2 ,we have $e_2 = x_2 - x_{2,d}$ .Define prescribed function $\varpi_2(t) = (2-0.01)e^{-t} + 0.01$ and performance error transformation function $\Phi_2(S_2) = \frac{e^{S_2} - e^{-S_2}}{e^{S_2} + e^{-S_2}}$ .Let $e_2 = \varpi_2 \Phi_2(S_2)$ .we have $S_2 = \Phi_2^{-1}\left(\frac{e_2}{\varpi_2}\right) = \varphi_2(b_2)$,where, $\varphi_2(b_2) = \frac{1}{2}\ln\frac{1+b_2}{1-b_2}$, $b_2 = \frac{e_2(t)}{\varpi_2(t)}$ .The derivatives of e_2 and $\Phi_2(S_2)$ are

$$\dot{e}_2 = \dot{x}_2 - \dot{x}_{2,d} = f_2 + g_2 u - \dot{x}_{2,d} \tag{13}$$

$$\dot{S}_2 = \frac{\dot{\varphi}_2}{\varpi_2}\left(\dot{e}_2 - \frac{\dot{\varpi}_2}{\varpi_2}e_2\right) \tag{14}$$

Substituting $\dot{e}_2$ into equation (14), we have

$$\dot{S}_2 = \frac{\dot{\varphi}_2}{\varpi_2}\left(f_2 + g_2 u - \dot{x}_{2,d} - \frac{\dot{\varpi}_2}{\varpi_2}x_2 + \frac{\dot{\varpi}_2}{\varpi_2}x_{2,d}\right) \tag{15}$$

Define Lyapunov function for the second subsystem $V_2 = \frac{1}{2}S_2^2$ .Taking the derivative of V_2 gets

$$\dot{V}_2 = S_2\dot{S}_2 = S_2\frac{\dot{\varphi}_2}{\varpi_2}\left(f_2 + g_2 u - \dot{x}_{2,d} - \frac{\dot{\varpi}_2}{\varpi_2}x_2 + \frac{\dot{\varpi}_2}{\varpi_2}x_{2,d}\right) \tag{16}$$

Choose virtual control law as

$$u = -\frac{1}{g_2}\left(f_2 - \dot{x}_{2,d} - \frac{\dot{\varpi}_2}{\varpi_2}x_2 + \frac{\dot{\varpi}_2}{\varpi_2}x_{2,d} + k_2\frac{\varpi_2}{\dot{\varphi}_2}S_2\right) \tag{17}$$

where, $k_2 > 0$ is a constant.

Substituting equation (18) into equation (17), we have $\dot{V}_2 = k_2 S_2^2 \le 0$ .Define Lyapunov function for the whole system $V = \frac{1}{2}S_1^2 + \frac{1}{2}S_2^2$. Taking the derivative of V gets $\dot{V} = S_1\dot{S}_1 + S_2\dot{S}_2$.

Substituting equation (10) into equation (15), we have

$$\dot{V} = -k_1 S_1^2 - k_2 S_2^2 \le -k_0 V \tag{18}$$

where, $k_0 = \min\{k_1, k_2\}$.

It proved that the system is stable in the sense of Lyapunov. At this point, the control quantity of missile model (7) based on backstepping controller with prescribed performance is

$$u = -\frac{J_z}{M_z^{\delta_z}}\left(\frac{M_z^{\alpha}\alpha + M_z^{\omega_z}\omega_z}{J_z} - \dot{x}_{2,d} + \frac{\dot{\varpi}_2}{\varpi_2}(x_{2,d} - x_2) + k_2\frac{w_2}{\dot{\varphi}_2}\varphi_2\right) \tag{19}$$

where, $x_{2,d} = -\frac{mg}{P\cos\alpha + C_y^{\alpha}}\left(-\frac{P\cos\alpha + C_y^{\alpha}}{mg}\cdot\frac{P\sin\alpha + C_y^{\alpha}\alpha}{mV} - \right.$

$$\left.\dot{y}_d - \frac{\dot{w}_1}{w_1}\left(\frac{P\sin\alpha + C_y^{\alpha}\alpha}{mg} - y_d\right) + k_1\frac{\dot{s}_1}{w_1}s_1\right) \tag{20}$$

4. Numerical Simulation

For the above model, set the expected load as $n_y^* = \sin(t)$. We set the initial conditions of simulation as follows: $n_y(0) = 0.5$, $\omega_z(0) = -1 rad/s$, $m = 1000kg$, $g = 9.8m/s^2$, $V = 800m/s$, $p = 50kN$, $C_y^\alpha = 15$, $M_z^\alpha = -15$, $M_z^{\omega_z} = -300$, $M_z^{\delta_z} = -10$, $J_z = 2500kg \cdot m^2$.

The correlative parameters of controller are chosen as $k_1 = 20$, $k_2 = 10$, $t = 0.01s$.

The simulation results are shown in Figures 1-4. Figure 1 shows the situation of system output overload tracking expected overload. It can be seen from the figure that the system output can track the expected track within 1s, and it can always keep stable track with the change of time. Figure 2 shows the situation of pitching angular velocity tracking pitching angular velocity commands, it has the characteristics of rapidity and stability. Figure 3 and figure 4 respectively represent the change of tracking error of overload and pitch angle velocity with time, and the dotted line in the figure represents the upper and lower bounds of prescribed error. The figure shows that the tracking error limited by the performance function, has a fast response speed, and the steady-state error is always kept in a small field of 0, which can meet the requirements of the system's prescribed steady and transient performance. Figure 5 shows the change of control quantity with time, the control quantity is always bounded, which indicates that the system is in the controllable range and can effectively prove the rationality of the controller design method.

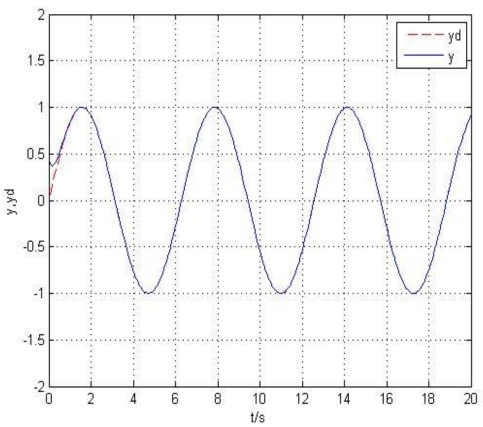

Figure 1.overload n_y tracking overload commands n_y^* (n_y^* —dashed line, n_y —solid line).

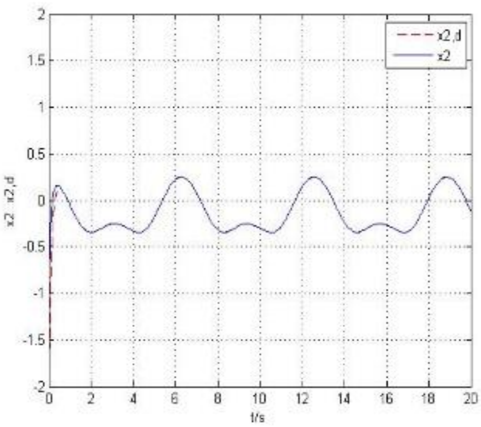

Figure 2. pitching angular velocity ω_z tracking pitching angular velocity commands ω_z^* (ω_z^* — dashed line, ω_z —solid line).

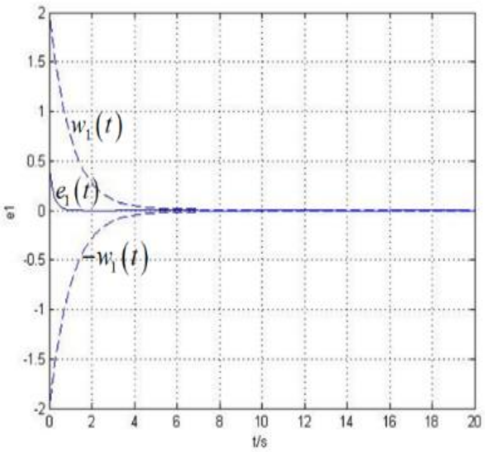

Figure 3. change of tracking error e_1 with time.

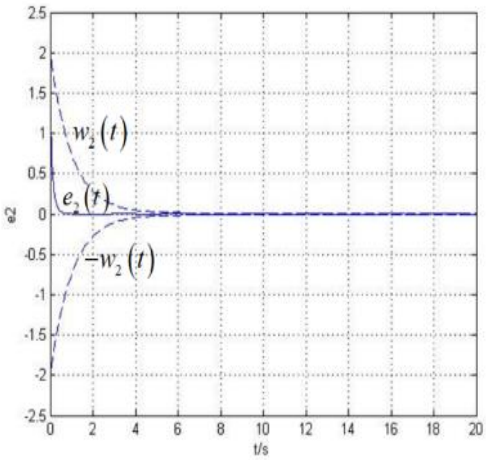

Figure 4. change of tracking error e_2 with time.

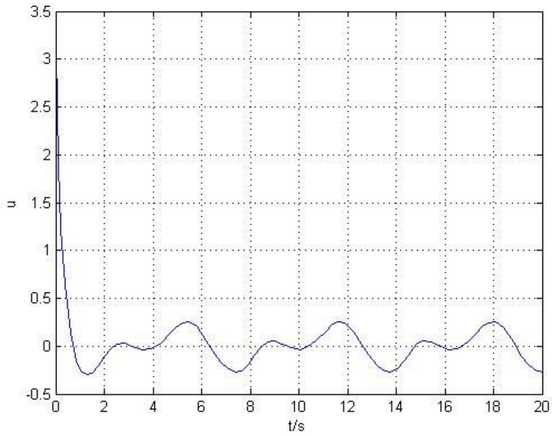

Figure 5. control amount u change with time.

5. Conclusions

Based on the missile pitch channel overload motion model, we have studied the design method of missile overload control system. By introducing performance function and error conversion function, we adopt the backstepping method with prescribed performance to design the missile overload controller. The given simulation results demonstrate the effectiveness of the proposed control method. The results show the missile system overload control with prescribed performance scheme can improve the transient and steady state performance of the system.

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