

Nonlinear elastic analysis of concrete beams based on the Smeared Crack Approach

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Abstract. In the present study, an analysis of plain and reinforced concrete beams under monotonic loading was made based on the Fixed Smeared Crack approach. The objectives of this research were to analyze the nonlinear behavior of the selected cases of analysis and to propose an alternative and simple model for the analysis of beams under service loadings, by means of Committee 435 of the American Concrete Institute. A brittle model for concrete and a linear-elastic behavior for steel reinforcement bars were considered. Results are presented through force-displacement curves and the sequence of cracking propagation. Also, a comparison of calculated instantaneous deflections of simply supported beams was made between the proposed model and other researches. It was verified that the proposed algorithm can predict adequately the cracking process and the deflections of beams subjected to service loadings, taking into account experimental results from other authors.

1. Introduction

Concrete structures undergo a high nonlinear behavior after cracking because the tensile strength decays and develops an anisotropic behavior [1]. Likewise, when the formation of micro-cracks entails in an important failure zone, the use of linear elastic models for structural analysis is insufficient [2]. Moreover, several authors have proposed complex models and methods that can simulate the behavior of concrete structures [3-6].

These cracking models can be categorized into two groups. The first of them, the Discrete Crack Approach, which evaluates cracking propagation as a physical phenomenon [6]. That is to say, it generates a geometrical discontinuity in the finite element mesh. This methodology was the first implemented for the analysis of cracking [7]. However, one disadvantage of this method is that it requires a constant recalculation of nodal connectivity between the elements [6]. Therefore, the computational cost of the method is high, because it employs sophisticated algorithms for the remeshing along the cracking process [8].

On the other hand, the Smeared Crack Approach, proposed originally by Rashid [3], evaluates cracking in a continuum along finite elements, in which the propagation of crack implies a localized modification of the constitutive equation [9]. In that regard, after the formation of a crack in an element, the material changes from an isotropic behavior to an orthotropic behavior [6]. Compared with the Discrete Crack Approach, this method follows the same principles as the finite element method based on displacements because the continuity of the displacement field is guaranteed in the system [10]. Likewise, since the simulation of cracking is in the finite element and not in their boundaries the direction of the cracks is not restricted [6].

In the analysis of beams under service loading, the ACI 435 Committee [11] mentions that the



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evaluation of deflections is as important as the calculation of the ultimate strength. Therefore, the Branson model [12] is suggested for the calculation of instantaneous elastic deflections in reinforced concrete beams, by means of the use of an effective inertia that takes into account the cracked inertia and the real inertia of the section and the cracking moment. Considering these parameters, limits for deflections are imposed for the design of reinforced concrete beams.

In this context, the present research deals with the analysis of unreinforced and reinforced concrete beams under monotonic loading for the evaluation of cracking, the ultimate strength and the deflections produced under service loading. The Smeared Crack Approach is used and the fragile model is selected for the evaluation of cracking, considering the fracture initiation perpendicular to the principal tensile stress when the stresses are greater than the ultimate tensile strength of the material. A linear elastic model for reinforcement bars is used. Finally, the obtained results are compared with other authors [12-15].

2. Method

In the present investigation the Smeared Crack Approach is used because it can evaluate cracking by altering the constitutive equation of the material [6].

This approach can be categorized into three groups: fixed, rotational and fixed multidirectional [6]. The principal difference between these lies the evaluation of the crack direction during the fracture process. While in the fixed model the direction of the crack remains invariant along the computational process, in the rotational model the crack direction may change in function of the principal stresses. On the other hand, a multidirectional model considers the direction of the crack by means of an angle of retention [8]. This approach is proposed with a combination of a failure criterion, the level of stress transfer after cracking and the softening model employed for the material which can be fragile, linear, multilinear and nonlinear [15]. This study employs a fragile softening model because it is capable to evaluate the structural degradation generated by cracking [6].

For the failure criterion, only the stresses of the cracks in the mode of fracture I are evaluated. This hypothesis holds true if compressive stresses are low [17]. Therefore, is optimal for the evaluation of beams, because the failure of the element is caused by the tensile stress produced.

For the evaluation of cracking is essential to differentiate the uncracked concrete from cracked concrete. For the first one it's assumed a linear elastic and isotropic behavior, governed by the following constitutive relation:

$$\mathbf{D}^{co} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (1)$$

Where E is the Young's Module of concrete and ν is the Poisson's module. In regard of the constitutive relation for the cracked material it's assumed that a cracked element lost all his mechanical properties, therefore its Young's Modulus is reduced to a negligible value in order to ensure mathematical stability during the computational process.

3. Results

3.1. Simple supported unreinforced notched concrete beam - Kormeling & Reinhardt [19]

First, to validate the results obtained from the proposed model they are compared with other computational model results obtained by Sena, et al. [13], which compared the results from their computational model with numerous constitutive relationships and the experimental results by Kormeling y Reinhardt [19]. The following figure shows the mesh implemented for the beam. It considers four-node quadrilateral isoparametric elements. The problem has a notch at the middle of the beam. Dimensions are expressed in millimeters. The beam has a width of 100 mm.

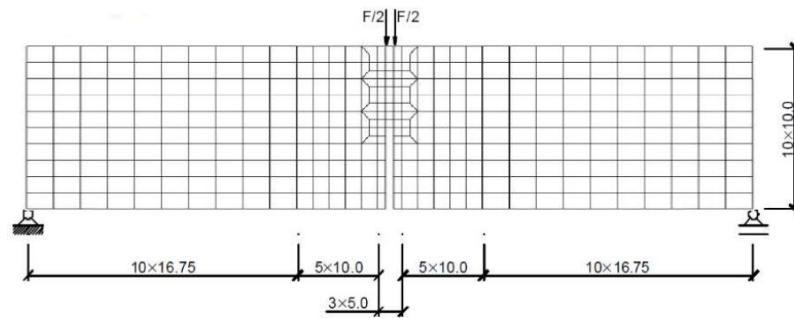


Figure 1. Proposed mesh.

For the analysis of the model a concrete density of 2400 kg/m³ was considered as well as a Poisson's module of 0.20 and an elasticity module of 20000.0 Mpa. For the parameters necessary to model crack it was considered an ultimate tensile strength of 2.4 Mpa.

For the solution of the softening problem, a displacement control in the point of loading was implemented as proposed by de Borst, Crisfield, Remmers y Verhoosel [20]. Thus, under the application of a vertical deflection of -1.00 mm the following force-deflection curve was obtained.

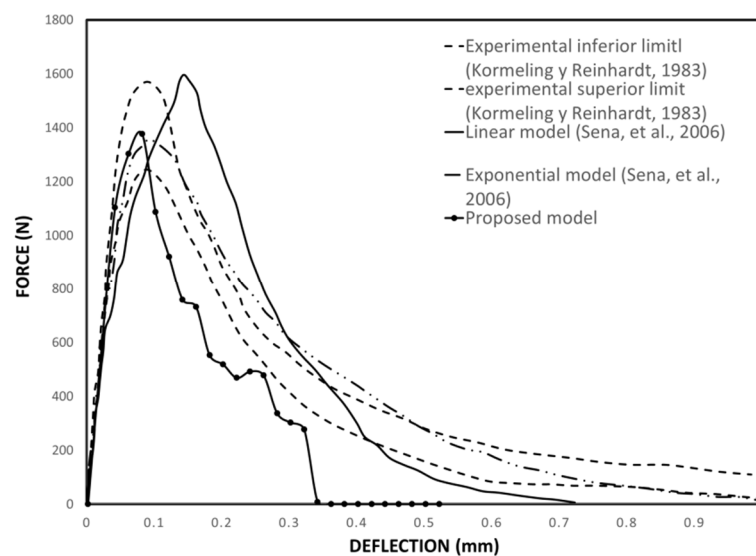


Figure 2. Force vs deflection curve at middle span of the beam.

The results show that the model can predict the maximum bearing capacity of the beam, as the calculated value is in the range of the experimental results of 1200N and 1600N approximately. Further, the progressive degradation of the bearing capacity of the beam is illustrated. However, this section of the curve decays abruptly because of the fragile constitutive relationship implemented.

Then, the progression of cracking is presented. The first group of crack appears when a displacement of -0.0225 mm is applied with a load of 586.39 N. On the other hand, the maximum bearing capacity of the beam was found under a deflection of -0.0823 mm, under a load of 1.376 kN approximately. It's showed that the peak force is within the experimental result range. This crack state can be observed in figure 3.

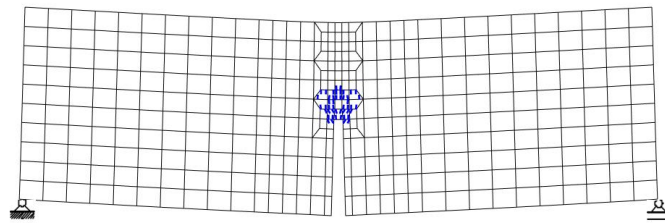


Figure 3. Fractured system under its maximum resistant load.

3.2. Simple supported reinforced concrete beam - Walraven [21]

Secondly, the reinforced beam experimentally tested by Walraven [21] was analysed. Numeric result obtained by de Borst y Nauta [14] were compared. The beam studied has a clear span of 0.45m and a width of 0.20 m. The reinforcement was composed of a single 20 mm diameter bar and two 14 mm diameter bars, both of them placed at 30 mm from the base of the beam. The dimensions, in meters, of the beam to be evaluated are presented in figure 4. Dimensions are presented in meters.

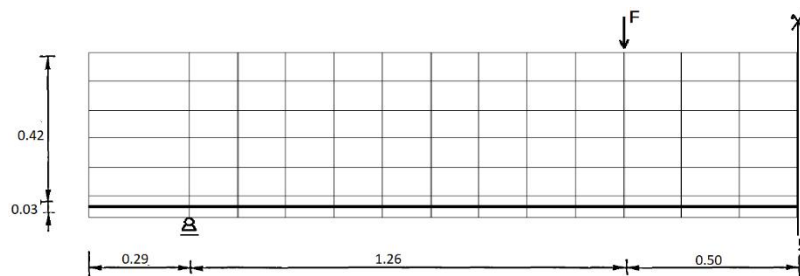


Figure 4. Case of study [14].

An elastic module of 28000.0 Mpa, a Poisson's module of 0.20 and an ultimate tensile strength of 2.5 Mpa were used. The reinforcement was assumed linear elastic with an elastic module of 210000 Mpa.

For the solution of the nonlinear, a force control was applied for an ultimate load of 80 kN, the following force vs deflection diagrams were obtained, for the mesh 1, proposed by de Borst y Nauta [14], and for the mesh 2, more refined, proposed by the present investigation. In the figure 5 the results of the force vs deflection diagram at the center of the beam are showed.

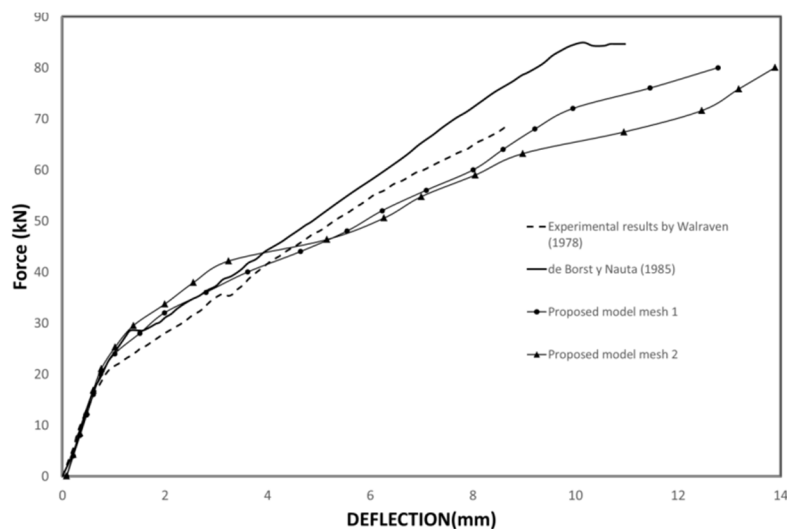


Figure 5. Force vs deflection curve at middle of the beam.

It's showed that the results obtained follow the same tendency as the results obtained by other authors. It is observed that the result has an almost linear behavior until a vertical deflection of approximately 0.8 mm and a force of 17 kN is reached. From this point, the beam loses his bearing capacity at the middle, because of the severity of the crack propagation. After surpassing this point, only the reinforcement provides stiffness to this part of the element. However, because the reinforcement its assumed linear elastic, the curve follows almost a linear tendency.

For the proposed mesh, figure 6 shows the cracked system when the first fracture forms. It develops when the beam has reached a vertical deflection of -0.5382 mm and a force of -14.01 kN.

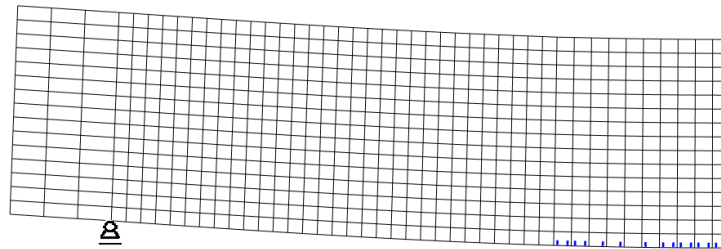


Figure 6. First cracks developed with the proposed mesh.

3.3. Unreinforced and reinforced concrete beams under service loadings - Branson [12] and Washa & Fluck [16]

For the evaluation of instant deflections, the ACI Committee 435 [11] suggests the use of the method proposed by Branson [12]. An effective inertia was proposed for the calculation of deflections. Moreover, a linear elastic analysis is performed for a unidimensional beam, considering the effective inertia for the computation of the deflection under service loading [12].

For the validation of the method, Branson [12] compared experimental results of tested beams and the experiments developed by Washa y Fluck [16] with the results from his calculation. A few of the analyzed beams were selected with the objective of comparing the result of the method proposed by [12] with the model proposed by this investigation. The properties of the selected beams can be found in [12]. The mesh employed for the analysis was proposed under guidelines for solving nonlinear problems for reinforced concrete by the Finite Element Method [22].

It is observed that the percentage of error obtained in the analysis in comparison with the experimental results is acceptable when compared with the results obtained by the recommendations of the ACI [11]. In the table 1 the calculated deflections by the proposed investigation and the Branson method [12] are showed.

Table 1. Comparative results.

N°	Reference	Beam	Exp Deflection	Newmark's Deflection	Effective inertia Deflection	Prop Model Deflection	Expvs Newmark's	Exps vs Effective inertia	Exp vs Prop Model
			mm	mm	mm	mm	%	%	%
1	[12]	SB-1	1.04	1.27	1.27	1.22	18%	18%	15%
2	[12]	SB-3	3.89	5.16	5.23	3.65	25%	26%	6%
3	[16]	A1,A4	13.46	15.49	15.75	13.94	13%	15%	3%
4	[16]	B1,B4	23.37	25.15	25.15	26.53	7%	7%	12%
5	[16]	C1,C4	40.13	44.45	44.45	46.30	10%	10%	13%
6	[16]	D1,D4	11.94	16.00	16.00	15.82	25%	25%	25%
7	[16]	B2,B5	24.89	25.91	25.91	26.08	4%	4%	5%
8	[16]	C2,C5	43.43	44.70	44.96	45.49	3%	3%	5%
9	[16]	D2,D5	14.22	16.26	16.51	16.19	13%	14%	12%
10	[16]	A3,A6	17.02	16.76	16.76	17.02	2%	2%	0%
11	[16]	B3,B6	26.42	25.91	25.91	26.90	2%	2%	2%
12	[16]	C3,C6	47.75	46.48	46.23	45.74	3%	3%	4%
13	[16]	D3,D6	17.78	16.76	16.76	16.49	6%	6%	8%

It is observed that the proposed model can analyze the propagation of cracks under incremental loading. Likewise, on top of calculating instant deflections and evaluate designs code criteria's, it is possible to verify the damage due to cracking in the element.

4. Conclusions

The results obtained by the computational model proposed in the present investigation were satisfactory by comparing force-deflection curves proposed by other authors. In this regard, the analysis showed that the model is capable of calculate the maximum bearing capacity of reinforced and reinforced concrete beams and present diagrams of the propagation of cracks, i.e. it can predict the localization and direction of the crack and the progressive failure of elements. On the other hand, the proposed model is capable of acceptably predicting the deflection of reinforced concrete beams under service loading. Moreover, its showed that the method used for the proposed model can predict cracking patterns under service loading. Because of this, it can be possible to stablish more design criteria's to control the damage produced by cracks.

However, due to the fact the constitutive relation employed by the proposed model is fragile, the evaluation of the degradation of the bearing capacity of the beams is overestimated. This disadvantage can be overcome by the implementation of more complex constitutive relations. It was observed that results were mesh dependent which can also be prevented by the implementation of more complex constitutive relations. This represent an oportunity of development of the presented model. Finally, numerical instability was observed during the development of the proposed model which are of common occurrence in softening materials. For this reason, more complex solution procedures like the Modified Riks Method enhanced with a local constrain equation or the Generalized Displacement Control Method are recommended for this type of problems.

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