Communications in Theoretical Physics

Commun. Theor. Phys. **72** (2020) 045401 (7pp)

https://doi.org/10.1088/1572-9494/ab6911

Testing the distance-duality relation with the baryon acoustic oscillations data and type Ia supernovae data

Jun Chen

School of Science, Kaili University, Kaili, Guizhou 556011, China

E-mail: 79892345@qq.com

Received 18 November 2019, revised 19 December 2019
Accepted for publication 24 December 2019
Published 6 March 2020

Abstract

In this letter, the cosmic distance-duality relation has been constrained with a model-independent method by combining the baryon acoustic oscillation (BAO) data and the type Ia supernova (SNe Ia) data. The results show that this relation is consistent with the observational data in the 68.27% error range, except for the instance of Union 2.1 plus BAO with the statistic errors only, where the relation is consistent with the observations in the 95.45% error range. To study the result of the uncertainty of the Hubble constant on the investigation of this relation,we treat the dimensionless Hubble constant h as a free parameter and get that the observational data favors the relation in the 68.27% error range. And then h has been marginalized and the results support that this relation is favored by the observations in the 68.27% error range too.

Keywords: Baryon acoustic oscillation, the cosmic distance-duality relation, type Ia supernovae

(Some figures may appear in colour only in the online journal)

1. Introduction

There is a very important relation in cosmology, and it is the distance-duality relation (DDR). It was founded in 1933 by Etherington [1], which is the relation between the angular diameter distance D_A and the luminosity distance D_L, and has the following form

$$\frac{D_L}{D_A(1+z)^2} - 1 = 0, \tag{1}$$

where z is the red-shift. There are three general hypotheses about the DDR: photons' number is constant, the Universe's space-time is denoted by Riemannian geometry and light travel along zero geodesics. Violation of the first hypothesis can study the cosmic opacity [2–9]. And violation of the next two hypotheses can study the exotic physics [10–14], Tiwari has constrained axion-like particles with the DDR in the paper [14]. Because the DDR does not relate with the matter and the field equations of Einstein, and influences the relationship of the observational data D_A and D_L, so it plays a very important role in modern cosmology and in cosmic observations [15–20]. Therefore, it is very requisite and worthy to check for the DDR.

Uzan *et al* studied the DDR [21] and got that this relation coincides with the Galaxy cluster observational data in the 68.27% error range and value of η is slightly smaller than 1. Rana *et al* [22] found that the DDR is consistent with observational data for the entire redshift range (0,2.418) in the 68.27% error range. Santos *et al* [23] discovered that the DDR is consistent with galaxy clusters observations and Hubble parameter observations in the 68.27% error range. Liao *et al* [24] got that the DDR is consistent with type Ia supernovae (SNe Ia) data and strong gravitational lensing observations in the 68.27% error range. Using type-Ia supernovae and ultra-compact radio data, Li and Lin [25] discussed the DDR and found that the DDR coincides with observations in the 95.45% error range. Lin, Li and Li [26] constrained the DDR and got that the DDR is true with the newer baryon acoustic oscillation (BAO) data and the type Ia supernovae (SNe Ia) data in the 68.27% error range. Holanda *et al* [27] have studied the DDR with supernovae Ia and strong gravitational lensing data and find that this relation is consistent the observations in the 68.27% error range. In the preceding time, the DDR was discussed too [28–44].

Lately, Wu *et al* [45] constrained the DDR with the Union2.1 data and five BAO data, and they found that the

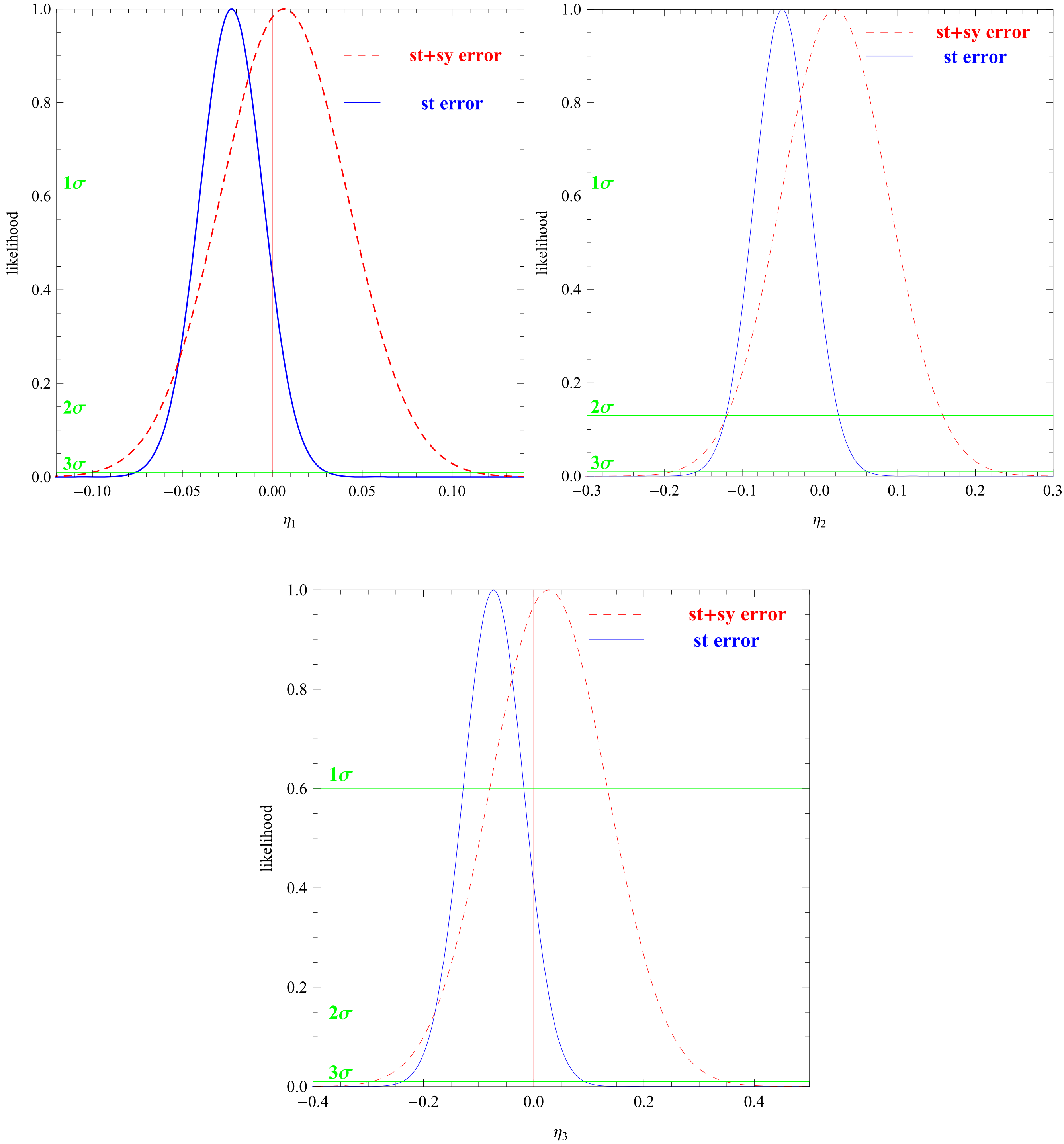

Figure 1. The likelihood distributions of η_1, η_2 and η_3 without (*st*) and with (*sy*) systematic errors from the Union2.1 plus BAO.

DDR is consistent with observational data in the 95.45% error range before h was marginalized. Now that more and new observations have been provided, we will check for the DDR with the observational data in this paper.

2. Method and samples

In order to check for the DDR with a model-independent way, one must get D_A and D_L at the same redshift from the different cosmic observations, and the DDR is true if the equation (1) is zero. The D_A data is chosen from the BAO [46–50] observational data. The BAO produces in the early universe and causes an over density of baryons at about the 150Mpc scale now. Because the measurement of the BAO is not related to the number of photons, it is regarded as the standard ruler. The nine data points have been listed in table 1. The D_L data are chosen from the standard candles, SNe Ia, include the Union2.1 SNe Ia [51] and the joint light-curve analysis SNe Ia (JLA) [52–54]. The JLA has 740 samples and the Union2.1

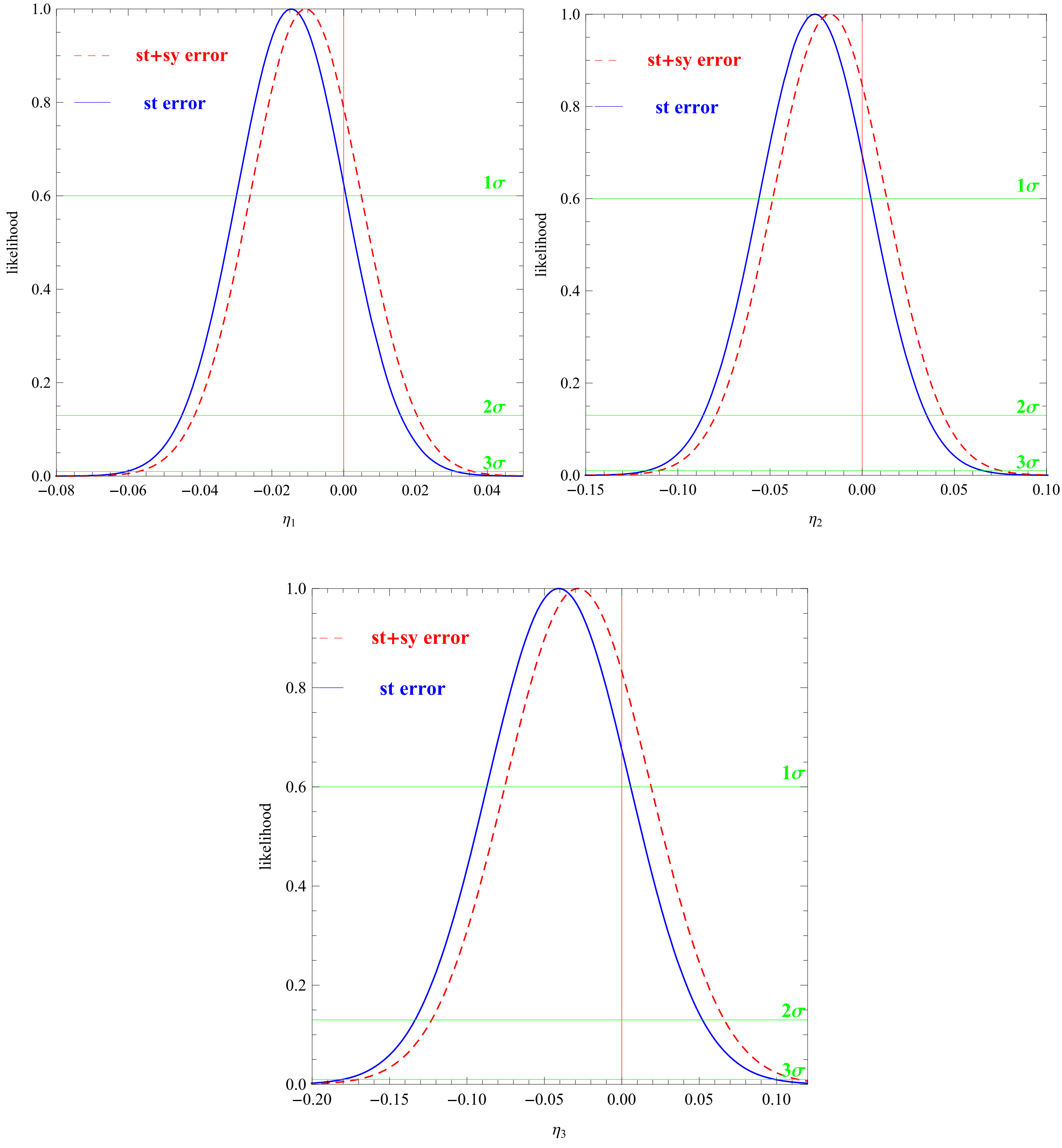

Figure 2. The likelihood distributions of η_1, η_2 and η_3 without and with systematic errors from the JLA plus BAO.

has 580 data points. Because one can only get the distance modulus μ directly from the type Ia supernovae data, so the D_L can be calculated through equation (2)

$$D_L = 10^{0.2(\mu-25)}. \tag{2}$$

In order to get the $D_L(z)$ from $\mu(z)$, all SNe Ia data in the region $[z - 0.005, z + 0.005]$ have been binned, and the computational formula is

$$\mu_{bin}(z) = \frac{\sum_i \mu_i \sigma_i^{-2}}{\sum_i \sigma_i^{-2}}, \tag{3}$$

and $\sigma^2_{\mu_{bin}}$ is

$$\sigma^2_{\mu_{bin}} = \frac{1}{\sum_i \sigma_i^{-2}}. \tag{4}$$

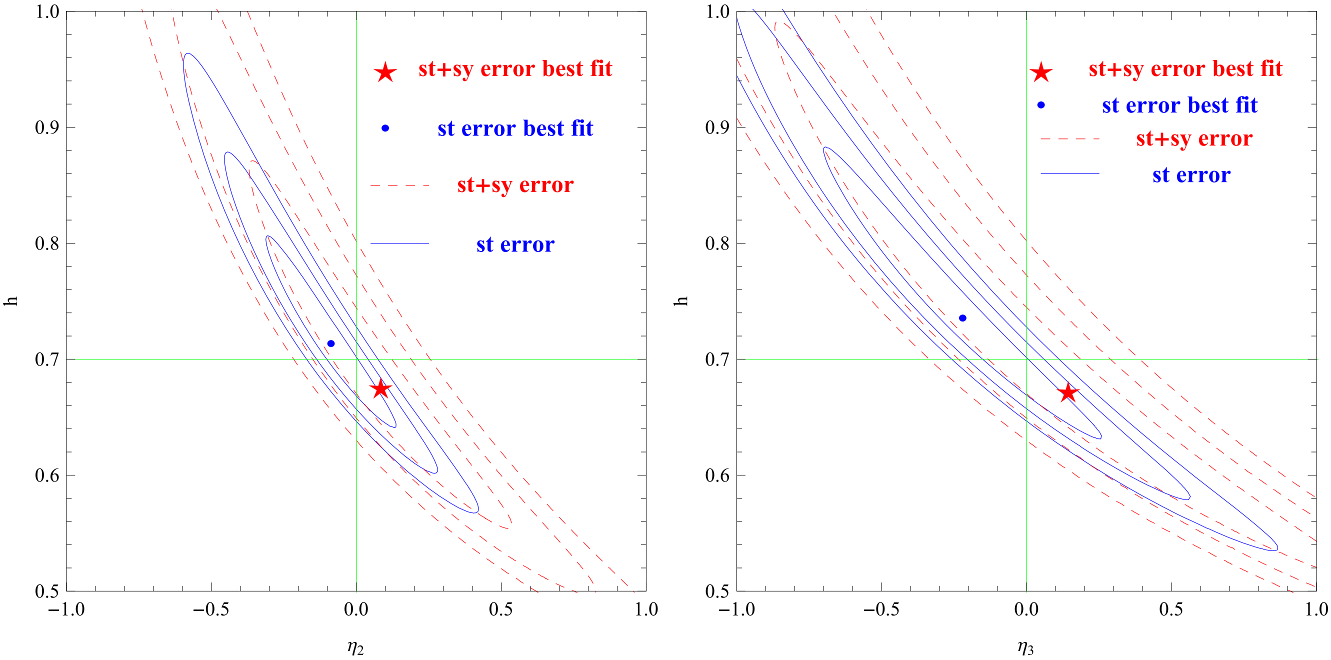

Figure 3. The contour plots of $h - \eta_2$ and $h - \eta_3$ from BAO plus Union2.1 observational data.

Here, $\sigma_{\mu_{bin}}$ is the error of the μ_i.

To check for the DDR with the observational data, three parametrizations of equation (1) have been chosen,

$$\frac{D_L}{D_A(1+z)^2} - 1 = \eta_1, \tag{5}$$

$$\frac{D_L}{D_A(1+z)^2} - 1 = \eta_2 z, \tag{6}$$

$$\frac{D_L}{D_A(1+z)^2} - 1 = \eta_3 \frac{z}{1+z}. \tag{7}$$

Here η_1, η_2 and η_3 are constants. The results of η_1, η_2 and η_3 will be null if the DDR is valid with the observational data.

To obtain the best fit results of η_1, η_2 and η_3, we calculate them by using $L \propto \mathrm{e}^{-\chi^2/2}$, with

$$\chi^2(\eta_j) = \sum_i \frac{(\eta_j - \eta_{j,obs}(z_i))^2}{\sigma^2_{\eta_{j,obs}}(z_i)}. \tag{8}$$

Here $j = 1$, 2, or 3, $\eta_{j,obs}(z_i)$ is the result gained from the observational data, and $\sigma_{\eta_{j,obs}}(z_i)$ is the error of $\eta_{j,obs}(z_i)$. The likelihood of η_j is $P = \mathrm{e}^{-\chi^2(\eta_j)/2}$.

Table 1. The nine baryon acoustic oscillation data.

z	$D_A(z)(Mpc)$	1σ	*References*
0.32	981	20	[46]
0.35	1050	38	[47]
0.38	1100	14 ± 8	[48]
0.51	1309	15 ± 9	[48]
0.61	1418	17 ± 10	[48]
0.44	1205	114	[49]
0.60	1380	95	[49]
0.73	1534	107	[49]
0.57	1380	23	[50]

Table 2. The best fit values of η and the 68.27% error range without systematic errors and with systematic errors.

η	Union2.1+BAO	JLA+BAO
$\eta_{1,st}$	-0.0227 ± 0.0177	-0.0146 ± 0.0147
$\eta_{2,st}$	-0.0485 ± 0.0364	-0.0256 ± 0.0304
$\eta_{3,st}$	-0.0730 ± 0.0550	-0.0407 ± 0.0464
$\eta_{1,sy}$	0.0066 ± 0.0354	-0.0106 ± 0.0155
$\eta_{2,sy}$	0.0193 ± 0.0694	-0.0176 ± 0.0309
$\eta_{3,sy}$	0.0270 ± 0.1069	-0.0283 ± 0.0473

3. Results

We first check for the η_1, η_2 and η_3 with the Union2.1, JLA and BAO data, and find that the likelihood distributions of η_1, η_2 and η_3 are shown in figures 1 and 2. The best fit values and errors are listed in table 2.

The DDR is consistent with the BAO plus Union2.1 data in the 95.45% error range if the systematic errors are not considered, and in the 68.27% error range if the systematic errors are considered. And the DDR is consistent with the BAO plus JLA data in the 68.27% error range whether the systematic errors are considered or not.

To obtain the distance modulus μ from the SNe Ia [51, 52], the reductive Hubble constant h ($H_0 = 100$ h km s^{-1} Mpc^{-1}, H_0 is Hubble constant) has been taken to be $h = 0.7$. Because there are minute differences in

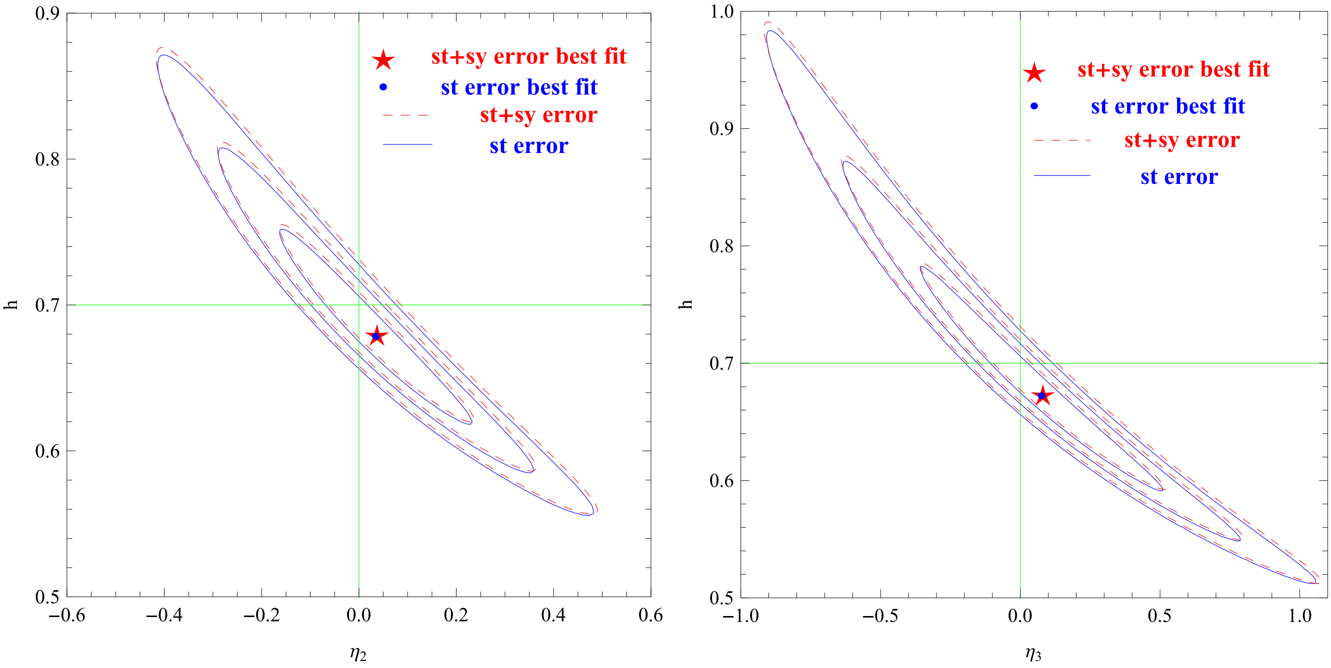

Figure 4. The contour plots of $h - \eta_2$ and $h - \eta_3$ from BAO plus JLA observational data.

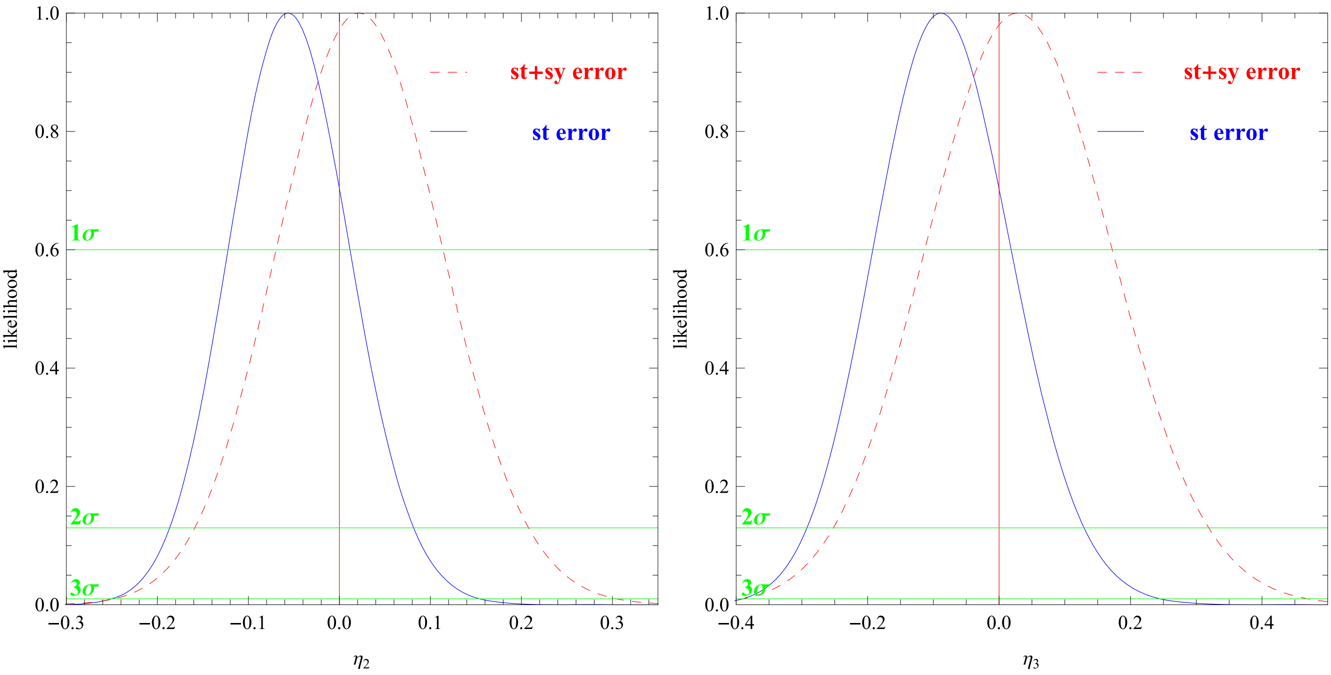

Figure 5. The likelihood functions of η_2 and η_3 with a marginalization over h from BAO plus Union2.1 observational data.

different observational h, for example, (0.678 ± 0.009 [55], 0.700 ± 0.022 [56, 57], 0.738 ± 0.024 [58], 0.6850 ± 0.0127 [52], $0.7612^{+0.0347}_{-0.0344}$ [59] and $0.672^{+0.012}_{-0.010}$ [60]), so we must consider the impact of $h = 0.7$. For the sake of discussing the influence of the uncertainty of h, we assume it is a free parameter. Because $D_L \propto \frac{1}{h}$, one can obtain $D_L = \frac{0.7}{h} 10^{0.2(\mu - 25)}$ [45].

And then, we constrain the DDR by allowing h to be a free parameter rather than a definite value. The results are shown in figures 3 and 4, and the best fit values are ($\eta_{2,st} = -0.0872$, $h = 0.714$), ($\eta_{2,sy} = 0.0840$, $h = 0.677$), ($\eta_{3,st} = -0.2203$, $h = 0.736$) and ($\eta_{3,sy} = 0.1432$, $h = 0.674$) with the Union2.1 plus BAO data; ($\eta_{2,st} = 0.0348$, $h = 0.679$), ($\eta_{2,sy} = 0.0369$, $h = 0.681$), ($\eta_{3,st} = 0.0763$, $h = 0.673$) and ($\eta_{3,sy} = 0.0806$, $h = 0.675$) with the JLA plus BAO data. So the DDR is consistent with the observational data in the 68.27% error range whether the systematic errors are considered or not while h is a free parameter.

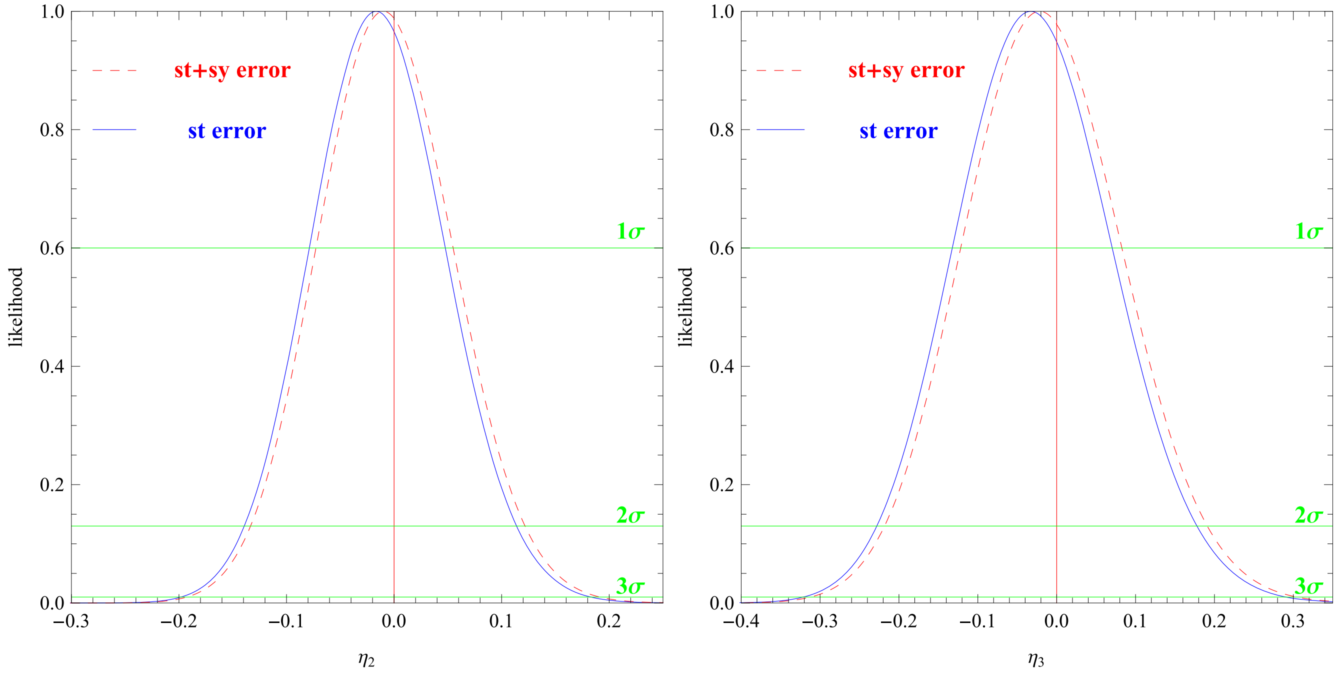

Figure 6. The likelihood functions of η_2 and η_3 with a marginalization over h from BAO plus JLA observational data.

To eliminate the influence of h, we marginalize the h. We assume that h is a Gaussian distribution, $\overline{P}(h) = \exp\left(-\frac{(h-h_{obs})^2}{2\sigma_{h_{obs}}^2}\right)$, and one can get $P(\eta_j)$ by calculating

$$P(\eta_j) = \int_{-\infty}^{+\infty} \overline{P}(h)\cdot P(\eta_j, h)\mathrm{d}h, \tag{9}$$

where $P(\eta_j, h) = \exp(-\chi^2(\eta_j, h)/2)$. The Hubble constant is chosen $H_0 = 70.0 \pm 2.2$ km s^{-1}Mpc^{-1} [56, 57] in this paper. The likelihood distributions of η_2 and η_3 are shown in figures 5 and 6. And find the best fit values and 68.27% error range are $\eta_{2,st} = -0.0564^{+0.0680}_{-0.0760}$, $\eta_{2,sy} = 0.0214^{+0.0924}_{-0.0909}$, $\eta_{3,st} = -0.0887^{+0.1071}_{-0.1035}$ and $\eta_{3,sy} = 0.0283^{+0.1439}_{-0.1414}$ with the Union2.1 plus BAO data; $\eta_{2,st} = -0.0167^{+0.0633}_{-0.0622}$, $\eta_{2,sy} = -0.0099^{+0.0648}_{-0.0627}$, $\eta_{3,st} = -0.0329^{+0.1033}_{-0.0995}$ and $\eta_{3,sy} = -0.0215^{+0.1041}_{-0.1002}$ with the JLA plus BAO data. Though the best fit values are not zero, but the DDR is accommodated in the 68.27% error range after h is marginalized, whether the systematic errors are considered or not.

4. Conclusion

The DDR is very important in modern cosmology because it is independent with any cosmic model, and it can reflect a lot of information about the Universe when one tests it.

In this letter, we check for the DDR with the new standard candles (SNe Ia) and the new standard rulers (BAO), the best fit values of η_j support that the DDR is valid to the observational data in the 68.27% error range, except for the instance of Union2.1 and BAO regardless of the systematic errors where the DDR is valid to observations in the 95.45% error range.

When we treat h as a free parameter, the DDR is consistent with observations in the 68.27% error range no matter how the systematic errors are included. So this constraint is dependent on the precise Hubble constant. And then, assuming h is a Gaussian distribution and making a marginalization over it, the likelihood functions of η_2 and η_3 are obtained, the best fit values of η_2 and η_3 are nearer zero, the DDR is consistent with observations in the 68.27% error range, whether systematic errors are considered or not. So, the more precise observations are expected to constrain the DDR in future.

Acknowledgments

This work was supported by the Foundation of the Guizhou Provincial Education Department of China under Grants No. KY[2016]104, the National Natural Science Foundation of China under Grants Nos. 11 465 011 and 11 865 018, the Foundation of Guizhou Provincial Science and Technology of China under Grants No. J[2014]2150.

References

[1] Etherington I M H 1933 *Philos. Mag.* **15** 761
[2] Li H K, Wu P X and Yu H W 2016 *Chin. Phys. Lett.* **33** 059801
[3] Liao K, Avgoustidis A and Li Z X 2015 *Phys. Rev.* D **92** 123539
[4] Lima J A S, Cunha J V and Zanchin V T 2011 *Astrophys. J. Lett.* **742** L26
[5] Holanda R F L and Busti V C 2014 *Phys. Rev.* D **89** 103517
[6] Nair R, Jhingan S and Jain D 2012 *J. Cosmol. Astropart. Phys.* **12** 028

[7] Chen J *et al* 2012 *J. Cosmol. Astropart. Phys.* **10** 029
[8] Chen J 2016 *Res. Astron. Astrophys.* **17** 175
[9] Chen J 2018 *Inter. J. Mod. Phys.* D **27** 1850054
[10] Aguirre A 1999 *Astrophys. J.* **525** 583
[11] Csaki C, Kaloper N and Terning J 2002 *Phys. Rev. Lett.* **88** 161302
[12] Bassett B A and Kunz M 2004 *Astrophys. J.* **607** 661
[13] Bassett B A and Kunz M 2004 *Phys. Rev.* D **69** 101305
[14] Tiwari P 2017 *Phys. Rev.* D **95** 023005
[15] Zhao Z 2019 *Commun. Theor. Phys.* **71** 1097
[16] Fu X Y, Wu P X, Yu H W and Zhou B J 2013 *Inter. J. Mod. Phys.* D **22** 1350025
[17] Zhang X and Huang Q G 2019 *Commun. Theor. Phys.* **71** 826
[18] Yu H, Lin Z C and Liu Y X 2019 *Commun. Theor. Phys.* **71** 991
[19] Fu X Y, Wu P X and Yu H W 2011 *Inter. J. Mod. Phys.* D **20** 1301
[20] Yang L M *et al* 2019 *Commun. Theor. Phys.* **71** 545
[21] Uzan J P, Aghanim N and Mellier Y 2004 *Phys. Rev.* D **70** 083533
[22] Rana A *et al* 2016 *J. Cosmol. Astropart. Phys.* **07** 026
[23] Santos S, Busti V C and Holanda R F L 2015 *J. Cosmol. Astropart. Phys.* **10** 061
[24] Liao K *et al* 2016 *Astrophys. J.* **822** 74
[25] Li X and Lin H N 2018 *Mon. Not. R. Astron. Soc.* **474** 313
[26] Lin H N, Li M H and Li X 2018 *Mon. Not. R. Astron. Soc.* **480** 3117
[27] Holanda R F L, Busti V C and Alcaniz J S 2016 *J. Cosmol. Astropart. Phys.* **03** 054
[28] Meng X H *et al* 2012 *Astrophys. J.* **745** 98
[29] Holanda R F L, Goncalves R S and Alcaniz J S 1606 *J. Cosmol. Astropart. Phys.* **2012** 022
[30] Li Z X, Wu P X and Yu H W 2011 *Astrophys. J. Lett.* **729** L14
[31] Holanda R F L, Lima J A S and Ribeiro M B 2010 *Astrophys. J. Lett.* **722** L233
[32] Nair R, Jhingan S and Jain D 2011 *J. Cosmol. Astropart. Phys.* **05** 023
[33] Lazkoz R, Nesseris S and Perivolaropoulos L 2008 *J. Cosmol. Astropart. Phys.* **07** 012
[34] Stern D *et al* 2010 *J. Cosmol. Astropart. Phys.* **1002** 008
[35] Cao S *et al* 2012 *J. Cosmol. Astropart. Phys.* **03** 016
[36] Holanda R F L, Carvalho J C and Alcaniz J S 2013 *J. Cosmol. Astropart. Phys.* **1304** 027
[37] More S *et al* 2016 arXiv:1612.08784 1
[38] De Bernardis F, Giusarma E and Melchiorri A 2006 *Inter. J. Mod. Phys.* D **15** 759
[39] Fu X Y and Li P C 2017 *Inter. J. Mod. Phys.* D **26** 1750097
[40] Chen Z X, Zhou B J and Fu X Y 2016 *Int. J. Theor. Phys.* **55** 1229
[41] Fu X Y, Wu P X, Yu H W and Li Z X 2011 *Res. Astron. Astrophys.* **11** 895
[42] Avgoustidis A 2010 *J. Cosmol. Astropart. Phys.* **10** 024
[43] Liao K, Li Z X, Ming J and Zhu Z H 2013 *Phys. Lett.* B **718** 1166
[44] Ruan C, Melia F and Zhang T 2018 *Astrophys. J.* **866** 31
[45] Wu P X *et al* 2015 *Phys. Rev.* D **92** 023520
[46] Cuesta A J *et al* 2016 *Mon. Not. R. Astron. Soc.* **457** 1770
[47] Xu X *et al* 2013 *Mon. Not. R. Astron. Soc.* **431** 2834
[48] Alam S *et al* 2017 *Mon. Not. R. Astron. Soc.* **721** 2617
[49] Blake C *et al* 2012 *Mon. Not. R. Astron. Soc.* **425** 405
[50] Lado S *et al* 2014 *Mon. Not. R. Astron. Soc.* **439** 3504
[51] Suzuki N *et al* 2012 *Astrophys. J.* **746** 85
[52] Betoule M *et al* 2014 *Astron. Astropart.* **568** A22
[53] Guy J *et al* 2007 *Astron. & Astrophys.* **466** 11
[54] Conley A *et al* 2011 *Astrophys. J. Suppl. Ser.* **192** 1
[55] Ade P A R *et al* 2016 *Astron. Astropart* **594** A13
[56] Hinshaw G *et al* 2013 *Astrophys. J. Suppl. Ser.* **208** 19
[57] Bennett C L *et al* 2013 *Astrophys. J. Suppl. Ser.* **208** 20
[58] Riess A G *et al* 2011 *Astrophys. J.* **730** 119
[59] Wang D and Meng X H 2017 *Astrophys. J.* **843** 100
[60] Abbott T M C *et al* 2018 *Mon. Not. R. Astron. Soc.* **480** 3879