Bubble translation driven by pulsation in a double-bubble system*

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The pulsation and translation of two cavitation bubbles are studied numerically in sound field. The results show that bubbles' pulsation driven by the sound makes them translate. Different pulsations lead to different translations. Two bubbles will be mutually attractive to each other if they pulsate in phase, while they will be repulsive if out of phase. Furthermore, the secondary Bjerknes force for small phase difference is attractive, and it becomes repulsive for other phase differences up to π phase difference due to the nonlinear effect, although the attractive strength between two bubbles is much larger than the repulsive strength. Finally, one bubble pulsation and the other bubble stationary make the bubbles repel each other.

Keywords: double bubble dynamics, pulsation, translation, phase difference

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1. Introduction

Tiny cavitation nuclei in a liquid can be activated by an ultrasonic wave and grow into visible bubbles by the naked eyes. The phenomenon is called the acoustic cavitation and the bubbles are named as the cavitation bubbles.[1] The cavitation bubbles not only pulsate in size, but also shake in position in the acoustic field. The acoustic field usually is a superpositon of the driving sound field and the sound field emitted by bubbles. Bubbles may have directional translational movement in addition to radial pulsation in the acoustic field due to the influence of acoustic radiation of other bubbles.[2] It is known that bubble dynamics includes the radial pulsation dynamics of the bubble and the bubble translation dynamics. The radial pulsation dynamics of the bubble is based on the classical Rayleigh–Plesset equation[3] and the commonly used Keller–Miksis equation[4] to study the internal mechanism of the bubble in the process of expansion, collapse and oscillation, while the main research content of bubble translation dynamics is the interaction force between bubbles. The interaction force between bubbles caused by the acoustic radiation generated by the bubble pulsation under ultrasonic waves is called the secondary Bjerknes force (SBF).[5,6] The SBF, leading to the translational motion of bubbles, determines the attraction and the rejection of bubbles, which is helpful for understanding the behavior of cavitation bubbles.

In recent years, many scholars are keen on the study of double bubbles[7–13] or multiple bubbles[14–20] to explore the interaction between bubbles in an acoustic field. Some researchers have studied the radial pulsation between the interacting cavitation bubbles. Liang *et al.*[21] discussed aspherical oscillation of two interacting bubbles at a fixed distance in an ultrasonic field. The translational motion of the interacting bubbles is also studied. Cui *et al.*[22] reported that the position of the cavitation bubble was measured experimentally and came to the conclusion that the position of cavitation bubble was constantly shaking. Doinikov[23] investigated numerically self-propulsion of a bubble pair in strong acoustic fields with pressure amplitude exceeding 1 bar. Oguz *et al.*[24] obtained that in certain parameter ranges, the SBF between the bubbles had a sign opposite to what would be expected on the basis of the linear theory of Bjerknes forces. In these studies, researchers mainly focused on the trajectory of the bubbles and the sign change of the SBF, however, the research on the cause of the bubbles' translation is insufficient.

In this paper, we study the dynamics of double interacting bubble in sound field to understand the physical causation of bubble translation. In Section 2, we calculate pressures of a double-bubble system (DBS). In Section 3, we give the specific dynamical equations of pulsation and translation to understand the source of bubble translation. In Section 4, we simulate numerically the pulsation of bubbles in various modes. In Section 5, we give the conclusion and discussion.

2. Pressures of a double-bubble system

There are two spherical bubbles with radii R and R' located at x and x' in an incompressible fluid with density ρ and sound velocity c (see Fig. 1). Two bubbles are driven by sound field including external driving sound field and radiated sound field due to bubbles' pulsation. For this DBS, the liquid pressure is nonspherical for both bubble centers, which is different from that for a single bubble system. In other words, the existence of the other pulsating bubble leads to the loss of the

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spherical symmetry of the liquid pressure, but it still remains axisymmetry for the axis through two bubble centers.

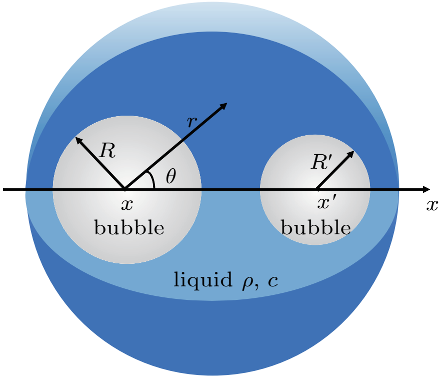

Fig. 1. Double bubbles in a liquid.

2.1. Pressure in liquid outside bubble

Usually, the sound field is described by the velocity potential φ. In an incompressible liquid, the velocity potentials of both single and double-bubble systems satisfy the Laplacian equation, but their solutions are essentially different due to their different boundary conditions. The velocity potential of the DBS is nonspherical and approximately solvable, unlike spherical and exactly solvable for the single bubble system. The velocity potential can be approximatively written as

$$\varphi \approx \left(\frac{A_0}{r}+B_0\right)P_0(\mu)+\left(\frac{A_1}{r^2}+B_1 r\right)P_1(\mu), \tag{1}$$

where r is a radial coordinate from the bubble center to the point in liquid (see Fig. 1), $\mu=\cos\theta$, the Legendre polynomials $P_0(\mu)=1$ and $P_1(\mu)=\mu$, and A_0, A_1, B_0, B_1 are the time-related coefficients (see Appendix), respectively. The nonspherical term on the right-hand side of Eq. (1) is mainly contributed to the existence of the other bubble, so that A_1 and B_1 contain variables of the other bubble. Once the velocity potential is given, we can obtain the pressure in the liquid through the generalized Bernoulli equation:

$$p_{\mathrm{L}}-p_\infty=-\rho\left(\frac{\partial\varphi}{\partial t}-\dot{x}\cdot\nabla\varphi\right)-\frac{\rho}{2}(\nabla\varphi)^2, \tag{2}$$

where $\dot{x}$ is the translation speed of the bubble center, and the pressure of far field,

$$p_\infty=p_0+p_{\mathrm{d}}(t), \tag{3}$$

where p_0 is the hydrostatic pressure, the driving acoustic pressure $p_{\mathrm{d}}(t)=-p_{\mathrm{a}}\sin(\omega t)$, with ω being the circular frequency and p_{a} the amplitude of the sound pressure. Substituting Eq. (1) into Eq. (2), the pressure in the liquid at bubble wall can be written as

$$p_{\mathrm{L,w}}=-\rho\left(\frac{\dot{A}_0}{R}+\dot{B}_0+\frac{\dot{R}^2}{2}-\frac{p_\infty}{\rho}+\frac{\dot{x}^2}{4}+\frac{3R'^2\dot{R}'\dot{x}}{2D^2}-\frac{3R'^3\dot{x}'\dot{x}}{2D^3}\right)$$
$$\times P_0(\mu)-\rho\left(\frac{\dot{A}_1}{R^2}+\dot{B}_1 R\right)P_1(\mu), \tag{4}$$

where D is the time-dependent distance between bubbles' centers and the overdot denotes the time derivative. Obviously, the liquid pressure of Eq. (4) is nonspherical and related to the polar angle θ due to the first-order velocity potential of Eq. (1). The first term on the right-hand side of Eq. (4) is spherically symmetric and the last term depends on the polar angle θ.

2.2. Pressure on the interface

There are also some additional pressures on the interface between the liquid and gas, such as the pressures due to surface tension and viscosity. Because the bubbles are assumed to be spherical balls, the interfacial pressure is spherically symmetric and can be expressed usually as

$$p_{\mathrm{I}}=\frac{2\sigma}{R}+\frac{4\eta\dot{R}}{R}, \tag{5}$$

where σ is the surface tension between the liquid and gas, and η is the viscosity of the liquid, respectively.

2.3. Pressure in gas inside bubble

From the above reason, the pressure in gas inside bubble is also spherically symmetric. According to the polytropic equation for the ideal gas, it can be written as

$$p_{\mathrm{G}}=\left(p_0+\frac{2\sigma}{R_0}\right)\left(\frac{R_0}{R}\right)^{3\kappa}, \tag{6}$$

where R_0 is the ambient radius, and κ is the polytropic exponent of the gas inside bubble, respectively.

3. Translation equation

The bubble wall is usually assumed to be geometrical surface with zero-mass, so that the mechanical equilibrium requires

$$p_{\mathrm{G}}-p_{\mathrm{I}}-p_{\mathrm{L,w}}=0. \tag{7}$$

Substituting Eqs. (4)–(6) into Eq. (7), we can obtain the dynamic equation including Legendre polynomials $P_0(\mu)$ and $P_1(\mu)$. According to the orthogonality of Legendre polynomials, we can finally obtain the two dynamic equations

$$\left(p_0+\frac{2\sigma}{R_0}\right)\left(\frac{R_0}{R}\right)^{3\kappa}-\frac{2\sigma}{R}-\frac{4\eta\dot{R}}{R}$$
$$+\rho\left(\frac{\dot{A}_0}{R}+\dot{B}_0+\frac{\dot{R}^2}{2}-\frac{p_\infty}{\rho}+\frac{\dot{x}^2}{4}+\frac{3R'^2\dot{R}'\dot{x}}{2D^2}-\frac{3R'^3\dot{x}'\dot{x}}{2D^3}\right)=0, \tag{8}$$

$$\rho\left(\frac{\dot{A}_1}{R^2}+\dot{B}_1 R\right)=0. \tag{9}$$

Substituting the coefficients A_0 and B_0 in Appendix into Eq. (8), the pulsation equation consistent with the literature[23] can be obtained. Substituting the coefficients A_1 and B_1 in Appendix, Equation (9) can be rewritten as

$$\begin{aligned}\frac{R\ddot{x}}{3}+\dot{R}\dot{x} = & -\frac{1}{D^2}\frac{\mathrm{d}}{\mathrm{d}t}\left(RR'^2\dot{R}'\right)\\ & +\frac{R'^2\left(RR'\ddot{x}'+R'\dot{R}\dot{x}'+5R\dot{R}'\dot{x}'-2R\dot{R}'\dot{x}\right)}{D^3}.\end{aligned} \tag{10}$$

It is easy to see that equation (10) is a dynamic equation describing the translation of the bubble. The right-hand side of Eq. (10) shows that the pulsation of bubbles causes their translation. The driving acoustic pressure $p_{\mathrm{d}}(t)$ does not drive the translation directly. No pulsation means no translation. Therefore bubbles' translation is driven by their pulsation.

It is worth noting that the second term on the right-hand side of Eq. (10) is slightly different from that in the literature,[23] but Eq. (10) is exactly same as that in the literature,[25] that is,

$$M\ddot{x}=3M\dot{v}-\dot{M}\left(\dot{x}-v'\right)+f_x, \tag{11}$$

where the virtual mass $M\equiv 2\pi\rho R^3/3$, a half of the mass of the displaced liquid,[17] $v'=-\frac{R'^2\dot{R}'}{D^2}+\frac{R'^3\dot{x}'}{D^3}$ is the liquid velocity generated by the other bubble at the bubble center, and $f_x=-12\pi\rho R^2\left(\dot{x}-v'\right)$ is the viscous drag,[26] respectively. Equation (10) does not contain the viscous drag, whereas it will be still considered in the numerical simulation in the next section.

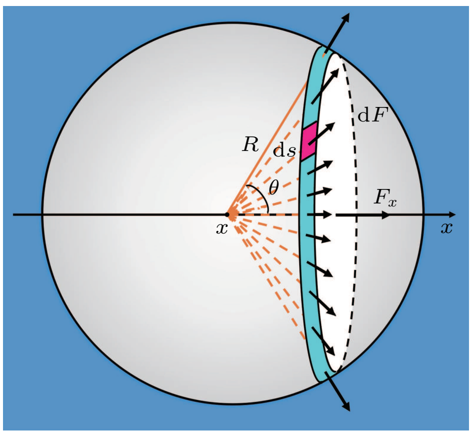

Fig. 2. Schematic diagram of force analysis of the bubble.

In fact, the translation dynamic equation (9) or (10) is derived from the inner product of P_1 over Eq. (7). In other words, equation (9) is equivalent to vanishing the inner product between $p_{\mathrm{L,W}}$ and P_1, because the interfacial pressure p_{I} and pressure in gas inside bubbles p_{G} are spherically symmetric and independent of P_1. The inner product means

$$\langle p_{\mathrm{L,W}}|P_1\rangle\equiv\int_0^{\pi}p_{\mathrm{L,W}}\cos\theta\sin\theta\mathrm{d}\theta\equiv-\frac{1}{2\pi R^2}F_x, \tag{12}$$

where F_x is the net force along the x-direction acted on the bubble wall. In Fig. 2, we show the net force F_x together with the force element $\mathrm{d}\boldsymbol{F}$ acted on the area element $\mathrm{d}\boldsymbol{s}$ on the bubble wall. They are related by

$$F_x=\int\mathrm{d}\boldsymbol{F}\cdot\boldsymbol{e}_x=-2\pi R^2\int_0^{\pi}p_{\mathrm{L,W}}\cos\theta\sin\theta\mathrm{d}\theta, \tag{13}$$

with $\boldsymbol{e}_x$ being the unit vector of the x-direction. Comparing Eq. (12) and Eq. (13), we can understand the physical meaning of F_x easily. Finally, the translation dynamic equation (9) or (10) means that the net force along the x-direction acted on the bubble wall vanishes for the gas bubble with zero-mass.

Of course, we can obtain both the pulsation and translation equations of the other bubble in the same way. Above all, sound wave drives the bubble pulsation, and the pulsation of one or two bubbles drives the translation of the bubbles.

4. Translation driven by pulsation

Before simulation, the drag force, $f_x/2\pi\rho R^2$, has been added into the right-hand side of Eq. (10).[23] To discuss the translation due to pulsation in DBS, we can use four combined ordinary differential equations to explore how the pulsation makes the bubble translate, of which the pulsation equations can be found in Ref. [23]. The MATLAB program package has been used to calculate the dynamical evolution of the two bubbles. We set the physical parameters in our proposed model as $\omega=2\pi\times 25$ kHz, $p_{\mathrm{a}}=1.31$ bar, $p_0=1.013$ bar, $\rho=998$ kg/m, $c=1500$ m/s, $\kappa=1.4$, $\sigma=0.0725$ N/m, $\eta=0.001$ Pa·s, and $R_{10}=R_{20}=5$ μm.

4.1. Pulsations in phase and out of phase

In strong sound field, cavitation bubbles pulsate in different phases subjected to their local pressure. In other words, there is a phase difference between any two pulsating bubbles. The typical phase differences, of course, are in phase and out of phase. We calculate the trajectories of two pulsating bubbles in phase [see Fig. 3(a)] and out of phase [see Fig. 3(b)], respectively. The result shows that the in-phase bubbles are mutually attractive, while the out-of-phase bubbles are repulsive. Both the strengths of attractive and repulsive SBFs decay with growing the distance D. In addition, the in-phase bubbles move faster than the out-of-phase bubbles in the same time interval, that is, the strength of attractive SBF is greater than the repulsive SBF.

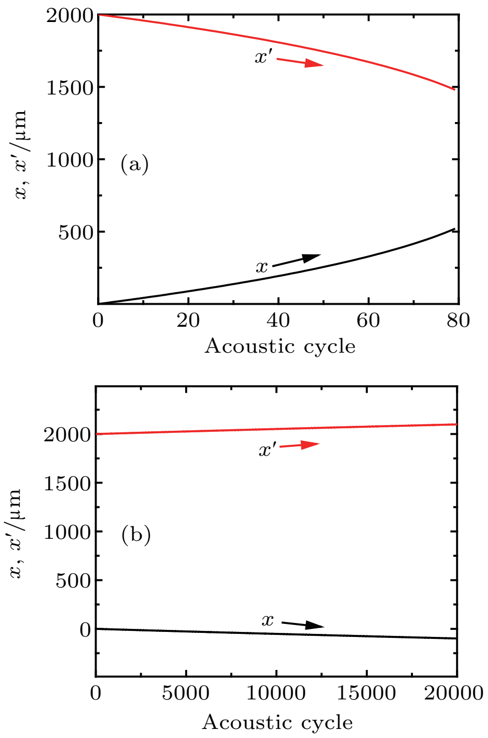

Fig. 3. Bubble translation driven by pulsation (a) in phase and (b) out of phase.

4.2. Pulsations in different phase

As mentioned above, the phases of two pulsating bubbles are extremely complex in a strong sound field, so that relationship of the phase difference is not only in-phase but also out-phase. Therefore, we calculate the SBF for more general phase-difference relation now. The SBF can be described by $B = -\frac{\rho}{4\pi D^2}\left\langle \dot{V}\dot{V}'\right\rangle e_r$, where V and V' are the volumes of the bubble and the other bubble, $\langle\rangle$ is the time average. The result has been plotted as a density figure, where the blue region shows the attractive SBF and the red region is repulsive, and the darker the color, the greater the strength of SBF [see Fig. 4(a)]. From Fig. 4(a), we can see that the bubbles are attractive to each other for the same phase (in phase) or small phase difference ($< 30^\circ$), and they become repulsive for large phase difference ($> 30^\circ$). When bubbles pulsate in the phase difference about 30°, they will keep an inertial translation, that is, $B = 0$.

The blue is darker than the red in Fig. 4(a), which shows that the strength of attractive SBF is greater than the repulsive SBF again although the red area is larger than the blue one. It is comprehensive that the strength of attractive SBF is larger than the repulsive one in physics. The nonlinear pulsation makes the in-phase oscillation appear for any time, while the out-of-phase one appear for less time. If the pulsation becomes linear, the strength of both attractive and repulsive SBFs will be equal to each other. In Fig. 4(b), we plot a similar figure for the two same bubbles driven by a small pressure amplitude (0.1 bar). Under such a weak driving sound, the radii of bubbles oscillate almost in sinusoidal mode. The expand and compress of bubble radius are symmetrical, leading to the fact that both the strength and area of the attractive (blue) and repulsive (red) SBFs are exactly the same as each other and the SBF vanishes at the phase difference 90° [see Fig. 4(b)].

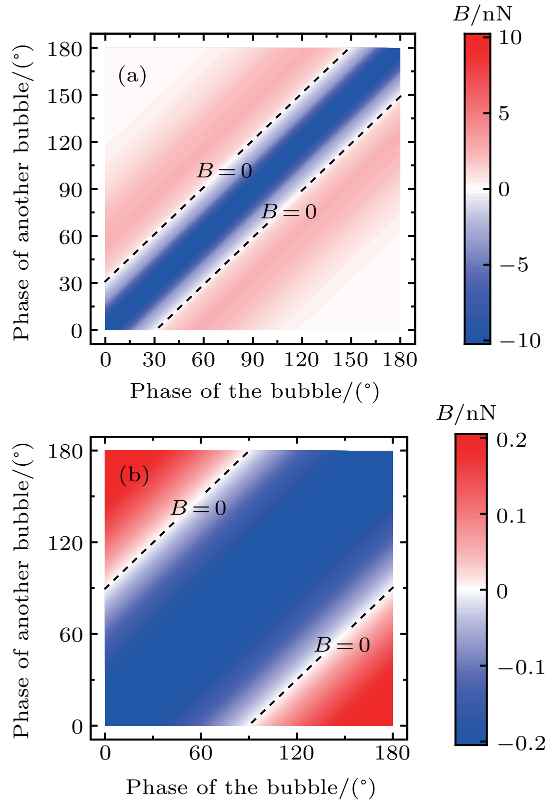

Fig. 4. Distributions of the secondary Bjerknes force of two pulsating bubbles in different phases for the driving amplitudes (a) 1.31 bar and (b) 0.1 bar.

4.3. One bubble pulsation

We consider two bubbles where the bubble pulsates and the other keeps stationary now. The calculation result has been shown in Fig. 5, in which both the bubbles move repulsively, which is in line with that obtained by Hay *et al.*[27] Compared the two pulsating bubbles mentioned above, the strength of SBF for the case of one bubble pulsation is much smaller than that for the case of both pulsating bubbles, although in Fig. 5 we have set the distance D to be one order of magnitude lower than that in Fig. 3.

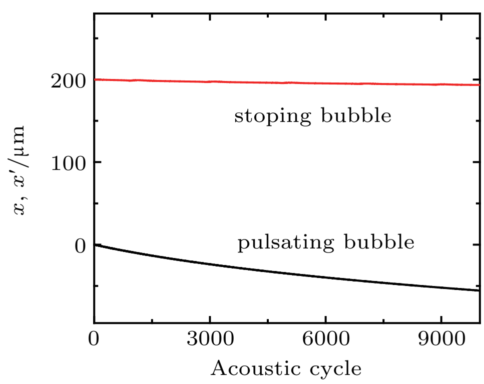

Fig. 5. Bubbles translation driven by one pulsating bubble.

This phenomenon is also understandable in the framework of dynamic Eq. (10). As $\dot{R}' = 0$, the main driving force in the order of $1/D^2$ vanishes, leading to the main driving force to become in the order of $1/D^3$.

5. Discussion and conclusion

In this paper, the pulsation and translation equations of double bubbles in a sound field are obtained by directly calculating the pressures. In this calculation process, the pulsation of bubbles is mainly driven by the external sound field and leads to their translation. In addition, the theoretical result we obtain is exactly the same as the dynamic equation of the virtual mass ball. To study the effect of pulsation on translation, these dynamical equations are used to simulate two bubbles pulsated in various modes. There are many pulsation modes of two bubbles, and the main one is pulsation in different phases, so we first calculate the effect of phase difference of two pulsating bubbles on the SBF. The numerical calculation shows that the attractive range of the phase difference, nearly from 0° to 30°, is much smaller than the repulsive one, nearly from 30° to 180°, but the strength of attractive SBF is greater than repulsive SBF. This asymmetry is caused by the nonlinear pulsation of bubbles. If the pulsation becomes linear, the strength of both attractive and repulsive SBFs will be the same as each other. Then, we calculate the case of one bubble pulsation. The simulation tells us that both bubbles move repulsively and strength of the SBF is much smaller than that for the case of both pulsating bubbles. Above all, both the calculation process and simulations show that bubble translation is driven by pulsation in the DBS.

Appendix A

The coefficients of Eq. (1) are written as[23]

$$A_0 = -R^2\dot{R}, \tag{A1}$$

$$A_1 \approx -\frac{R^3}{2}\left(\dot{x} + \frac{R'^2\dot{R}'}{D^2} - \frac{R'^3\dot{x}'}{D^3}\right), \tag{A2}$$

$$B_0 \approx \frac{-R'^2\dot{R}'}{D} + \frac{R'^3\dot{x}'}{2D^2}, \tag{A3}$$

$$B_1 \approx -\frac{R'^2\dot{R}'}{D^2} + \frac{R'^3\dot{x}'}{D^3}. \tag{A4}$$

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