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Podolsky electrodynamics from a condensation of topological defects

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Abstract – In this paper we demonstrate the arising of higher-derivative contributions to the effective action of electrodynamics on the basis of the generalized Julia-Toulouse mechanism and explicitly show that the complete effective action arising within this methodology is nonlocal.

Introduction. – The nonlocality is treated now as an important ingredient in field theory models. Being initially introduced in order to take into account finite-size effects in phenomenology allowing to rule out ultraviolet divergences [1], it began recently to acquire strong interest within other contexts, especially, within gravity, where there is a strong hope that a nonlocal extension could allow to construct a gravity model both renormalizable (or even ultraviolet finite) and ghost-free, see discussion in [2]. The key idea of nonlocal field theories looks like as follows. While the quadratic action of usual higher-derivative theories is described by a polynomial function of the d'Alembertian operator $\Box$ which can be expanded in primitive multipliers, and it allows the arising of new, ghost degrees of freedom [3], one can consider a class of theories where, instead of the polynomial function of the d'Alembertian operator, the quadratic action is characterized by an essentially non-polynomial, so-called entire function of $\Box$, for example the exponential one, which does not admit expansion in primitive multipliers and hence does not generate new degrees of freedom, see [2] and references therein. Various aspects of nonlocal field theories have been studied, including the exact solutions within the gravitational (mostly cosmological) context (see, *e.g.*, [4–8]) and explicit calculations of loop corrections in usual and supersymmetric theories [9–11]. Therefore, it is natural to expect that nonlocality can imply interesting physical effects within other contexts as well. As one of the possible applications of nonlocal field theories, in this paper we propose their application to condensation of topological defects which is an important phenomenon within the condensed matter context (we note that the application of the quantum field theory approach to the condensed matter is now intensively developed, giving origin to the methodology of analog models [12]).

In this letter, we propose a generalization of the Julia-Toulouse (JT) mechanism to a nonlocal case. Indeed, it is well known [13] that the Julia-Toulouse mechanism is based on the coupling of the gauge field to some extra fields (topological defects) represented by some sources J^μ. The idea behind the JT mechanism is that a proliferation of the topological defects in a system, such that they become dynamical fields (condensation), drives the system to a phase transition. As will be formally presented in the next section, superconductors can be seen as a good example to describe this idea. In the case of a superconductor, a proliferation of defects (vortices) can drive the system from a superconductivity state to a free Maxwell theory. In order to achieve this result we introduce a so-called activation term $J^\mu \mathcal{O} J_\mu$, and the integration over the topological defects implies the arising of new terms modifying the original gauge theory. Initially, in [13] and further in [14], the operator $\mathcal{O}$ was suggested to describe only the low-energy fluctuations, that is, being proportional to a unit operator thus giving a Thirring-like interaction. In this paper we take into account not only low-energy fluctuations but we also consider the UV fluctuations and show that as a consequence, the condensation of topological defects drives the Maxwell electrodynamics to a massive nonlocal Podolsky electrodynamics.

This letter is organized as follows: in the next section, we, based on [15], provide a general description of the condensation of topological defects in regular superconductors; in the third section, we show how the mechanism used to describe superconductors can be used to obtain nonlocal Podolsky electrodynamics; in the fourth section, we show how the nonlocal JT mechanism can also be applied in the Chern-Simons theory to generate a higher-derivative CS theory; finally, in the fifth section we present our comments and conclusions.

Phase transition and the condensation of topological defects. – In this section we describe the mechanism presented in [15] to explain how a condensation of topological defects can drive a system to a phase transition. Let us consider a Euclidean action describing the electromagnetic field interacting with an external source with electric charge q

$$S_{em}=\int \mathrm{d}^4x\left(\frac{1}{4}F_{\mu\nu}F_{\mu\nu}-iqA_\mu J_\mu\right), \tag{1}$$

where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and J_μ is a classical current. The partition function of the system coupled with an external auxiliary current j_μ, carrying charge e, summed over the classical configurations of the external sources J_μ reads

$$\begin{aligned} Z[j]&=\sum_J\int\mathcal{D}A\delta[\partial_\mu J^\mu]e^{-\int\mathrm{d}^4x\left(\frac{1}{4}F_{\mu\nu}F_{\mu\nu}-iqA_\mu J_\mu\right)}\\ &\quad\times e^{-ie\int\mathrm{d}^4x\ A_\mu j_\mu}\\ &=\sum_J\int\mathcal{D}A\mathcal{D}\theta e^{-\int\mathrm{d}^4x\left(\frac{1}{4}F_{\mu\nu}F_{\mu\nu}-iq\left(A_\mu+\frac{1}{q}\partial_\mu\theta\right)J_\mu\right)}\\ &\quad\times e^{-ie\int\mathrm{d}^4xA_\mu j_\mu}. \end{aligned} \tag{2}$$

In order to maintain explicitly the gauge invariance we inserted a delta function requiring the classical source to be conserved and exponentiated it with the help of an auxiliary field θ. The gauge symmetry is realized as

$$\begin{aligned} A_\mu&\to A_\mu+\partial_\mu\chi,\\ \theta&\to\theta-q\chi. \end{aligned} \tag{3}$$

As a consequence of gauge invariance (current conservation) we have that

$$J^\mu(x)=\int\mathrm{d}\tau\frac{\mathrm{d}y^\mu}{\mathrm{d}\tau}\delta^4(x-y(\tau)). \tag{4}$$

The summation over J in (2) represents the sum over all the possible worldlines $y(\tau)$ of charges. In (2) we did not consider dynamics of charges yet, but soon we will supplement the action with the corresponding term given by $S(y(\tau))$. For many point charges we have a sum over the worldlines of all the charges. For a continuous distribution of charges we would have a continuous source, whose sum over different ensemble configurations is defined by a path integral weighted by an action $S(J)$. The condensation is an operation that maps an ensemble of 1-currents into an ensemble of 1-forms. This operation specifies a physical process that connects different theories, describing the system in different phases. For example, in (2) if we had $J_\mu=0$ as the only configuration this would give us the free Maxwell theory. Another example is to consider J_μ as a continuous field ($\sum_J\to\int\mathcal{D}J$), as a result we have that J_μ turns into a Lagrange multiplier forcing the gauge field to vanish, which is just the Meissner effect in a perfect superconductor (with zero penetration length). Supplementing (2) with contact terms to the classical current. The contact terms are on the action S_J. The new partition function reads

$$\begin{aligned} Z[j]&=\sum_J\int\mathcal{D}A\mathcal{D}\theta e^{-\int\mathrm{d}^4x\left(\frac{1}{4}F_{\mu\nu}F_{\mu\nu}-iq\left(A_\mu+\frac{1}{q}\partial_\mu\theta\right)J_\mu\right)}\\ &\quad\times e^{-ie\int\mathrm{d}^4xA_\mu j_\mu}e^{-S_J}. \end{aligned} \tag{5}$$

Following the notation introduced in [15], we have that the allowed local terms for the current looks like

$$S_J=\frac{1}{2m^2}J_\mu J^\mu+\frac{1}{2m^4}J_\mu\Box J^\mu+\cdots, \tag{6}$$

where m is the parameter which determines the electromagnetic penetration length in the superconductor. Here we suggest that the effective action of any physical variable including currents can be represented in the form of the derivative expansion [16], which effectively means that the low-energy effective action can be represented as a power series in $\frac{\Box}{m^2}$, or, which is the same, in $\frac{p^2}{m^2}$ within the momentum representation, where the momentum square p^2 is assumed to be small, to ensure the validity of the perturbative description. In a whole analogy with the nonlocal field theory, we naturally assume all terms in the derivative expansion to be proportional to various negative degrees of the same energy scale, as occurs for example in nonlocal gravity [2] and other nonlocal field theory models. In our case, this energy scale is presented by m, the mass scale inverse to the electromagnetic penetration length.

From (4), one can express the current as $J_\mu=\epsilon_{\mu\nu\lambda\rho}\partial^\nu\Sigma^{\lambda\rho}$. In this sense $J_\mu J^\mu$ can be treated as a kinetic term. Considering only the low-energy fluctuations in (6), in the condensate phase and integrating (10) in J we have

$$\begin{aligned} Z[j]&=\int\mathcal{D}A\int\mathcal{D}\theta\ e^{-\int\mathrm{d}^4x\left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{q^2m^2}{2}\left(A_\mu+\frac{1}{q}\partial_\mu\theta\right)^2\right)}\\ &\quad\times e^{-ie\int\mathrm{d}^4xA_\mu j_\mu}. \end{aligned} \tag{7}$$

We note that some aspects related to the generating functional in nonlocal theories have been discussed also in [17,18], where the loop corrections in nonlocal theories have been discussed. However, unlike these papers, in the present paper we consider an essentially nonperturbative approach. Thus, we have obtained the action for the electromagnetic response in a superconductor with penetration length $\sim 1/m$. The condensation of currents drove the system to a phase transition. In [15], the authors showed that the dilution/condensation process can also be described in a dual point of view. The dual of the charge condensation described here is the dilution of the equivalent defects (vortices) in the superconductor. Meaning that if the vortices dilute (and charge condensation occurs), the system becomes a perfect superconductor, as in (7), or if the vortices condensate (and charge dilution takes place), we recover the free Maxwell theory. As is known, the superconductivity phase of a material can be destroyed by proliferation of vortices, and, as is shown in [15], the picture presented here describes this scenario. The dual picture of (7) can be obtained by introducing

$$\int \mathcal{D}\eta \; \delta[J^\mu(x) - \eta^\mu(x)] = 1 \tag{8}$$

in the path integral (10), using the Poisson identity [19]

$$\sum_J \delta[J^\mu(x) - \eta^\mu(x)] = \sum_K e^{i2\pi \int \mathrm{d}^4x \eta^\mu K_\mu} \tag{9}$$

we have that

$$\begin{aligned} Z[j] = \sum_K \int & \mathcal{D}\eta \mathcal{D}A \mathcal{D}\theta \\ & \times e^{-\int \mathrm{d}^4x \left(\frac{1}{4} F_{\mu\nu} F_{\mu\nu} - iq\left(A_\mu + \frac{1}{q}\partial_\mu\theta + \frac{2\pi}{q}K_\mu\right)\eta_\mu \right)} \\ & \times e^{-ie\int \mathrm{d}^4x A_\mu j_\mu} e^{-S_\eta}. \end{aligned} \tag{10}$$

In order to fully understand the role of K_μ we will chose $S_\eta = 0$. In this case, an integration in η gives us

$$A_\mu = -\frac{1}{q}(\partial_\mu\theta + 2\pi K_\mu). \tag{11}$$

From this it can be seen that if K_μ dilutes we have the Meissner effect in a perfect superconductor with zero penetration length and if it condensates we have free Maxwell theory with K playing the role of the gauge field. Also from (11), we can see how K represents the vortices of the system. K is a 1-current, and, as such, it defines a localized line in space. This means that A_μ is restricted to flux filaments defined by K (vortices). From that we have

$$\oint_C \mathrm{d}x^\mu A_\mu = \frac{2\pi}{q} \oint_C \mathrm{d}x^\mu K_\mu = \frac{2\pi n}{q}, \tag{12}$$

where $\oint \mathrm{d}x^\mu K_\mu = n \in \mathbb{N}$ is a linking number between the line C and the line defined by K. Equation (12) is also a link between our approach and the Higgs field in the Higgs mechanism. The phase of the Higgs fields, as a phase, presents a discontinuity. It has a $2\pi n$ jump every time it goes around a closed curve. In our approach K stands for this jump experienced by the phase. In our formulation θ is a regular field and

$$\partial_\mu\theta' = \partial_\mu\theta + 2\pi K_\mu, \tag{13}$$

where θ' is the phase of the Higgs field [20]. This analogy only holds in the diluted phase, where we are summing over delta functions (localized lines in space) represented by K [19].

The Poisson identity (9) is the key idea to implement and to understand the dual scenario: from (9), as an example, it can be seen that if we have $J_\mu = 0$ as the only configuration, this identity will be reduced to $\delta(\eta_\mu) = \int \mathcal{D}K e^{2\pi i \int \mathrm{d}^4x \eta^\mu K_\mu}$. This means that for a charge dilution ($J_\mu = 0$) the result will be that the K condensates ($\sum_K \to \int \mathcal{D}K$). If the J proliferates ($\sum_J \to \int \mathcal{D}J$) this makes the l.h.s. of (9) go to 1, forcing $K = 0$ on the r.h.s. of the equality. This points to the fact that K_μ are defects which are dual to the charges J_μ. Due to (8), the dual version of (6) reads

$$S_\eta = \frac{1}{2m^2}\eta_\mu\eta^\mu + \frac{1}{2m^4}\eta_\mu\Box\eta^\mu + \cdots. \tag{14}$$

By considering the lowest energy fluctuation in η in (14), as in the previous dual case with J, an integration in η_μ yields

$$\begin{aligned} Z[j] = \sum_K \int & \mathcal{D}A \int \mathcal{D}\theta \\ & \times e^{-\int \mathrm{d}^4x \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{q^2 m^2}{2}\left(A_\mu + \frac{1}{q}\partial_\mu\theta + \frac{2\pi}{q}K_\mu\right)^2 \right)} \\ & \times e^{-ie\int \mathrm{d}^4x A_\mu j_\mu}. \end{aligned} \tag{15}$$

Podolsky electrodynamics as an emergent theory. – From the previous section, it can be seen that by considering low-energy fluctuations of η_μ we are able to describe the electromagnetic response in a superconductor. In this section our discussion starts from eq. (14) and explores what is the gauge emergent system by considering high energy fluctuations of η_μ. Considering high energy fluctuations of η_μ means that

$$\begin{aligned} S_\eta &= \frac{1}{2m^2}\eta_\mu\eta^\mu + \frac{1}{2m^4}\eta_\mu\Box\eta^\mu + \cdots \\ &= \frac{1}{2m^2}\eta_\mu \left[\sum_{n=0}^{\infty} \left(\frac{\Box}{m^2} \right)^n \right] \eta_\mu \\ &= \frac{1}{2m^2}\eta_\mu \mathcal{F}\left(\Box/m^2\right)\eta_\mu. \end{aligned} \tag{16}$$

The integration in η_μ generates

$$\begin{aligned} Z[j] = \sum_K \int & \mathcal{D}A \int \mathcal{D}\theta \\ & \times e^{-\int \mathrm{d}^4x \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{q^2 m^2}{2\mathcal{F}(\Box/m^2)}\left(A_\mu + \frac{1}{q}\partial_\mu\theta + \frac{2\pi}{q}K_\mu\right)^2 \right)} \\ & \times e^{-ie\int \mathrm{d}^4x A_\mu j_\mu}. \end{aligned} \tag{17}$$

From (17), it is clear that if K condensates, which corresponds to integration over K, K_μ becomes the gauge field of the theory and the Maxwell theory is recovered. If K dilutes we have as a result the Podolsky theory for A_μ, as will be shown.

By keeping K diluted and using (13), we can define $A'_\mu = A_\mu + \frac{1}{q}\partial_\mu\theta'$. This redefinition leaves the electromagnetic tensor invariant. In this scenario the action reads

$$S = \int \mathrm{d}^4x \left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A'_\mu \frac{q^2m^2}{2\mathcal{F}\left(\Box/m^2\right)}A'_\mu\right). \quad (18)$$

It is interesting to note the following effect. Under an appropriate change of variables, the nonlocality in the kinetic term can be transferred to the vertices (or mass term), and vice versa. It can be done in the following manner.

Let us consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_m\phi e^{\frac{\Box}{\mu^2}}\partial^m\phi - V(\phi). \quad (19)$$

Here, the nonlocality is concentrated in a kinetic term. Let us do the change of variables

$$e^{\frac{\Box}{2\mu^2}}\phi \to \tilde{\phi}. \quad (20)$$

Our Lagrangian takes the form

$$\mathcal{L} = \frac{1}{2}\partial_m\tilde{\phi}\partial^m\tilde{\phi} - V(e^{-\frac{\Box}{2\mu^2}}\tilde{\phi}). \quad (21)$$

So, the potential becomes nonlocal instead of the kinetic term (in certain cases when the kinetic term looks like $\phi\Box\hat{T}\phi$, and the potential term looks like $((\hat{T})^{1/2}\phi)^n$, where $\hat{T}$ is an operator introducing the nonlocality, *e.g.*, $\hat{T} = e^{\frac{\Box}{2\mu^2}}$ as in the example above, we can remove the nonlocality both from the kinetic and potential term, but these cases are trivial).

A similar situation takes place for other field theory models, including the electromagnetic field. The model (18), under the replacement $\mathcal{F}(\Box/m^2)^{-1/2}A'_\mu \to \tilde{A}_\mu$ becomes

$$S = \int \mathrm{d}^4x \left(\frac{1}{4}F_{\mu\nu}\mathcal{F}\left(\Box/m^2\right)F^{\mu\nu} - \frac{q^2m^2}{2}A'_\mu A'_\mu\right). \quad (22)$$

Thus we have obtained the nonlocal generalization of the famous Podolsky term explicitly looking like [21]

$$\mathcal{L}_{Podolsky} = \frac{1}{2}a^2\partial_\lambda F^{\lambda\mu}\partial^\nu F_{\nu\mu} \simeq \frac{1}{2}a^2F_{\mu\nu}\Box F_{\mu\nu}, \quad (23)$$

where the sign $\simeq$ denotes the on-shell equivalence. This Lagrangian has been originally introduced in order to eliminate divergences in the electron self-energy and the vacuum polarization current, and actually constitutes the first known example of the higher-derivative regularization. We note that within all our replacements by the rule $\phi\Box\hat{L}\phi$, where $\hat{L}$ is the nonlocal operator, and the same rule for A_μ, no extra contributions to the effective actions are generated. Indeed, when we carry out these transformations, although they are nonlocal, they are linear in fields, so, in the generating functional we get only the extra multiplier $\det\hat{L}^{1/2}$, and since it does not depend on any fields, it yields only a field-independent additive term in the effective action which clearly can be neglected. So, we generated the essentially nonlocal Podolsky electrodynamics with the action

$$\begin{aligned} Z[j] &= \sum_K \int \mathcal{D}A \int \mathcal{D}\theta \\ &\times e^{-\int \mathrm{d}^4x\left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4m^2}\partial^\mu F_{\mu\nu}\left[\sum\limits_{n=0}^{\infty}\left(\frac{\Box}{m^2}\right)^n\right]\partial_\beta F^{\beta\nu} - \frac{q^2m^2}{2}A'_\mu A'_\mu\right)} \\ &\times e^{-ie\int \mathrm{d}^4x A_\mu j_\mu}, \end{aligned} \quad (24)$$

where an integration by parts was used to obtain the result. In (16), we can suggest that the energy scale characterizing the derivative expansion is m_p. This would have provided the following action in the exponent of (24):

$$\begin{aligned} S = \int \mathrm{d}^4x \bigg(&\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4m_p^2}\partial^\mu F_{\mu\nu} \\ &\times \left[\sum_{n=0}^{\infty}\left(\frac{\Box}{m_p^2}\right)^n\right]\partial_\beta F^{\beta\nu} - \frac{q^2m^2}{2}A'_\mu A'_\mu\bigg). \end{aligned} \quad (25)$$

For $n = 0$, the action (25) is the same as the Maxwell+Higgs+Podolsky one treated in [22]. It was pointed out there that, for the case $m_p = m$, the solution of the equations of motion for (25) reads as $A \sim e^{\pm f(m)|x|}$, where $f(m)$ is a function of m^2, and $f(m)^{-1}$ is responsible for the London penetration length as in (7). By considering higher-order derivatives and suggesting that $m_p = m$, as we do in (16), we do not vary the form of the solutions for our equations of motion but $f(m)$ changes. This means that by considering higher derivatives in (16) we modify the London penetration value.

It is worth mentioning that if we considered only the first two terms in the expansion (16), the resulting action would be the local Podolsky action, which would be equivalent to only consider the term $n = 0$ is the sum in (24). Therefore we show that the Podolsky electrodynamics can be obtained using a mechanism that was originally used to describe phase transitions due to the condensation/proliferation of topological defects.

It is interesting to note that the higher-derivative Podolsky term can be generated as well within the usual perturbative approach. Let us start with the usual Lagrangian of the spinor field coupled to the electromagnetic one,

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - eA\!\!\!/ - m)\psi. \quad (26)$$

The one-loop effective action of the gauge field is immediately written in the form of the fermionic determinant,

$$\Gamma^{(1)} = i\mathrm{Tr}\ln(i\partial\!\!\!/ - eA\!\!\!/ - m). \quad (27)$$

We consider the simplest contribution to it generated by the two-point function of A^μ,

$$\Gamma_2^{(1)} = -\frac{e^2}{2}\int \mathrm{d}^4p A_\mu(-p)\Pi^{\mu\nu}(p)A_\nu(p), \quad (28)$$

where

$$\Pi^{\mu\nu}(p) = \mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\gamma^\mu \frac{1}{\not{k}-m}\gamma^\nu \frac{1}{\not{k}+\not{p}-m}. \qquad (29)$$

It is well known that the effective action itself is nonlocal being an infinite series in derivatives of the external fields, or, as is the same, in the external momentum p. While the contribution of the second order in an external momentum has become a paradigmatic result in QED describing the wave function renormalization of the gauge field, the higher-order results have not been discussed up to now. So, we expand (29) up to the fourth order and find

$$\Pi_4^{\mu\nu}(p) = \mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\gamma^\mu \frac{1}{\not{k}-m}\gamma^\nu \frac{1}{\not{k}-m}\not{p}\frac{1}{\not{k}-m} \times\not{p}\frac{1}{\not{k}-m}\not{p}\frac{1}{\not{k}-m}\not{p}\frac{1}{\not{k}-m}. \qquad (30)$$

The trace and integral can be calculated explicitly. We arrive at

$$\Pi_4^{\mu\nu}(p) = \frac{4p^2}{15m^2(4\pi)^2}(p^\mu p^\nu - p^2\eta^{\mu\nu}). \qquad (31)$$

The corresponding contribution to the effective action is

$$\Gamma_{2,4}^{(1)} = -\frac{e^2}{15m^2(4\pi)^2}F_{\mu\nu}\Box F^{\mu\nu}. \qquad (32)$$

This term is finite as it must be. It has just the desired Podolsky form. In principle, the contributions to the effective action involving sixth and higher even orders in momenta can be obtained as well, so one can write down the complete one-loop two-point function as

$$\Gamma_2^{(1)} = e^2 F_{\mu\nu}\left(\sum_{n=0}^{\infty} c_n\left(\frac{\Box}{m^2}\right)^n\right)F^{\mu\nu}, \qquad (33)$$

where the zero order is the known renormalized QED result. In principle, the function

$$f(\Box) = \sum_{n=0}^{\infty} c_n\left(\frac{\Box}{m^2}\right)^n$$

can be defined, where c_n are some numbers; however, apparently this function can be found only order by order but not in the closed form.

Higher-derivative Chern-Simons term. – In this section, we show that by extending the mechanism presented in the previous section, for a Chern-Simons (CS) theory, and following the prescription in the second section, the corresponding emergent theory, obtained along the same lines as eq. (22), is a higher-derivative CS theory. For the CS theory the analog of (1) is

$$S_{cs} = \int \mathrm{d}^3x\left(\frac{\kappa}{2}\epsilon_{\mu\nu\rho}A_\mu\partial_\nu A_\rho - iqA_\mu J_\mu\right), \qquad (34)$$

and, since the CS action is gauge invariant, (4) still holds. Following the prescriptions presented in the previous sections we arrive at

$$S_{cs} = \int \mathrm{d}^3x\Big(\frac{\kappa}{2}\epsilon_{\mu\nu\rho}A_\mu\partial_\nu\mathcal{F}\left(\Box/m_{cs}^2\right)A_\rho - \frac{q^2m^2}{2}A'_\mu A'_\mu\Big). \qquad (35)$$

This is the analog of (22) for the CS theory.

Comments and conclusions. – We have succeeded to generalize the Julia-Toulouse mechanism to a case of a nonlocal contact term. One should note that, *a priori*, there are no restrictions on the form of the contact term within this approach. We demonstrated explicitly that in this case one can generate a nonlocal generalization of the electrodynamics whose action is an infinite series in derivatives, so that one has, besides the usual Maxwell term, also the Podolsky term, and higher-order terms. We showed explicitly that the Podolsky term can be generated as a quantum correction as well, being finite, so we can see that perturbative and nonperturbative approaches for obtaining new terms are equivalent in a certain sense as has been claimed in [14]. We showed also that if we consider that the theory is characterized by only one energy scale in (16), as is assumed in [15], the solutions of the equations of motion are of the form $e^{\pm f(m)|x|}$, meaning that, by considering high-order derivatives in (16) we are actually changing London's penetration length in the superconductivity phase. Actually, we demonstrated that the Julia-Toulouse approach opens broad possibilities for obtaining new effective theories with higher-derivative operators. It could be interesting to generalize this approach for Lorentz-breaking case since, earlier, the Julia-Toulouse methodology has been successfully applied in the Lorentz-breaking case in the three-dimensional theory [14]. Especially, it is interesting to study the impacts of the dimension-six terms considered in [23–26], within the Julia-Toulouse approach. Besides the possible applications within Lorentz symmetry breaking scenarios, this approach can also be used to show how magnetic permeability arises from a condensation of topological defects. We expect to conduct these studies in forthcoming papers.

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