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Improved perturbed nonlinear Schrödinger dynamical equation with type of Kerr law nonlinearity with optical soliton solutions

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Abstract

In this paper we have presented the analytical treatment of integrable improved perturbed nonlinear Schrödinger equation with type of Kerr law nonlinearity by using a newly proposed technique extended modified auxiliary equation mapping method. By the implementation of this method we have obtained a variety of some new and quite general form of exact traveling wave solutions in which we are including periodic, doubly periodic, combined, dark, bright, half dark, half bright, using three parameters which is the main key difference of our newly proposed method. For detailed dynamical physical description of our newly found results we have presented them with graphical representation using Mathematica 10.4 to explain in a more efficient manner the behavior of different physical structures of solutions.

Keywords: mathematical physics, nonlinear optics, mathematical methods

(Some figures may appear in colour only in the online journal)

1. Problem formulation and introduction

To study the extraction of optical solitons is one of excited and most fascinating branch of applied physics and mathematical studies with potential applications in most of scientific and engineering disciplines including nuclear physics, nonlinear optics, plasma physics, optical communication systems, nano-fiber technology, electromagnetism, chemistry, mathematical physics, biomedical problems, many other physical and natural sciences [1–36].

At present time the technology based on optical fiber system has the key importance to transfer information, for example in the transformation of telecommunication networks and other internet sources [1]. Which are frequently used in biomedical studies biological studies. Because of which, in coming recent years it is more important to study the extraction of optical solitons as they are considered as the basic ingredients for information transfer from a mathematical perspective.

Recently, to extract exact solutions for partial differential equations have been received considerable attention of many researchers in mathematics. Many researchers including mathematicians, physicists, biologists have studied extensively integrable Improved Perturbed Nonlinear Schrödinger equation with type of Kerr law nonlinearity which was first time proposed by Biswas [35], which is a well known governing model with potential applications in optical and quantum mechanics. If we go back in past to read the literature we will observe that with the application of different numerical techniques the approximated solutions have obtained for this well known governing model but the present research is the motivation to obtain a variety of some new and quite general form of exact traveling wave solutions in a more compact form using our newly proposed technique. [4, 5]. Therefore our research work is the motivation to study the dynamical structures of optical solitons for integrable improved perturbed nonlinear Schrödinger equation with temporal evolution. As a consequences we found some new families of solitons including periodic, doubly periodic, bright, half bright, dark, half dark, combined, family of kink type solutions using three parameters that is actually the main key point of our proposed new method.

Also, the sufficient and necessary constraint conditions for the existence of optical soltions are carried out during the mathematical derivation.

At present time a number of some new very efficient, powerful and more accurate methods which are dealing with analytical studies just have been introduced in the literature with the help of different new mathematical softwares for example Maple, Mathematica 10.4 and Matlab. Like the jacobi elliptic function method [6], extended Fan sub-equation method [7, 8], Kudryashov method, the Bäcklun transform method [9, 10] ,the tanh-function method [14] , homogenous balance method [11], inverse scattering method [15, 16],the truncated expansion method [12, 13] and many more [37–41].

In section second the main steps of our newly developed method was described in detail. In section three we have presented the implemented results of our newly developed technique for improved perturbed nonlinear Schrödinger equation. In section four a more detailed physical description with respect to graphs of our results and discussion of the obtained results is given. While in section five some of concluding remarks were given.

2. Extended modified auxiliary equation mapping method

Suppose we are considering a more general form of nonlinear complicated partial differential equation(NLPDE) which is there to represent any nonlinear wave problem with the help of a set of independent variables $S_x = \{x_0 = t,\ x_1,\ x_2,\ \ldots\ldots\ldots\ldots,\ x_p\}$ with function $\zeta(x,\ t)$ as dependent function given in the following

$$T(\zeta,\ \zeta_t,\ \zeta_{xi},\ \zeta_{xixj},\ldots\ldots) = 0, \tag{1}$$

Here T represents a polynomial function with its argument $\zeta(x,\ t)$ with nonlinear terms and its partial derivatives.

To determine exact traveling wave solutions we will use linear transformation given in the following,

$$\zeta = \zeta(\xi),\ \ \xi = \sum_{i=0}^{l} \chi_i x_i, \tag{2}$$

where χ_i, $i = 0,\ 1\ \ \ldots\ \ldots\ldots l,$. are any constants. Further by the help of above linear transformation (6) into (1) we will have the below mentioned 'nonlinear ordinary differential equation (NLODE)',

$$Q(\zeta,\ \zeta',\ \zeta'',\ \zeta''',\ldots\ldots) = 0, \tag{3}$$

Then in the next step we are assuming that $\zeta(\xi)$ will be expressed into the general solution which is given as below as series of $\psi(\xi)$,

$$\zeta = \zeta(\xi) = \textstyle\sum_{j=0}^{n} a_j \psi^j(\xi) + \sum_{j=-1}^{-n} b_{-j}\psi^j(\xi) + \sum_{j=2}^{n} c_j \psi^{j-2}(\xi)\psi'(\xi) + \sum_{j=1}^{n} d_j \left(\dfrac{\psi'(\xi)}{\psi(\xi)}\right)^j, \tag{4}$$

where the a_j, b_j, c_j, d_j are arbitrary constants which are need to be determined later, and here $\psi(\xi)$ is there with satisfying the following generalized solution which is given into the form of a series of $\psi(\xi)$ with three parameters:

$$\psi'^2 = \left(\frac{d\psi}{d\xi}\right)^2 = \mu_1\psi^2(\xi) + \mu_2\psi^3(\xi) + \mu_3\psi^4(\xi), \tag{5}$$

Here in above expression μ_1, μ_2, and μ_3 are any constants to be determined later. To determine v in form of explicit, we will follow the steps given in the following one by one:

1. Firstly we will determine the non-negative integer 'n' by balancing between the terms of highest order non-linearity and highest order partial derivative in equation (5) of equation (6).
2. Further by putting the value of equation (6) with equation (7) into (5), and by the collection of with all those terms of same power ${\psi'}^k(\xi)\psi^j(\xi)$ $(j = 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ \ldots\ \ldots.n,\ \ k = 0,\ 1)$, and by setting them to zero, we will find out a system of algebraic equations, and then by the help of an appropriate mathematical software, we will obtain a set of values with constants a_j, b_j, c_j, d_j.
3. By putting all the above mentioned values of arbitrary constants into the equation (6) then we will find out the required form of results of equation (3).

3. Improved perturbed nonlinear Schrödinger equation with type of Kerr law nonlinearity

The focus of this section is to apply our newly developed technique to check the precision of our method. In the consequence of which we will obtain a variety of some new exact traveling wave results of Improved Perturbed Nonlinear Schrödinger equation with type of Kerr law Nonlinearity which is a well known model to study optical solitons in optical communication systems. The dimensionless form first time presented and studied by Biswas [35, 36, 42] of our model is given by

$$iu_t + au_{tx} + bu_{xx} + cF(|u|^2)u = i\alpha u_x + i\lambda(|u|^{2m}u)_x + i\upsilon(|u|^{2m})_x u. \tag{6}$$

In the above model the coefficients represented by a and b takes the introduction of dispersion terms, while on the left hand side the first term is there to show the linear evolution term. The coefficient of a represents the improved term which is there to take the introduction of stability analysis to the NLSE excluding this term the problem otherwise will be an ill-posed problem. And the coefficient of b shows the usual group velocity dispersion(GVD).

When we talk about right hand side there are some perturbation terms included in the model NLSE for example α is there to show the coefficient of inter modal dispersion, while λ is the representation of self steeping constant which is necessary to present there to avoid from the formation of shock waves. While the coefficient υ is the representation of non linear dispersion term. While m is the representation of power law nonlinearity parameter which is present to represent the the full nonlinearity factor which has been considered on a generalized setting. Its important to mention here that all these perturbation terms are of Himltonian type and hence in the finally speaking the perturbed NLSE is given by (6) which is regarded to be integrable. The coefficient of c mentioned on the left hand side is the representation of non Kerr law nonlinearity term which has been presented by the functional F. In my present research work we will deal only with Kerr law nonlinearity which is in other words some time called as the cubic nonlinearity. While this type of nonlinearity actually initiates only when a particular light wave in an optical fiber system is subjected to nonlinear responses. In this case:

$$F(u) = u. \tag{7}$$

So, the equation (6) takes the form as mentioned below:

$$iu_t + aq_{tx} + bu_{xx} + c(|u|^2)u = i\alpha u_x + i\lambda(|u|^2u)_x + i\upsilon(|u|^2)_x u. \tag{8}$$

Further by the consideration of below mentioned linear wave transformation

$$u(x, t) = Q(\xi)e^{i\phi},\ \phi = -\kappa x + \omega t,\quad \text{and while } \xi = x + \lambda_1 t \tag{9}$$

Where λ_1, κ, and ω are arbitrary constants. Now by using the above mentioned linear transformation in equation (8), and with the decomposition into real and imaginary parts respectively we are having the following equations:

$$(a\lambda_1 + b)\left(\frac{\partial^2 Q(\xi)}{\partial\xi^2}\right) + (c - \lambda\kappa)Q^3(\xi) + (\kappa\omega a - \kappa^2 b - \alpha\kappa - \omega)Q(\xi) = 0. \tag{10}$$

And

$$(\lambda_1 - \kappa\lambda_1 a + \omega a - 2\kappa b - \alpha)\left(\frac{\partial Q(\xi)}{\partial\xi}\right) - (3\lambda + 2\upsilon)Q^2(\xi)\left(\frac{\partial Q(\xi)}{\partial\xi}\right) = 0. \tag{11}$$

Equation (10) can be rewritten as

$$R_1\left(\frac{\partial^2 Q(\xi)}{\partial\xi^2}\right) + R_2 Q^3(\xi) + R_3 Q(\xi) = 0. \tag{12}$$

where $R_1 = (a\lambda_1 + b)$, $R_2 = (c - \lambda\kappa)$, and $R_3 = (\kappa\omega a - \kappa^2 b - \alpha\kappa - \omega)$ By the principal of homogeneous balance, making a quite balance between Q'' and Q^3 terms in equation (12), we will obtain $n = 1$. And then by using our technique equation (12) takes the general solution as given below:

$$Q(\xi) = a_0 + a_1\psi(\xi) + \frac{b_1}{\psi(\xi)} + d_1\frac{\psi'(\xi)}{\psi(\xi)}. \tag{13}$$

While here $\psi(\xi)$ is there with satisfying the following equations with its mentioned partial derivatives:

$$\psi'^2 = \left(\frac{d\psi}{d\xi}\right)^2 = \mu_1\psi^2(\xi) + \mu_2\psi^3(\xi) + \mu_3\psi^4(\xi); \tag{14}$$

$$\psi''(\xi) = \mu_1\psi(\xi) + \frac{3}{2}\mu_2\psi^2(\xi) + 2\mu_3\psi^3(\xi); \tag{15}$$

$$\psi'''(\xi) = (\mu_1 + 3\mu_2\psi(\xi) + 6\mu_3\psi^2(\xi))\psi'(\xi). \tag{16}$$

By substituting equation (13) using equation (14) into equation (12), and by the collection of all coefficients of same powers $\psi'^k(\xi)\psi^j(\xi)$ (‘$k = 0, 1$’. ‘$j = 0, 1, 2, 3, 4, 5, 6, \ldots$ ......n’) and by putting them to zero will give us a new system of algebraic equations, using an appropriate mathematical software like Maple Mathematica, and distinct families of newly and more generally developed exact travelling wave solutions with different values of constants a_0, b_1, d_1 a_1 with frequency are found, by substituting them in equation (13) we will found our solutions of Improved Perturbed Nonlinear Schrödinger equation with type of Kerr law Nonlinearity as listed below.

Family 1:

$$a_0 = \pm\frac{i\sqrt{\Delta_1}}{\sqrt{\Delta_2}},\ b_1 = d_1 = 0,\ a_1 = \pm\frac{i\Delta_3\mu_2}{2\sqrt{\Delta_1}\sqrt{\Delta_2}},\quad \mu_1 = \frac{2\Delta_1}{\Delta_3},\ \mu_3 = \frac{\Delta_3\mu_2^2}{8\Delta_1}. \tag{17}$$

then by putting all above mentioned values into equation (12) using the equation (14), in this family we will have the following mentioned results with the help of our newly developed method, the following results of equation (6) in this family.

$$u_1(x, t) = -\left(\frac{i[2(ac + b^2c^2) + b^2(\mu_1 - c\beta_1(2bc^2 + 3\mu_1) + \epsilon(1 - 3c\beta_1)\mu_1 \tanh\left[\frac{1}{2}\sqrt{\mu_1}(\omega + \xi_0)\right])]}{2\sqrt{\Delta_1}\sqrt{\Delta_2}}\right)e^{i\phi}$$

$$\text{Where } \mu_1 > 0,\ \epsilon = \pm 1,\ \mu_2^2 - 4\mu_1\mu_3 = 0. \tag{18}$$

$$u_2(x, t) = i\left(\frac{\left(-4\Delta_1 + b^2(1 + \frac{\epsilon \sinh[\sqrt{\mu_1}(\omega + \xi_0)]}{\delta + \cosh[\sqrt{\mu_1}(\omega + \xi_0)]})(1 - 3c\beta_1)\mu_2\sqrt{\frac{\mu_1}{\mu_3}}\right)}{4\sqrt{\Delta_1}\sqrt{\Delta_2}}\right)e^{i\phi}$$

$$\text{Where } \mu_1 > 0,\ \mu_3 > 0,\ \mu_2 = -\sqrt{4\mu_1\mu_3}. \tag{19}$$

Here in above solutions 'ϵ' and 'δ' are can be of any choices of 1 or -1, with any appropriate choice of $(\epsilon, \delta) = (1, 1), (-1, 1), (1, -1), (1, 1)$.

$$u_3(x, t) = i\left(\frac{\left(-2\Delta_1 - b^2(1 + \frac{\epsilon(\sqrt{1 + p^2}\,\delta + \cosh[\sqrt{\mu_1}(\omega + \xi_0)])}{p + \sinh[\sqrt{\mu_1}(\omega + \xi_0)]})(1 - 3c\beta_1)\mu_1\right)}{2\sqrt{\Delta_1}\sqrt{\Delta_2}}\right)e^{i\phi}$$

$$\text{Where } \mu_1 > 0,\ (\epsilon, \delta) = (1, 1), (1, -1), (-1, 1), (1, 1). \tag{20}$$

Here $\Delta_1 = (-ac - b^2c^2 + b^3c^3\beta_1)$, $\Delta_2 = (\beta - bc\beta_2)$, $\Delta_3 = (b^2 - 3b^2c\beta_1)$, While here '$p$' and '$\xi_0$' are any arbitrary constants.

Family 2:

$$d_1 = \pm i\frac{\sqrt{\frac{2}{3}}\sqrt{\Delta_4}}{\sqrt{\Delta_5}},\ b_1 = a_0 = a_1 = 0,\ \mu_1 = \frac{3\Delta_6}{2\Delta_4}, \tag{21}$$

then by putting all above mentioned values into equation (12) using the equation (14), in this family we will have the following mentioned results with the help of our newly developed method, the following results of equation (6) in this family.

$$u_4(x, t) = \left(i\frac{\epsilon\ \mathrm{sech}\left[\frac{1}{2}\sqrt{\mu_1}(\omega + \xi_0)\right]^2\sqrt{\Delta_4}\sqrt{\mu_1}}{\sqrt{6}\sqrt{\Delta_5}(1 + \epsilon \tanh\left[\frac{1}{2}\sqrt{\mu_1}(\omega + \xi_0)\right])}\right)e^{i\phi}$$

$$\text{Where } \mu_1 > 0,\ \epsilon = \pm 1,\ \mu_2^2 - 4\mu_1\mu_3 = 0. \tag{22}$$

$$u_5(x, t) = i\left(\frac{\sqrt{\frac{2}{3}}\sqrt{\Delta_4}\,\epsilon(1 + \delta\cosh[\sqrt{\mu_1}(\omega + \xi_0)])\sqrt{\mu_1}}{\sqrt{\Delta_5}(\delta + \cosh[\sqrt{\mu_1}(\omega + \xi_0)])(\delta + \cosh[\sqrt{\mu_1}(\omega + \xi_0)] + \epsilon\sinh[\sqrt{\mu_1}(\omega + \xi_0)])}\right)e^{i\phi}$$

$$\text{Where } \mu_1 > 0,\ \mu_2 = -\sqrt{4\mu_1\mu_3}. \tag{23}$$

Here in above solutions 'ϵ' and 'δ' are can be of any choices of 1 or -1, with any appropriate choice of $(\epsilon, \delta) = (1, 1), (-1, 1), (1, -1), (1, 1)$.

$$u_6(x, t) = i\left(\frac{\sqrt{\frac{2}{3}}\sqrt{\Delta_4}\,\epsilon(-1 - \sqrt{1 + p^2}\ \delta\cosh[\sqrt{\mu_1}(\omega + \xi_0)] + p\sinh[\sqrt{\mu_1}(\omega + \xi_0)])\sqrt{\mu_1}}{\sqrt{\Delta_5}(p + \sinh[\sqrt{\mu_1}(\omega + \xi_0)])(p + \sqrt{1 + p^2}\,\delta\ \epsilon + \epsilon\cosh[\sqrt{\mu_1}(\omega + \xi_0)] + \sinh[\sqrt{\mu_1}(\omega + \xi_0)])}\right)e^{i\phi}$$

$$\text{Where } \mu_1 > 0,\ (\epsilon, \delta) = (1, 1), (1, -1), (-1, 1), (1, 1). \tag{24}$$

Here $\Delta_4 = (b + a\lambda_1)$, $\Delta_5 = (c - \lambda\kappa)$, '$p$' and '$\xi_0$' are arbitrary constants.

Family 3:

$$d_1 = \pm\frac{\sqrt{\Delta_4}}{2\sqrt{\Delta_5}},\ b_1 = \mu_2 = \mu_3 = a_1 = 0,\ a_0 = \pm i\frac{\sqrt{\Delta_6}}{2\sqrt{\Delta_5}},\ \mu_1 = -\frac{\Delta_6}{\Delta_4}, \tag{25}$$

then by putting all above mentioned values into equation (12) using the equation (14), in this family we will have the following mentioned results with the help of our newly developed method, the following results of equation (6) in this family.

$$u_7(x, t) = -i\left(\frac{2\sqrt{\Delta_6} + \frac{\epsilon\ \mathrm{sech}\left[\frac{1}{2}\sqrt{\mu_1}(\omega+\xi_0)\right]^2\sqrt{\Delta_4}\sqrt{\mu_1}}{(1+\epsilon\ \tanh[\sqrt{\mu_1}(\omega+\xi_0)])}}{4\sqrt{\Delta_5}}\right)e^{i\phi}$$

$$\textit{Where}\ \mu_1 > 0,\ \ \epsilon = \pm 1,\ \ \mu_2^2 - 4\mu_1\mu_3 = 0. \tag{26}$$

$$u_8(x, t) = -i\left(\frac{\sqrt{\Delta_6} + \frac{\epsilon(1+\delta\cosh[\sqrt{\mu_1}(\omega+\xi_0)])\sqrt{\Delta_4}\sqrt{\mu_1}}{(\delta+\cosh[\sqrt{\mu_1}(\omega+\xi_0)])(\delta+\cosh[\sqrt{\mu_1}(\omega+\xi_0)])+\epsilon\ \sinh[\sqrt{\mu_1}(\omega+\xi_0)])}}{2\sqrt{\Delta_5}}\right)e^{i\phi}$$

$$\textit{Where}\ \mu_1 > 0,\ \ \mu_2 = -\sqrt{4\mu_1\mu_3}. \tag{27}$$

Here in above solutions 'ϵ' and 'δ' are can be of any choices of 1 or -1, with any appropriate choice of $(\epsilon, \delta) = (1, 1)$, $(-1, 1)$, $(1, -1)$, $(1, 1)$.

$$u_9(x, t) = -i\left(\frac{\sqrt{\Delta_6} + \frac{\epsilon\ (-1-\sqrt{1+p^2}\,\delta\cosh[\sqrt{\mu_1}(\omega+\xi_0)]+p\ \sinh[\sqrt{\mu_1}(\omega+\xi_0)])\sqrt{\Delta_4}\sqrt{\mu_1}}{(p+\sinh\ [\sqrt{\mu_1}(\omega+\xi_0)])(p+\sqrt{1+p^2}\,\delta\ \epsilon+\epsilon\ \cosh[\sqrt{\mu_1}(\omega+\xi_0)]+\sinh\ [\sqrt{\mu_1}(\omega+\xi_0)]))}}{2\sqrt{\Delta_5}}\right)e^{i\phi}$$

$$\textit{Where}\ \mu_1 > 0,\ \ (\epsilon, \delta) = (1, 1), (1, -1), (-1, 1), (1, 1). \tag{28}$$

Here $\Delta_6 = (-\alpha\kappa - b^2\kappa^2 + \kappa\omega a)$, $\Delta_4 = (b + a\lambda_1)$, $\Delta_5 = (c - \lambda\kappa)$, '$p$' and '$\xi_0$' are arbitrary constants.

Family 4:

$$d_1 = -i\frac{\sqrt{\Delta_4}}{\sqrt{2}\sqrt{\Delta_5}},\ \ b_1 = a_0 = 0,\ \ \mu_1 = \frac{2\Delta_6}{\Delta_4},\ \ a_1 = i\frac{\sqrt{\Delta_4}\sqrt{\mu_3}}{\sqrt{2}\sqrt{\Delta_5}}. \tag{29}$$

then by putting all above mentioned values into equation (12) using the equation (14), in this family we will have the following mentioned results with the help of our newly developed method, the following results of equation (6) in this family.

$$u_{10}(x, t) = \left(-i\sqrt{\Delta_4}\frac{\left(\epsilon\ \mathrm{sech}\left[\frac{1}{2}\sqrt{\mu_1}(\omega+\xi_0)\right]^2\sqrt{\mu_1}\mu_2 + 2\mu_1\sqrt{\mu_3}\left(1+\epsilon\ \tanh\left[\frac{1}{2}\sqrt{\mu_1}(\omega+\xi_0)\right]\right)^2\right)}{2\sqrt{2}\sqrt{\Delta_5}\mu_2(1+\epsilon\ \tanh\left[\frac{1}{2}\sqrt{\mu_1}(\omega+\xi_0)\right])}\right)e^{i\phi}$$

$$\textit{Where}\ \mu_1 > 0,\ \ \epsilon = \pm 1,\ \ \mu_2^2 - 4\mu_1\mu_3 = 0. \tag{30}$$

$$u_{11}(x, t) = i\left(\frac{\sqrt{\Delta_4}(-2\epsilon\ (1+\delta\cosh[\sqrt{\mu_1}(\omega+\xi_0)])\pi + (\delta+\cosh[\sqrt{\mu_1}(\omega+\xi_0)]+\epsilon\ \sinh[\sqrt{\mu_1}(\omega+\xi_0)])^2\pi)}{2\sqrt{2}\sqrt{\Delta_5}(\delta+\cosh[\sqrt{\mu_1}(\omega+\xi_0)])(\delta+\cosh[\sqrt{\mu_1}(\omega+\xi_0)]+\epsilon\ \sinh[\sqrt{\mu_1}(\omega+\xi_0)])}\right)e^{i\phi}$$

$$\textit{Where}\ \pi = \sqrt{\mu_1},\ \ \mu_1 > 0,\ \ \mu_2 = -\sqrt{4\mu_1\mu_3}. \tag{31}$$

Here in above solutions 'ϵ' and 'δ' are can be of any choices of 1 or -1, with any appropriate choice of $(\epsilon, \delta) = (1, 1)$, $(-1, 1)$, $(1, -1)$, $(1, 1)$.

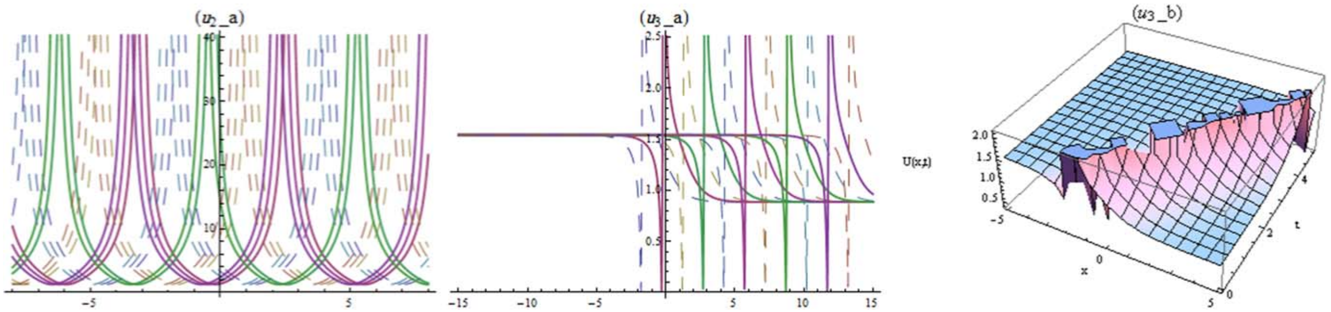

Figure 1. The physical representation of (19) and (20). here (q_2-a) represents 2nd dimensional graph of (19) with $\xi_0 = .3$, $\epsilon = .8$, $a = .2$, $b = .5$, $c = -1$, $\mu_1 = -1.2$, $\mu_2 = -1.8$, $\mu_3 = 1.9$, $\beta = 2.8$, $\beta_1 = 1.8$, $\beta_2 = 1$, $\delta = 1.9$ in intervals (0, 10), (−15, 15) as type of bright soliton. And (q_3-a) shows 2nd dimensional graph of (20) as type of bright soliton with different shape in intervals (0, 10), (−15, 15) with $\xi_0 = -1.0$, $\epsilon = 1.8$, $a = .2$, $b = 1.5$, $c = 1$, $\mu_1 = 1.8$, $\mu_2 = -1.9$, $\mu_3 = .9$, $\beta = -2.8$, $\beta_1 = .2$, $\beta_2 = 1$, $\delta = -1.9$, $p = -1$. And (q_3-b) represents 3rd dimensional graph of (20) with $\xi_0 = -1.0$, $\epsilon = 1.8$, $a = .2$, $b = 1.5$, $c = 1$, $\mu_1 = 1.8$, $\mu_2 = -1.9$, $\mu_3 = .9$, $\beta = -2.8$, $\beta_1 = .2$, $\beta_2 = 1$, $\delta = -1.9$, $p = -1$ in intervals (0, 5), (−5, 5) as bright soliton.

$$u_{12}(x,t) = \left(\left(i\frac{\sqrt{\Delta_4}\sqrt{\mu_1}}{\sqrt{2}\sqrt{\Delta_5}}\right)\right.$$
$$\times\left(\frac{\epsilon\,(1+\sqrt{1+p^2}\,\delta\cosh[\sqrt{\mu_1}(\omega+\xi_0)] - p\sinh[\sqrt{\mu_1}(\omega+\xi_0)])}{(p+\sinh[\sqrt{\mu_1}(\omega+\xi_0)])(p+\sqrt{1+p^2}\,\delta\,\epsilon+\epsilon\cosh[\sqrt{\mu_1}(\omega+\xi_0)]+\sinh[\sqrt{\mu_1}(\omega+\xi_0)])}\right.$$
$$\left.\left.-\frac{\left(1+\frac{\epsilon\,(\sqrt{1+p^2}\,\delta+\cosh[\sqrt{\mu_1}(\omega+\xi_0)])}{p+\sinh[\sqrt{\mu_1}(\omega+\xi_0)]}\right)\sqrt{\mu_1}\sqrt{\mu_3}}{\mu_2}\right)\right)e^{i\phi}$$
$$\textit{Where } \mu_1 > 0,\ \mu_3 > 0,\ (\epsilon, \delta) = (1, 1), (1, -1), (-1, 1), (1, 1). \tag{32}$$

Here $\Delta_4 = (b + a\lambda_1)$, $\Delta_5 = (c - \lambda\kappa)$, '$p$' and '$\xi_0$' are arbitrary constants.

4. Graphical or physical representation of the results

In this section we are going to represent the graphical representation of our newly found distinct families of exact travelling wave results in which we are including combined functions, hyperbolic functions, rational functions, trigonometric functions with different appropriate figures (Figures 1, 2, 3 and 4) to have a complete understanding of graphical description of improved perturbed nonlinear Schrödinger equation with type of Kerr law nonlinearity using Mathematica 10.4.

5. Results and discussion

This section is going to highlight the similarities and differences of our newly found a variety of results which we just have found by the help of our newly developed method with those solutions which are present already in the literature obtained by different authors using different mathematical techniques. The first and main key point to obtain new families of results of our method is the main body or structure of our modified new proposed solution (4), which is totally different and new structure using three parameters which is the main key difference of our newly proposed method, further to understand the dynamics of our obtained solutions more completely the graphical structure by using quite different and new sets of values of constants a_j, b_j, c_j, d_j either by using the software Mathematica 10.4 or Maple are given, Also equation (5) gives us distinct forms of exact traveling wave new results which are including rational functions, combined functions, trigonometric functions, and hyperbolic functions. In the following we have listed some same results which we have obtained by our method in comparison of other results obtained by other techniques already available in literature.

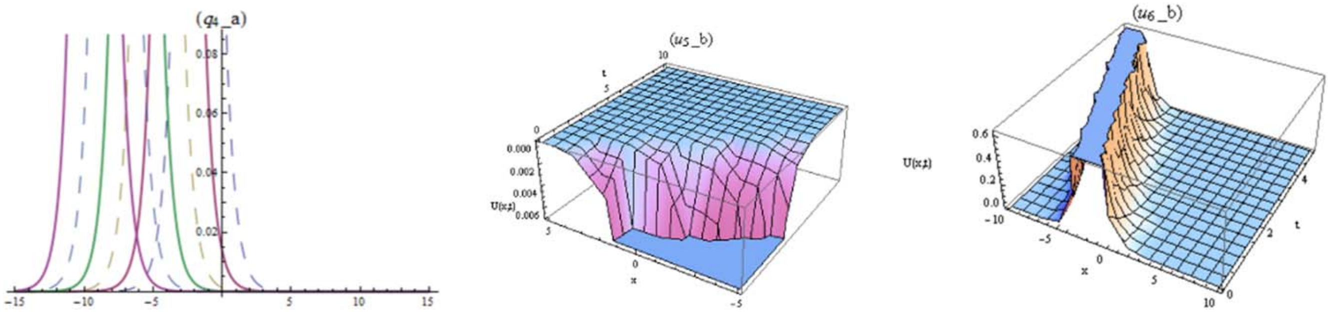

Figure 2. The physical representation of (22), (23) and (24). here (q_4-a) represents 2nd dimensional graph of (22) with $\xi_0 = 1.3$, $\epsilon = 1.8$, $a = 2$, $b = 1.5$, $c = 2$, $\mu_1 = 2.5$, $\mu_2 = 1.9$, $\mu_3 = 2$, $\beta = 2$, $\beta_1 = 1.8$, $\beta_2 = 1$ in intervals (0, 5), (−5, 5) as type of bright soliton of different shape. And (q_5-b) shows 3rd dimensional graph of (23) as type of half bright soliton with different shape in intervals (0, 5), (−5, 5) with $\xi_0 = 3.8$, $\epsilon = 2.9$, $a = 2.8$, $b = 1.5$, $c = -1$, $\mu_1 = 1.6$, $\mu_2 = 1.8$, $\mu_3 = 1.9$, $\beta = 2.8$, $\beta_1 = 1.8$, $\beta_2 = 1$, $\delta = -1.9$. And (q_6-b) represents 3rd dimensional graph of (24) with $\xi_0 = 1$, $\epsilon = 1.8$, $a = 2$, $b = 1.5$, $c = 3$, $\mu_1 = 2.5$, $\mu_2 = -1.6$, $\mu_3 = -2$, $\beta = -2.8$, $\beta_1 = 2.8$, $\beta_2 = 1.5$, $\delta = 1.9$, $p = 1$ in intervals (0, 5), (−5, 5) as bright soliton.

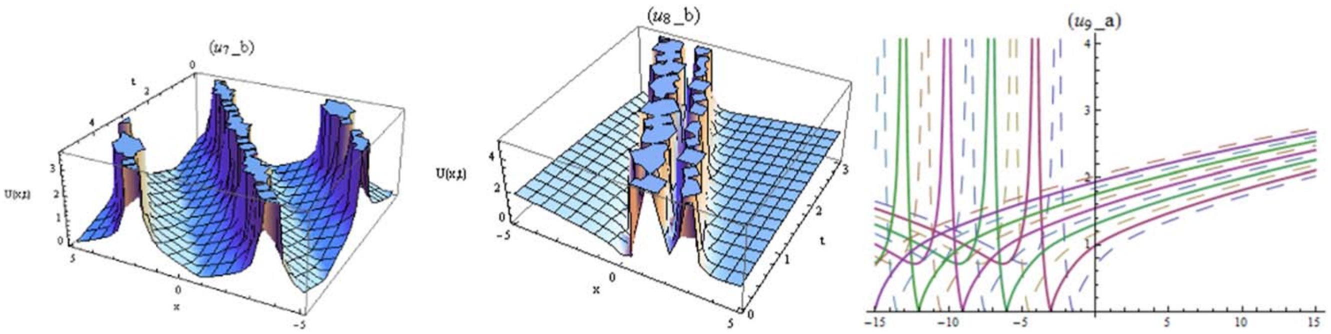

Figure 3. The graphical representation of (26), (27) and (28). here (q_7-b) represents 3rd dimensional graph of (26) with $\xi_0 = -1$, $\epsilon = 1.5$, $a = -2$, $b = -1.5$, $c = -1$, $\mu_1 = -1.2$, $\mu_2 = -1.3$, $\mu_3 = 1.8$, $\beta = -2.5$, $\beta_1 = -1.8$, $\beta_2 = -1$ in intervals (0, 5), (−5, 5) as type of periodic soliton of different shape. And (q_8-b) represents 3rd dimensional graph of (27) as type of doubly periodic bright soliton with different shape in intervals (0, 3), (−5, 5) with $\xi_0 = -1.8$, $\epsilon = -1.9$, $a = -2.8$, $b = -1.5$, $c = 1$, $\mu_1 = 1.6$, $\mu_2 = 1.9$, $\mu_3 = 1.9$, $\beta = 1.8$, $\beta_1 = -1.8$, $\beta_2 = 1$, $\delta = -1.9$. And (q_9-a) represents 3rd dimensional graph of (28) as bright soliton of different shape with $\xi_0 = 1.9$, $\epsilon = 1.8$, $a = 2$, $b = 1.5$, $c = 1.9$, $\mu_1 = 1.7$, $\mu_2 = 1.6$, $\mu_3 = 2$, $\beta = 1.8$, $\beta_1 = 2.8$, $\beta_2 = 1.5$, $\delta = 1.9$, $p = 1$ in intervals (0, 10), (−15, 15).

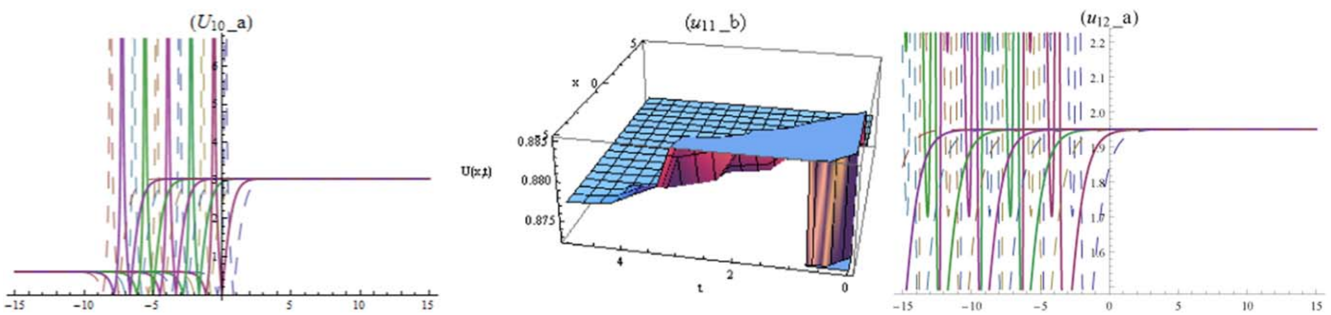

Figure 4. The physical representation of (30), (31) and (32). here (q_{10}-a) represents 2nd dimensional graph of (30) with $\xi_0 = -1.6$, $\epsilon = 1.5$, $a = 2.9$, $b = 3$, $c = -1.9$, $\mu_1 = 1.9$, $\mu_2 = -1.9$, $\mu_3 = 1.8$, $\beta = -2.5$, $\beta_1 = -1.8$, $\beta_2 = 1.8$ in intervals (0, 10), (−15, 15) as type of bright soliton with quite different shape. And (q_{11}-b) shows 3rd dimensional graph of (31) as type of half bright soliton with different shape in intervals (0, 5), (−5, 5) with $\xi_0 = 3.5$, $\epsilon = 2$, $a = 3$, $b = -1.9$, $c = 3.5$, $\mu_1 = 1.9$, $\mu_2 = 3$, $\mu_3 = -1.9$, $\beta = -1.9$, $\beta_1 = 1.9$, $\beta_2 = -1$, $\delta = -1$. And (q_{12}-a) represents 2nd dimensional half bright soliton with different structure of (32) with $\xi_0 = 3$, $\epsilon = 1.9$, $a = -2$, $b = -1.9$, $c = 1.9$, $\mu_1 = 1.9$, $\mu_2 = 1.7$, $\mu_3 = 2.9$, $\beta = 1.8$, $\beta_1 = -2.8$, $\beta_2 = 1.8$, $\delta = -1.9$, $p = 1$ in intervals (0, 10), (5, 5).

Results obtained by *Improved Sub* − ODE *Method*:

- As it was claimed in Improved Sub-ODE Method that when $h_0 = 0$ and $h_1 = 0$, the general elliptic equation degenerated into improved sub-equation auxiliary ordinary differential equation and the authors [19, 20] listed all the solutions under this improved sub-equation but our results for the same equation Improved Perturbed Nonlinear Schrödinger equation with type of Kerr law nonlinearity are different in complex domain which are more general in compact form with a straight forward and with a more simplified way

calculated, which are quite hard to discuss in complex domain. So we have found some new variety of exact travelling wave solitons in a complex domain.
- The extraction of exact solutions in a complex domain for nonlinear improved perturbed nonlinear model with respect to time is of key importance with potential applications in optical and quantum physics. Which is not calculated yet by any other technique, so our obtained results are totaly new with quite different structures.
- A variety of new optical solitons is extracted here first time in complex domain using a single technique using three parameters which is the main key difference of our newly proposed method.
- It is important to mention here that we have checked all solutions by using Mathematica.

Results obtained by *New Mapping Method*:

- we made a detailed comparison of our newly obtained results with the results obtained by *New Mapping Method* mentioned in [19, 20], all listed solutions are in real domain, while our results are totaly different and newly obtained in complex domain.

Results obtained by *Extended Auxiliary equation Method*:

- we made a detailed comparison of our newly obtained results with the results calculated by *Bernoulli Sub* – ODE *Method* mentioned in [42], all listed solutions are in real domain, but our results are quite different and new in complex domain.

In the last, its quite important to describe here that the results found in [19, 20] are totaly different from our obtained variety of solutions, So our solutions are completely new and more general in a compact form with in complex domain. From the above comparison we can conclude that we have obtained a variety of new optical solitons which are more general and has not been extracted before, from which it shows that our method is more effective, reliable, very helpful, quite straight forward with less computational time to study more analytically other complicated nonlinear partial differential models.

6. Concluding remarks

In this work we have successfully presented a new method to study more analytically other complicated nonlinear partial differential equations(NLPDES), which are appearing in quantum mechanics, nano-technology, mathematical physics, chemistry, nonlinear optics, molecular biology, plasma physics, elastic media, and in different engineering disciplines. As an application we have implemented our newly developed technique on integrable improved perturbed nonlinear Schrödinger equation with type of Kerr law nonlinearity with potential applications in optical and quantum mechanics, in the consequences of that we have obtained some new variety of more general families of analytic travelling wave solutions which are actually highly useful to study the qualitative features of several phenomena more accurately and also have a profound impact in the development and improvement of some quite new mathematical softwares which are highly helpful and very useful to find out the numerical solutions of other complicated partial differential equations and also to make a comparison with analytical solutions. To represent a detailed graphical structure of our obtained solutions we have shown them with graphs explaining with three parameters using the software Mathematica 10.4 which gives the physical interpretation more clearly. Also the computational work and efficiency of the method demonstrates the reliability, straightforwardness, and simplicity of the method.

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