Influence of magnetic field on a curved circular plate and flat plate lubricated with non-Newtonian fluid

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Abstract. This paper presents a Analytical study of the effect of magnetic field on the performance of flat and curved circular plate with Non-Newtonian fluids. From the Stokes theory a Modified Reynolds equation is derived. The solution for pressure, load capacity and squeeze time is obtained to study the influence of transverse magnetic field and couple stress parameter on these characteristics. The non-dimensional pressure, load and squeeze film time is computed numerically and presented graphically. The results predicts that, the effect of Hartmann number M_0 and Non-Newtonian fluid l^* is improved in pressure, load carrying capacity significantly whereas delay in squeezing time as compared to the non-magnetic(NMC) and Newtonian(NC) cases.

1. Introduction

The squeeze film mechanism exhibit a vital role in number of practical problems like gears, rolling elements, jet engines, machine tools ,human joints, bearings and bio-lubrication. Pertaining to their applications, several investigators have been studied squeeze film lubrication theoretically and experimentally. Cameron [1] analysed the squeeze film between parallel plates. The squeeze film bearing of cylinder plate system was analyzed by Hamrock [2]. Lin[3] presented MHD squeeze film for finite plates, author reported that presence of magnetic field provides an increase in the response time and value of load carrying capacity as compared to classical non-conducting lubricant case. The study of Magneto-hydrodynamic squeeze film performance between curved annular plates was studied by Lin and Lu [4] and concluded that due to applied magnetic field there is an increase in pressure and load carrying capacity and the response time. Effect of roughness on magneto-hydrodynamic squeeze film between finite rectangular plates was studied by Buzurke et al [5] and it was concluded that effect of magnetic field which increases the performance of squeeze film lubrication as compared to non-conducting lubricants. Further theoretical studies are found in the MHD squeeze film bearings by Kuzma [6], Shukla [7]. They concluded that, the use of an electrically conducting lubricant in the presence of transverse magnetic field results in a better performance of squeeze film bearings.

The generalized form of couple stress theory was presented by Stokes [8] which allows for polar effects as the presence of body couples and couple stresses. Couple stresses on the characteristics of finite journal bearing is studied by Lin [9] this study showed improvement of load carrying capacity and decrease in friction due to the presence of additives of fluid. Wang et al [10] analysed the dynamically loaded journal bearing, and author concluded that presence of couple stress which increase the oil film pressure and thickness with decrease in side leakage flow. Squeeze film thrust bearings lubricated couple stresses is studied by Ramanaiah and Sarkar [11] and noticed that the influence of couple stress enhance the load capacity whereas decrease in the response time. Kashinath [12] made an analysis on squeeze film on parallel plates, The circular stepped plate with non-newtonian fluids were studied by Naduvinamani and Siddangouda [13] it shows that the significant increase in load capacity and delayed squeeze film time due to the use of couple stress lubricant as comparing with Newtonian fluids.

MHD couple stress on triangular plates was analysed by Kashinath [14]. Fathima et al [15] analysed the different types of plates with MHD couple stress. MHD couple stress between two parallel plates were analysed by Shalini et al [16]. Magneto-hydrodynamics couple stress circular stepped plates is studied by Naduvinamani et al [17]. From these studies we observed that the improved in load capacity and decrease in squeeze film time by the effect of Hartmann number and couple stress. So for, the author is not aware of the influence of magnetic field on a flat plate and curved circular plates with Non-Newtonian fluids. Hence in this paper an investigation made to study the influence of magnetic field on a curved circular and flat plate with non-Newtonian fluids.

2. Mathematical Formulation and Solutions

Figure 1 is the geometrical representation of squeeze film lubrication between two plates the upper plate approaches to the lower plate with a uniform squeezing velocity $\partial h_m / \partial t$. Let h is the film thickness taken is of the form.

$$h = h_m \exp(-\alpha r^2) \qquad 0 \le r \le a \tag{1}$$

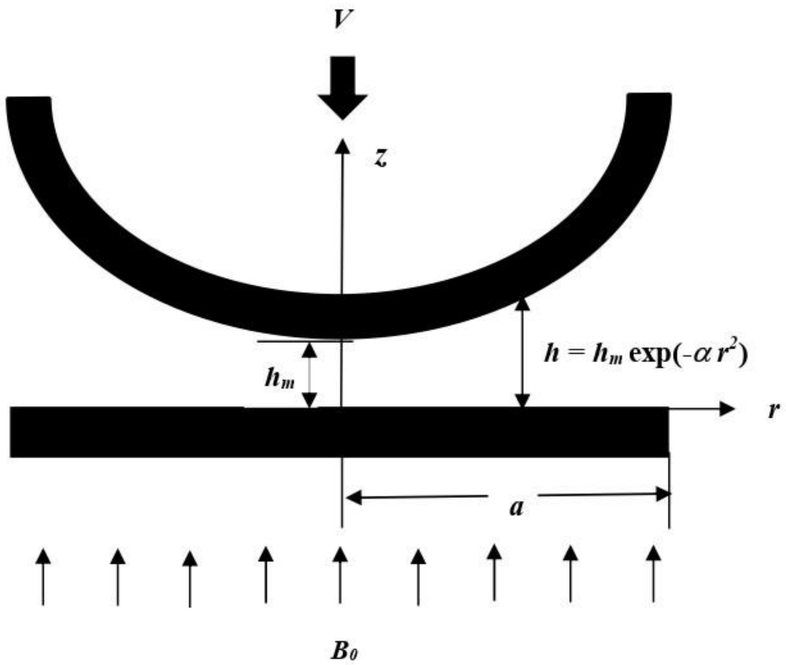

Figure 1: Schematic diagram flat plate and curved circular plate

Where, α is the curvature parameters plate and h_m is minimum film thickness. The magnetic field B_0 is applied perpendicular to the plate. Incompressible couple stress fluid is considered as a lubricant in this system. The continuity equation and MHD momentum equations with electrically conducting fluids in polar form is given by

$$\mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} - \sigma B_0^2 u = \frac{\partial p}{\partial r} \tag{2}$$

$$\frac{\partial p}{\partial z} = 0 \tag{3}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \tag{4}$$

Where, σ is electrical conductivity, p be the fluid film pressure in the region, μ be the viscosity, and η is a new material constant for the non-Newtonian fluid.

The relevant boundary conditions for the velocity field are,

For the upper plate surface $z = h$

$$u = 0 \qquad \frac{\partial^2 u}{\partial z^2} = 0 \qquad w = V = -\frac{\partial h_m}{\partial t} \tag{5a}$$

For the lower plate surface z = 0

$$u = 0 \qquad \frac{\partial^2 u}{\partial z^2} = 0 \qquad w = 0 \tag{5b}$$

The velocity equation u is obtained by evaluating equation (2) with the conditions as

$$u = \left\{ \left(g_1 - g_2\right) - 1 \right\} \frac{h_{m0}^2}{\mu M_0^2} \frac{\partial p}{\partial r} \tag{6}$$

Where, $g_1 = g_{11},\ g_2 = g_{12}$ for $4M_0^2 l^2 / h_{m0}^2 < 1$ (7a)

$g_1 = g_{21},\ g_2 = g_{22}$ for $4M_0^2 l^2 / h_{m0}^2 = 1$ (7b)

$g_1 = g_{31},\ g_2 = g_{32}$ for $4M_0^2 l^2 / h_{m0}^2 > 1$ (7c)

Where M_0 is the Hartmann number defined by $M_0 = B_0 h_{m0} \left(\sigma/\mu\right)^{1/2}$ with B_0 as the magnetic field and l is the couple stress parameter given by $l = (\eta / \mu)^{1/2}$, The corresponding relations in equations (7a), (7b) and (7c) are given in **Index A.**

Using the value of u obtained in equation (6) to the equation (4) and use the boundary conditions 5(a) and 5(b) for integrating across the film thickness. The MHD couple stress Reynolds type equation for film pressure gained of the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{ rS(h,l,M_0)\frac{\partial p}{\partial r} \right\} = \mu V \tag{8}$$

Where

$$S(h,l,M_0)=\begin{cases}\dfrac{h_{m0}^2}{M_0^2}\left\{\dfrac{2l}{(A^2-B^2)}\left(\dfrac{B^2}{A}\tanh\dfrac{Ah}{2l}-\dfrac{A^2}{B}\tanh\dfrac{Bh}{2l}\right)+h\right\} & \text{for} \quad 4M_0^2l^2/h_{m0}^2<1\\ \dfrac{h_{m0}^2}{M_0^2}\left\{\dfrac{h}{2}Sech^2\left(\dfrac{h}{2\sqrt{2}l}\right)-3\sqrt{2}l\tan h\left(\dfrac{h}{2\sqrt{2}l}\right)+h\right\} & \text{for} \quad 4M_0^2l^2/h_{m0}^2=1\\ \dfrac{h_{m0}^2}{M_0^2}\left\{\dfrac{2l}{M_0}\left\{\dfrac{\left(A_2Cot\theta-B_2\right)SinB_2h-\left(B_2Cot\theta+A_2\right)SinhA_2h}{CosB_2h+CoshA_2h}\right\}+h\right\} & \text{for} \quad 4M_0^2l^2/h_{m0}^2>1\end{cases}$$

Introducing the dimensionless parameters given below

$$r^*=\frac{r}{a},\ h^*=\frac{h}{h_{m0}},\ l^*=\frac{2l}{h_{m0}},\ \beta=\alpha a^2,\ P^*=-\frac{h_{m0}^3p}{\mu a^2V}$$

into the equation (8). The non-dimensional Reynolds equation is obtained in the form

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}\left\{r^*F(h^*,l^*,M_0)\frac{\partial P^*}{\partial r^*}\right\}=-1 \tag{9}$$

Where,

$$F(h^*,l^*,M_0)=\begin{cases}\dfrac{1}{M_0^2}\left\{\dfrac{l^*}{(A^{*2}-B^{*2})}\left(\dfrac{B^{*2}}{A^*}\tanh\dfrac{A^*h^*}{l^*}-\dfrac{A^{*2}}{B^*}\tanh\dfrac{B^*h^*}{l^*}\right)+h^*\right\} & \text{for} \quad 4M_0^2l^{*2}<1\\ \dfrac{1}{M_0^2}\left\{\dfrac{h^*}{2}Sech^2\left(\dfrac{h^*}{\sqrt{2}l^*}\right)-\dfrac{3l^*}{\sqrt{2}}tanh\left(\dfrac{h^*}{\sqrt{2}l^*}\right)+h^*\right\} & \text{for} \quad 4M_0^2l^{*2}=1\\ \dfrac{1}{M_0^2}\left\{\dfrac{l^*\left(A_2^*Cot\theta^*-B_2^*\right)SinB_2^*h^*-l^*\left(B_2^*Cot\theta^*+A_2^*\right)SinhA_2^*h^*}{M_0\left(CosB_2^*h^*+CoshA_2^*h^*\right)}+h^*\right\} & \text{for} \quad 4M_0^2l^{*2}>1\end{cases}$$

$$h^*=h_m^*\exp(-\beta r^{*2}),\ A^*=\left\{\frac{1+\left(1-M_0^2l^{*2}\right)^{1/2}}{2}\right\}^{1/2},\ B^*=\left\{\frac{1-\left(1-M_0^2l^{*2}\right)^{1/2}}{2}\right\}^{1/2}$$

The boundary condition for the squeeze film pressure are given by

$$\frac{dP^*}{dr^*}=0 \quad \text{at} \quad r^*=0 \tag{10a}$$

$$P^*=0 \quad \text{at} \quad r^*=0 \tag{10b}$$

Integrating the equation (9) as a function of r^* and applying the boundary conditions (10a) and (10b) , the non-dimensional squeeze film pressure is obtained in the form

$$P^* = -\frac{1}{2}\int_1^{r^*} \frac{r^*}{F(h^*, l^*, M_0)} dr^* \tag{11}$$

The load carrying capacity w is given by

$$w = 2\pi \int_0^a p^* dx \tag{12}$$

The dimensionless load capacity is of the form

$$W^* = \frac{W h_{m0}^3}{2\pi\mu a^4 \left(-dh_m/dt\right)} = -\frac{1}{2}\int_0^1 \left\{ \int_1^{r^*} \frac{r^*}{F(h^*, l^*, M_0)} dr^* \right\} r^* dr^* \tag{13}$$

The non-dimensional squeeze film time is given by

$$T^* = \frac{W h_{m0}^2}{2\pi\mu a^4} t = -\frac{1}{2}\int_1^{h_1^*} \left[\int_0^1 \left\{ \int_1^{r^*} \frac{r^*}{F(h^*, l^*, M_0)} dr^* \right\} r^* dr^* \right] dh_m^* \tag{14}$$

3. Results and Discussion

The effect of magnetic field on between two plates with non-dimensional couple stress l^* is shown in this paper. The parameter M_0 is the Hartmann number and $l^* \left(= 2l/h_{m0} \right)$ where $l = (\eta/\mu)^{1/2}$ arises due to the presence of small polar additives in the lubricant. Hence in this analysis the following range of values of non-dimensional parameters are used to analyze the bearing performance.

a) Hartmann number M_0 = 0 to 6.
b) Couple stress parameter l^* = 0 to 0.6.
c) Curvature parameter β = 0 to 1.

3.1 Pressure

Figure 2 describes the variation of dimensionless pressure P^* is plotted versus r^* for distinct values of M_0 with l^* = 0.3 and β = 0.8. It is noticed that the pressure P^* is increases due to increase values of Hartmann number M_0 . figure 3 presents the variation of dimensionless film pressure P^* as a function r^* for different values of couple stress l^* with M_0 = 3 and β = 0.8. It is observed that the non-dimensional P^* is improved with increase values of couple stress l^* . The variation of pressure P^* versus r^* various values of curvature parameter β with M_0 = 3 and l^* = 0.3 is presented in the figure 4. It is found that the dimensionless pressure P^* is more significant for ascending values of β .

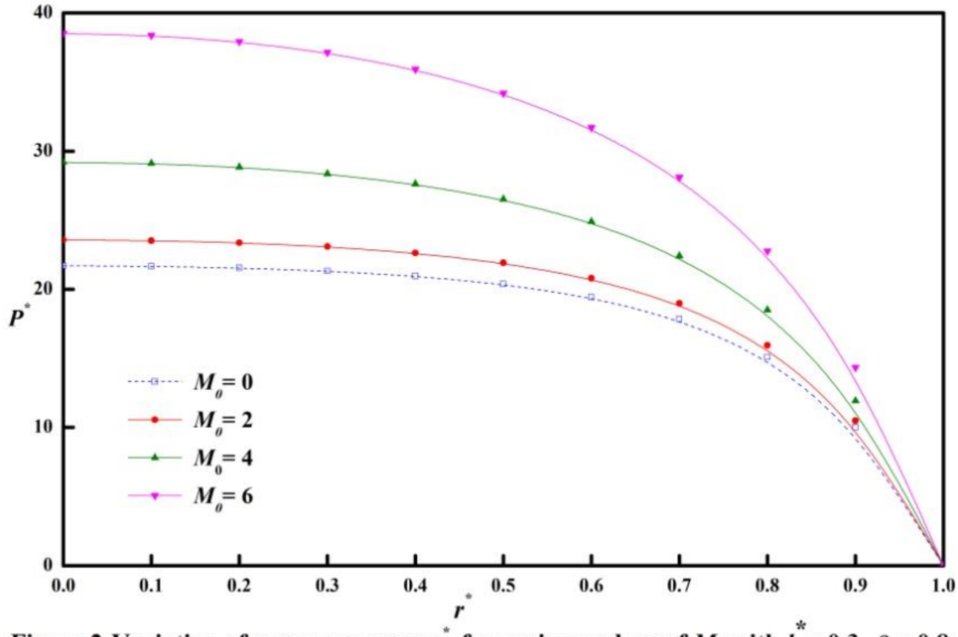

Figure 2:Variation of pressure versus r^* for various values of M_0 with $l^* = 0.3$, $\beta = 0.8$

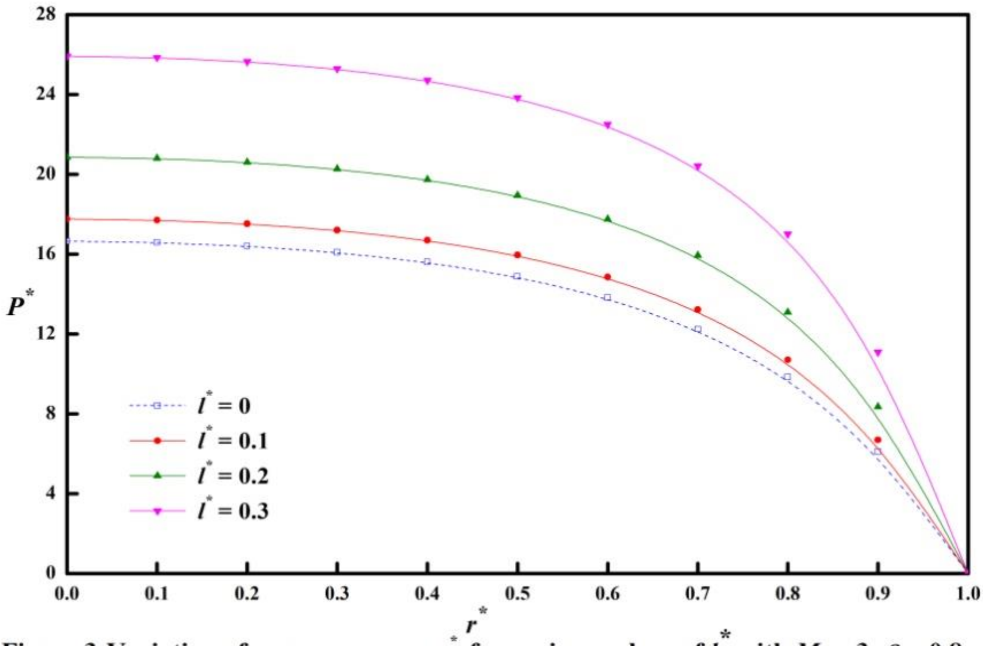

Figure 3:Variation of pressure versus r^* for various values of l^* with $M_0 = 3$, $\beta = 0.8$

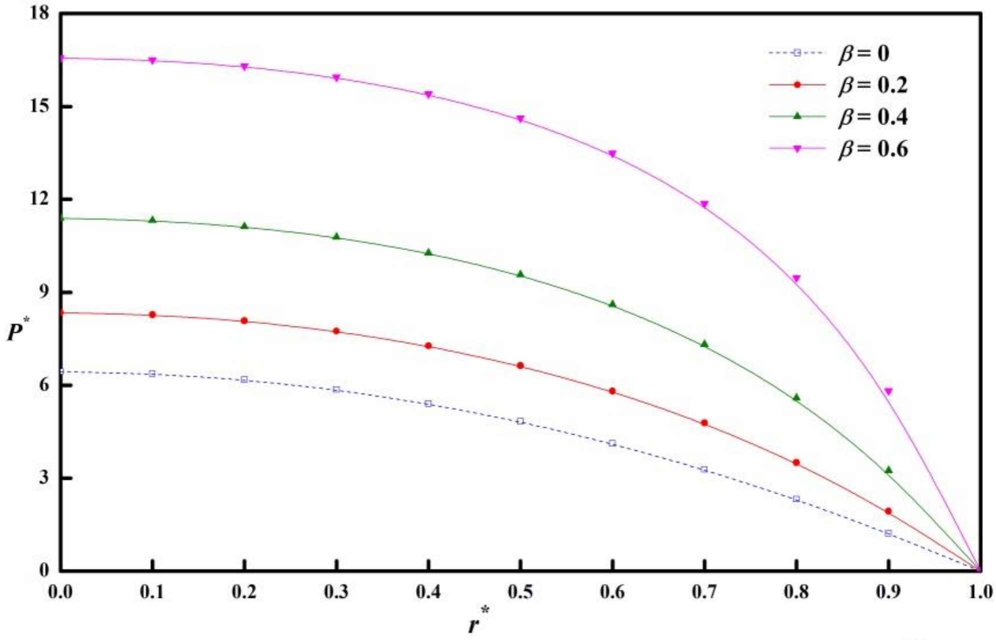

Figure 4:Variation of pressure versus r^* for various values of β with $M_0 = 3$, $l^* = 0.3$

3.2 Load carrying capacity

Figure 5 depicts the change of dimensionless W^* with curvature parameter β for various values M_0 with l^* =0.3. It is noticed that that the applied magnetic field M_0 for ascending values provides the more load capacity W^*. figure 6 shows the variation of dimensionless load carrying capacity W^* with curvature parameter β for distinct values of couple stress l^* with $M_0 = 3$. It is found that the couple stress lubricant is enhancing the load carrying capacity as compared to the Newtonian case. The couple stress fluid is creates the more pressure build in the fluid film region, and this causes the more load carrying capacity.

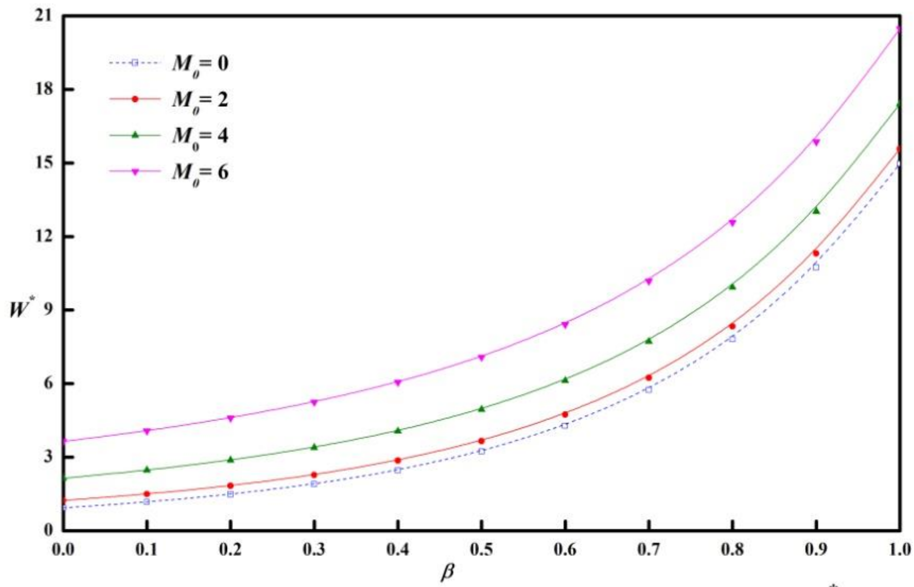

Figure 5:Variation of Load carrying capacity versus β for various values of M_0 with $l^* = 0.3$

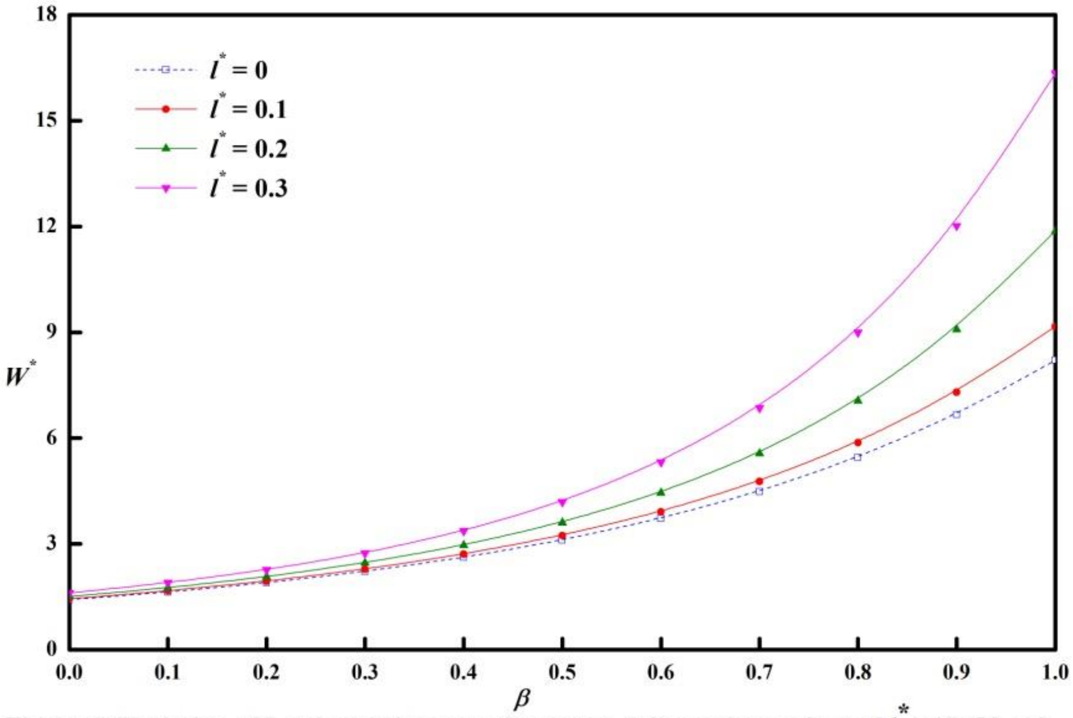

Figure 6: Variation of Load carrying capacity versus β for various values of l^* with $M_0 = 3$

3.3 Squeeze film time

The dimensionless squeeze time T^* versus h_m^* for various values of M_0 with l^* = 0.3 and $\beta = 0.8$ is shown in figure 7. It found that, the squeezing time is more significant because of magnetic field as compared to the non-magnetic case. figure 8 describes the variation of dimensionless squeeze film T^* with h_m^* for different values of couple stress l^* with M_0 = 3 and $\beta = 0.8$. It is noticed that, the effect of couple stress l^* provides an increase in the squeeze film time as observing to non- Newtonian case. Variation of non-dimensional squeeze film time T^* versus film thickness h_m^* for different values of β with M_0 = 3 and l^* =0.3 is presented in figure 9. This shows that the squeezing time increases as increase values of β .

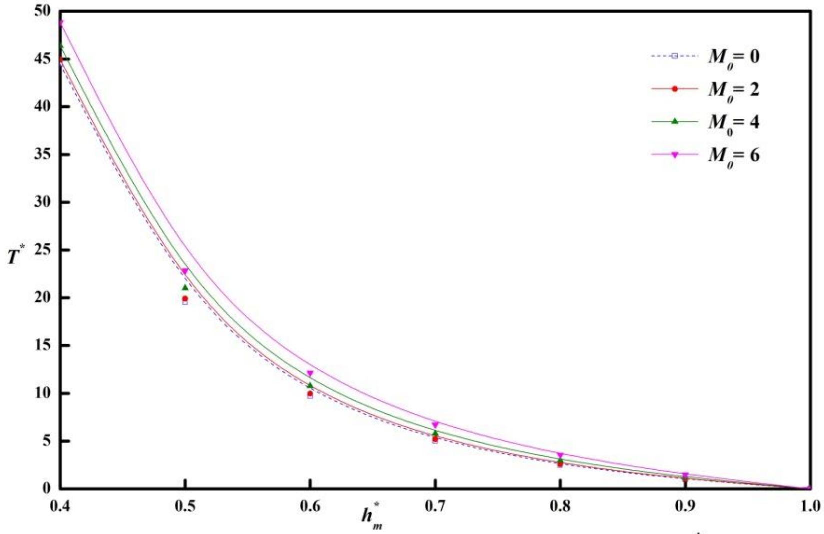

Figure 7:Variation of squeezing time versus h_m^* for various values of M_0 with $l^* = 0.3$, $\beta = 0.8$

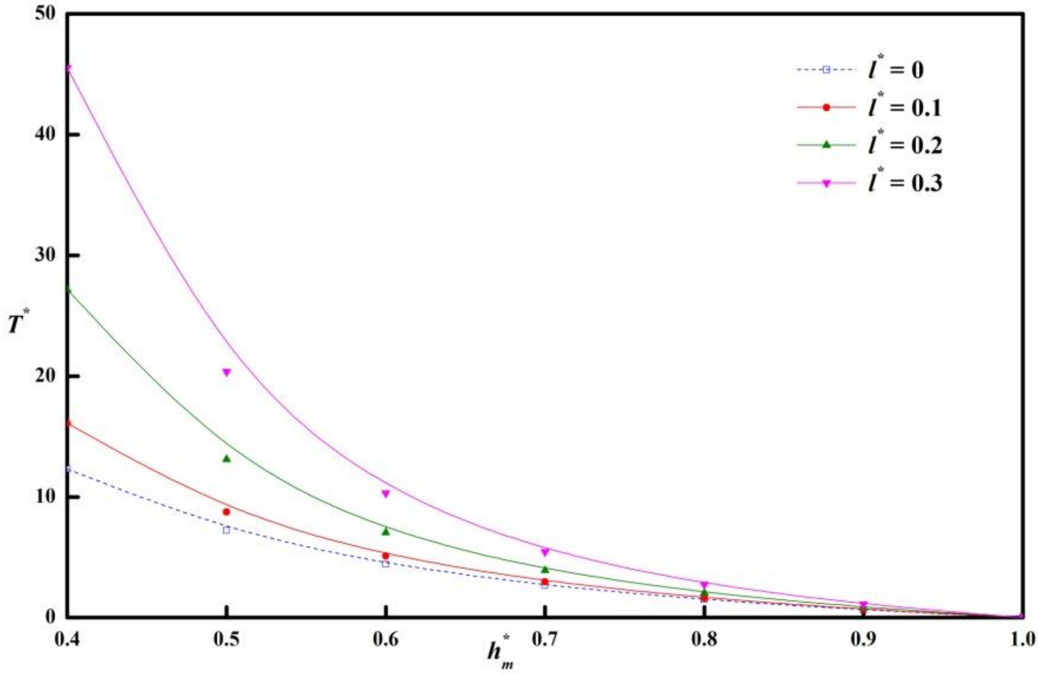

Figure 8: Variation of squeezing time versus h_m^* for various values of l^* with $M_0 = 3$, $\beta = 0.8$

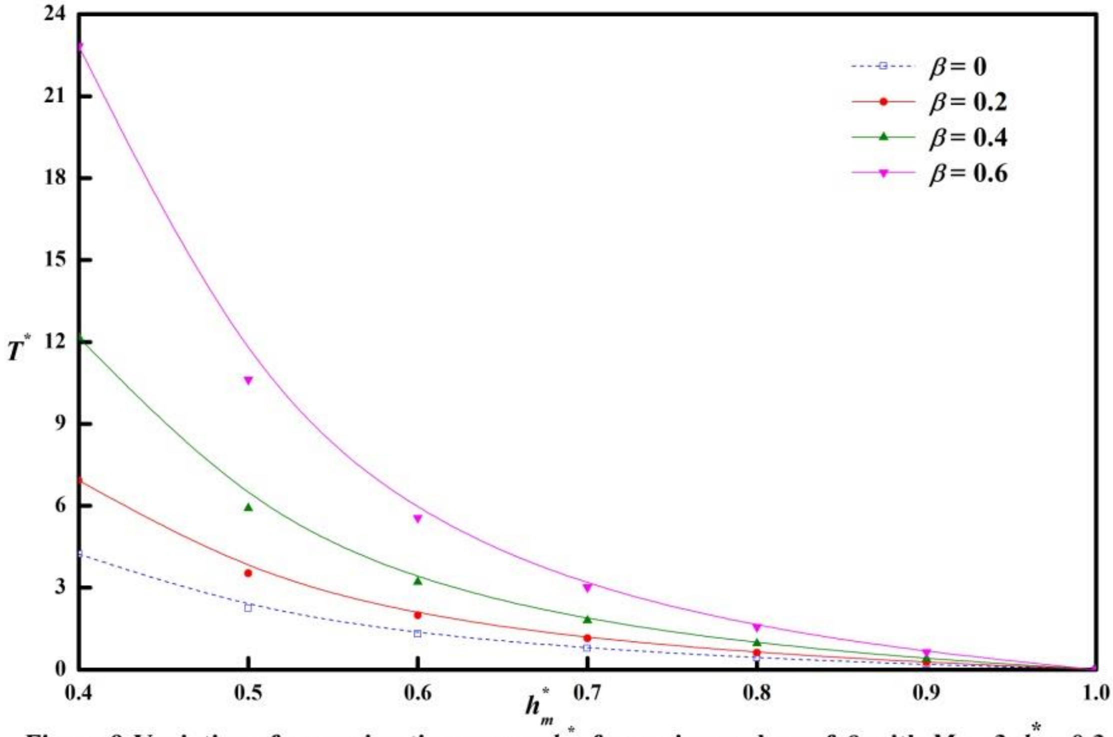

Figure 9: Variation of squeezing time versus h_m^* for various values of β with $M_0 = 3$, $l^* = 0.3$

4. Conclusion

The effect of transverse magnetic field between two plates with couple stress fluid is studied on the basis of Stokes couple stress theory.

The following interpretation drawn from the results and discussion.

- The increase in pressure, load capacity and the squeeze film time is found in increase of magnetic field as compared with non-magnetic case.
- The increase in the pressure, load carrying capacity and lengthen the squeeze film time due to use of non-Newtonian lubricant as compared with Newtonian case.
- Non-dimensional Pressure, load carrying capacity and squeeze film time is improved for the increase values of β .

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Nomenclature

a Bearing length

B_0 applied magnetic field

h film thickness

h_m minimum film thickness

h^* non-dimensional film thickness $\left(=h/h_{m0}\right)$

l couple -stress parameter $\left(\eta/\mu\right)^{1/2}$

l^* non-dimensional couple stress parameter $\left(2l/h_{m0}\right)$

M_0 Hartmann number $\left(=B_0 h_{m0}(\sigma/\mu)^{1/2}\right)$

p pressure in the film region

P^* non-dimensional pressure ($\left(=-\frac{h_{m0}^3 p}{\mu a^2 V}\right)$)

r radial coordinates

r^* non-dimensional radial coordinate

t squeeze film time

T^* non-dimensional squeezing time $\left(=\frac{W h_{m0}^2}{2\pi\mu a^4} t\right)$

u, v velocity components in film region

W load carrying capacity

W^* non-dimensional load carrying capacity $\left(=-\frac{W h_{m0}^3}{2\pi\mu a^4\left(-dh_{m0}/dt\right)}\right)$

η material constant characterizing couple stress

μ viscosity coefficient

σ electrical conductivity

Appendix A:

$$g_{11}=\frac{A^2}{\left(A^2-B^2\right)}\frac{\cosh\left\{B\left(2z-h\right)/2l\right\}}{\cosh\left(Bh/2l\right)}$$
(A1a)

$$g_{12}=\frac{B^2}{\left(A^2-B^2\right)}\frac{\cosh\left\{A\left(2z-h\right)/2l\right\}}{\cosh\left(Ah/2l\right)} \tag{A1b}$$

$$A=\left\{\frac{1+\left(1-4M^2 l^2/h_m^2\right)^{\frac{1}{2}}}{2}\right\}^{\frac{1}{2}} \qquad B=\left\{\frac{1-\left(\;-\;M^2\;/\;_{m}1\right)^{\frac{1}{2}}}{2}\right\}^{\frac{1}{2}} \tag{A1c}$$

$$g_{21}=\frac{2\cosh\left\{\left(z-h\right)/\sqrt{2l}\right\}+2\cosh\left(z/\sqrt{2l}\right)}{2\left\{\cosh\left(h/\sqrt{2l}\right)+1\right\}} \tag{A2a}$$

$$g_{22} = \frac{\left(z/\sqrt{2l}\right)\sinh\left\{\left(z-h\right)/\sqrt{2l}\right\} + \left\{\left(z-h\right)/\sqrt{2l}\right\}\sinh\left(z/\sqrt{2l}\right)}{2\left\{\cosh\left(h/\sqrt{2l}\right)+1\right\}} \tag{A2b}$$

$$g_{31} = \frac{\cos B_2 z \cosh A_2\left(z-h\right) + \cosh A_2 z \cosh B_2\left(z-h\right)}{\cosh A_2 h + \cosh B_2 h} \tag{A3a}$$

$$g_{32} = \frac{\cot\theta\left\{\sinh A_2 z \sinh B_2\left(z-h\right) + \sin B_2 z \sinh A_2\left(z-h\right)\right\}}{\cosh A_2 h + \cosh B_2 h} \tag{A3b}$$

$$A_2 = \sqrt{M/lh_m}\cos\left(\theta/2\right)\ , B_2 = \sqrt{M/lh_m}\sin\left(\theta/2\right),\ \theta = \tan^{-1}\left(\sqrt{4l^2M^2/h_m^2-1}\right) \tag{A3c}$$