Estimation of the dispersion parameter in a population dynamics model of the coffee berry borer (*Hypothenemus hampei*)

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Abstract. The main pest insect in coffee countries is the coffee berry borer (*hypothenemus hampei*), which impacts the coffee crops negatively. In this work, we estimate the dispersion parameter on a partial differential equation to model the population distribution in space of the coffee berry borer. To estimate the parameter, we use a technique based on the analytical resolution of the partial differential, data collected from a coffee farm to set up initial and boundary conditions, and the Bootstrap method. Results allow us to estimate the dispersion parameter with a coefficient of variation of 13%.

1. Introduction

Agriculture in Colombia has been significant for the economy for many years. The monocultures characterize the agricultural activity by region (sugar, cane, coffee, flowers, cotton, etc.), protruding coffee production as the activity that has given greater international recognition, thanks to the high quality of the grain [1-3]. However, one of the biggest threats in the production of coffee worldwide is the coffee berry borer (*Hypothenemus hampei*), a beetle native from Africa with size of a pin's head [4]. This pest is challenging to eradicate because it continues to reproduce and disperse when emerging from ripe fruits located in the soil [5,6]. Consequently, different Colombian agencies are interested in understanding the cycle of life of the coffee berry borer, particularly, the mechanisms of dispersion of the beetle in the coffee farms [7]. Several mathematical models have been proposed [8], however, to calibrate these models, it is required to improve and update estimates of the parameters used.

Therefore, in this paper, we estimate the dispersion parameter on a partial differential equation to model the population distribution in space of the coffee berry borer. The methodology to estimate the parameter is based on the analytical resolution of the partial differential, the data collected from a coffee farm and the Bootstrap method.

2. Mathematical model

To propose a model for the distribution in space of the coffee berry borer, we assume an isolated environment. Additionally, we consider the transect walk across the coffee farm as a data collection strategy. Therefore, we consider a model in a one-dimensional space as shown in Equation (1):

$$\left\{\begin{array}{c} \frac{\partial \mathrm{B}}{\partial \mathrm{t}} = \alpha \frac{\partial^2 \mathrm{B}}{\partial x^2} + \varepsilon \mathrm{B}, x \in \Omega \text{ and } \mathrm{t} \in \mathrm{J} \\ \left.\frac{\partial \mathrm{B}}{\partial x}\right|_{x=0} = 0 = \left.\frac{\partial \mathrm{B}}{\partial x}\right|_{x=H} \\ \mathrm{B}\,(0, \mathrm{x}) = \mathrm{B0}\,(\mathrm{x})\,; \end{array}\right\}, \tag{1}$$

where $\mathrm{B} \equiv \mathrm{B}\,(\mathrm{t}, \mathrm{x})$ represents coffee berry borer concentration at $\mathrm{x} \in \Omega = \{0, \mathrm{H}\}$ and time $\mathrm{t} \in \mathrm{J} = (0, \mathrm{L})$, , the parameter α is the diffusivity (dispersion coefficient) and ε the intrinsic growth rate; assuming an initial distribution of the coffee beetle given by $\mathrm{B}(0, \mathrm{x}) = \mathrm{B0}(\mathrm{x})$.

Equation (1) has only one solution and its analytical solution is given by Equation (2) is explicitly found it following the methodology proposed by Myint-U [9].

$$\begin{gathered} \mathrm{B}(\mathrm{t}, \mathrm{x}) = \frac{\mathrm{a}_0}{2} + \sum_{\mathrm{n}=1}^{\infty} \mathrm{a}_{\mathrm{n}} \exp\left(\alpha\left[\varepsilon - \left(\frac{\mathrm{n}\pi}{\mathrm{H}}\right)^2\right]\mathrm{t}\right)\cos\left(\frac{\mathrm{n}\pi\mathrm{x}}{\mathrm{H}}\right) \\ \mathrm{a}_{\mathrm{n}} = \frac{2}{\mathrm{H}}\int_0^{\mathrm{H}} \mathrm{B}_0(\mathrm{x})\cos\left(\frac{\mathrm{n}\pi\mathrm{x}}{\mathrm{H}}\right)\,\mathrm{dx} \text{ with } \mathrm{n} = 0,1,2, \ldots \end{gathered} \tag{2}$$

Furthermore, $\mathrm{B}\,(\mathrm{t}, \mathrm{x})$ is absolutely convergent for $\mathrm{t} \geq 0$ and $0 \leq \mathrm{x} \leq \mathrm{H}$.

3. Field data

To obtain the field data, we seek the advice of an expert collector to quantify the infestation of the coffee berry borer in the crops, in a coffee farm divided into 14 cultivation batches, from which specific data were obtained in time and space. The batches are denoted with $\mathrm{i} = 1, \ldots, 14$. See Figure 1.

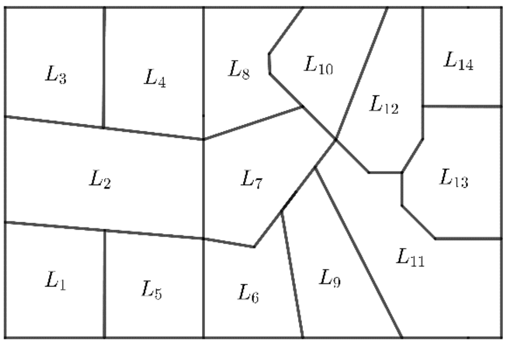


Figure 1. Studied farm batches' diagram.

Table 1. Percentage values of coffee beetle infestation for each batch.

	Coffee berry borer percentage by batch (%)													
Date	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}	L_{11}	L_{12}	L_{13}	L_{14}
12/02/2013	3	4.4	2.8	2.4	4.5	3.7	3.7	4	3.3	2.9	4.1	2	0.8	4.5
03/07/2013	2.8	7.6	4.1	1.9	2.8	3	3.8	2.5	4.6	4.3	1.1	0.3	2.3	3.2
23/08/2013	2.9	2.8	4.1	2.3	2.9	2.2	2.7	2.5	2.1	4.5	2.1	3.5	3.5	2.3
24/10/2013	11.4	3.5	3.8	3.9	4.8	1.6	5.1	17.9	5.5	14.3	2.8	2	1.4	3.9
21/10/2014		27.4	32.4	13.9	13.1	16.8	21.1		11.4		9.1	9	18.6	
01/12/2014		7.9	6.2									5	6.5	
23/12/2014		4.1			6.9	4.1	9.7		6.1		4.9			
06/02/2015		4.2	5.3	4.8	4.6	4.6	2.8				3.7	4.5	7	
11/05/2015		1.2	0.1	0.8	2	2	2		3.9		2.6	5.3	5.9	
18/07/2015		4.2	4.7	5.7	6.1	5.6			5.2			8.3	3.8	
07/11/2015	0.6	0.6	3.7		4	3.9			4.7	3.5		3.5	3.2	3

Table 1 shows the percentage of coffee beetle infestation obtained from the transect walk across the coffee farm at the fourteen batches, in different moments in time. Since the data obtained are not uniformly distributed over time, we use a polynomial interpolation technique (Lagrange interpolating polynomials) to obtain the missing data and generate a uniform data sample for the 34 months in which the data were obtained.

We define the parameters H, L and ε from Equation (1), as follows: H is the upper limit of the interval that represents the spatial domain, L is the upper limit of the interval that represents the temporal variable and ε is the intrinsic rate of growth of the coffee berry borer.

To estimate H and L, we took the dimensions of the farm and the time elapsed during the process of data collection (34 months), finally we obtained $H = 3.5$ and $L = 8$. Additionally, we used the intrinsic rate of growth parameter $\varepsilon = 0.003771$, according to Fernández [4].

According to the batches' distribution in the coffee farm (Figure 1), in which the data was collected, a graph is constructed (Figure 2) that represents the batches and its adjacencies.

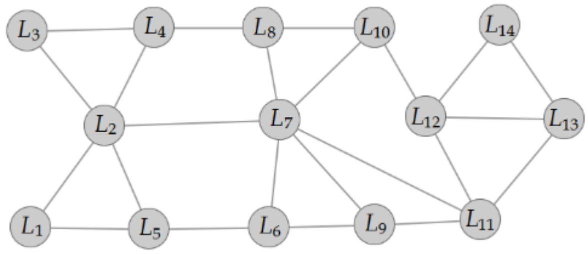


Figure 2. Representation of coffee farm for adjacencies batches.

Next, some possible routes (Hamiltonian paths) are chosen randomly to determine the initial condition $B_0(x)$ [10], taking into account the first infestation data collected. Figure 3 shows some particular examples.

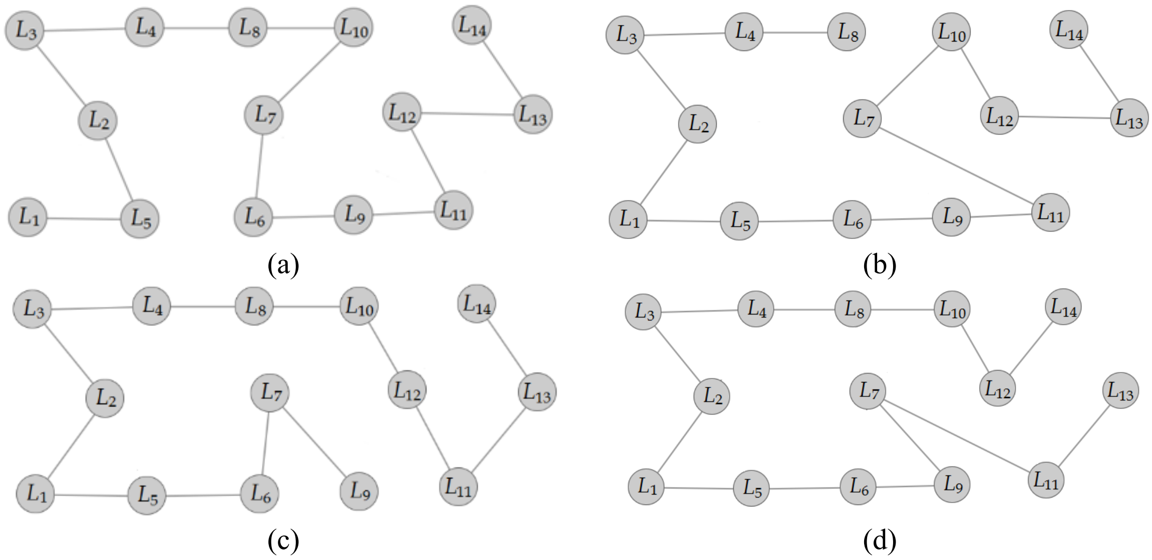


Figure 3. Hamiltonian patches chosen. (a) Patch 1, (b) Patch 2, (c) Patch 3 and (d) Patch 4.

4. Sample estimation and results

We know that the solution to the problem in Equation (1) is the function given by the formula in Equation (3) [11,12]:

$$B(t,x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[\frac{2}{H}\int_0^H B_0(x)\cos\left(\frac{n\pi x}{H}\right)dx\right]\exp\left(\alpha\left[\varepsilon-\left(\frac{n\pi}{H}\right)^2\right]t\right)\cos\left(\frac{n\pi x}{H}\right)$$
$$\text{where: } a_0=\frac{2}{H}\int_0^H B_0(x)dx \tag{3}$$

Then, we formulate an algebraic system of equations that depend on α (dispersion coefficient), to find the different values of it using the field data. In addition, taking advantage of the fact that the solution of Equation (1) converges absolutely. We take only the sufficient terms of the series to obtain the required information. Specifically, using the first five terms of the series, the approximate solution is shown in Equation (4):

$$B\,(t,x)\approx\frac{a_0}{2}+\sum_{n=1}^{5}\left[\frac{2}{H}\int_0^H B_0(x)\cos\left(\frac{n\pi x}{H}\right)dx\right]\exp\left(\alpha\left[\varepsilon-\left(\frac{n\pi}{H}\right)^2\right]t\right)\cos\left(\frac{n\pi x}{H}\right) \tag{4}$$

With $H = 3.5$, $L = 8.25$ and $\varepsilon = 0.003771$. Given (t_i, x_j) a field data in time i and space j, we derive the system in Equation (5):

$$B(t_i,x_j)=\frac{a_0}{2}+\sum_{n=1}^{5}\left[\frac{2}{H}\int_0^H B_0(x)\cos\left(\frac{n\pi x}{H}\right)dx\right]\exp\left(\alpha\left[\varepsilon-\left(\frac{n\pi}{H}\right)^2\right]t_i\right)\cos\left(\frac{n\pi x_j}{H}\right)$$
$$C_0=\frac{a_0}{2},\ C_n=\frac{2}{H}\left[\int_0^H B_0(x)\cos\left(\frac{n\pi x}{H}\right)dx\right]\cos\left(\frac{n\pi x_j}{H}\right) \text{ and } K_n=\left[\varepsilon-\left(\frac{n\pi}{H}\right)^2\right]t_i \tag{5}$$

With $n = 1, \dots 5$.Thus Equation (5) takes the form of Equation (6):

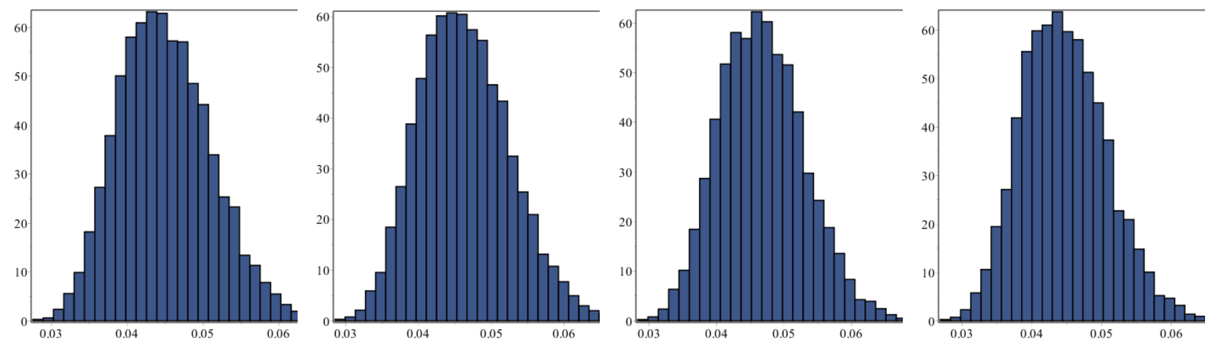
$$B(t_i,x_j)=C_0+C_1\exp(k_1\,\alpha)+C_2\exp(k_2\,\alpha)+C_3\exp(k_3\,\alpha)$$
$$+C_4\exp(k_4\,\alpha)+C_5\exp(k_5\,\alpha) \tag{6}$$

Which it represents the $i-th$ equation of the algebraic system of equations for α. Given that the values of C_m and k_m are known, this algebraic system is solved (numerically) to obtain different values of α. Then, when we solve the system of equations numerically, we obtain a certain amount of values for α, which by means of the Bootstrap method [13] generates a generous amount of random samples. This allow us to estimate the following sample distribution of the parameter α.

Table 2 summarizes some results obtained when applying the Bootstrap method from Maple, sampling with 7500 iterations. Note from the third and fourth column of table, the values corresponding to the mean and variance statistics, allowing to conclude that the value of the dispersion parameter (on average) is around $\alpha = 0.045$ and the variance around 3×10^{-5}. These results allow us to estimate the dispersion parameter with a coefficient of variation of 13%. Additionally, a frequency histogram (Figure 4), allows to guess about the probability function of the α parameter, being (in its behavior) very close to a normal distribution.

Table 2. Several Hamiltonian patches and Bootstrap values.

Sequence	Patch	Mean	Variance
$[L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8, L_9, L_{10}, L_{11}, L_{12}, L_{13}, L_{14}]$	1	0.04502	0.0000377
$[L_8, L_4, L_3, L_2, L_1, L_5, L_6, L_9, L_{11}, L_7, L_{10}, L_{12}, L_{13.}L_{14}]$	2	0.04637	0.0000396
$[L_9, L_7, L_6, L_5, L_1, L_2, L_3, L_4, L_8, L_{10}, L_{12}, L_{11,}L_{13}, L_{14}]$	3	0.04675	0.0000398
$[L_{13}, L_{11}, L_7, L_9, L_6, L_5, L_1, L_2, L_3, L_4, L_8, L_{10}, L_{12}, L_{14}]$	4	0.04465	0.0000371



(a) (b) (c) (d)

Figure 4. Frequency histograms of Bootstrap values obtained from patches at Figure 3. (a) Patch 1, (b) Patch 2, (c) Patch 3 and (d) Patch 4.

5. Conclusion

In this work we obtained an estimation of the dispersion coefficient of the coffee berry borer in a model based on partial differential equations, using the analytical solution of classic diffusion model with initial and boundary values. Subsequently, we use the field data collected in a coffee farm to determine different parameter values for dispersion constant; and finally, we estimate the dispersion coefficient under the resampling process called Bootstrap. These results allow us to estimate the dispersion parameter with a coefficient of variation of 13%. The methodology used in this work to obtain the estimation is general and it can be extended to estimate parameters in other models based on partial differential equations. Besides, each step in the methodology may be interchangeable for a different technique, for instance, the truncation of the analytical solution given by the Fourier series can be changed for a finite difference approximation. These changes will be a subject of further study.

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