

Temperature Cross in Three-Fluid Cryogenic Heat Exchangers

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ABSTRACT. In multi-fluid heat exchangers, local equalization of fluid temperatures occur(s) at certain location(s) within the heat exchanger, depending on the chosen values of the operating condition parameters and design parameters. This is referred to as a ‘temperature cross’ between the concerned fluids. The formation of the temperature cross leads to a situation wherein there is reversal of heat transfer in the heat exchanger section on one side of the cross. A three-fluid cryogenic heat exchanger, involving thermal interaction between all the three-fluids - hot, cold and intermediate - is investigated for the effect of the temperature cross, for the different flow arrangements. Non-dimensional governing equations are formulated to account for ambient heat-in-leak and longitudinal wall conduction and solved using finite element method. An algorithm, using MATLAB, has been used to determine the position of the temperature cross. The variation of the temperature cross is investigated for varying values of the Thermal Resistance Ratio between intermediate-cold fluids and hot-cold fluids (HI).

NOMENCLATURE

i_h, i_i & i_c	Directional constants as defined in Eqs. (1)-(3)
\dot{m}	Mass flow rate, (kg/s)
\dot{Q}	Heat transfer rate, (W)
c_p	Specific heat at constant pressure, J/kg-K
T	Temperature (K)
C	Heat capacity rate of the fluids defined by the product of \dot{m} and c_p (W/K)
P	Wetted perimeter for any contact area (m)
P_1, P_2, P_3, P_4	Wetted perimeters corresponding to areas A_1, A_2, A_3, A_4 respectively
L	Heat exchanger length (m)
L_e	Effective length of heat exchanger as defined by $L/\text{number of elements}$
A	Surface area for heat transfer as defined by the product of P and L_e (m ²)
A_1, A_2, A_3, A_4	Areas as illustrated in Fig. 2
U	Overall heat transfer coefficient (W/m ² -K)
U_1, U_2, U_3, U_4	Overall heat transfer coefficients as illustrated in Fig. 2
H_1, H_2, H_3	Ratio of thermal resistances as defined in Table 1
R_1, R_2	Ratio of heat capacity rates as defined in Table 1
NTU	Overall number of transfer units as defined in Table 1



n	Local NTU for each fluid in contact with a specified wall as defined in Table 1
x	Axial co-ordinate (m)
X	Non-dimensional axial co-ordinate as defined in Table 1

Greek

θ	Dimensionless temperature as defined in Table 1
λ	Longitudinal wall conduction factor as defined in Table 1

Subscripts

a	Ambient
h	Hot fluid
c	Cold fluid
i	Intermediate fluid
W1, W2, W3, W4	Separating walls as illustrated in Fig. 2
in	Inlet
out	Outlet

1. INTRODUCTION

In the gas processing and petrochemical industries, the development of cryogenic temperatures also requires the exchange of thermal energy between three or more fluids or fluid flows. In cryogenics and chemical processing units, such as air separation systems, helium-air separation units, hydrogen purification and liquefaction, ammonia gas synthesis, and others, extensive use of three-fluid and multi-fluid heat exchangers is found. Longitudinal wall conduction and ambient heat-in-leak are two significant parameters which degrade the performance the heat exchangers working in this temperature range, and need to be accounted for while writing the governing equations.

The fluid with the highest mean temperature is referred to as the 'hot fluid' in the three-fluid heat exchanger, while the one with the lowest mean temperature is referred to as the 'cold fluid'. The other fluid is the 'intermediate fluid'.

1.1. Formation of Temperature Cross

In multi-fluid heat exchangers, local equalization of fluid temperatures occur(s) at certain location(s) within the heat exchanger [1], depending on the chosen values of the operating condition parameters and design parameters. This is referred to as a 'temperature cross' between the concerned fluids. The formation of the temperature cross leads to a reversal of heat transfer, from the intended direction, in a section of the heat exchanger, either before or after the temperature-cross points depending on flow directions, for e.g., cold fluid will start heating the intermediate fluid, on one side of the heat exchanger, as seen in Figure 1. This means that the corresponding heat transfer areas are wasted. Hence, there is an eventual need to know about the position(s) of the temperature cross, to improve the heat exchanger design.

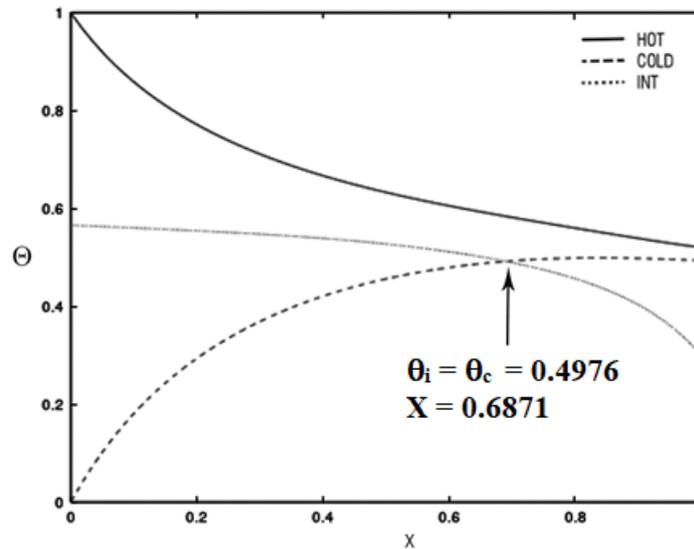


Figure 1. Three-fluid heat exchanger having 3-thermal communications: Depiction of the temperature cross for flow arrangement P4. Values of non-dimensional parameters: $R_1=2$, $R_2=1.25$, $\theta_{i,in}=0.3$, $\theta_a=1$, $Ntu=1$, $H_1=1.5$, $H_2=2$, $H_3=0.1$, $\lambda=0.08$.

2. MODEL FORMULATION

In the present paper, a parallel flow three-fluid heat exchanger, involving three-thermal interactions, of the type shown in Figure 2, is investigated for the effect of temperature cross. The model introduced in this paper for the three-fluid heat exchanger is a general model that can be generalized to all three-fluid, single pass, parallel flow heat exchangers taking into account all potential thermal interactions and flow arrangements. The fluid-pipe configuration for the heat exchanger is shown in Figure 2 and is similar to the one suggested by Aulds and Barron [2]. Each fluid interacts with the other two fluids. Four different flow arrangements are generated depending on the corresponding direction of flow of each fluid – P1, P2, P3 & P4 - are possible [1,3,4,5,6]. These are provided in Figures 3 – (a) – (d).

For analysis, the following assumptions are made:

- (a) The heat exchanger is in a steady state.
Basis: The heat exchanger is a steady state device for most of the time after a brief period of unsteady behaviour during the initial and final moments.
- (b) All properties are constant with time and space.
Basis: Properties are more or less constant at most conditions and only change drastically near the critical point and also during phase change, which have not been considered for the present analysis.
- (c) Within a stream the temperature distribution is uniform in the transverse direction and equal to the average temperature of the fluid.
Basis: The temperature distribution is fairly uniform along the heat exchanger length after the thermal and hydrodynamic entry lengths, in most situations.
- (d) There is no heat source or sink in the heat exchanger or in any of the fluids.
Basis: There is no heat source or sink unless there is local heating or cooling along the heat exchanger length, which have not been considered for the present analysis.
- (e) There is no phase change in the fluid streams.
Basis: For simplicity and in order to ensure that fluid properties are constant, phase change has not been considered for the present analysis.
- (f) The heat transfer area is constant along the length of the heat exchanger.
Basis: Most heat exchangers are made of tubes or pipes of constant cross section.

V. Krishna [9] et al, have adopted an analytical method – method of decoupling transformations – to solve the governing equations. In this paper, the differential governing equations have been solved adopting the FEM technique, primarily to obtain the temperature distribution of the fluids. Further an algorithm, using MATLAB, is written to determine the position of the temperature cross between fluids.

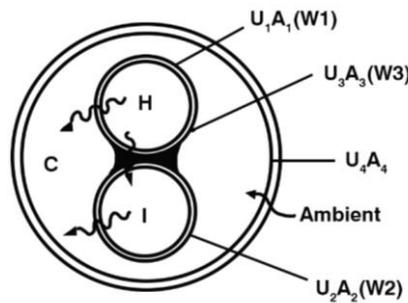


Figure 2. Tubular arrangement of the 3-fluid heat exchanger with 3-thermal communications with longitudinal wall conduction and heat-in-leak from the ambient.

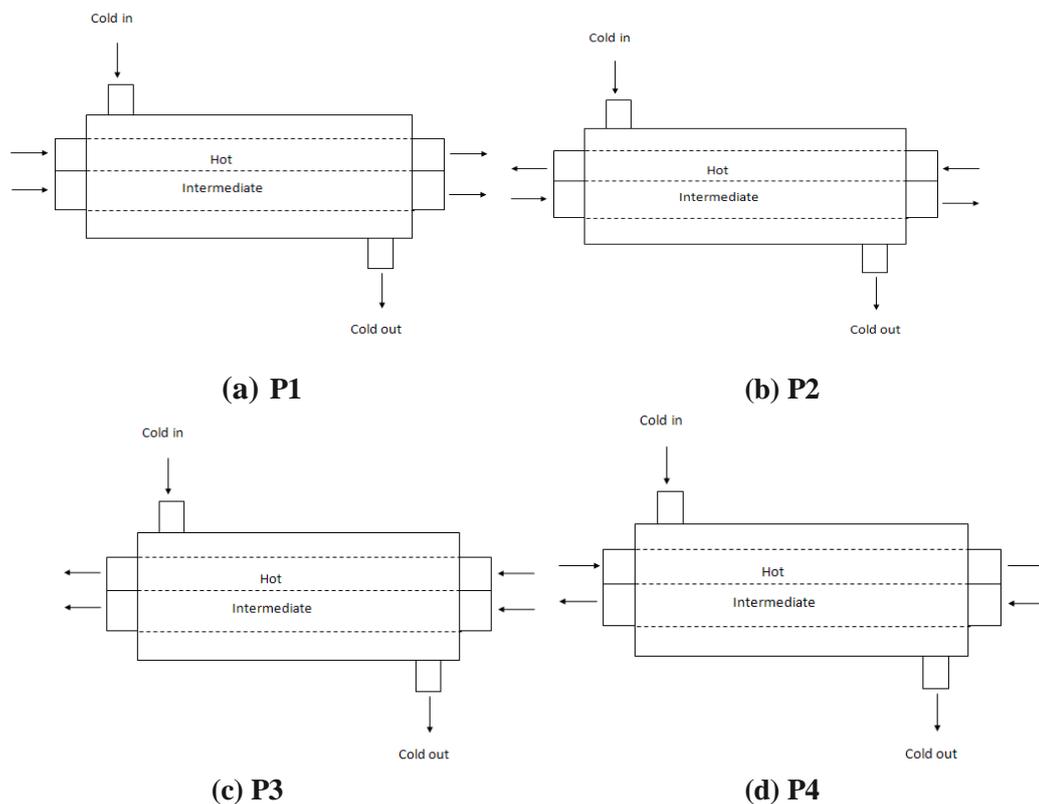


Figure 3. Flow arrangements.

2.1. Governing Equations

A set of non-dimensional parameters are identified and defined to assess the performance of the heat exchanger, the details of which are provided in Table (1).

In terms of energy balances for a differential element, the non-dimensional governing equations for three fluids and three walls are written as follows:-

$$\text{Hot Fluid:} \quad i_H \frac{d\theta_H}{dX} + n_{ih}(\theta_H - \theta_{W1}) + n_{3h}(\theta_H - \theta_{W3}) = 0 \quad (1)$$

$$\text{Intermediate Fluid:} \quad i_I \frac{d\theta_I}{dX} - n_{3i}(\theta_{W3} - \theta_I) + n_{2i}(\theta_I - \theta_{W2}) = 0 \quad (2)$$

$$\text{Cold Fluid:} \quad i_C \frac{d\theta_C}{dX} - n_{ic}(\theta_{W1} - \theta_C) - n_{2c}(\theta_{W2} - \theta_C) - n_4(\theta_a - \theta_C) = 0 \quad (3)$$

$$\text{Wall-1:} \quad \frac{\lambda_1}{R_2} \frac{d^2\theta_{W1}}{dX^2} + n_{1h}(\theta_H - \theta_{W1}) - \frac{n_{1c}}{R_2}(\theta_{W1} - \theta_C) = 0 \quad (4)$$

$$\text{Wall-2:} \quad \lambda_2 \frac{d^2\theta_{W2}}{dX^2} + \frac{n_{2i} R_2}{R_1}(\theta_I - \theta_{W2}) - n_{2c}(\theta_{W2} - \theta_C) = 0 \quad (5)$$

$$\text{Wall-3:} \quad \frac{\lambda_3}{R_2} \frac{d^2\theta_{W3}}{dX^2} + n_{3h}(\theta_H - \theta_{W3}) - \frac{n_{3i}}{R_1}(\theta_{W3} - \theta_I) = 0 \quad (6)$$

In the above equations for the three fluids, directional constants (i_H , i_I & i_C) are incorporated into the governing equations in order to make them applicable to all four flow arrangements. The values taken are +1 for the x direction positive and -1 for the x direction negative.

The notations for the different walls are:

W1 - wall in-between the hot fluid and the cold fluid

W2 - wall in-between the intermediate fluid and the cold fluid

W3 - wall in-between the hot fluid and the intermediate fluid

For the non-dimensional analysis, the local ntu of each pair of fluids across a given wall is assumed to be the same according to Kroeger's analysis [7], i.e.

$$\left. \begin{array}{l} \text{Across wall 1: } n_{1h} = n_{1c} = n_1 \\ \text{Across wall 2: } n_{2c} = n_{2i} = n_2 \\ \text{Across wall 3: } n_{3h} = n_{3i} = n_3 \end{array} \right\} \quad (7)$$

The local ntu of each fluid is correlated with the overall NTU as shown below:-

$$\text{NTU} = n_1 \left[\frac{R_2}{1+R_2} \right] = n_2 \left[\frac{R_2}{H_1(R_1+R_2)} \right] = n_3 \left[\frac{R_2}{H_2(1+R_1)} \right] = \frac{n_4}{H_3} \quad (8)$$

Table 1. Details of the Heat Exchanger Non-Dimensional Parameters.

Symbol	Expression	Particulars	Physical Significance
θ	$\frac{T - T_{c,in}}{T_{h,in} - T_{c,in}}$	Non-Dimensional Temperature	Temperature in non-dimensional terms
X	$\frac{x}{L}$	Non-Dimensional Length	Length in non-dimensional terms
R_1	$\frac{C_h}{C_i}$	Heat Capacity Ratio	Ratio of heat capacities between hot and intermediate fluids
R_2	$\frac{C_h}{C_c}$	Heat Capacity Ratio	Ratio of heat capacities between hot and cold fluids (R2)
H_1	$\frac{U_2 A_2}{U_1 A_1}$	Thermal Resistance ratio	Ratio of the thermal resistances between intermediate - cold fluids and hot - cold fluids
H_2	$\frac{U_3 A_3}{U_1 A_1}$	Thermal Resistance ratio	Ratio of the thermal resistances between hot - intermediate fluids and hot - cold fluids
H_3	$\frac{U_4 A_4}{U_1 A_1}$	Thermal Resistance ratio	Ratio of the thermal resistances between cold fluid - ambient and hot - cold fluids
NTU	$\frac{U_1 A_1}{C_c}$	Number of Transfer Units	Measure of thermal size

λ	$\frac{kA_c}{C_c L_e}$	Longitudinal wall conduction parameter	Measure of a wall's longitudinal heat conductivity
n_{1h}	$\left(\frac{hA_1}{C_h}\right)_{H-W1}$	Local ntu for hot fluid in contact with wall 1	Measure of the convective heat transfer between hot fluid and wall 1
n_{1c}	$\left(\frac{hA_1}{C_c}\right)_{W1-C}$	Local ntu for cold fluid in contact with wall 1	Measure of the convective heat transfer between wall 1 and cold fluid
n_{3h}	$\left(\frac{hA_3}{C_h}\right)_{H-W3}$	Local ntu for hot fluid in contact with wall 3	Measure of the convective heat transfer between hot fluid and wall 3
n_{3i}	$\left(\frac{hA_3}{C_i}\right)_{W3-I}$	Local ntu for intermediate fluid in contact with wall 3	Measure of the convective heat transfer between wall 3 and intermediate fluid
n_{2i}	$\left(\frac{hA_2}{C_i}\right)_{I-W2}$	Local ntu for hot fluid in contact with wall 2	Measure of the convective heat transfer between intermediate fluid and wall 2
n_{2c}	$\left(\frac{hA_2}{C_c}\right)_{W2-C}$	Local ntu for cold fluid in contact with wall 2	Measure of the convective heat transfer between wall 2 and cold fluid
n_4	NTU^*H_3	Conductance factor for ambient with cold fluid	Measure of heat transfer from the ambient to the cold fluid

Table 2. Boundary conditions for non-dimensional fluid temperatures for the various flow arrangements.

Flow arrangement - P1			
X	θ_h	θ_c	θ_i
0	1	0	$\theta_{i, in}$
1	-	-	-
Flow arrangement - P2			
X	θ_h	θ_c	θ_i
0	-	0	$\theta_{i, in}$
1	1	-	-
Flow arrangement - P3			
X	θ_h	θ_c	θ_i
0	-	0	-
1	1	-	$\theta_{i, in}$
Flow arrangement - P4			
X	θ_h	θ_c	θ_i
0	1	0	-
1	-	-	$\theta_{i, in}$

Table 3. Adiabatic Wall Boundary Conditions.

X	Boundary Condition
0	$\left[\frac{d\theta_{wi}}{dX}\right]_{X=0} = 0$
1	$\left[\frac{d\theta_{wi}}{dX}\right]_{X=1} = 0$

Where: $i = 1, 2$ and 3 for the corresponding three walls

3. FEM TECHNIQUE

3.1. Method of Weighted Residuals and Matrix Description

Using the method of weighted residuals (Lewis et al, [8]), the differential governing equations (1) – (6) are written as:

$$\text{Hot Fluid:} \quad \int_0^1 W \left\{ i_H \frac{d\theta_H}{dX} + n_{H-W1}(\theta_H - \theta_{W1}) + n_{H-W3}(\theta_H - \theta_{W3}) \right\} dX = 0 \quad (9)$$

$$\text{Intermediate Fluid:} \quad \int_0^1 W \left\{ i_I \frac{d\theta_I}{dX} - n_{W3-I}(\theta_{W3} - \theta_I) + n_{I-W2}(\theta_I - \theta_{W2}) \right\} dX = 0 \quad (10)$$

$$\text{Cold Fluid:} \quad \int_0^1 W \left\{ i_C \frac{d\theta_C}{dX} - n_{W1-C}(\theta_{W1} - \theta_C) - n_{W2-C}(\theta_{W2} - \theta_C) - n_4(\theta_a - \theta_C) \right\} dX = 0 \quad (11)$$

$$\text{Wall-1:} \quad \int_0^1 W \left\{ \frac{\lambda_1}{R_2} \frac{d^2\theta_{W1}}{dX^2} + n_{H-W1}(\theta_H - \theta_{W1}) - \frac{n_{W1-C}}{R_2}(\theta_{W1} - \theta_C) \right\} dX = 0 \quad (12)$$

$$\text{Wall-2:} \quad \int_0^1 W \left\{ \lambda_2 \frac{d^2\theta_{W2}}{dX^2} + \frac{n_{I-W2} R_2}{R_1}(\theta_I - \theta_{W2}) - n_{W2-C}(\theta_{W2} - \theta_C) \right\} dX = 0 \quad (13)$$

$$\text{Wall-3:} \quad \int_0^1 W \left\{ \frac{\lambda_3}{R_2} \frac{d^2\theta_{W3}}{dX^2} + n_{H-W3}(\theta_H - \theta_{W3}) - \frac{n_{W3-I}}{R_1}(\theta_{W3} - \theta_I) \right\} dX = 0 \quad (14)$$

Assuming a linear variation of hot, intermediate and cold fluid temperatures in a single element, the following equations give the temperature at any point along the length of fluid flow:

(a) For the positive x direction

$$\left. \begin{aligned} \theta_h &= N_1\theta_{h,in} + N_2\theta_{h,out} \\ \theta_i &= N_1\theta_{i,in} + N_2\theta_{i,out} \\ \theta_c &= N_1\theta_{c,in} + N_2\theta_{c,out} \end{aligned} \right\} \quad (15)$$

(b) For the negative x direction

$$\left. \begin{aligned} \theta_h &= N_1\theta_{h,out} + N_2\theta_{h,in} \\ \theta_i &= N_1\theta_{i,out} + N_2\theta_{i,in} \\ \theta_c &= N_1\theta_{c,out} + N_2\theta_{c,in} \end{aligned} \right\} \quad (16)$$

Assuming the temperature of the walls is given by a linear variation:

$$\left. \begin{aligned} \theta_{W1} &= N_1\theta_{W1,in} + N_2\theta_{W1,out} \\ \theta_{W2} &= N_1\theta_{W2,in} + N_2\theta_{W2,out} \\ \theta_{W3} &= N_1\theta_{W3,in} + N_2\theta_{W3,out} \end{aligned} \right\} \quad (17)$$

Here N_1 and N_2 are shape functions defined by:

$$\left. \begin{aligned} N_1 &= 1 - X \\ N_2 &= X \end{aligned} \right\} \quad (18)$$

After the substitution of these approximations in equations (9) - (14), a suitable weighted parameter (W) is defined to obtain a set of algebraic equations. In Galerkin's method, shape functions - N_1 and N_2 are taken as the weighted parameters.

The heat exchanger is discretized into a number of elements. The discretized governing equations for each element are written in matrix form:

$$[K]\{\Theta\} = \{f\} \quad (19)$$

$[K]$ is the local stiffness matrix for each element, $\{\Theta\}$ is the non-dimensional temperature vector and $\{f\}$ gives the loading terms. Assembling the local stiffness matrix for all the elements leads to the formation of the global stiffness matrix. Boundary conditions are then enforced on the global stiffness matrix and the loading vector. The equations are solved using MATLAB to obtain dimensionless temperatures along the length of the heat exchanger.

3.2. Validation of the FEM Technique

Shrivastava and Ameal [3] have provided the results for a 3-fluid, single pass, parallel flow heat exchanger model with 3-thermal communications taking into account - all thermal interactions and flow arrangements possible. They have presented the results for flow arrangement P2, showing the effect of various non-dimensional design and operating parameters such as H_1 , H_2 , R_1 , R_2 , NTU and $\theta_{i,in}$ on the non-dimensional temperature distributions of the three fluid streams and their respective effectivenesses. They have assumed no thermal interaction with the ambient with the heat exchanger completely insulated. The current model is compared to that of Shrivastava and Ameal [3, 4], neglecting the effect of ambient. Figure 4 shows the comparisons of the non-dimensional temperature distribution of the hot fluid for different instances of H_2 and they fit perfectly.

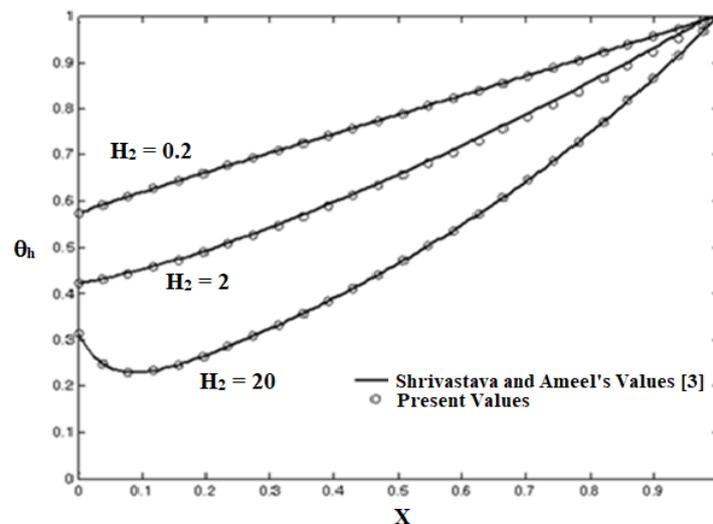


Figure 4. Effect of H_2 on the hot fluid temperature profile for a 3-fluid heat exchanger neglecting ambient heat-in-leak.

Comparison of current model - FEM values with Shrivastava and Ameal's values [3].

Values of other non-dimensional parameters: $R_1=2$, $R_2=1.25$, $H_1=1.5$, $NTU=1$, $\theta_{i,in}=0.5$, $\theta_a=0$.

4. DETERMINATION OF THE TEMPERATURE CROSS

The temperature cross for any pair of fluids is determined, by comparing the temperature differences between them at any two consecutive points at a fixed distance interval. If the difference across the two points is lesser than 0.0001, then it suggests the existence of a temperature cross, and the point is appended to a cross list. The procedure repeated to arrive at the exact location of the crosses. The flow chart for this algorithm is provided in Figure 5. The algorithm, written in MATLAB, is used to determine position of the temperature cross for different values of thermal resistance ratio between the hot-cold fluid pair to intermediate-cold fluid pair, H_1 . However, this model may be used to determine temperature cross for any varying design/operating parameter.

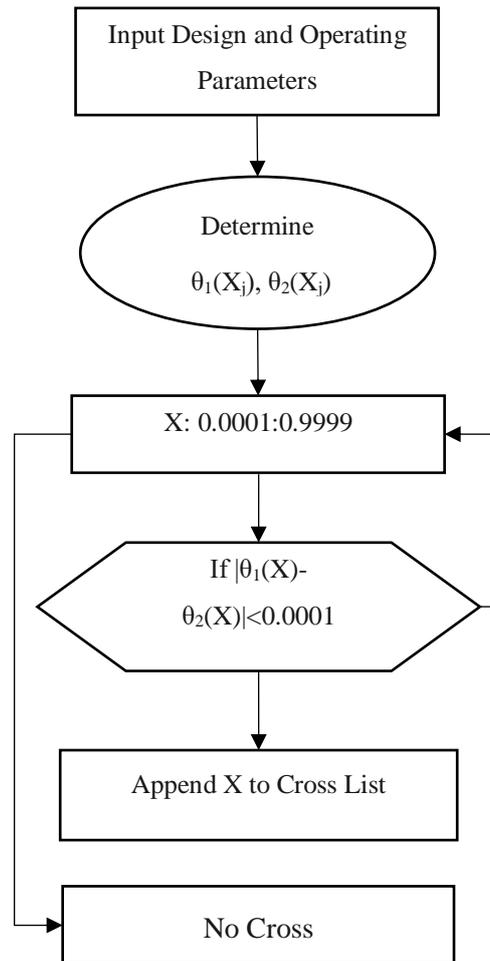


Figure 5. Flow chart for the algorithm for determination of temperature cross.

5. RESULTS AND DISCUSSION

The governing equations are solved by FEM and implemented in MATLAB In order to obtain the non-dimensional temperatures of the three fluids.

The position of the temperature cross for the different fluid pairs is determined for different values of ratio of thermal resistances between intermediate - cold fluids and hot - cold fluids (H_1). The value of the parameter heat-in-leak H_3 is taken as 0.1 and the parameter of the longitudinal conduction of the wall λ has been assumed to be the same for all the separating walls and taken as 0.08. The non-dimensional ambient temperature θ_a is taken as 1. These are indicative values, which were used in earlier studies on heat exchangers [10, 11]. The different boundary conditions for the three fluids in the four flow arrangements are provided in Table 2.

Effect of Thermal Resistance Ratio between intermediate - cold fluids and hot - cold fluids (H_1) on Temperature Cross

The temperature cross phenomenon for different fluid combinations in the various flow arrangements is examined for different values for the operating condition parameter H_1 , the Ratio of thermal resistance of hot and cold fluids. The value of H_1 is varied from 0.15 – 1.5, in steps of 0.15 and from 1.5 – 15, in steps of 1.5; and the position of the temperature cross is determined for each value. The variation in the position of the temperature cross with changing values of H_1 have been presented in for the P3 arrangement in Figures 6 and 7.

The following general observations are noted: - (i) No temperature crosses are observed for the P2 arrangement with change in H_1 values. (ii) All temperature crosses involve the intermediate fluid.

H_1 is the heat transfer resistance ratio between the hot – cold fluids and the intermediate – cold fluids V. Krishna [9] et al. An increase in H_1 results in a decrease in heat transfer resistance between the cold and intermediate fluids relative to the other resistance Shrivastava and Ameer [3]. Due to this the disparity between cold and the intermediate fluid temperature distributions is reduced and the temperature distribution of the cold fluid tends to follow that of the intermediate fluid. This gives rise to steeper temperature gradients for both the fluids at their entrances, with increased values of H_1 , as observed in Figures 6 and 7 The steeper gradients and the tendency of the cold fluid profile to follow the intermediate fluid profile leads to the position of the temperature cross shifting towards the cold fluid inlet of the respective flow arrangement.

For the P3 arrangement, temperature crosses occur for every value of H_1 between $X = 0.0348$ ($H_1=0.15$) and $X = 0.1202$ ($H_1 = 15$) and the position of the cross moves towards $X = 1$ (cold fluid inlet), as H_1 is increased. Three temperature crosses occur between the intermediate and the hot fluids at $X = 0.5574$ ($H_1 = 0.15$), $X = 0.6662$ ($H_1 = 0.3$) and $X = 0.9002$ ($H_1=0.45$).

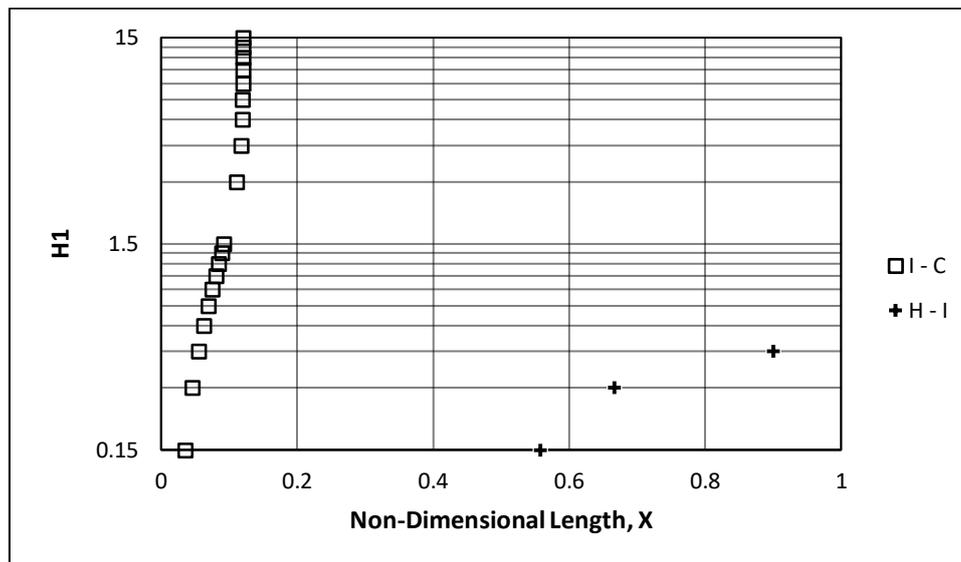


Figure 6. Variation of the Temperature Cross for different values of H_1 in flow arrangement P3.

Values of other non-dimensional parameters are - $\frac{c_i}{c_h}=0.5$, $\frac{c_c}{c_h}=0.8$, $H_3=0.1$, $\Theta_a=1$, $NTU=1$, $H_2=2$, $\Theta_{i,in}=0.3$, $\lambda=0.08$.

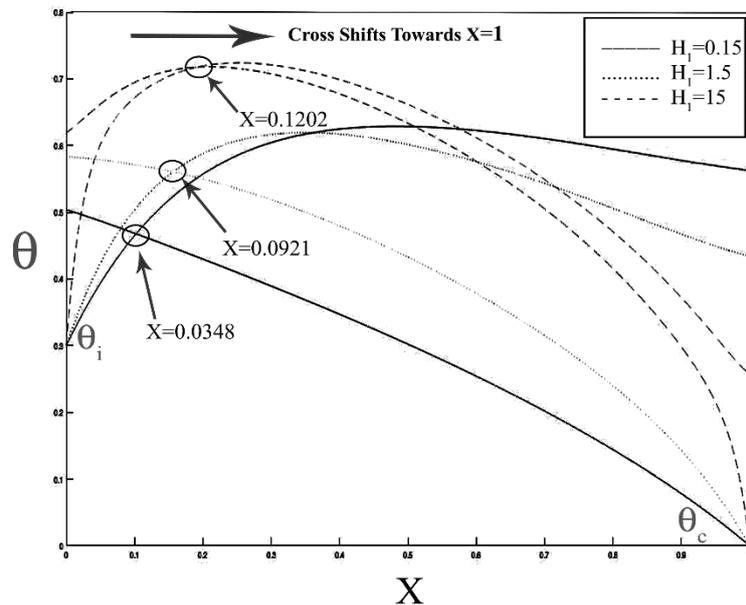


Figure 7. Magnified view of crosses occurring for H_1 values 0.15, 1.5 and 15 for P3 arrangement.

6. CONCLUSIONS

In multi-fluid heat exchangers, local equalization of fluid temperatures occur(s) at certain location(s) within the heat exchanger, referred to as a ‘temperature cross’ between the concerned fluids. The formation of the temperature cross leads Heat transfer reversal in the heat exchanger section on one side of the cross leading to the fluid which has to be heated having higher temperatures than the fluid which has to be cooled. This means that the corresponding heat transfer areas are wasted leading to reduced effectiveness of the heat exchanger. Hence, there is an eventual need to know about the position(s) of the temperature cross, to improve the heat exchanger design.

A three-fluid cryogenic heat exchanger model involving thermal interaction between all the three-fluids and subjected to heat-in-leak from the surroundings and longitudinal conduction in the separating walls, is examined for the effect of the temperature cross. Four non-dimensional design parameters and four non-dimensional operating parameters which influence the heat exchanger performance are identified, including those that account for the heat-in-leak from the surroundings and longitudinal conduction in the separating walls. Non-dimensional governing equations are formulated and solved using the FEM technique and implemented in MATLAB to obtain the temperature profiles of all the fluids. The solution technique is validated by comparing the solutions obtained with those published in the literature and found to match perfectly.

Out of the eight non dimensional parameters, the effect of each of H_1 on the P3 arrangement is studied and it observed that increasing values of H_1 cause the cross to move towards $X=1$.

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