An approximate analytical solution of non linear partial differential equation for water infiltration in unsaturated soils by combined Elzaki Transform and Adomian Decomposition Method

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Abstract. The aim of the concerned paper is to describe the behaviour of the water infiltration problems in unsaturated soils. Governing equation of this phenomenon is known as Richards' equation. The solution of the Richards' equation has been found by Elzaki Adomian Decomposition Method. This method gives a solution in terms of convergent series. Comparison of the approximate solutions and exact solutions have been found here. MATLAB and MATHEMATICA are used to obtain numerical and graphical representation.

1. Introduction

Many important phenomena occuring in field of engineering and science are frequently modeled through ordinary or partial differential equations. Ideally one hopes to find the exact solutions of these equations. Researchers use analytical and numerical methods to find closed form solutions of these equations. But every time it is not possible to find exact solution of problem. The process of water penetrating into the soil is infiltration and it is applied in both hydrology and soil sciences. The rate of infiltration is influenced by the condition of the soil surface, vegetative cover, and soil properties including porosity, hydraulic conductivity, and moisture content. Modelling of multi-phased flow through porous media presents an important problem of practical interest for geotechnical and geo-environmental engineering. Buckingham [1] obtained equations first and then Richards [2] derived equations which are used for defining fluid flow through porous media which were based significantly on semi-empirical equations. To define the analytical solution of Richards' equation is not easy task because of some limitations. Even though some authors [4, 3, 6, 10, 11, 7, 12, 16] have been defined the solution of governing nonlinear Richards' equation by different methods [23, 8, 9].

In the present study, Elzaki Adomian Decomposition Method (EADM) [21] has been applied to solve the problem of one-dimensional infiltration of water in unsaturated soil. Elzaki Adomian Decomposition Method (EADM) is a combination of Elzaki Transform [22, 24] and Adomian Decomposition Method. EADM gives the solution in form of a convergent series.

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2. Richards' equation

The equation first proposed by Richards [2] to describe non-saturated flow in soils, known as Richards' equation, is based on Buckingham's [1, 13] studies at the beginning of the 20^{th} century. Richards' equation is basically a general partial differential equation describing water movement in unsaturated soils [14]. 3 main forms of considered equation explained in the article like the mixed formulation, the h-based formulation and the θ-based formulation, where h is the weight based pressure potential and θ is the volumetric water content. Combination of Darcy's law and the continuity equation is given by Richards' equation. Here one dimensional infiltration of water in the vertical direction in an unsaturated soil may be derived by invoking Darcy's law and the continuity equation, as follows

$$q = -K\frac{\partial H}{\partial z} = -K\frac{\partial (h+z)}{\partial z} = -K\left(\frac{\partial h}{\partial z}+1\right), \tag{1}$$

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z}, \tag{2}$$

where hydraulic conductivity is given by K, H is head equivalent of hydraulic potential, flux density and time are given by q and t respectively. Putting equation (1) in equation (2), mixed form of Richards' is:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K\left(\frac{\partial h}{\partial z}+1\right)\right] \tag{3}$$

In above equation (3), soil water content (θ) and pore water pressure head (h) are 2 independent variables. To obtain the solution of the equation, constitutive relations are needed to describe interdependence between pressure, saturation and hydraulic conductivity. However, it is possible to remove either θ or h by assuming the concept of differential water capacity, defined as the derivative of the soil water retention curve:

$$C(h) = \frac{d\theta}{dh} \tag{4}$$

Replacing equation (4) in equation (3), h-based expression of Richards' equation is defined.

$$C(h) \times \frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left(K\frac{\partial h}{\partial z}\right) + \frac{\partial K}{\partial z} \tag{5}$$

We are producing here a phrase is called pore water diffusivity (D). It can be given as the ratio of the hydraulic conductivity (K) to the differential water capacity (C). Therefore θ-based form of Richards' equation may be defined. So that D is given as

$$D = \frac{K}{C} = \frac{K}{\frac{d\theta}{dh}} = K\frac{dh}{d\theta} \tag{6}$$

Here note that pore water diffusivity (D) and hydraulic conductivity (K) both are dependent on θ (moisture content). From equations (3) and (6), Richards' equation is given as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial \theta}{\partial z}\right) + \frac{\partial K}{\partial z} \tag{7}$$

Here dependent parameters are given by D and K. Both are difficult to estimate. Different kind of models have been suggested to calculate the above mentioned parameters. But Van Genuchten model [18] and Brook's and Corey's model [15] are more useful than other models. Van Genuchten model matches experimental data but the functional structure of the model is

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very complex. So it's impossible to apply it in analytical solution procedure. Whereas Brooks and Corey's model has a more precise definition. It has been assumed within this research work. As per Brook's and Corey's model, some relations are given to define D and K.

$$D(\theta) = \frac{K_s}{\alpha\lambda\left(\theta_s - \theta_r\right)}\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^{2+\frac{1}{\lambda}} \tag{8}$$

$$K(\theta) = K_s\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^{3+\frac{2}{\lambda}} \tag{9}$$

where saturated conductivity, residual water content and saturated water content are given by K_s, θ_r and θ_s respectively. Experimentally obtained parameters are given by a and λ. Brooks and Corey determined λ as pore-size distribution index [15]. More manipulation of Brooks and Corey's model yields the equations (10) and (11).

$$D(\theta) = D_0\left(n+1\right)\theta^m,\ m \geq 0 \tag{10}$$

$$K(\theta) = K_0\theta^k,\ k \geq 1 \tag{11}$$

where constants are given by K_0, D_0 and k which are representing soil properties such as pore-size distribution and particle size. In this presentation of D and K, θ is scaled between 0 and 1 and diffusivity is normalized so that $\forall m,\ \int\limits_0^1 D(\theta)\,d\theta = 1$ [11]. Based on Brook's and Corey's representation of D and K, various analytical and numerical solutions to the Richards' equation have been investigated. In equations (10) and (11), putting $n = 0$ and $k = 2$ which provides the classic Burgers' equation. Several authors have done work on Burgers' equation [17, 19]. By applying the traveling wave technique [11], instead of time and depth, another variable that could be a linear combination of them is found. Tangent-hyperbolic function is usually applied to solve these transform equations. Thus θ - based Richards' equation in order of $(n, 1)$ is obtained as [11]:

$$\theta_t + \alpha\theta^n\theta_z - \theta_{zz} = 0 \tag{12}$$

Exact solution of equation (12) is given as

$$\theta(z,t) = \left(\frac{\gamma}{2} + \frac{\gamma}{2}\tanh\left(\left[A_1(z - A_2 t)\right]\right)\right)^{\frac{1}{n}} \tag{13}$$

where $A_1 = -\frac{\alpha n + n|\alpha|}{4(1+n)}\gamma \quad (n \neq 0),\ A_2 = \frac{\gamma\alpha}{(1+n)}$
Here arbitrary constants are α and γ. Both constants are selected as 1 [10] in this paper. Now initial condition are given by assuming $t = 0$. In the available work, nonlinear θ-based Richards' equation has been studied.

The objective of this paper is to obtain an approximate analytical solution of governing nonlinear Richards' equation by combination of Elzaki transform and Adomian Decomposition Mehtod which is given by equation (7) and supported by equations (10) and (11).

3. Analysis of Elzaki Adomian Decomposition Method

Consider [20] a general nonlinear nonhomogeneous PDE

$$LH(x,t) + RH(x,t) + NH(x,t) = g(x,t), \tag{14}$$

$$H(x,0) = m(x),\ H_t(x,0) = f(x), \tag{15}$$

where the second order linear differential operator is given by $L = \partial^2/\partial t^2$, the linear differential operator of order less than L is given by R, N is the nonlinear differential operator and $g(x,t)$ is supply term.
Apply Elzaki transform to equation (14)

$$E\left[LH(x,t)\right] + E\left[RH(x,t)\right] + E\left[NH(x,t)\right] = E\left[g(x,t)\right] \tag{16}$$

Here E denotes Elzaki transform. Using the property of the Elzaki transform in (16)

$$E\left[H(x,t)\right] = p^2E\left[g(x,t)\right] + p^2m(x) + p^3f(x) - p^2E\left[RH(x,t) + NH(x,t)\right] \tag{17}$$

Apply inverse Elzaki transform to equation (17)

$$H(x,t) = G(x,t) - E^{-1}\left\{p^2E\left[RH(x,t) + NH(x,t)\right]\right\} \tag{18}$$

where $G(x,t)$ is emerging from supply term and the prescribed initial conditions.
An infinite series solution of equation (17) is

$$H(x,t) = \sum_{n=0}^{\infty} H_n(x,t) \tag{19}$$

The nonlinear operator is

$$NH(x,t) = \sum_{n=0}^{\infty} A_n \tag{20}$$

where A_n are the Adomian polynomials which can be calculated from formula (21)

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i H_i\right)\right]_{\lambda=0} \qquad n = 0, 1, 2, ... \tag{21}$$

substituting (19) and (20) into (18),

$$\sum_{n=0}^{\infty} H_n(x,t) = G(x,t) - E^{-1}\left[p^2E\left[\left(R\sum_{n=0}^{\infty} H_n(x)\right) + \left(\sum_{n=0}^{\infty} A_n\right)\right]\right] \tag{22}$$

Collecting the results on both sides of equation (22)

$$\begin{aligned}
H_0(x,t) &= G(x,t), \\
H_1(x,t) &= -E^{-1}\left[p^2E\left[RH_0(x,t) + A_0\right]\right], \\
H_2(x,t) &= -E^{-1}\left[p^2E\left[RH_1(x,t) + A_1\right]\right], \\
H_3(x,t) &= -E^{-1}\left[p^2E\left[RH_2(x,t) + A_2\right]\right], \\
&\;\;. \\
&\;\;. \\
&\;\;.
\end{aligned} \tag{23}$$

In general,

$$H_{n+1}(x,t) = -E^{-1}\left[p^2E\left[RH_n(x,t) + A_n\right]\right] \tag{24}$$

The solution is given as

$$H(x,t) = H_0(x,t) + H_1(x,t) + H_2(x,t) + H_3(x,t) + ... \tag{25}$$

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4. Implementation of EADM to solve the Richards' equation

In this part, we apply EADM to find the solution of Richards' equation. For the sake of convenience two different cases of n are considered.

4.1. Case 1: if $n = 1$

Here we solve the Richards equation by proposed method when $n = 1$. So the equation (12) can be written in the form

$$\frac{\partial \theta}{\partial t} + \theta \frac{\partial \theta}{\partial z} - \frac{\partial^2 \theta}{\partial z^2} = 0 \tag{26}$$

with the initial condition

$$\theta(z, 0) = \left(\frac{1}{2} + \frac{1}{2}\tanh\left(-\frac{z}{4}\right)\right) \tag{27}$$

Applying Elzaki transform on both sides of equation (26),

$$\frac{E\left[\theta(z,t)\right]}{v} - v\theta(z,0) + E\left[\theta\frac{\partial \theta}{\partial z}\right] - E\left[\frac{\partial^2 \theta}{\partial z^2}\right] = 0$$

After applying initial condition and inverse Elzaki Transform, we get

$$\theta(z,t) = \left(\frac{1}{2} + \frac{1}{2}\tanh\left(-\frac{z}{4}\right)\right) - E^{-1}\left[vE\left\{\theta\frac{\partial \theta}{\partial z}\right\}\right] + E^{-1}\left[vE\left\{\frac{\partial^2 \theta}{\partial z^2}\right\}\right]$$

Using Adomian Decomposition method, we have

$$\sum_{n=0}^{\infty}\theta_n(z,t) = \left(\frac{1}{2} + \frac{1}{2}\tanh\left(-\frac{z}{4}\right)\right) - E^{-1}\left[vE\left\{\sum_{n=0}^{\infty}A_n(z,t)\right\}\right] + E^{-1}\left[vE\left\{\sum_{n=0}^{\infty}\theta_{nzz}(z,t)\right\}\right] \tag{28}$$

Comparing the results on both sides of equation (28),

$$\begin{aligned}
\theta_0(z,t) &= \left(\frac{1}{2} + \frac{1}{2}\tanh\left(-\frac{z}{4}\right)\right), \\
\theta_1(z,t) &= E^{-1}\left[vE\left\{\theta_{0zz}(z,t) - A_0\right\}\right], \\
\theta_2(z,t) &= E^{-1}\left[vE\left\{\theta_{1zz}(z,t) - A_1\right\}\right], \\
\theta_3(z,t) &= E^{-1}\left[vE\left\{\theta_{2zz}(z,t) - A_2\right\}\right], \\
&\;\;. \\
&\;\;. \\
&\;\;.
\end{aligned} \tag{29}$$

Here A_n is Adomian polynomial and it represents the nonlinear term $\theta\frac{\partial\theta}{\partial z}$ and computed using the following formula (21).
Some few Adomian polynomials are given as,

$$\begin{aligned}
A_0 &= \theta_0\theta_{0z} \\
A_1 &= \theta_1\theta_{0z} + \theta_0\theta_{1z} \\
A_2 &= \theta_2\theta_{0z} + \theta_1\theta_{1z} + \theta_0\theta_{2z} \\
A_3 &= \theta_3\theta_{0z} + \theta_2\theta_{1z} + \theta_1\theta_{2z} + \theta_0\theta_{3z} \\
&\;\;. \\
&\;\;. \\
&\;\;.
\end{aligned} \tag{30}$$

Using Adomian polynomials (30) and the iteration formulas (29), we obtain

$$\begin{aligned}
\theta_0(z,t) &= \left(\frac{1}{2} + \frac{1}{2}\tanh\left(-\frac{z}{4}\right)\right) \\
\theta_1(z,t) &= \frac{1}{16}\sec h\left(\frac{z}{4}\right)^2 t \\
\theta_2(z,t) &= \frac{1}{128}\sec h\left(\frac{z}{4}\right)^2 \tanh\left(\frac{z}{4}\right) t^2
\end{aligned} \tag{31}$$

The approximate solution is

$$\theta(z,t) = \left(\frac{1}{2} + \frac{1}{2}\tanh\left(-\frac{z}{4}\right)\right) + \frac{1}{16}\sec h\left(\frac{z}{4}\right)^2 t + \frac{1}{128}\sec h\left(\frac{z}{4}\right)^2 \tanh\left(\frac{z}{4}\right) t^2 + ... \tag{32}$$

Table 1. Comparison between the solutions obtained by different methods and absolute error for n= 1 and t=1.

z	EADM	HPM [6]	DTM [6]	Exact	Error=|Exact - EADM|
0	0.5625	0.5625	0.5625	0.562177	0.000323
1	0.43809	0.43809024	0.43823114	0.437823	0.000267
2	0.320934	0.3209337	0.32111924	0.320821	0.000113
3	0.222672	0.22267245	0.22281962	0.2227	2.8E-05
4	0.14795	0.14795014	0.1480376	0.148047	9.7E-05
5	0.0952425	0.09524247	0.1480376	0.0953495	0.000107

Table 2. Comparison between the solutions obtained by different methods and absolute error for n= 1 and t=3.

z	EADM	HPM [6]	DTM [6]	Exact	Error=|Exact - EADM|
0	0.6875	0.6875	0.6875	0.679179	0.008321
1	0.569981	0.5699813	0.5737855	0.562177	0.007804
2	0.441954	0.44195411	0.44697828	0.437823	0.004131
3	0.320928	0.32092823	0.32490192	0.320821	0.000107
4	0.220438	0.22043756	0.22279881	0.2227	0.002262
5	0.145161	0.14516129	0.1463338	0.148047	0.002886

Table 3. Comparison between the solutions obtained by different methods and absolute error for n= 1 and t=5.

z	EADM	HPM [6]	DTM [6]	Exact	Error=|Exact - EADM|
0	0.8125	0.8125	0.8125	0.7773	0.0352
1	0.716262	0.71626155	0.73387361	0.679179	0.037083
2	0.585689	0.58568895	0.608949	0.562177	0.023512
3	0.442867	0.44286656	0.46126328	0.437823	0.005044
4	0.312916	0.3129156	0.32384731	0.320821	0.007905
5	0.209947	0.20994707	0.21537534	0.2227	0.012753

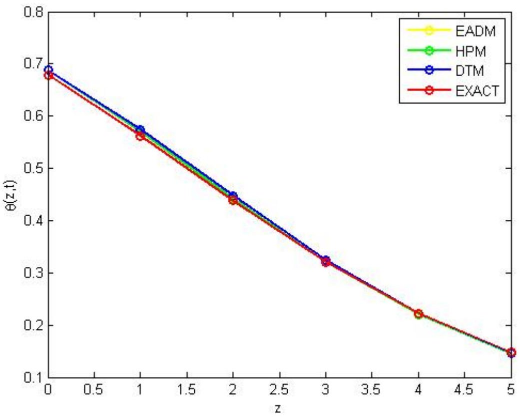

Figure 1. Comparison between the solution obtained by different methods and exact solution solution (n = 1, t = 3)

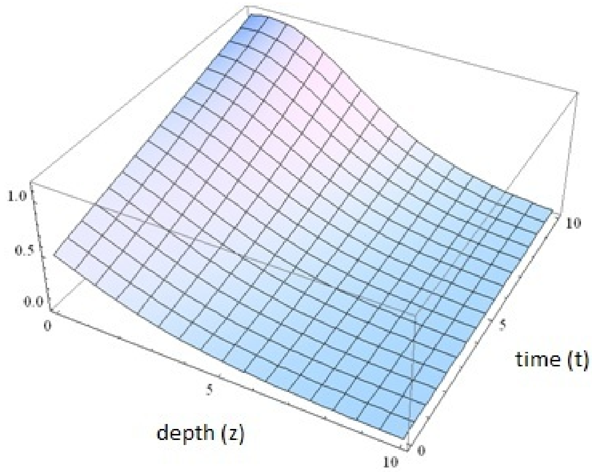

Figure 2. 3D behaviour of an approximate solution

Here Table 1, Table 2 and Table 3 show the comparison of numerical solutions obtained with z = 0, 1, 2, 3, 4,5 for t =1, 3 and 5 by Elzaki Adomain Decomposition Method (EADM), Homotopy Perturbation Method (HPM), Differential Transform Method (DTM) and Exact solutions. Also the error between exact solutions and the solutions defined by EADM has been found in the above mentioned tables. Figure 1 displays the comparison between the solution defined by various methods and exact solution (n = 1, t = 3). 3D behaviour of $\theta(z,t)$ for n = 1 is represented by Figure 2.

4.2. Case 2: if $n = 2$

Solving Richards equation here by EADM for $n = 2$. So the equation (12) can be written as

$$\frac{\partial \theta}{\partial t} + \theta^2 \frac{\partial \theta}{\partial z} - \frac{\partial^2 \theta}{\partial z^2} = 0 \tag{33}$$

with the initial condition

$$\theta(z,0) = \left(\frac{1}{2} + \frac{1}{2}\tanh\left(-\frac{z}{3}\right)\right)^{\frac{1}{2}} \tag{34}$$

After applying Elzaki transform on both sides of equation (33) along with initial condition, we get

$$E\left[\theta(z,t)\right] - v^2\left(\frac{1}{2}+\frac{1}{2}\tanh\left(-\frac{z}{3}\right)\right)^{\frac{1}{2}} + vE\left[\theta^2\frac{\partial\theta}{\partial z}\right] - vE\left[\frac{\partial^2\theta}{\partial z^2}\right] = 0 \tag{35}$$

Apply inverse Elzaki Transform,

$$\theta(z,t) = \left(\frac{1}{2}+\frac{1}{2}\tanh\left(-\frac{z}{3}\right)\right)^{\frac{1}{2}} - E^{-1}\left[vE\left\{\theta^2\frac{\partial\theta}{\partial z}\right\}\right] + E^{-1}\left[vE\left\{\frac{\partial^2\theta}{\partial z^2}\right\}\right]$$

Applying decomposition technique,

$$\sum_{n=0}^{\infty}\theta_n(z,t) = \left(\frac{1}{2}+\frac{1}{2}\tanh\left(-\frac{z}{3}\right)\right)^{\frac{1}{2}} - E^{-1}\left[vE\left\{\sum_{n=0}^{\infty}A_n(z,t)\right\}\right] + E^{-1}\left[vE\left\{\sum_{n=0}^{\infty}\theta_{nzz}(z,t)\right\}\right] \tag{36}$$

Comparing on both sides of equation (36), we have

$$\begin{aligned}
\theta_0(z,t) &= \left(\frac{1}{2}+\frac{1}{2}\tanh\left(-\frac{z}{3}\right)\right)^{\frac{1}{2}}, \\
\theta_1(z,t) &= E^{-1}\left[vE\left\{\theta_{0zz}(z,t) - A_0\right\}\right], \\
\theta_2(z,t) &= E^{-1}\left[vE\left\{\theta_{1zz}(z,t) - A_1\right\}\right], \\
\theta_3(z,t) &= E^{-1}\left[vE\left\{\theta_{2zz}(z,t) - A_2\right\}\right], \\
&\;\;. \\
&\;\;. \\
&\;\;.
\end{aligned} \tag{37}$$

Some Adomian polynomials for the nonlinear $\theta^2\frac{\partial\theta}{\partial z}$ are given as,

$$\begin{aligned}
A_0 &= \theta_0^2\theta_{0z} \\
A_1 &= 2\theta_0\theta_1\theta_{0z} + \theta_0^2\theta_{1z} \\
A_2 &= 2\theta_0\theta_2\theta_{0z} + \theta_0^2\theta_{1z} + 2\theta_0\theta_{2z} + \theta_1^2\theta_0 \\
&. \\
&. \\
&.
\end{aligned} \tag{38}$$

Using Adomian polynomials (38) and the iteration formulas (37), we get

$$\begin{aligned}
\theta_0(z,t) &= \left(\frac{1}{2}+\frac{1}{2}\tanh\left(-\frac{z}{3}\right)\right)^{\frac{1}{2}} \\
\theta_1(z,t) &= -\frac{\sec h\left[\frac{z}{3}\right]^4\left(-1+3\tanh\left[\frac{z}{3}\right]\right)}{72\sqrt{2-2\tanh\left[\frac{z}{3}\right]}\left(-1+\tanh\left[\frac{z}{3}\right]\right)}t \\
\theta_2(z,t) &= \frac{\sec h\left[\frac{z}{3}\right]^9\left(\cosh\left[\frac{2z}{3}\right]-\sinh\left[\frac{2z}{3}\right]\right)\left(-29\cosh\left[\frac{z}{3}\right]+7\cosh\left[z\right]+99\sinh\left[\frac{z}{3}\right]-9\sinh\left[z\right]\right)}{2592\sqrt{2-2\tanh\left[\frac{z}{3}\right]}\left(-1+\tanh\left[\frac{z}{3}\right]\right)^3}\frac{t^2}{2}
\end{aligned} \tag{39}$$

The approximate solution is

$$\theta(z,t)=\left(\frac{1}{2}+\frac{1}{2}\tanh\left(-\frac{z}{3}\right)\right)^{\frac{1}{2}}-\frac{\sec h\,\left[\frac{z}{3}\right]^4\left(-1+3\tanh\,\left[\frac{z}{3}\right]\right)}{72\sqrt{2-2\tanh\,\left[\frac{z}{3}\right]}\left(-1+\tanh\,\left[\frac{z}{3}\right]\right)}t+$$
$$\frac{\sec h\,\left[\frac{z}{3}\right]^9\left(\cosh\,\left[\frac{2z}{3}\right]-\sinh\,\left[\frac{2z}{3}\right]\right)\left(-29\cosh\,\left[\frac{z}{3}\right]+7\cosh\,[z]+99\sinh\,\left[\frac{z}{3}\right]-9\sinh\,[z]\right)}{2592\sqrt{2-2\tanh\,\left[\frac{z}{3}\right]}\left(-1+\tanh\,\left[\frac{z}{3}\right]\right)^3}\frac{t^2}{2}+... \tag{40}$$

Table 4. Comparison between the solutions obtained by different methods and absolute error for n= 2 and t=1.

z	EADM	HPM [6]	DTM [6]	Exact	Error=|Exact - EADM|
0	0.700287	0.74529927	0.74523612	0.745203	0.044916
1	0.58141	0.6251658	0.62523967	0.625046	0.043636
2	0.465194	0.49773364	0.4978617	0.497658	0.032464
3	0.361456	0.38025275	0.38034966	0.380234	0.018778
4	0.273559	0.28255609	0.28260789	0.28257	0.009011
5	0.202672	0.20649731	0.20652044	0.206522	0.00385

Table 5. Comparison between the solutions obtained by different methods and absolute error for n= 2 and t=3.

z	EADM	HPM [6]	DTM [6]	Exact	Error=|Exact - EADM|
0	0.704652	0.81513698	0.81329556	0.812869	0.108217
1	0.576126	0.71035216	0.71234069	0.707107	0.130981
2	0.461495	0.58473648	0.58830312	0.582446	0.120951
3	0.376334	0.45747802	0.46020619	0.456737	0.080403
4	0.303302	0.34499388	0.34645962	0.345258	0.041956
5	0.235861	0.25425481	0.25491158	0.254891	0.01903

Table 6. Comparison between the solutions obtained by different methods and absolute error for n= 2 and t=5.

z	EADM	HPM [6]	DTM [6]	Exact	Error=|Exact - EADM|
0	0.733023	0.87624497	0.86708837	0.867373	0.13435
1	0.566559	0.79520155	0.80437883	0.780588	0.214029
2	0.430311	0.67841825	0.69543543	0.666837	0.236526
3	0.367854	0.54435032	0.55749704	0.539758	0.171904
4	0.322919	0.41690713	0.4240044	0.417475	0.094556
5	0.267762	0.30994731	0.31313539	0.312686	0.044924

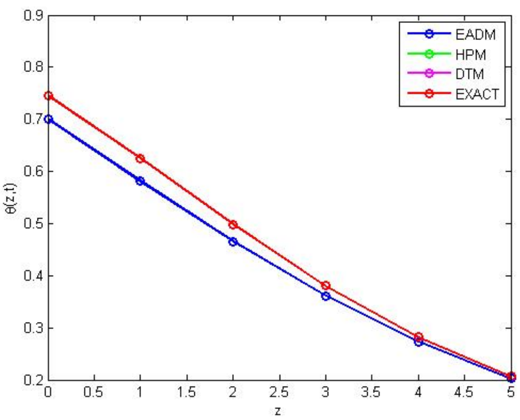

Figure 3. Comparison between the solution obtained by different methods and exact solution (n = 2, t = 1)

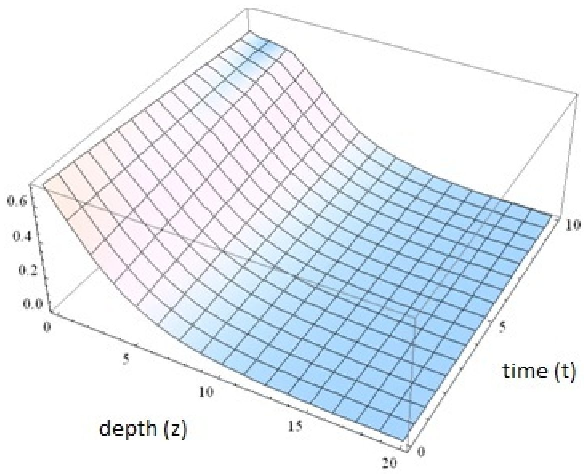

Figure 4. 3D behaviour of an approximate solution

Here Table 4, Table 5 and Table 6 display the comparison of numerical solutions obtained with z = 0, 1, 2, 3, 4,5 for t =1, 3 and 5 by Elzaki Adomain Decomposition Method (EADM), Homotopy Perturbation Method (HPM), Differential Transform Method (DTM) and Exact solutions. Also the error between exact solutions and the solutions defined by EADM has been found in the above mentioned tables. Figure 3 displays the comparison between the solution defined by various methods and exact solution ($n = 2$, $t = 1$). 3D behaviour of moisture content $\theta(z, t)$ for n= 2 is represented by Figure 4.

5. Conclusion

An approximate analytical solution of Richards' equation has been successfully obtained here by Elzaki Adomian Decomposition Method. Richards' equation is employed for modeling infiltration in unsaturated soils. Here note that good agreement can be observed between the numerical results obtained using EADM, HPM, DTM and exact solutions. Also error between EADM and exact solutions has been found here and it is negligible. Thus we can say that the proposed methodology is extremely reliable and efficient to find the analytical and numerical solutions of nonlinear problems.

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