Geometry optimization studies on nonplanar wingtip devices for typical transport aircraft

Manjunath I B[1,] Venkatesh M Kulkarni[1], Balaraman P[2]

[1]Department of Mechanical Engineering,VTU- Center for Post Graduate studies, Kalaburagi-585105,India
[1]Department of Mechanical Engineering,VTU- Center for Post Graduate studies, Kalaburagi-585105,India
[2] Department of Mechanical Engineering, SASTRA,Deemed to be University, Thanjavur, Tamilnadu,India

Author1: manju.bhairappa@gmail.com
Author2: vmkulkarni@vtu.ac.in
* Corresponding author email: manju.bhairappa@gmail.com

Abstract. Wings are the main lift generating sources for an aerospace system. Wing design is a complex process that involves selection of many aerofoil design parameters, wing design requirements and considerations. The main focus in this study is to validate nonplanar wingtip design methodologies and make a numerical estimation and comparison of aerodynamic characteristics of non planar wing tip devices and the associated induced drag.Naturally the high pressure on the bottom surface of the wing causes streamlines from underneath the wing to wrap around the tip to the top, thereby equalizing the pressure difference at the tip. This creates the rotating flow commonly known as a wing tip vortex. The current study investigates and assesses the possibility of retrofitting a non planar wing tip device for the rectangular tapered wing configuration without adding any additional weight to the main wing structure. There are basically three case studies. The first one is a validation of predicted experimental data against numerical simulation using FLUENT® tool for classical NACA 23015 aerofoil.
The second one involves taking into consideration possible mounting location of the nonplanar wingtip device on the main wing for smooth operation and gradual lift generation process.
A third case study is comparison of 'wing alone experimental data' against 'wing with nonplanar winglet'data computed using FLUENT® in three dimensional space. Additionally estimation of numerical optimum load distribution is done for one of the optimum nonplanar winglet geometry using MATLAB tool. Finally comparison of MATLAB results with experimental loading is done for clarity.

1. Introduction

The current investigation undertaken is for a relatively small aircraft operating in the range of Reynolds number around 20 million and cruise Mach number of 0.6.The geometric parameters of the winglet investigated have been the winglet span, cant angle, sweep and taper ratio in terms of their influence on drag, induced drag, and overall lift to drag ratio (range) as well as wing root bending

moments. The results of the research from the indicated survey show that winglet span and cant angle offer the highest gains in terms of performance while taper ratio and sweep angle have a minor contribution. In general, winglets provide an increase in aerodynamic efficiency in terms of overall L/D[1]; however, some of the configurations result in large wing root bending moments and increased weight that would make the option of nonplanar wingtip device impractical.

The purpose of the current study is to assess the possibility of adapting nonplanar wingtip devices to light transport aircraft, the SARAS, by using the present-day capabilities of computational fluid dynamics. Use is being made of a solver program, FLUENT® from ANSYS Inc in conjunction with open source XFLR and MATLAB program.

This study basically involves optimizing a given geometry of nonplanar wingtip configuration for improving the aerodynamic efficiency in terms of optimum overall lift to drag ratio with reduced minimum induced drag and the additional structural loading.

Wind-tunnel tests produce flow fields of good quality and accurate measurements, but only to about 15 spans downstream, and that only in unusual facilities. There is renewed interest in wake surveys by Brune 1994, de Bruin et al 1996[2], helped by computer-driven systems and by arrays of 5-hole probes. Details of the near-wing flow field matter because some features survive in the far wake. Two industrial incentives are (a) to attribute drag or lift changes to local geometry changes and (b) to define separate components for the drag, usually the "viscous" drag and the "induced" or "vortex" drag, which scale differently with Reynolds number. Unfortunately, that distinction remains too ambiguous, efforts to date base the distinction on some equations of lifting-line theory, which are defeated by separation and viscosity. "Apparent induced drag" becomes "apparent viscous drag" as the survey plane is moved downstream by Saffman 1974[3], whereas the total drag calculated is quite accurate. This ambiguity creates an opportunity for a theoretical leap forward; the definitive theory would deal with multiple nonplanar lifting surfaces.

A concern in wind tunnels is vortex meandering, which artificially diffuses the time-averaged vorticity. The strategies devised to correct for this effect by Devenport et al 1996[4] may not be definitive. In particular, the probability distribution function of the vortex position is often assumed to be Gaussian. However, meandering caused by a large-scale mode of oscillation in the tunnel could give the vortex the shape of a sine wave, in which case the PDF would be M-shaped instead of bell-shaped. There are also concerns about residual turbulence and stratification, and the analysis is limited to flow visualization and particle-image velocimetry (PIV), which is less mature than 5-hole probes. The extrapolation to flight situations requires extreme care. Flight tests contain the complete physics and are essential to the verification and validation of any prediction or control method.

Deliberate tests such as tower fly-bys, by Garodz & Clawson 1993[5] are expensive, if only because of the revenue potential of an airliner for a day, and allow little control over atmospheric conditions (a problem NASA is addressing through an extended test campaign, (Vicroy et al 1997) [6]. Using commercial flights is cost-effective, but restricted to a narrow range of flight conditions. Quantitative measurements are difficult, and often impossible out of ground effect, as the instrumentation is on the ground (Rudis et al 1996, Vaughan et al 1996) [7]. In a typical test, an airplane with a span of 40 m flies 80 m above ground level; the vortices have an initial descent velocity of 2 m/s and are followed for 100s. These numbers speak for themselves, but abusive generalizations have been made from such tests. Flow visualization, by condensation or smoke, is very valuable but not fully reliable. In particular, condensation disappears when the minimum temperature increases past a threshold value that does not depend only on the vortex characteristics. Many statements in the literature regarding vortices being "destroyed" or "cut" are probably erroneous.

A predictive ATC technology will, clearly, draw on many sources of knowledge. The physical model will be semi-empirical and may be very complex, more than Greene's 1986[8]; the system is likely to use weather predictions and may depend on real-time measurements of the vortices. Progress will depend on effective efforts with each of the above tools and on constant discussions aimed at clear concepts and consensus. The axial flow has a rich behaviour, may sustain small-scale turbulence, and could also be essential in detecting trailing vortices from behind. It has surprised many of us that the velocity relative to the atmosphere may be directed towards the airplane ("wake-like," as behind a

nonlifting drag-producing body) but also away from it ("jet-like") (Brown 1973)[9]. However, most of the experts believe all the vortices on one side of the plane of symmetry do merge, based on visualizations and on the rarity of "multiple hits" on instrumented towers. Also, simulations today are not conclusive for such subtle effects of turbulence and of viscous velocity defects. If the vortices do not merge, the aspect of the "mature" wake is quite different. Instead of descending in a quasi-steady manner, two or more vortex pairs "tumble down" together. The wake is not followed by an oval of fluid (and contaminants) from the initial altitude; instead, it periodically exchanges fluid with the atmosphere (Spalart 1996) [10].One of the most useful approaches to the solution of this tip vorticity problem is the method of restricted variations. Munk, Jones [11], and others used this approach for solving many induced-drag-related problems. Note that the vertical extent of the system near the tips is the critical parameter and that although the box plane represents the absolute minimum solution, many other concepts provide very similar drag reductions and show that spanwise camber is most effective near the tip (Lowson 1990)[12].It should be noted that these results represent inviscid solutions. Some studies have also included the effect of lift-dependent section drag in the optimization (Rokhsaz 1992, Kroo 1984) [13]. Although the section polar usually has a small effect on the optimal loading, the role of even constant section drag has quite different effects for planar and nonplanar wings.

For nonplanar wings, however, if the projected area and lift are fixed adding nonplanar area adds to the total drag. The optimal lift distribution [14], [15] is then a function of the viscous drag, and there is an optimal winglet height. Studies at NASA Langley that compared these two concepts with a constrained root-bending moment concluded that winglets were preferred over span extensions (Heyson et al 1977) [16].Span extensions have the disadvantage of causing a large aircraft wing root bending moment. Studies with somewhat different constraints suggested that the two approaches were almost identical in these respects (Jones & Lasinski 1980) [17], [18]. The currently accepted view is that the complexity of the structural model and constraints limits the general applicability of any such conclusions. The evaluation of optimal winglet height and dihedral depends on the details of the wing structure, whether the wing is gust critical or maneuver critical, whether large regions of the wing are sized based on a minimum skin gauge, and whether the design is new or a modification of an existing design. In the evaluation of wing tip device advantages must be undertaken for each design and include an array of multidisciplinary considerations. These include the effect on aeroelastic deflections and loads, flutter speed, aircraft trim, stability and control effects (especially lateral characteristics), and off-design operation, as well as effects on maximum lift, and finally, marketing considerations. There is no clear answer to the optimal configuration, and even when winglets are adopted, the geometries vary widely. The MD-11 uses a winglet not unlike that described by Whitcomb (1976) [19].This "vortex diffuser" concept was studied by Lockheed in the 1980s (Hackett 1980) [20], and although it is recognized that the vortex drag reduction is independent of the longitudinal position, some advantages are claimed for this aft positioning of the nonplanar surface.

2. Governing equations

- **Steady Continuity equation in three dimension**

$$\nabla.V = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

- **Steady Momentum equation in three dimension**

$$\rho\left(\frac{\partial}{\partial t} + \nabla.V\right)u = -\frac{\partial p}{\partial x} + \mu\nabla^2 u + \rho f_x \tag{2}$$

$$\rho\left(\frac{\partial}{\partial t} + \nabla.V\right)v = -\frac{\partial p}{\partial y} + \mu\nabla^2 v + \rho f_y \tag{3}$$

$$\rho\left(\frac{\partial}{\partial t} + \nabla.V\right)w = -\frac{\partial p}{\partial z} + \mu\nabla^2 w + \rho f_z \tag{4}$$

3. Modeling and analysis

The parametric modeling, analysis, numerical mesh investigations, validation studies are carried out using ICEM CFD and ANSYS FLUENT®,XFLR with MATLAB for all the two dimensional and three dimensional parametric geometry configurations. This study is done   in order to have high level of confidence in the configuration design stage of the aircraft design phase. The semispan geometric model configuration dimensions are given in table 1 below taken from figures 1(a),1(b),1(c) respectively.

3.1 Mesh independence study

 The mesh independence studies clearly show no variation in the lifting characteristics by doubling or halving the grid size in the orthogonal directions influencing the wing geometry region (wall boundary in case of viscous simulation). The associated mesh parameters for the above case study are taken from the Table 3 below.

Mesh independence study is very important and vital in order to characterize the lift coefficient appropriately and also finetune the grid so as to ensure and accelerate the fall of residuals within the first 200 to 250 iterations and ensure correct solution with minimum computational error.

Table 1. Table below showing geometric configuration of wing along with nonplanar wingtip taken from figure 3(a).

S No	Wing alone geometry configuration details (Non dimensional)	Total plan form of Rectangular and Tapered wing
1	Full Span	7.25
2	Aspect ratio	8
3	Chord (MAC)	0.895
4	Plan form area	6.5

Table 2. Table highlighting the geometry parameter and NACA aerofoil details of a nonplanar wingtip device used on the high aspect ratio wing.

Non Planar Wingtip configuration		
Partitions half-span	0.15	0.15
Partitions sweep(radians)	-0.5236	-0.5236
Partitions taper	0.79	0.7
Partitions inner airfoil (NACA)	23012	23012
Partitions outer airfoil (NACA)	23012	23012

Table 3. Below table gives insight into geometric and grid parameters with three-dimensional structured mesh statistics for the winglets as shown in figure 1(b) and 1(c). The table also provides certain details on simplified computational scheme and recommendations for "lift characterization" of the high aspect ratio rectangular tapered wing and nonplanar wingtip devices in three dimensional space.

S N o	Recommended grid and solver parameters used as per AIAA and FLUENT® CFD solver	
1	Domain topology and type	C-grid, Algebraic, Structured Multiblock
2	Domain extents in three-dimensional space	X1=-10 X2=20 Y1=-10 Y2=10 Z1=0 Z2=28
3	Total cell count	202156(medium grid)
4	Turbulence models used: for turbulent steady flow	One equation Spalart Almaras, with residual setting-10e-5 for conservation of mass, momentum and energy
5	Cell quality	Skewness range=0.1 to 0.35 Aspect ratio range=10 to 110
6	Wall Y+ plus range used	High=230 Low=5
7	Boundary conditions Elements/Stretching factor/cell wall distance used in boundary layer	Inlet=velocity. Outlet=pressure outlet. Symmetry=symmetry. Wall=No slip 15 elements normal to wall and 50 along the mean chord Stretching/Expansion ratio=1.2 Cell wall distance=0.00045 to 0.0006

Figure1 (a). Schematic showing the geometry of the rectangular tapered wing planform.

| **Figure1 (b).** Schematic showing the full planform geometry of high aspect ratio rectangular tapered wing with nonplanar winglet with negative twist. | **Figure 1(c).** Schematic showing the full planform geometry of high aspect ratio rectangular tapered wing with nonplanar winglet with positive twist. |

4. Results and discussions

In this section we disccuss the three case studies separately as follows

4.1 *Case study 1. Validation of the flow over an Aerofoil*

We first study here the flow over a NACA 23015 aerofoil for which standard wind tunnel results are easily available.

Figure 1(d). Comparison of "Lift and pitching moment coefficients" at Mach Number 0.3,Re=6 million for baseline NACA23015.

Figure1(d) gives for this aerofoil, the details of several pressure distributions over the range of incidences, corresponding to different regions of the lift curve along with the path lines typical for the three different regions of the flow, viz., the linear region; the region preceding stall where the lift curve departs from the linearity, with reduced slope; and the region beyond stall where substantial

flow separation exists over the aft portion of the aerofoil. All the three regions are indicated with the arrows. These pictures are obtained from the commercial software FLUENT®. As a validation for this application standard experimental results are also indicated for the lift and pitching moment curves.

The aerofoil investigations made in the above study lead to verification and validation of the experimental lifting and pitching moment characteristics for the classical NACA23015 aerofoil as depicted from figure 1(d).

4.1.1 Case study 2. Wing alone-experiment and Nonplanar winglets at different locations steady state numerical results.

In this section the "wing alone-wind tunnel" results are compared with the results for two sets of "wing with nonplanar winglet configurations". Winglet considered here for subsonic flow study and investigation is, a low aspect ratio wing geometry of short span and chord with suitable combination of dihedral (cant angle), twist and sweep mounted at 1.5units (*4.1.2. Case study 2*) and 2.0 units (*4.1.3. Case study 2*) on upper surface as shown in figures 2(a), 2(b), 2(c) and 2(d) respectively.

Can we position or deploy winglets anywhere/any place along the span on the wing ? What are the consequences?

Yes we can use. Due care and validation studies supported by additional windtunnel tests is highly recommended for investigation and for fruitful results in terms of overall efficiency of the wing.Following two case studies are presented here for clarity and understanding of the flow physics around winglets.In these two case studies the winglets are not positioned at tips.In these two cases the wing shapes are geometrically   same in shape and   dimensions. Comments are being made and presented under conclusion section for case studyA2. (The results for two cases are as shown from figures 2(e) and 2(f) respectively).
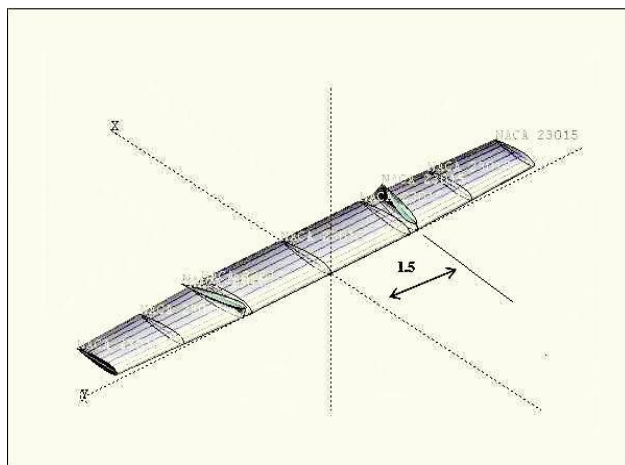

Figure 2(a). Figure above showing the nonplanar winglet positioned at 1.5units from the root chord along the span.
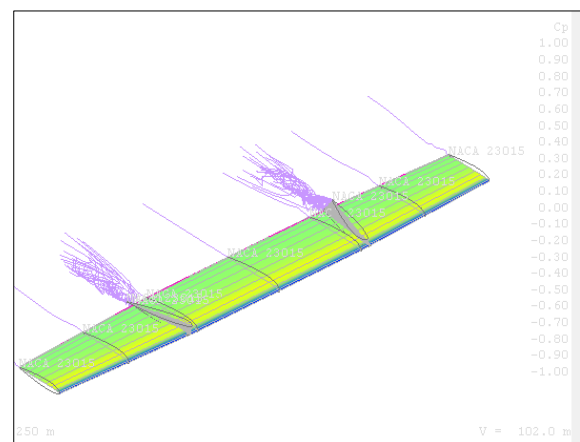

Figure 2(b). Figure above showing the nonplanar winglet positioned at 1.5units from the root chord along the span with
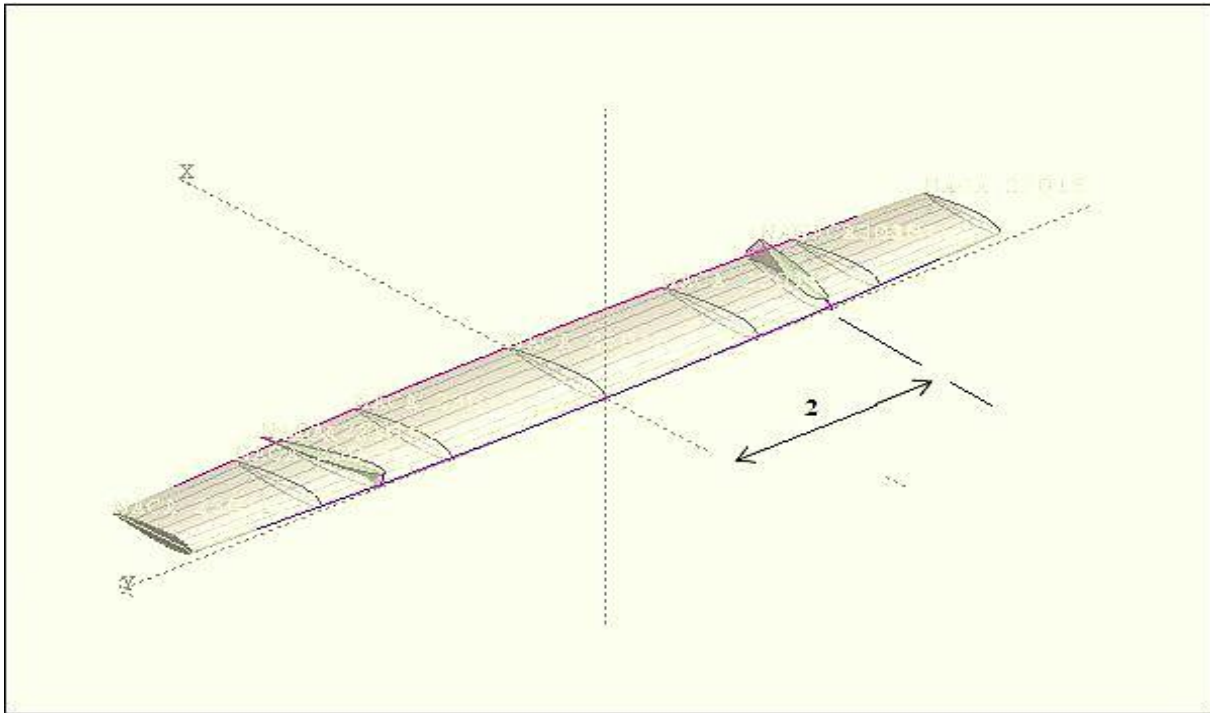"(-Cp) Pressure coefficient distribution" and streamlines.

Figure2 (c). Figure above showing the nonplanar winglet positioned at 2.0 units from the root chord along the span.
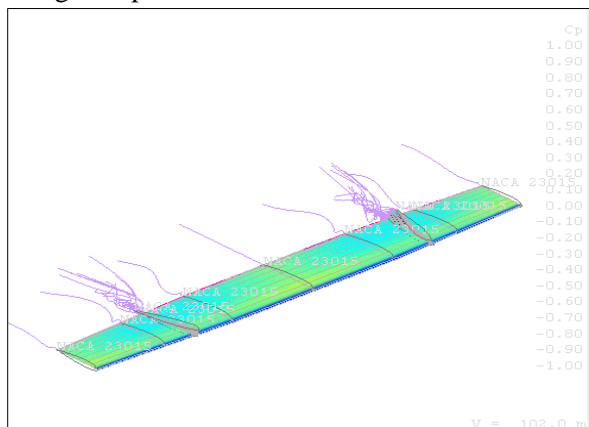

Figure2 (d). Figure above showing the nonplanar winglet positioned at 2units from the root chord along the span with "(Cp) Pressure coefficient distribution" and streamlines.
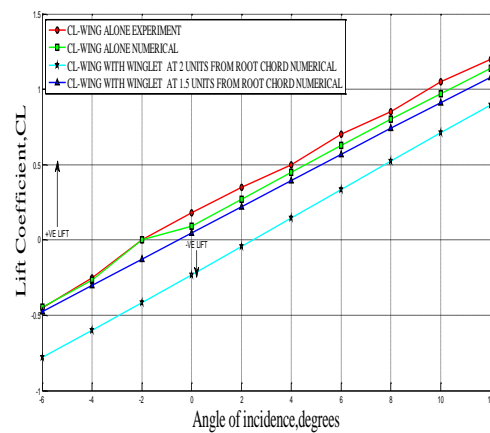

Figure2 (e). Comparison of "Lift coefficient ,CL" at Mach Number 0.3 for wing alone case with nonplanar wingtips located at 1.5units and 2units along the span of high aspect ratio wings.
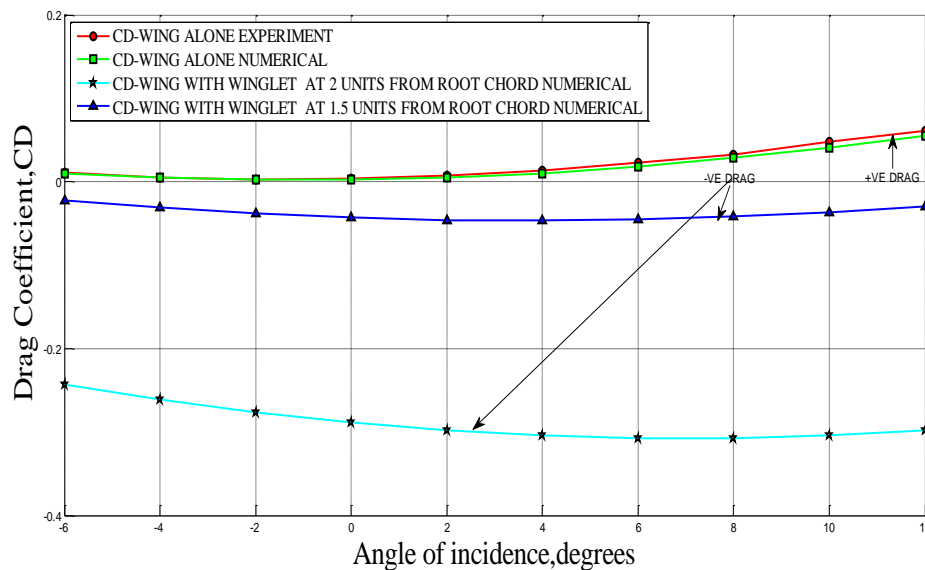
Figure2 (f). Comparison of "Drag coefficients, CD" at Mach Number 0.3 for experimental wing alone case with nonplanar wingtips located at 1.5units and 2units along the span of high aspect ratio wings.

4.1.2. Case study 2.

One of the winglet positioned at 1.5units from the wing root chord along the span as shown in figure2 (a).Additional clarity can be seen from the graphs shown in figures 2(d) and 2(e) respectively with representation from the arrows.

- Negative lift and negative drag coefficients with large variation of pitching moment (not shown) not acceptable as seen from figures 2(e) and 2(f) with arrows respectively.
- Lift curve slope (dCL/dα) falls gradually hence leads to loss of lift at low and high angle of incidences so nonplanar wingtip devices are not preferred at location 1.5units along the wing span.
- Large vortex in the downwash just behind the winglet is seen slighty different than in case of winglets with a case positioned at 2.0 units from the wing root chord.

4.1.3. Casestudy 2.

One of the winglet positioned at 2units from the root chord along the span as shown in figure2(c).

- Leading edge flow gets disturbed around the location of 2units, so the suction peak along the span gets affected that leads to negative lift with very high drag which is unacceptable as seen from figures 2(e) and 2(f) with arrows pointing in the positive and negative directions of the respective drag and lift curves. This also results into large variations in pitching moments for the above configuration.
- Lift curve slope (dCL/dα) falls rapidly hence sudden loss of lift at high angle of incidences.
- Large vortex and non uniform disturbed flow is observed in the downwash just behind the nonplanar winglet as shown from figure 2(d).

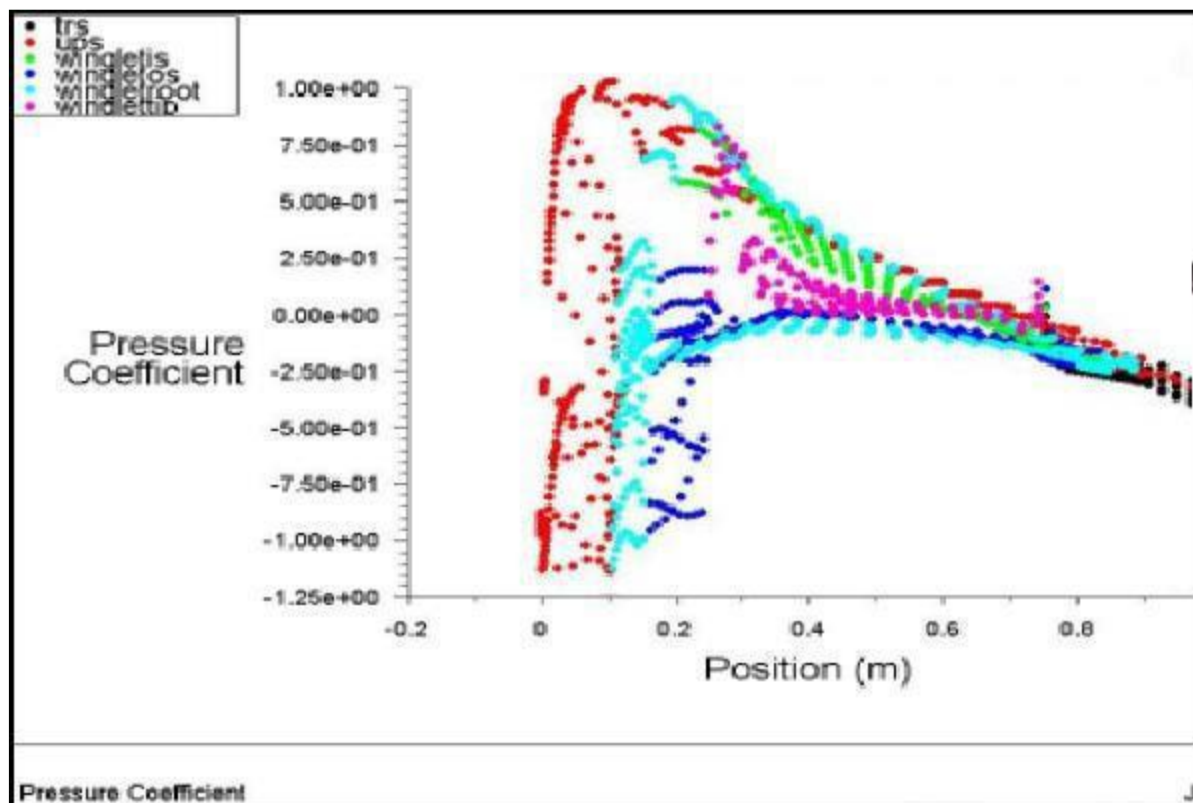### 4.1.4 Case study 3. Wing alone-experiment and Nonplanar Winglet steady state numerical results.

Figure3 (a). Schematic of subsonic pressure coefficient distribution data over the nonplanar wingtip configuration
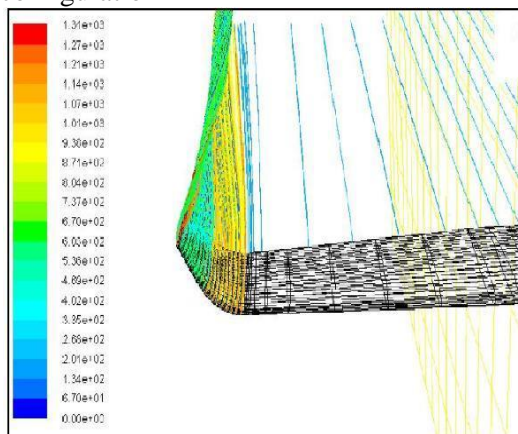

Figure3 (b). Schematic of the flow path lines for the flow over nonplanar wingtip configuration. The nonplanar wingtip is blended to the main wing at its tip clear from above figure.
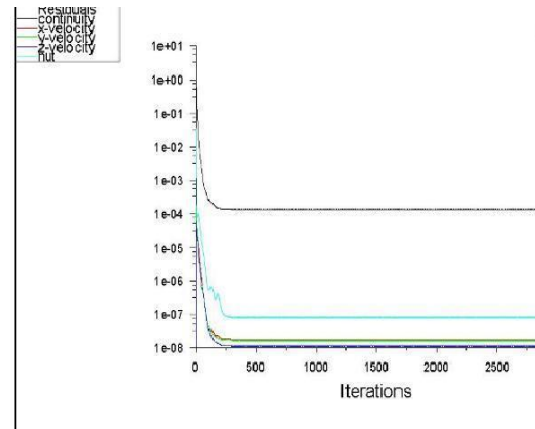

Figure3 (c). Schematic of the residual history for the "wing with nonplanar wingtip configuration" solution.

Figures 3(a), (b), (c) provide very important information on flow path lines over the nonplanar wingtip, the residual history for pressure, velocity and other turbulence quantities, subsonic pressure coefficient. These quality informations are achieved through the use of data provided from table 1 for the nonplanar wingtip device and the wing alone configurations respectively.
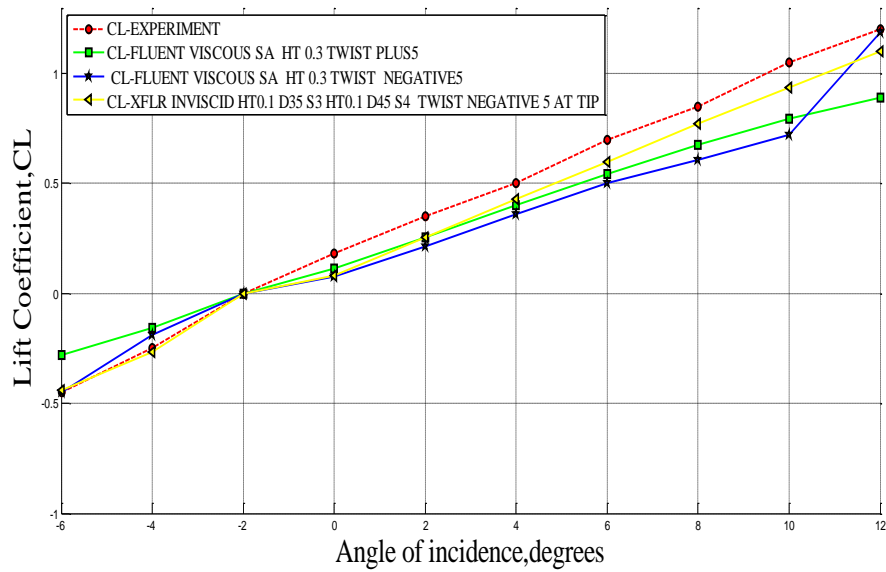
Figure3 (d). Comparison of "Lift coefficient" at Mach Number 0.3 for "wing alone" case with "wing with nonplanar wingtip".
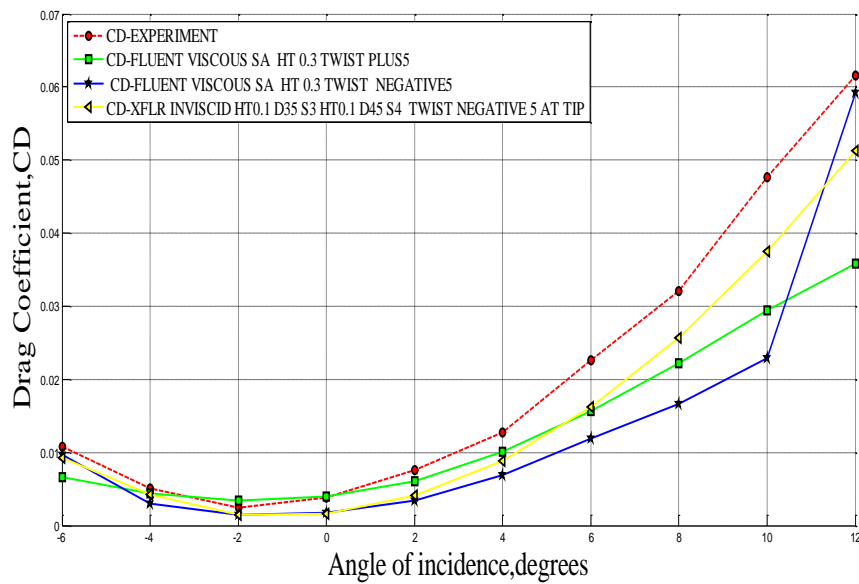

Figure3 (e). Comparison of "Drag coefficient" at Mach Number 0.3 for wing alone case with "wing with nonplanar wingtip".
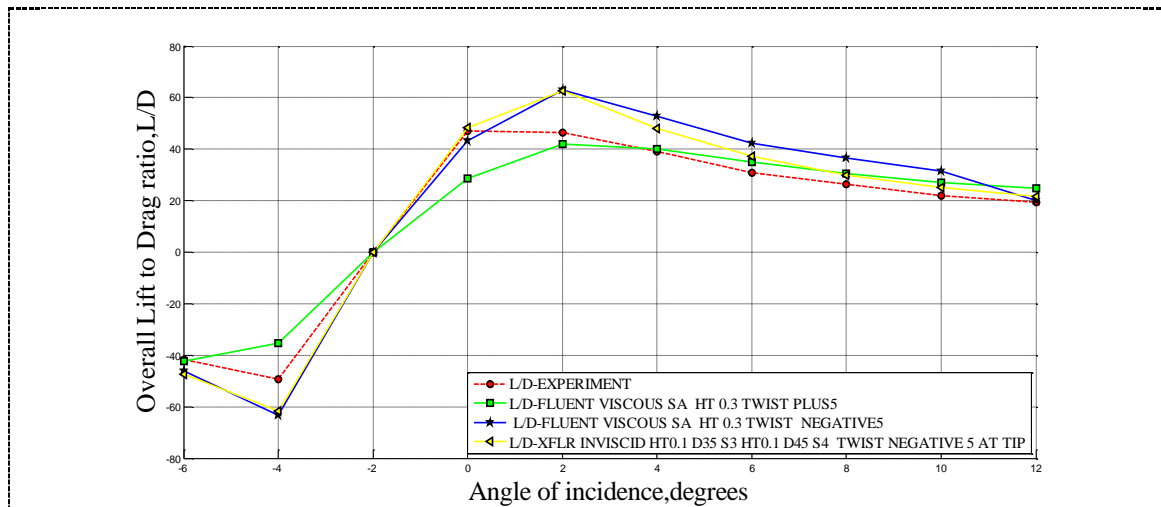
Figure3 (f). Comparison of "Over all Lift to Drag ratio" at Mach Number 0.3 for wing alone case with "wing with nonplanar wingtip".
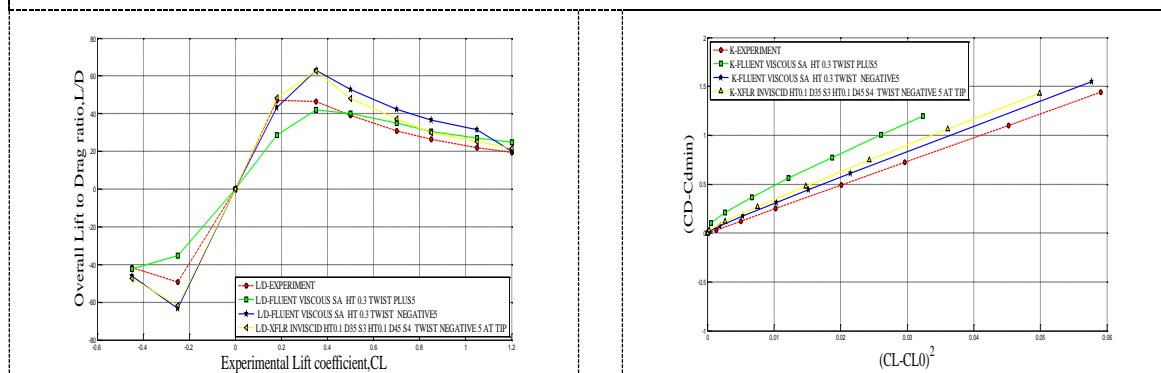

Figure3 (g). Comparison of "CL-Experiment Vs Overall Lift to Drag ratio" at Mach Number 0.3 for wing alone with "wing with nonplanar wingtip".

Figure3 (h). Comparison of "K" the subsonic induced drag factor at Mach Number 0.3 for the nonplanar wingtips and wing alone experimental configuration.

In this section the "wing alone-wind tunnel results" are compared with the results obtained for two of the numerical "wing with nonplanar wingtip configurations" figures 3(a), 3(b), 3(c), 3(d), 3(e), 3(f), 3(g), 3(h) and comments are being made about the improvements obtained in terms of overall efficiency(L/D),average induced drag factor "K_{avg}" for the case of wing with nonplanar wingtip devices in the conclusion section.

The final overall geometric dimensions of the optimum nonplanar wingtip configuration are, tip aerofoil thickness (around 12%), height of the wingtip (around 110mm with tip pointing upword, optimum average cant angle of 40degrees with negative twist of 5 degree at the winglet root and tip respectively) are finalized, these dimensions are purely based on two of the important observed facts that are clear from the graphs shown in figures 3(d),3(e), 3(f), 3(g), 3(h). It is very evident from curves shown, figure 3(d) and figure 3(e), there is large variation of lift curve slope at high angle of incidences beyond 10 degrees,(dCL/dα) for both the cases (of positive twist and negative twists shown in figure1(b), figure1(c)) corresponding to the winglet height of 0.3.

- The first one is the subsonic induced drag factor"K"as shown by Figure. 3(h) from case study and the second is the reduced combined wing loading as depicted by the results in Figure3(i), 3(j), 3(k) under the case study 3 respectively. The graph lines shown in green indicate the

wing loadings over span (uniform load caused due to lift), shear and root bending moments are slightly lower than the experimental data, this is due to the presence of nonplanar winglet at the tip i.e. wingtip redistribute the load on the wing by altering the pressure(Cp) on upper and lower surfaces respectively.

- The use of nonplanar wingtip device on the wing has improved the overall lift to drag ratio by about 13% to 15% for the incidence range from 0 degrees to 4 degrees of incidence corresponding to about 8 to 9 % reduction in average induced drag factor (K_{avg}) from experimental wingalone case.

- So reduction in height of the non planar wingtip device is preferred here for the drag reduction and augment higher overall lift to drag ratio (L/D) without additional weight penalties.

- The combined loadings obtained by the above methodology may be used as input by the structural engineers and designers using Finite Element Methods and other techniques. The loads are important and pivotal in deciding the material, optimum skin thickness of the aerodynamic shape etc.

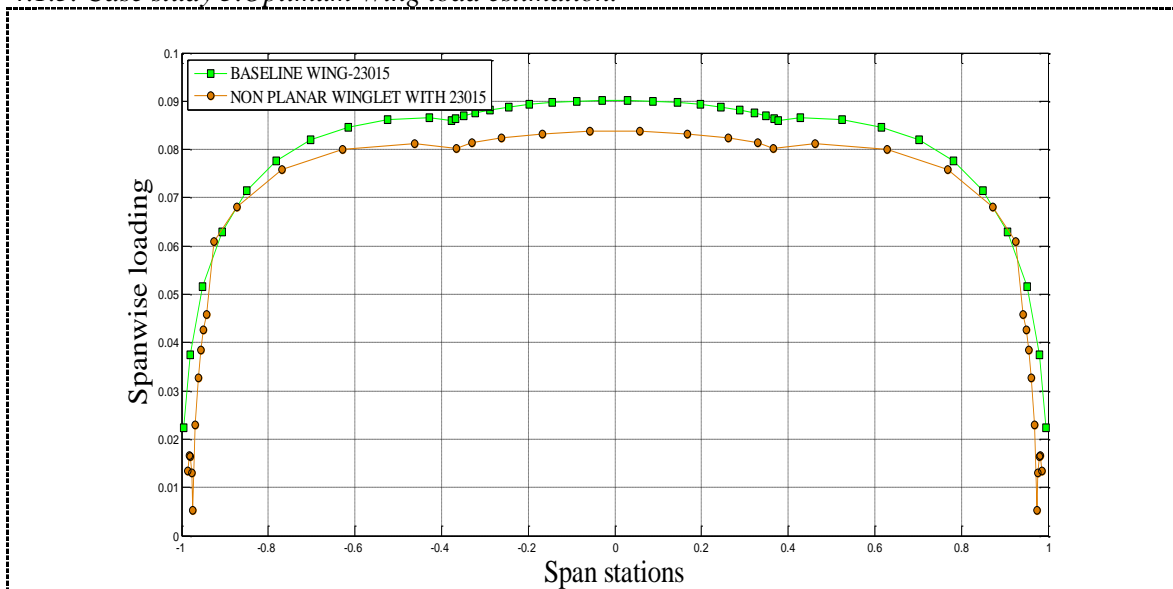*4.1.5. Case study 3.Optimum Wing load estimation.*

Figure3 (i). Comparison of wing alone experimental spanwise wing loading with optimum nonplanar wingtip for the configurations in figure 1(b) and figure 3(a) respectively.
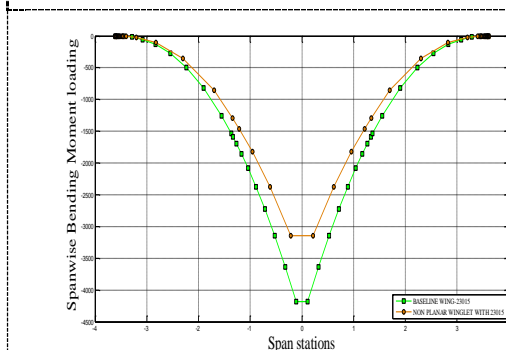

Figure3 (j). Comparison of wing alone spanwise bending moment with planar wingtip for the configurations shown in figure 1(b) and figure 3(a) respectively.
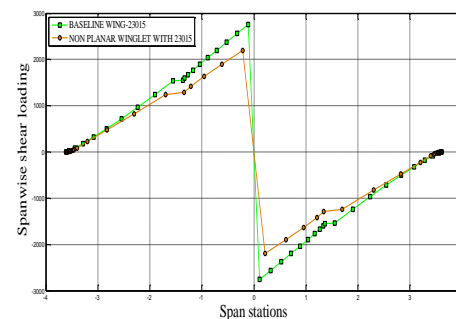

Figure3 (k). Comparison of wing alone spanwise Shear loading with nonplanar wingtip for the configurations in figure 1(b) and figure3 (a) respectively.

This section also gives comparison and clarity on the estimated wing loading between "experimental wing alone case" and the "numerical wing with nonplanar wingtip" cases shown in figures 3(i), 3(j) and 3(k) respectively.

5. Conclusions

Apart from the above mentioned two case studies additional tests on non planar winglets have also been dealt with, and clarity was drawn to use the winglets at the tip only. Use of wingtip devices for the high aspect ratio wings in the current study at the tips is found to be more efficient in handling the tip vortex and reduce the tip losses. The winglet heights are chosen ranging from 0.3m (300mm) to 0.11m.(110mm) from existing literature for design of nonplanar winglets. The optimized height of the nonplanar winglet from current investigation is restricted to and around 110mm with the tip aerofoil as 12% thick.

Inappropriate positioning and deployment of nonplanar wingtip devices leads to large variation of pressure difference on upper and lower surfaces of the main wing that result in loss of overall efficiency (L/D), of high aspect ratio wings considered here in the current studies.

Findings from the present research methodology can be used for (a) Better configuration design and redesign of five digit thick aerofoils for specific purpose and aerodynamic application. (b) Optimum structural design of the associated wing components like lug joints, stringers, spars etc. (c) Blended wing body planform configurations studies ,for subsonic aerodynamic applications etc. The parametric studies carried out on two of the nonplanar winglet systems through numerical simulations have shown interesting and meaningful results.The performance of winglets interms of improving overall 'L/D'were found to be sensible for the incidence range up to the nonlinear leg of the lift curve when compared with the clean wingalone configuration design.

In this investigation the nonplanar wingtip configurations to specifically suit the requirements of SARAS aircraft have been conceptually realized with clarity from experimental data. These two design configurations (with lower induced drag and reduced wing loading) identified guaranteed to perform better than the wing alone configuration (with higher induced drag and higher wing loading) that is currently being considered for the aircraft without much of the additional weight penalties. The present numerical investigations and the literature studies have indicated that the performance of this current wing with nonplanar wingtips (with same weight) would be superior to that of the wing alone configuration.

References
[1] Anderson, J.D. Introduction To Flight, 6th Edition, Mc-Graw Hill co, New York, 1985.
[2] Brune G W. *Quantitative low-speed wake surveys. J. Aircraft.* **31(2):249–55**, 1994.
[3] Govindaraju S P, Saffman P.G. *Flow in a turbulent trailing vortex. Phys. Fluids* **14(10):2074–80**, 1971.
[4] Devenport W J, Rife MC, Liapis SI, Follin G J. *The structure and development of a wing-tip vortex. J. Fluid Mech.* **312:67–106**, 1996.
[5] Garodz L J, Clawson K L.*Vortex wake characteristics of B757–200 and B767–200 aircraft using the tower fly-by technique*.NOAA Tech. Mem. ERL ARL—199 Silver Spring, MD: Air Resources Lab, 1993.
[6] Vicroy D, Brandon J, Greene G, Rivers R, Shah G, et al. *Characterizing the hazard of a wake vortex encounter*. AIAA Paper **97–0055**, 1997.

[7] Rudis, R.P, Burnham DC, Janota P. *Wake vortex decay near the ground under conditions of strong stratification and wind shear.* Presented at Advis. Group Aerospace. Res. Dev. (AGARD) Symposium,Trondheim, Norway, **11:1–10**, 1996.

[8] Greene, G C. *An approximate model of vortex decay in the atmosphere. J. Aircraft.***23 (7):566–73**, 1986.

[9] Brown, C E. *Aerodynamics of wake vortices.AIAA J*. **11(4):531–36**, 1973.

[10] Spalart, P R. *On the motion of laminar wingwakes in a stratified fluid. J. Fluid Mech.***327:139–60**,1996.

[11] Munk ,M. 1923.*The minimum induced drag of aerofoils.* NACA Rep. **121,** NACA, Hampton, Va, 1996.

[12] Lowson, M. V. *Minimum induced drag for wings with spanwise camber. AIAA Journal of Aircraft.***27:627–31**, 1990.

[13] Rokhsaz K. *A brief survey of wing tip devices for drag reduction,* SAE 932574,Society Automotive. Engg.,Warrendale, Pa, 1993.

[14] Mangler, W.*The lift distribution of wings with end plates.* NACA TM **856**, NACA, Hampton, Va, 1938.

[15] Maskell, E C. *Progress towards a method for the measurement of the components of the drag of a wing of finite span.* RAE TR **72232**, R. Aircraft. Establishment. Farnborough, UK, 1973.

[16] Heyson, H. H., Riebe, G. D. and Fulton, C. *Theoretical parametric study of the relative advantages of winglets and wing tip extensions.* NASA TMX-**74003**, 1977.

[17] Jones, R .T. *The spanwise distribution of lift for minimum induced drag of wings having a given lift and a given bending moment.*NACA TN **2249**, NACA Hampton, V A, 1950.

[18] Jones R T, Lasinski TA. *Effect of winglets on the induced drag of ideal wing shapes. NASA TM 81230, NASA,Ames Research. Centre.*, Moffett Field, California, 1980.

[19] Whitcomb, R. T. *A design approach and selected wind tunnel results at high subsonic speeds for wing-tip mounted winglets.* NASA TND-**8260**, 1976.

[20] Hackett, J.E. *Vortex drag reduction by aft-mounted diffusing vanes.* ICAS paper **80-13.4**, 1980.