

# On the nonlinear control of a single-phase current source converter for sinusoidal voltage generation

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**Abstract.** Colombian power system is being transformed for the large-scale integration of renewable energy resources and energy storage systems; all of these are integrated by power electronic converters controlled by voltage and current sources. In addition, for regulatory policies it is required that the alternating current networks that feeds linear and nonlinear loads, can provide pure sinusoidal voltage and current forms in terms of frequency and amplitude. To guarantee it, in this paper the design of a nonlinear controller for a single-phase current source converter is explored, which is switched through a pulse-width modulated signal for providing sinusoidal voltages on linear loads. It is applied a feedback nonlinear control design on the dynamical model of the converter by using its averaged representation. The proposed control strategy allows guaranteeing asymptotic stability in the sense of Lyapunov for closed-loop operation. In steady state conditions the voltage behavior on the linear load evidences a sinusoidal form with an estimation error lower than 0.667%, which can be considered negligible for any practical power system application. All the simulations are conducted via MATLAB software 2017b licensed by Universidad Tecnológica de Bolívar, Colombia.

## 1. Introduction

Power electronic converters are essential components of modern power systems; they allow the integration of classical and renewable generation as well as energy storage devices or nonlinear loads [1–3], etc. These can be classified as voltage source converters and current source converters [4]. The first one typically interconnects renewable generation and energy storage technologies working with a differential of potential via capacitor links; the second one works with currents flowing through inductor links. In specialized literature it is common to find control approaches for voltage source converters in comparison with current source converters, since modulate voltage is considered an easy task when compared with modulation of current [5]. Due to this, here we present a nonlinear control strategy for pulse width modulated-current source converters (PWM-CSC) since it represents a challenge in comparison with classical voltage source converters [6].

Literature reports multiple control approaches applied in PWM-CSC for sinusoidal voltage regulation in linear loads such as time-domain analysis [7], energy shaping and damping injection controllers [6,8]; nonlinear feedback proportional controllers [9], nonlinear pulse-width-modulated (PWM) modulators [10], and robust control [11], among others. Unlike these works, we propose in this research a nonlinear controller with proportional integral actions that minimizes the steady state error at the same time that



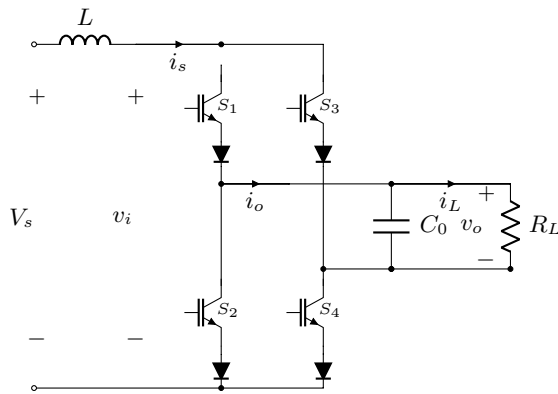
Lyapunov's stability is guaranteed [12]. Numerical simulations in MATLAB/Simulink allow to verify the effectiveness of the proposed approach to generate sinusoidal signals for resistive loads.

In the Colombian power system context, renewable energy resources and energy storage system are being impelled by regulatory polices such as the law 1715 related to the integration of renewables into the power system [13]. In this sense, due to the nature of these technologies i.e., generation and storage, power electronic converters must be used for their integration to power system; this implies that control techniques are required to guarantee their stable operation with minimum harmonic distortion in the case of alternating current applications [14, 15]. For doing so, in this research the main contribution is related to the possibility of generating sinusoidal voltages to supply linear loads with minimum steady state errors by guaranteeing asymptotic stability condition in the sense of Lyapunov for closed-loop operation. Observe that the proposed approach can be applicable in the rural areas by providing electrical service from renewables such as wind and photovoltaic, in conjunction with battery energy storage systems to support sinusoidal voltage and currents in linear and nonlinear loads [4].

Regarding the above, section 2 presents the dynamical model of the PWM-CSC; section 3 shows the control design via a nonlinear feedback approach; section 4 presents the main characteristics of the MATLAB/simulink implementation and the problem parametrization; section 5 shows all the simulation results and discussions; followed by the conclusions and future works in section 6.

## 2. Dynamical model

The PWM-CSC generates a bilinear dynamical model with two state variables associated with the current flowing through the inductor and capacitor, interconnected to the direct current (DC) and alternating current (AC) side of the converter respectively. In Figure 1 the internal configuration of a PWM-CSC is depicted for providing sinusoidal voltage to linear loads.



**Figure 1.** Electrical connection of the PWM-CSC for single-phase voltage regulation in linear loads.

Observe that in Figure 1 it is presented the physical connection of the forced commutated switches, creating the PWM-CSC which are fed by a constant voltage source connected in series with the inductor in the dc side. In the ac side, a capacitor and the resistive load are connected in parallel.

To obtain the nonlinear dynamical model that represents the PWM-CSC let us suppose that the switching frequency is high enough (10 kHz or more) to use an averaging modeling representation. If we apply second Kirchhoff's law to the closed-loop trajectory formed by the inductor at the dc side of the converter and the first Kirchhoff's law to the node that connects the capacitor in the ac side, then, Equation (1) and Equation (2) present the complete dynamical model.

$$L_s \frac{d}{dt} i_s = V_s - R_s i_s - m v_o, \quad (1)$$

$$C_o \frac{d}{dt} v_o = m i_s - R_L^{-1} v_o, \quad (2)$$

where  $i_s$  is the current through the inductor of the dc side,  $v_o$  is the voltage across the capacitor in the ac side,  $L_s$  and  $R_s$  are the resistive and inductive components of the inductor of the dc side,  $C_o$  is the capacitance value in the ac side,  $V_s$  is the constant voltage source that provides energy to the PWM-CSC,  $R_L$  is the resistive constant load, and  $m$  represents the modulation index that works as control input. Note that  $u$  is in the interval of -1 to 1.

The dynamical system presented in Equation (1) and Equation (2) is bilinear, since it contains the product between state variables and the control input. To obtain an standard state variables representation, let us define the state variables  $x_1 = i_s$  and  $x_2 = v_o$  and the control input as  $u = m$ , which produces the Equation (3) and Equation (4).

$$L_s \dot{x}_1 = V_s - R_s x_1 - u x_2, \quad (3)$$

$$C_o \dot{x}_2 = u x_1 - R_L^{-1} x_2, \quad (4)$$

The objective of control for the PWM-CSC is to enhance a desired sinusoidal state for the voltage profile in the ac side, *i.e.*,  $x_2^* = V_m \sin(\omega t + \theta)$ ; where  $V_m$  is the peak of the voltage profile,  $\omega$  is the angular frequency and  $\theta$  represents the phase angle.

To address this control objective, the next section we will propose a nonlinear feedback controller that guarantees asymptotic convergence of  $x_2$  and the bounded performance of  $x_1$  for all  $t \geq 0$ .

### 3. Nonlinear controller design

In this section we explore the possibility of developing a nonlinear feedback controller with proportional integral action that ensures the controllability and sinusoidal form of the voltage output profile [12].

To design the nonlinear controller for providing sinusoidal service to linear loads using a PWM-CSC, let us suppose the current through the inductor in the dc side of the converter is a positive definite function, *i.e.*,  $x_1(t) > 0$ ,  $\forall t > 0$ . This was previously proved by [6], where the general structure of  $x_1(t)$  is defined in Equation (5).

$$x_1(t) = I_{sd} + I_{sh} \sin(2\omega t + \varphi), \quad (5)$$

where  $I_{sd}$  is a constant positive current (direct current value) and  $I_{sh}$  models the ripple function, which oscillates with the double of the frequency of the ac side, and  $\varphi$  is the phase angle. Note that for positive definiteness,  $I_{sd} > I_{sh}$ .

Observe that the solution of the inductor current reported in Equation (5) is presented in the numerical results through the ordinary differential equation package available in MATLAB simulink.

To propose a nonlinear control law that makes  $x_2$  asymptotically stable [7], let us recur to the previous assumption to obtain the control law defined in Equation (6).

$$u = \frac{1}{x_1} \left( C_o \dot{x}_2^* + R_L^{-1} x_2^* - k_p (x_2 - x_2^*) - k_i \int_0^t (x_2 - x_2^*) dt \right), \quad (6)$$

where  $k_p$  and  $k_i$  are positive constants related to proportional and integral control actions, respectively.

Now, if we substitute Equation (6) in Equation (4), then, the ordinary differential expression defined in Equation (7).

$$C_o \dot{\tilde{x}}_2 = -(k_p + R_L^{-1}) \tilde{x}_2 - k_i \int_0^t \tilde{x}_2 dt, \quad (7)$$

where  $\tilde{x}_2$  is defined as  $x_2 - x_2^*$ ; which clearly implies that if  $\tilde{x}_2$  tends to zero, then,  $x_2$  tends to  $x_2^*$ .

The analytical solution of Equation (7) is easily achievable by any linear solution method of ordinary differential equations, which produces Equation (8).

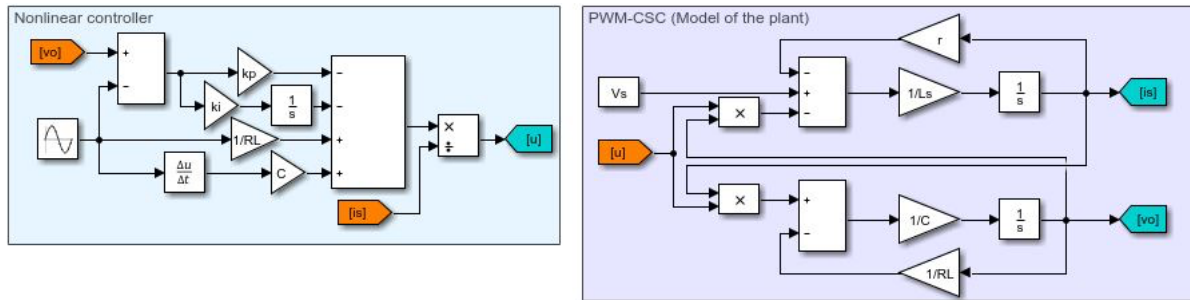
$$\tilde{x}_2(t) = k_1 e^{-t \frac{a - \sqrt{a^2 - 4bc}}{2c}} + k_2 e^{-t \frac{a + \sqrt{a^2 - 4bc}}{2c}}, \quad (8)$$

where  $a = k_p + R_L^{-1}$ ,  $b = k_i$  and  $c = C_0$ , and,  $k_1$  and  $k_2$  are real constants related with initial conditions. To guarantee that  $\tilde{x}_2(t)$  tends to zero, then,  $a > 2\sqrt{bc}$ ; this implies that the voltage at the capacitor in the ac side converges asymptotically to each sinusoidal reference.

Note that the proposed controller can be reduced to a proportional controller without compromising the asymptotic convergence, by making  $k_i = 0$ . In addition, for practical purposes  $C_0$  and  $R_L$  during the implementation of the control law defined in Equation (6) can be considered as the expected values of these parameters, this is,  $\bar{C}_0$  and  $\bar{R}_L$ ; notwithstanding, the asymptotic convergence of the voltage profile continues being preserved.

#### 4. Simulink implementation and test system parameters

The numerical implementation of the PWM-CSC is performed by using its averaged model defined by Equation (1) and Equation (2) through the MATLAB/Simulink package as presented in Figure 2.



**Figure 2.** Simulink implementation of the model of the converter and the proposed controller.

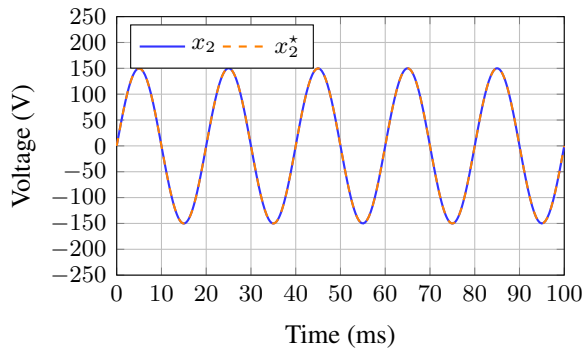
Observe that in Figure 2 it is implemented the dynamical model of the PWM-CSC using block diagrams and the control law designed via nonlinear feedback analysis. This implementation is easily developed using the product (division), sum, derivative, integral and gain blocks.

For this implementation, the following parameters were considered:  $L_s = 10$  mH,  $r = 1$   $\Omega$ ,  $C_0 = 200$   $\mu$ F,  $R_L = 50$   $\Omega$ , and  $V_s = 48$  V. In the case of the sinusoidal voltage reference, we assume a magnitude about  $V_m = 150$  V with a frequency of  $f = 50$  Hz. All these values have been taken from [6]. The proportional and integral gains used during the simulation are  $k_p = 5$  and  $k_i = 2$ , which have been obtained after a trial and error search. Additionally, the initial conditions for the current through the inductor and the voltage at the capacitor were set as  $i_s(0) = 25$  A and  $v_0(0) = 0$  V, respectively.

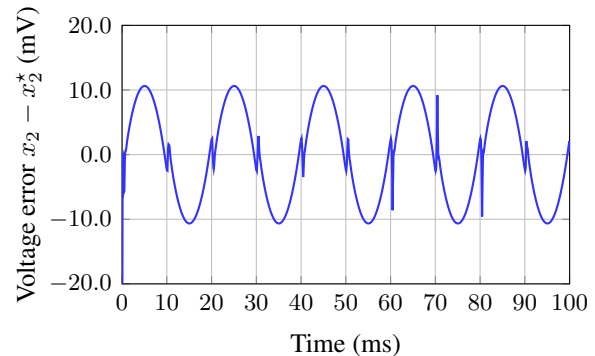
#### 5. Numerical results

The simulations were carried out in a desk-computer with an Intel(R) Core(TM) i5 – 3550 processor at 3.50 GHz, 8 GB RAM, running a 64-bits Windows 7 Professional operating system by using MATLAB/Simulink 2017a.

In Figure 3 and Figure 4 are presented the comparison between the voltage output and its reference as well as the absolute error between both signals when  $R_L = 50$   $\Omega$ . Note that from results in Figure 3 and Figure 4, we can affirm that: i) the voltage performance at the capacitor has sinusoidal form (see Figure 3), which is the objective of the PWM-CSC when integrated to linear loads at the ac side; and ii) The estimation error is about 1 V as depicted in Figure 4, which clearly shows the advantage of the proposed nonlinear controller to follow time-varying references.

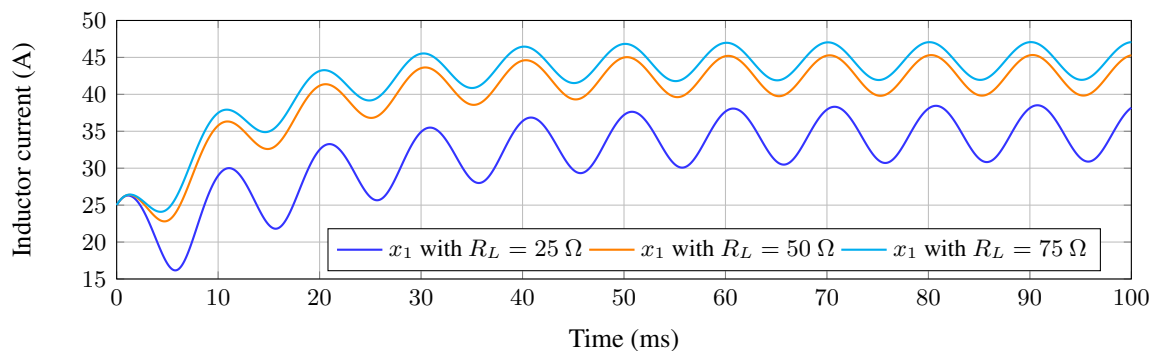


**Figure 3.** Comparison between voltage output and reference voltage signal.



**Figure 4.** Tracking error between voltage output and reference voltage signal.

Figure 5 presents the current performances in the load when changes from  $25 \Omega$  to  $75 \Omega$ . It is important to mention that when the resistive load increases, then the total current flowing through the inductor increases too, while the amplitude of the oscillation decreases.



**Figure 5.** Current behavior in the dc side of the converter for different resistive loads

In addition, we can observe that the current flowing through the inductor shows an oscillation with sinusoidal form at the double of the frequency as expected for inverter applications, since the instantaneous power oscillates at this frequency, as reported in [7]. Finally, this simulation confirms that the current through the inductor behaves as defined in Equation (5); which confirms that the proposed nonlinear controller is stable in the sense of Lyapunov.

## 6. Conclusion and future work

A nonlinear control design for output voltage regulation in single-phase linear loads using a current-source converter was presented in this paper. A stability analysis was performed on the proposed controller, evidencing asymptotic stability in the sense of Lyapunov. The output voltage form was sinusoidal as required in the control objective, with an estimation error lower than 1 V (0.667%), considered as negligible for any practical power application. The main advantage of the proposed feedback nonlinear control design lies in the possibility to present a nonlinear proportional-integral controller for a current-source converter, that allows tracking sinusoidal trajectories with minimum error without recurring to space transformations of the dynamical model of the converter, since the proposed approach works directly with its nonlinear dynamics to propose a nonlinear control design.

As future work, it will be feasible to apply passivity-based control techniques with proportional-integral gains for obtaining global asymptotic stable controllers taking the advantage of the port-Hamiltonian open-loop structure of the PWM-CSC; in addition, these controllers could be extended to nonlinear loads, such as constant power or loads modeled with exponential or polynomial representations.

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