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Convection heat transfer in a porous medium saturated with an Oldroyd B fluid - A Review

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Abstract. In this review paper, the important milestones in model studies such as Darcy and Brinkman on heat transfer through porous medium were summarized. Mathematical expressions pertaining to models were studied to understand the response of the Oldroyd B fluid flowing through a porous medium with a finite element boundary conditions. Research papers on Linear stretched sheet and circular tube flow models gave the clear picture of the extent of work carried out by the heat transfer researchers. Handful of verticals are identified as research gaps which still remains unexplored. Hence Provides an opportunity to carryout in-depth analysis for complete understanding of heat transfer thorough a Oldroyd B fluid filled porous media.

1.Introduction

Non-Newtonian fluids widely used in industrial processes and in our daily life, for example, oil industries, in chemicals, food processing like custard, honey, starch suspensions toothpaste, corn starch, paint, blood, shampoo and many more. Fluid’s elasticity or known as Boger fluids is the particular characteristic with constant viscosity of some non-Newtonian liquids behaves like an elastic solid when stretched out but creates an effect in the fluid where it flows like a liquid.

Non-Newtonian fluids are belonging to viscoelastic class having viscus as well as elastic characteristics and a mixture of a solvent dispersed in polymer. Examples includes paints, shampoos, biological fluids, industrial chemicals, clay coating, DNA suspensions, paints, cements, certain oils, drilling muds, sludge, food products, paper pulp, slurries and aqueous foams, grease which does not follow Newton’s law of viscosity. The common use and interested for industries and academic research are the viscoelastic Maxwell fluid, the viscoelasti Jeffrey’s fluid, the viscoelastic Oldroyd B fluid, the viscoelastic Carreau fluid, and many more.

Non-Newtonian fluids are commonly divided into three groups: viscoelastic, time-dependent and time-independent[1], here are some of the Prominent examples are presented in Table 1.

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Table 1: Examples of a non-Newtonian rheological models.

Model	Equation
Power-Law	$\mu = C\dot{\gamma}^{n-1}$
Ellis	$\mu = \frac{\mu_0}{1+(\frac{\tau}{\tau_{1/2}})^{\alpha-1}}$
Carreau	$\mu = \mu_\infty + \frac{\mu_0-\mu_\infty}{[1+(\dot{\gamma}t_c)^2]^{\frac{1-n}{2}}}$
Herschel-Bulkley	$\tau = \tau_0 + C\dot{\gamma}^n (\tau > \tau_0)$
Maxwell	$\tau + \lambda_1 \frac{\partial\tau}{\partial t} = \mu_0\dot{\gamma}$
Jeffreys	$\tau + \lambda_1 \frac{\partial\tau}{\partial t} = \mu_0(\dot{\gamma} + \lambda_2\frac{\partial\dot{\gamma}}{\partial t})$
Upper Convected Maxwell	$\tau + \lambda_1 \overset{\nabla}{\tau} = \mu_0\dot{\gamma}$
Oldroyd-B	$\tau + \lambda_1 \overset{\nabla}{\tau} = \mu_0(\dot{\gamma} + \lambda_2\overset{\nabla}{\dot{\gamma}})$
Godfrey	$\mu(t) = \mu_i - \Delta\mu'(1-e^{-t/\lambda'}) - \Delta\mu''(1-e^{-t/\lambda''})$
Stretched Exponential Model	$\mu(t) = \mu_i + (\mu_{\inf} - \mu_i)(1-e^{-(t/\lambda_s)^c})$

The Oldroyd-B constitutive model is the type of non-Newtonian fluids that describes the behaviour of some visco-elastic fluids. The Oldroyd-B fluid cannot describe shear thinning or shear thickening but stress relaxation, creep and the normal stress differences can be described. In past, modest researchers published works concerning the boundary layer analysis in an Oldroyd-B liquid, compared with its Newtonian counterpart. Bhatnagar et al. [2] studied about the linearly stretched sheet with stream-wise pressure gradient with boundary layer formation in Oldroyd- B fluid.

A porous medium is a solid material which contains a tiny pore filled with a fluid and continuous with each other which includes natural substances like rocks, soil, zeolites, and biological tissues and wood. Thermal convection (forced, free, mixed etc.) in a fluid-saturated with porous medium have been received greater attention in the past few decades due to the wide applications in food processing, nuclear waste disposal, geothermal energy utilization, oceanography and petroleum industries [3].

In this review paper, the works on Darcy model and brinkman model using Oldroyd B fluid saturated with porous medium is been presented.

2. Literature Review

2.1 Darcy Model

The heat transfers in porous medium and viscoelastic fluid flow received much attention in engineering fields, such as enhanced oil recovery, composite manufacturing process and textiles coating. Studies had focused much on the modelling of the time-dependent motions. Darcy's law is the linear Newtonian model that relates the local pressure gradient in direction of flow to the fluid superficial velocity through the viscosity of the fluid and the permeability of the medium. It is the simplified model to describe the flow in a porous media. Darcy's law is the empirical relation and derived from Navier Stokes equation via homogenization using capillary bundle model. The momentum balance equation of a fluid phase is basic for theoretical analysis. Since Darcy's law contains only viscous term so it is applicable to only laminar flow at low Reynolds number (Re<1) and neglects the heat transfer, boundary effects and whose validity is restricted totlaminar, isothermal, purely viscous, incompressible Newtonian flow. As the velocity increases, the flow enters a nonlinear regime and the inertial effects are no longer negligible. [4]

M.G. Alishaev is the first person to translate from Russian to English, "For the calculation of delay phenomenon in filtration theory" problems with an unsteady filtration through a porous medium, which uses Darcy's law and assumes that the equilibrium between pressure gradient and velocity is achieved instantly. This study is useful for studying the filtration of non- Newtonian oils, polymer solutions, mixtures, emulsions and so forth.[5].

Khuzhayorov et al. [6] proposed the macroscopic filtration law for describing transient linear viscoelastic fluid flow in a porous medium by upscaling the heterogeneity scale description. Tan and Masuoka [7] investigated Stokes' first problem for viscoelastic fluid in the porous half-space. Ehlers and Markert [8] suggested a model to explain the mechanical behaviour of hydrated soft tissues as a materially incompressible binary medium of one linear viscoelastic porous solid skeleton. Beg et al. [9] and Khani et al. [10] studied using non- Darcy porous media with thermophysical effects on the heat transfer of a third-grade viscoelastic fluid.

The instability or stability of fluid flow in a porous media serves important part of porous media fluid mechanics. In particular, the interaction between shearing forces and buoyancy on the stability of fluid flow in a vertical layer of the porous medium has been the subject for intensive research interest as many technological phenomena and geophysical are maintained by their forces.

2.2 Heat Transfer through Darcy model for convection in a porous medium saturated with Oldroyd-B fluid

B.M. Shankar & I.S. Shivakumara [11] studied the stability of the conduction regime of natural convection in porous vertical slab saturated with a Oldroyd-B fluid by using modified Darcy's law. They solved eigenvalue problem using extracted Darcy-Rayleigh number with respect to wave number for different values of physical parameters and Chebyshev collocation method and his observations stated that the basic flow is unstable for visco-elastic fluids despite the basic state being the same for Newtonian and Oldroyd-B fluids. They found that the viscoelasticity parameters exhibit both the stabilizing and destabilizing influence on the systems. They realized that the current system is more unstable compared to a Oldroyd-B fluid indicated by the results of Maxwell fluid study.

Hamza [12] investigated the unsteady magneto hydrodynamic (MHD) flowfof an Oldroyd-B fluid through attube filled with a porous media by applying the uniformccircular magnetic field perpendicular to a flow direction. He solved governing equations by modified Darcy's law counting the resistance representations. In his work it was discussed theeeffects of various parameters on flow characteristics such assnon-Newtonian flow and permeability. By comparing the velocitypprofile of Newtonian fluid and Maxwell fluidwwith Oldroyd fluid, He found that thevvelocity distribution increased with the increase of permeabilitypparameter of the porous medium, but decreased with the increased magnetic and the frequency parameters.

I.S. Shivakumara and S Suresh Kumar [13] performed the linear stability studies of a Oldroyd B fluid saturated with horizontal porous layer depends on the quadratic drag vertical through-flow with a boundary as porous. In their main objective they investigated the convective instability combined with an in-compressive binary visco-elastic fluid in a Cartesian coordinate system such that origin is at the bottom of the layer and Z axis is vertically upward. By the modified Forchheimer-extended Darcy model, the quadratic drag in through flow was experimented. The results reveal that The oscillatory convection is possible even if $\Lambda > 1$ and $M\tau > 1$.

Jinhu Zhao [14] investigated modified fractional Darcy's law on heat transfer of natural convection in a porous medium. Finite difference method combined with LI algorithm was developed graphically and analysed the parameters such as velocity, temperature boundary layers, average skin friction coefficient, Prandtl number and Nusselt number. The heat transfer, temperature and velocity boundary layers is influenced by Prandtl number. The purpose of this work is to investigate characteristics of generalized Oldroyd B fluid flow through a porous medium and heat transfer.

B. Khuzhayorov [6] was concerned fluid flow in porous media, in deriving a macroscopic infiltration law for describing transient linear viscoelastic, which expressed in Fourier space and a generalised Darcy's law with a dynamic permeability tensor valids at low Reynolds and Deborah numbers. By taking a bundle of capillary tubes, the flow of Oldroyd B fluid through it gives a set of analytical results which shows that negative apparent density may occur.

2.3 Governing equations of Darcy model

Darcy law is experimented on a steady state unidirectional flow in a uniform medium states the proportionality between pressure difference and the flow rate is expressed as [15]

$$u = -\frac{k}{\mu} \cdot \frac{\partial p}{\partial x} \tag{1}$$

where $\frac{\partial p}{\partial x}$ is pressure gradient, k being specific permeability and μ is the dynamic viscosity of the fluid.

For 3 Dimension equation reduces to

$$v = -\frac{k}{\mu} \cdot \nabla p \tag{2}$$

In the case of an isotropic medium the Eq. (2) simplifies to

$$\nabla p = -\frac{\mu}{k} \cdot v \tag{3}$$

On the basis of the equation of an Oldroyd-B fluid, the Modified-Darcy–Oldroyd model for describing both retardation and relaxation phenomena was suggested [16-17]

$$\left(1 + t_m \frac{\partial}{\partial t}\right) \nabla p = -\frac{\mu}{K}\left(1 + t_p \frac{\partial}{\partial t}\right) \mathrm{V},$$

where t_p and t_m are strain and stress relaxation time constants, K the permeability of the porous medium, μ the effective fluid viscosity, p the pressure, and V the Darcian velocity. When tm = tp = 0 the above equation can be simplified to Darcy's law as in equation .3. By analogy with Oldroyd-B constitutive relationships, the following phenomenal Model, which relates the pressure drop, body force and velocity for a viscoelastic fluid in a Darcy porous medium, is considered [16-18]:

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) (\nabla p - \rho \vec{g}) = -\frac{\mu}{K}\left(1 + \lambda_2 \frac{\partial}{\partial t}\right) \vec{q} \tag{4}$$

where q = *(u, v, w)* is a velocity vector, μ is the viscosity of the fluid, p is the pressure, ρ is the fluid density, K is the permeability, $\lambda 1$ and $\lambda 2$ are stress relaxation and strain retardation time constants, respectively. If λ1 and $\lambda 2$ are assumed to the same value, the Eq. is simplified to Darcy's law. Therefore Eq. (4) can be regarded as an approximate form of an experimental momentum equation for the Oldroyd-B fluid through a porous medium. The Oberbeck– Boussinesq approximation is implored and the model is completed by the adding continuity and energy balance equations along with the equation of state,

$$\nabla . \vec{q} = 0 \tag{5}$$

$$\alpha \frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \kappa \nabla^2 T \tag{6}$$

$$\rho = \rho_0 \{1 - \beta (T - T_0)\} \tag{7}$$

where T is temperature, β is the thermal expansion coefficient, κ is the thermal diffusivity, ρ_0 is the density at reference temperature T = T0 (at the middle of the channel), and α is the ratio of heat capacities.

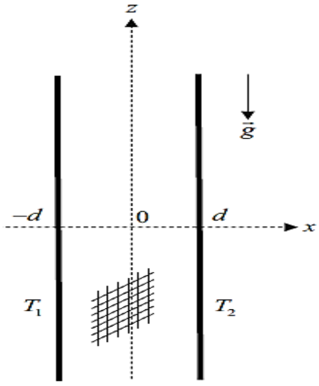

Figure 1: Sketch of porous channel

The quantities are made dimensionless by scaling velocity by κ/d, time by d2/κ, length by d, pressure by κμ/K, and temperature by T (= T2 − T1) to get

$$\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\nabla P - R_{\mathrm{D}} T\hat{k}\right) = \left(1+\Lambda_2\frac{\partial}{\partial t}\right)\vec{q} \tag{8}$$

$$\alpha\frac{\partial T}{\partial t} + (\vec{q}\cdot\nabla)\,T = \nabla^2 T \tag{9}$$

where $R_D = \rho_0 g\beta\nabla T\ Kd/\mu\kappa$ is the Darcy–Rayleigh number, $\Lambda_1 = \lambda_1\kappa/d^2$ is the relaxation parameter, and $\Lambda_2 = \lambda_2\kappa/d^2$ is a retardation parameter. Taking curl on both sides of Eq. (5) to eliminate the pressure term and introduces the stream function ψ (x, z, t),

$$(u, 0, w) = \left(-\frac{\partial\psi}{\partial z}, 0, \frac{\partial\psi}{\partial x}\right) \tag{10}$$

the governing Eqs. (8) and (9) become

$$R_{\mathrm{D}}\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} = \left(1+\Lambda_2\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}\right) \tag{11}$$

$$\alpha\frac{\partial T}{\partial t} + \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial z} - \frac{\partial\psi}{\partial z}\frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}. \tag{12}$$

The isothermal boundaries are an impermeable, and the appropriate boundary conditions are considered

T =−1 at x =−1 for ψ = 0,
T =1 at x = 1 for ψ = 0, [19]

2.4 Darcy Brinkman Model

An alternative to Darcy's equation is what is commonly called as Brinkman's equation. With inertial terms omitted and take the form [15]

$$\nabla P = -\frac{\mu}{K}\mathbf{v} + \tilde{\mu}\nabla^2\mathbf{v}.$$

Two viscous terms are there in this equation. The first is the Darcy term and the second is similar to the Laplacian term which appears in Navier–Stokes equation. The coefficient μ is an effective viscosity Brinkman set μ and equal to each other, but in general it is not true.

2.5 Heat Transfer through Darcy- Brinkman law of convection in a porous medium saturated with Oldroyd-B fluid

Qiulei Sun et.al [20] carried out the nonlinear analysis on chaotic convection through porous medium with Oldroyd B fluid where temperature was modulated. Darcy-Brinkman model was considered in their study considering the four dimensional truncated Galerkin expansion to address stability with a varied Darcy Rayleigh number. With the three external parameters viz phase angle Φ, frequency, Chaotic Darcy- Brinkman convection of an Oldroyd-B fluid in porous media is analysed by the establishment of a four-dimensional system through truncated Galerkin expansion. The effect of Darcy- Rayleigh number on the convection is studied numerically in two conditions for k1 and k1 with different Darcy number. the motionless solution loses its stability and the other equilibrium point takes over when R exceeds for this case $\Gamma < (\sigma R_{cr}+1)/ [\sigma R_{cr}(1-\Lambda)]$.

B M Shankar [21] performed studies on Horizontal layer of porous medium in brinkman model to perform stability investigation of effective viscosity chebyshev collocation method was used to drive modified Orr- Sommerfield equation. Based on critical Reynolds number and Critical wavenumber with critical wave speed hydrodynamic stability was analysed. In their study it was documented as the number of terms in the equation

$$\psi(z) = \sum_{j=0}^{N} T_j(z)\psi_j$$

increases the flow pattern remains uniform and accuracy improved between 7-10 digits for a number of terms N (80-100).

Chen Yin [22] A two-layer fluid over porous medium saturated with Oldroyd B fluid when heated at the bottom result in stable thermal Convection on comparison with stationary convection, the oscillatory Convection is located at a lower depth ratio. In this test it is found that critical Reynolds number for stationery Convection is greater than oscillatory Convection. For solving eigenvalue problems, chebyshev-tau QZ algorithm were used. Binomial nature of neutral curves, other porous properties, strain retardation, stress relaxation time and boundary conditions for both stationary Convection and oscillator Convection are considered by varying death ratios. It stated that critical Rayleigh numbers for oscillator conventional are lower than stationary Convection.

NIU Jun [23] investigated on constant heat flux on square box open top porous model filled with Oldroyd B fluid. Higher heat transfer rate during fluid flow destabilizes by increasing retardation time. The complicated flow pattern results were tabulated with flow bifurcation with increase of relaxation time and decrease of retardation time

J. H. Merkin [23] discussed boundary layer flow and heat transfer over a permeable stretching sheet and shrinking surface embedded in porous medium using Brinkman. In this paper they considered prandtl number (pr), mass flux parameter s, for suction $s > 0$, for an impermeable surface $s=0$ and for blowing $s < 0$. They also considered other parameters related to porous medium, a wall velocity λ, velocity ratio m.

Wenchang Tan [24] applied the modified Darcy's law for a viscoelastic fluid extended to Oldroyd-B fluid filled porous media. The effect of viscoelasticity on the unsteady flow in porous media was investigated in their work. Fourier sine transform was used as it involves Y dependent steady state solution which is different than that of the clear fluid. Steady state solution of stokes first problem can be obtained by $u(y) = -e^{by}$. For stokes first problem, velocity oscillations at a fixed distance are clearly seen for a short relaxation time.

B.M Shankar [25] A vertical layer of porous medium was considered and the effect of inertia on the stability of buoyancy driven parallel shear flow is analysed. For describing the flow in a porous medium, Lapwood–Brinkman model with fluid viscosity was used and Chebyshev collocation method is employed for eigenvalue problem. The criticalaDarcy–Rayleigh number RDc, the critical wave number ac and the critical wave speed cc are computed over a wide range of values of the Darcy–Prandtl number PrD and the Darcy number. Inertia creates an Instability; this shows destabilizing effect stationary mode and it exhibits a dual behaviour traveling-wave mode.

2.6 Governing equations of Darcy-Brinkman model

Qiulei sun [20] has considered horizontal porous medium with infinitely extended with a layer of depth with two impermeable plates cooled from and heated from below saturated with Oldroyd B fluid and in time -periodic manner

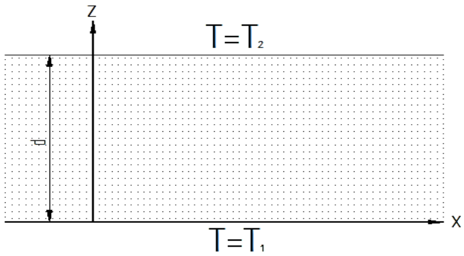

Figure 2: A saturated Layer of Porous Oldroyd b Fluid

The governing equations of the system are given as

$$\vec{q} = (u, v, w)$$

$$\left(1+\bar{\lambda}_1 \frac{\partial}{\partial x}\right)\left[\frac{\rho_0}{\varepsilon}\left(\frac{\partial \vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}\right)\right] = -\nabla p + \left(1+\bar{\lambda}_2 \frac{\partial}{\partial x}\right)\left[-\frac{\mu}{k}u+\mu_e \nabla^2 u\right]$$

3. Literature Summary

3.1 Results & Conclusions

The effect of increasing ratio of heat capacities has a stabilizing effect on the system. The results for the Maxwell fluid exhibit that the system is more unstable compared to Oldroyd-B type of viscoelastic fluids[11]. Based on the Modified- Darcy–Brinkman–Maxwell model, linear convective stability of a Maxwell fluid layer in a porous medium heated from below has been analysed and it is possible to derive the modified-Darcy–Brinkman–Maxwell model by macroscopic averaging techniques, but only after making a closure that incorporates some empirical material [7].

3.2 Research Gaps

From the above literature study it was noted that heat transfer analytics was the prime area of interest in the modern industrial applications involving fluid flow situations. The main areas which can be explored further for an in-depth study are listed below.

- Instable flow of viscoelastic fluids can be modelled to higher degree FEM equations.
- Non linearity in convective Heat transfer in a porous medium may be modelled as a mathematical case study.
- Skin friction factor of a Oldroyd B fluid can be found for the circular tube flow, parallel plate and stretching sheet.

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