PAPER

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A resonant cavity method for measuring electrical permittivity in the GHz region

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Abstract

Resonant cavity perturbation as a means to measuring permittivity has been used since the early 1940s. It is attractive because it is non-contact, non-destructive and experimentally simple. Only recently, however, has the cost of a small analyser board become acceptably low that experiments can be carried out in schools and colleges. Using the TE 1,0,1 cavity mode in a rectangular cavity, measurements of permittivity for distilled water, methanol and PTFE in the microwave region of 1 GHz are presented and these agree well with published data. A granular sugar sample is also investigated to confirm measurements carried out with a compact Lecher Line apparatus

Keywords: cavity, resonator, dielectric, permittivity

1. Introduction

The measurement of permittivity in a Lecher Line apparatus has recently been discussed [1]. In this case it was noted that the wavelength of a microwave wave was reduced while passing through a material so that a phase change could be detected when a comparison was made to a reference signal.

The resonant method is very similar. An electromagnetic wave is trapped in a metal box as illustrated in figure 1

The mode patterns are exactly similar to standing wave patterns on a skipping rope with the red line, in figure 1(a), being the skipping rope. One loop for the mode TE 1,0,1, two loops for the mode TE 1,0,2 and three loops for the TE 1,0,3 mode. If a sample is centrally placed in the cavity then only those frequencies that generate modes with odd numbers can be used as these will then provide regions of maximum electric field at the sample. Although this work uses a cavity resonating in the TE 1,0,1 mode other workers [2] have used a cavity operating in the TE 1,0,11 mode to obtain optimum results.

It is hoped that students will gain a greater understanding of resonant structures and appreciate that materials may have a 'lossy' part to their dielectric constant. Of course, microwave cookers, operating at 2.45 GHz, depend very much on this property of water [3].

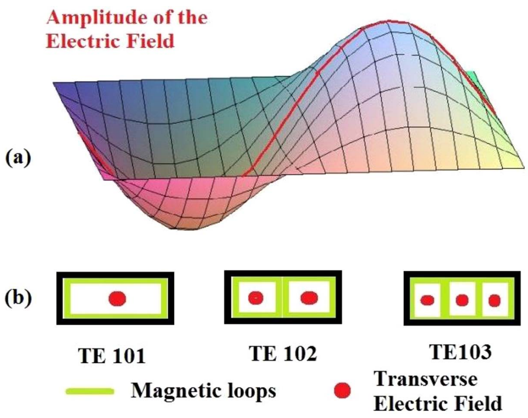


Figure 1. (a) A 2D wave profile for the electric field in the TE 1,0,2 mode. (b) Symbolic diagrams showing three mode patterns, red circles signify regions of maximum electric field.

2. Theory

The basic idea behind the cavity resonance method is that the wavelength of a wave is reduced when passing through a dielectric material as was explained in [1].

Thus any sample placed in the cavity will ‘squeeze’ the trapped wave so that it no longer neatly fits into the cavity. Only when the wave is changed to a longer wavelength (lower frequency) will the ‘fit’ be restored. Therefore, if a microwave signal is passed through such a resonant structure the transmission will be expected to have the form given in figure 2.

For the empty cavity we will have trace A and traces B and C will be obtained when two other samples are placed in a central position in the TE 1,0,1 cavity. In both cases we see a shift to lower frequencies and this is easily measurable as Δf. The other parameter that can be introduced is the Q-factor and this is a measure of the sharpness of the resonant peak. Again, the Q-factor can be measured. If δ is the band width of a peak at a position 3 dB down from the maximum transmission then:- Q-factor = f/δ where f is the frequency at resonance.

The perturbation theory of the resonant cavity is fully developed in [2] and the following equations are obtained:

If V is the volume of the cavity, v is the volume of the sample and permittivity is written

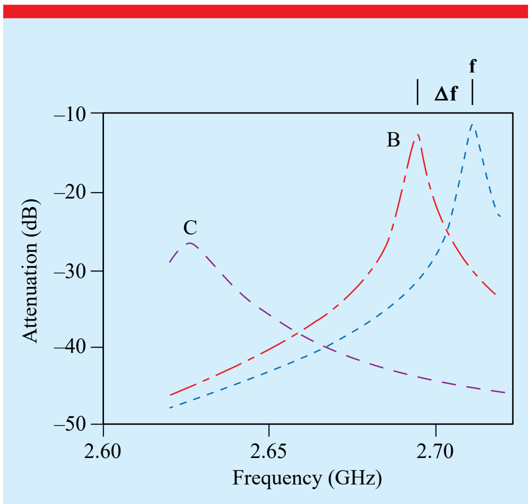


Figure 2. Transmission through a resonant cavity structure.

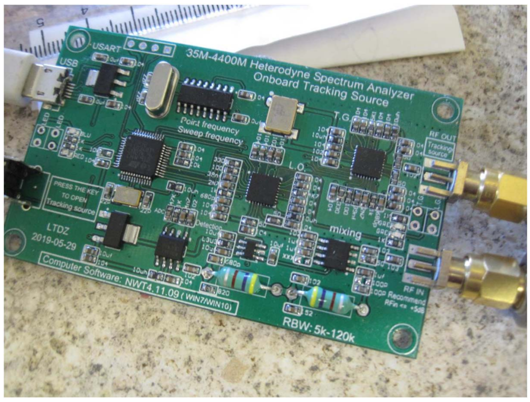


Figure 3. The photograph shows an analyser, purchased from Banggood (www.Banggood.com). The white lead is the USB cable.

in its complex form, namely, $\varepsilon^* = \varepsilon' - j\varepsilon''$ then we have:

$$\varepsilon'' = 1 + \frac{V.\Delta f}{2.v.f} \tag{1}$$

$$\varepsilon'' = \frac{V}{4.v}(1/Q_{sample} - 1/Q_{empty}). \tag{2}$$

Equation (1) allows the real (‘normal dielectric constant’) permittivity to be measured. However, if there is an interaction between the electromagnetic radiation and the sample which involves heating then the sample is said to be ‘lossy’ and

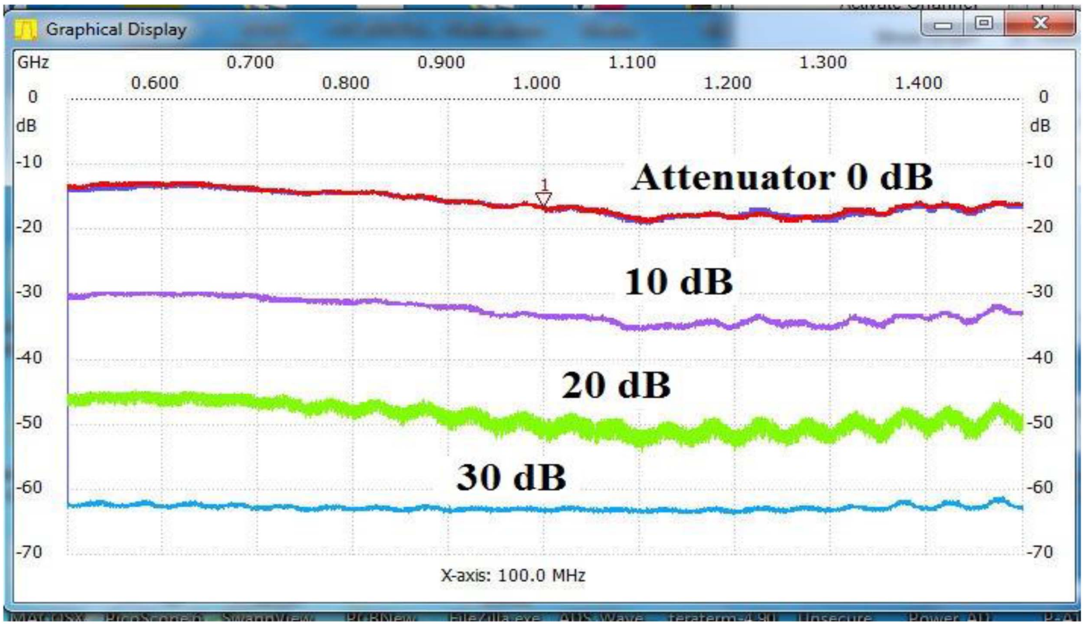


Figure 4. Attenuator measurements over the range 500 MHz to 1.5 GHz.

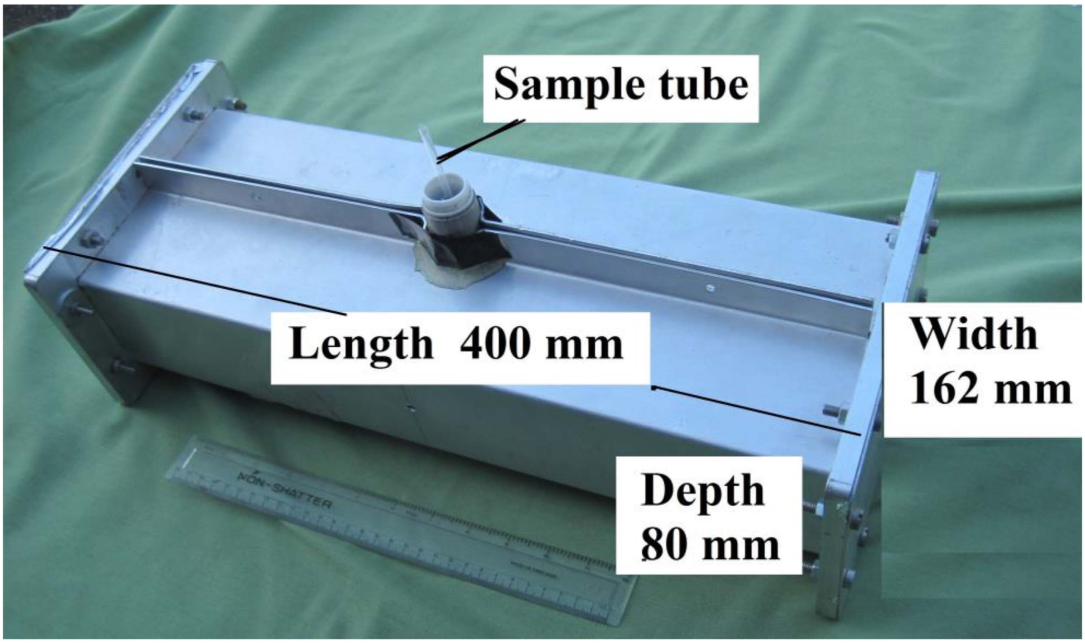


Figure 5. A cavity constructed from aluminium sheet with resonant frequency close to 1 GHz. (As an observation—although the analyser (figure 3) has been miniaturized, the laws of physics dictate the size of the cavity.)

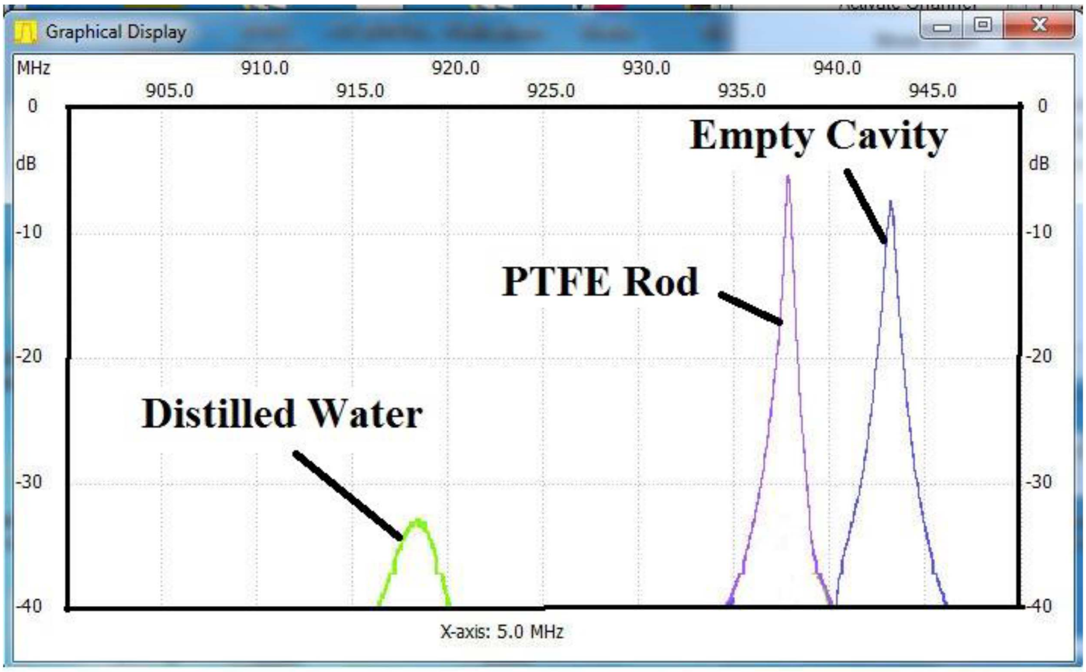


Figure 6. Resonant peaks for an empty cavity, a PTFE rod and distilled water in a tube.

Table 1. Permittivity data for several materials. (The ambient temperature for all measurements was 20 °C.)

Sample	Sample diameter (cm)	Real epsilon	Imaginary epsilon
PTFE	1.50 (±0.01)	1.99 (±0.05)	Zero
Granular sugar	1.60 (±0.01)	1.75 (±0.05)	0.017 (±0.005)
Distilled water	0.38 (±0.01)	77 (±2)	3.5 (±0.4)
Methanol	0.38 (±0.01)	27 (±1)	8.1 (±0.8)

equation (2) is used to calculate the imaginary permittivity. If samples extend for the whole of the depth of the cavity then one can change the volume ratio to an area ratio. For small samples, however, say, a ceramic bead or a grain kernel, then a depolarising factor has to be included in equations (1) and (2), [4].

Full details and limitations of the perturbation method are given in [2].

There is a simple relationship between frequency and wavelength for free-space electromagnetic waves, namely, $\lambda_{\text{free-space}}$ (cm) = 30/f (GHz). But in the case of ***guided*** waves one has to use the ***guide wavelength*** and this is given by:

$$\lambda_{guide} = \frac{\lambda_{free-space}}{\sqrt{1-(\lambda_{free-space}/\lambda_{cut-off})^2}}$$

where $\lambda_{\text{cut off}}$ is twice the broad dimension of the waveguide. A detailed derivation of this formula is given in the textbook by Cross [5].

3. Apparatus

Now that a low cost analyser is available the experiment is very simple; the unit is connected to a resonant cavity by coaxial cables. The analyser is illustrated in figure 3.

The analyser was purchased as a pcb unit but was mounted in an enclosure for protection' Software was down-loaded from the Banggood website. In good faith, the author feels that this analyser would be a worthwhile investment for any laboratory but measurements with a calibrated attenuator showed that it is necessary to calibrate the attenuation scale; figure 4.

The calibrated attenuator, with steps of 10 dB, was buffered with two fixed 7 dB attenuators at the input and output ports. One can see that the analyser registers approximately—60 dB whereas the true attenuation is only 30 dB. (why additional noise appears on the 20 dB trace remains a mystery at this present time).

It is customary to use lengths of standard wave-guide in the construction of a cavity; indeed, the investigators in [2] have fabricated their resonators from a length of WG 12 wave-guide (inner dimensions 47.5 mm $\times$ 22.2 mm). However, for a low frequency resonator such wave-guides are costly so the present cavity was fabricated from aluminium sheet (1 mm thick) as given in figure 5.

It was made in two halves so that folding the metal sheet was easily achieved. The fundamental mode of resonance is TE 1,0,1.

End plates had centrally place connectors with small antennas so that the wave was launched into the cavity space. A circular hole was drilled centrally in the top of the cavity to allow samples to be inserted. (Further details regarding the cavity are given in the appendix.)

4. Results

Transmission measurements were made on a variety of samples and graphs for a PTFE sample and a water sample are given in figure 6.

We see frequency shifts in the resonance peaks when samples are placed in the cavity in a manner suggested by figure 2. The software provides zoom facilities so that the frequency shifts and Q-factors can be determined with reasonable accuracy.

Table 1 records the measured permittivities.

Data for granular sugar agrees with that given previously [1] and the present measurements shows that there is a measurable imaginary epsilon value. The real epsilon value for PTFE agrees with published data but the present measurements cannot detect a significant loss component. The data for methanol and distilled water agree with data presented by Vijay *et al* [6].

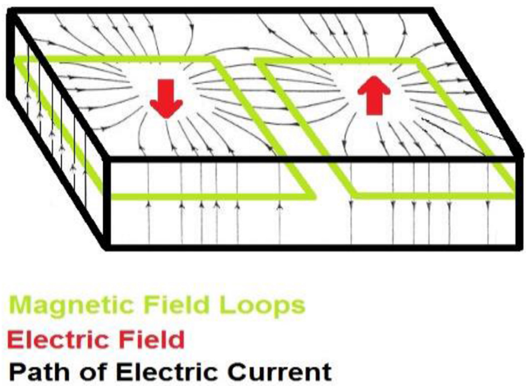
5. Conclusions

Construction of a microwave cavity and the purchase of a low cost analyser has allowed permittivity measurements to be carried out for PTFE, granular sugar, methanol and distilled water. The cavity techniques is non-contacting and non-destructive and experimentally simple to carry out.

It has to be admitted that measurements are only made at a single frequency; in the present case, 0.95 GHz. However, further data could be obtained by using higher order modes or indeed, making another cavity with different dimensions.

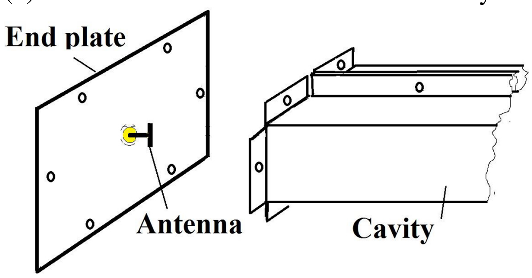
Appendix

(a) More details of field lines in a TE 1,0,2 mode cavity:



Note, there are no current paths ACROSS the broad face of the cavity so it can be made in two halves and then screwed together; electrically to two parts may be insulated from each other.

(b) Construction details of the resonant cavity:



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Frank Thompson joined the Physics School at Manchester Polytechnic, UK after spending some years working in oil prospecting in Libya. In the late 1990s he early retirement, and he has done consultancy work since then. At present, he is working at the MACE Centre, Manchester University.