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Effect of the new extended uncertainty principle on black hole thermodynamics

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Abstract – In this paper we study the thermodynamic properties of the Schwarzschild black hole using a new form of the extended uncertainty principle. By calculating the corrected mass-temperature relation, we investigate the limit of lower bound for the extended-uncertainty-principle–corrected black hole temperature. Also, we obtain the extended-uncertainty-principle–corrected entropy and heat capacity. We compare the behaviors of the usual form and the corrected form of thermodynamic properties for the static black hole.

Introduction. – The ordinary Heisenberg uncertainty principle is obtained as $\Delta X \Delta P \geqslant \hbar/2$ and by a deformation of the ordinary Heisenberg uncertainty principle also known as the generalized uncertainty principle (GUP), the concept of minimum measurable length into quantum mechanics is introduced [1–31]. Also, in order to study both the velocity of light and Planck energy as universal constants, another kind of deformation of the Heisenberg algebra known as Doubly Special Relativity (DSR) has been proposed [32–35].

Recently extending the Heisenberg uncertainty principle has been proposed in the context of Extended Uncertainty Principle (EUP), which describes the concept of minimum measurable momentum into quantum mechanics [36–43]. Also, the Hawking temperatures of the Schwarzschild-(anti-) de Sitter black holes under EUP have been reproduced, while it has been shown that GUP increases the Hawking temperature [6–8]. In ref. [9], by defining the new GUP, the authors have proposed the existence of a minimum observable momentum. The GUP-corrected energy of the quantum harmonic oscillator for all energy levels to first- and second-order perturbation is calculated [13]. In refs. [14] and [15], the deformed Lifshitz gauge theory based on GUP and the deformation of the Heisenberg algebra, which are consistent with both the GUP and DSR theory, are analyzed, respectively. We can see in ref. [38] that the uncertainty principle derivation of the Hawking temperature can be extended to (anti-) de Sitter-like black holes and the thermodynamics of the black holes is investigated starting from the Uncertainty Principle of string theory and non-commutative geometry. Also, in ref. [39] the connection between the extended generalized uncertainty principle (EGUP) and triply special relativity is studied. In ref. [40], by introducing EGUP with the area theorem, the correction value of black hole entropy for the three types of space-time is calculated. In ref. [41], the thermodynamics of the Friedmann-Robertson-Walker (FRW) universe under GUP and EUP has been obtained. In ref. [42], the measurable properties of a black hole horizon are modified by introducing a large mass scale correction to the Schwarzschild metric inspired by EUP.

In this paper we study the thermodynamic properties of the Schwarzschild black hole by using the new form of the EUP. This paper is organized as follows: In the next section a new form of EUP has been introduced. In the third section we discuss the black hole thermodynamic properties. In the subsection "The EUP-corrected mass-temperature relation" we obtain the corrected mass-temperature relation for EUP black hole and minimal temperature of black hole. In the subsection "The EUP black

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hole heat capacity" the EUP black hole heat capacity has been calculated. In the subsection "The EUP black hole entropy" by calculating the corrected heat capacity, we find the EUP black hole entropy. Finally, a conclusion is presented in the last section.

The new extended uncertainty principle. – In this section, in order to investigate the concept of minimum measurable momentum into quantum mechanics, we have considered the following new form of deformed Heisenberg algebra for one spatial dimensional case which is called EUP:

$$[X,P] \geqslant \frac{i\hbar}{1-q|X|}, \tag{1}$$

where q is a parameter and we choose $0 \leqslant q \leqslant 1$, the EUP is then given by

$$\begin{aligned}\Delta X \Delta P &\geqslant \frac{\hbar}{2}\left\langle \frac{1}{1-q\,|X|}\right\rangle \\ &\geqslant \frac{\hbar}{2}[1+q\langle|X|\rangle+q^2X^2+q^3|X|X^2+q^4(X^2)^2+\cdots] \\ &\geqslant \frac{\hbar}{2}\left[-q(\Delta X)+q(\Delta X)+1+q\,\langle|X|\rangle+\cdots\right] \\ &\geqslant \frac{\hbar}{2}\left[-q(\Delta X)+\frac{1}{1-q(\Delta X)}\right]. \end{aligned} \tag{2}$$

The momentum for this new EUP is given by

$$\Delta P \geqslant \frac{\hbar}{2}\left[-q+\frac{1}{\Delta X(1-q(\Delta X))}\right], \tag{3}$$

by solving the above quadratic equation, the EUP gives the minimal momentum as

$$\Delta P \geqslant \frac{3}{2}q\hbar. \tag{4}$$

Black hole thermodynamics. –

The EUP-corrected mass-temperature relation. By characterizing the momentum uncertainty, for any massless quantum particle near the Schwarzschild black hole horizon with mass M, its temperature can be written as

$$T = \frac{c\Delta P}{\kappa}, \tag{5}$$

where c is the speed of light and κ is the Boltzmann constant. Using this equation and the obtained minimal momentum, we can calculate the lower bound for the black hole temperature as follows:

$$T \geqslant T_{\min} = \frac{3}{2}\frac{cq\hbar}{\kappa}; \tag{6}$$

by defining the Schwarzschild radius of the black hole as

$$r_S = \frac{2GM}{c^2}, \tag{7}$$

where G is the Newton universal gravitational constant we can consider the position uncertainty of a particle as the order of the Schwarzschild black hole radius near the black hole horizon,

$$\Delta X = \gamma r_S, \tag{8}$$

where γ is a scale factor, r_s is the Schwarzschild radius. Regarding the thermodynamics, the properties of black hole remnants have been extensively studied in ref. [44]. In ref. [12], the minimum masses of Schwarzschild black hole have been obtained from GUP and the authors have compared the mass of Schwarzschild black hole in the presence of generalized uncertainty principle with the obtained mass of Schwarzschild black hole from the ordinary Heisenberg uncertainty principle. Here, by inserting the eq. (5) and the eq. (8) into eq. (3), the relationship corresponding to the black hole mass has been obtained as

$$\frac{\hbar}{2}\left(-\frac{2\gamma qGM}{c^2}+\frac{1}{1-\frac{2\gamma qGM}{c^2}}\right) = \frac{2\gamma GM\kappa T}{c^3}; \tag{9}$$

by solving the above equation for M, we get

$$M = \frac{c^2}{4G\gamma q}\left[1-\left(1-\frac{4c\hbar q}{c\hbar q+2\kappa T}\right)^{\frac{1}{2}}\right]; \tag{10}$$

by introducing m_p as the Planck mass and considering the relationship $(m_P c)^2 = \frac{\hbar c^3}{G}$, eq. (10) reduces to

$$M = \frac{(m_P c)^2}{2\gamma(c\hbar q+2\kappa T)}, \tag{11}$$

and in the absence of correction due to EUP, the above equation reduces to $M = \frac{(m_P c)^2}{4\gamma\kappa T}$, where our obtained result for the black hole mass is consistent with the previous obtained result of other works in the literature when $q=0$ [36].

By comparing the black hole mass in the absence of correction due to EUP with the Hawking temperature $T_H = \frac{(m_P c)^2}{8\pi\kappa M}$, we can obtain $\gamma = 2\pi$. Then the total form of the mass-temperature relation can be rewritten as

$$M = \frac{c^2}{8\pi Gq}\left[1-\left(1-\frac{4c\hbar q}{c\hbar q+2\kappa T}\right)^{\frac{1}{2}}\right], \tag{12}$$

and for a small value of q, the black hole mass can be extended as

$$M = \frac{(m_P c)^2}{8\pi\kappa}\left(\frac{1}{T}-\frac{c\hbar q}{2\kappa T^2}+\frac{(c\hbar q)^2}{4\kappa^2 T^3}+\cdots\right), \tag{13}$$

as we can see for the limit of value $q=0$ or in the absence of correction due to EUP, we reach $M = \frac{(m_P c)^2}{8\pi\kappa T} = \frac{\hbar c^3}{8\pi G\kappa T}$ which is consistent with the previously obtained result for the usual form of the black hole mass in ref. [36].

The EUP black hole heat capacity. We can investigate the heat capacity of the black hole under EUP. The EUP-corrected heat capacity relationship can be calculated as

$$C = c^2\frac{\mathrm{d}M}{\mathrm{d}T} = -\frac{m_p^2 c^4\kappa}{2\pi(2\kappa T+c\hbar q)^2}. \tag{14}$$

In order to verify obtained relationship for the EUP-corrected heat capacity, we can calculate eq. (14) for a small value of q and then compare its limit value for $q=0$ and the usual form of the black hole heat capacity. For a small value of q we have

$$C=-\frac{m_p^2c^4}{8\pi\kappa}\left[\frac{1}{T^2}-\frac{c\hbar q}{\kappa T^3}+\frac{3(c\hbar q)^2}{\kappa^2T^4}+\cdots\right], \tag{15}$$

and for $q=0$, eq. (15) reduces to $C=-\frac{m_p^2c^4}{8\pi\kappa T^2}$ that is consistent with the previous obtained result for the usual form in the absence of correction due to EUP [36].

The EUP black hole entropy. We can determine the EUP black hole entropy by using the first law of black hole thermodynamics,

$$\begin{aligned} S &= c^2\int_{M(T_{\min})}^{M(T)}\frac{\mathrm{d}M}{T}=\int_{T_{\min}}^{T}C(T)\frac{\mathrm{d}T}{T},\\ &=-\frac{(m_pc)^2\kappa}{8\pi q^2\hbar^2}\left[0.15+4\mathrm{Ln}(2)+\frac{4c\hbar q}{c\hbar q+2\kappa T}\right.\\ &\quad\left.+4\mathrm{Ln}\left(\frac{\kappa T}{c\hbar q+2\kappa T}\right)\right]+S_0, \end{aligned}\tag{16}$$

where S_0 is the integration constant. Expanding the black hole entropy relationship eq. (16) for a small value of q, we get

$$\begin{aligned} S &= S_0-\frac{0.15(m_pc)^2\kappa}{8\pi q^2\hbar^2}+\frac{\left(m_pc^2\right)^2\kappa}{16\pi(\kappa T)^2}-\frac{\left(m_pc^2\right)^2(c\hbar q)\,\kappa}{24\pi(\kappa T)^3}\\ &\quad+\frac{3\left(m_pc^2\right)^2(c\hbar q)^2\kappa}{128\pi(\kappa T)^4}+\cdots, \end{aligned}\tag{17}$$

and by fixing the value of S_0 as $S_0=\frac{0.15(m_pc)^2\kappa}{8\pi q^2\hbar^2}$, the total corrected entropy form of the EUP black hole can be rewritten as

$$\begin{aligned} S &= -\frac{(m_pc)^2\kappa}{8\pi q^2\hbar^2}\left[4\mathrm{Ln}(2)+\frac{4c\hbar q}{c\hbar q+2\kappa T}\right.\\ &\quad\left.+4\mathrm{Ln}\left(\frac{\kappa T}{c\hbar q+2\kappa T}\right)\right]. \end{aligned}\tag{18}$$

For a small value of q we get

$$S=\frac{\left(m_pc^2\right)^2\kappa}{8\pi\kappa T}\left[\frac{1}{2\kappa T}-\frac{c\hbar q}{3(\kappa T)^2}+\frac{3(c\hbar q)^2}{16(\kappa T)^3}+\cdots\right], \tag{19}$$

and in the absence of correction due to EUP, eq. (19) reduces to $S=\frac{\left(m_pc^2\right)^2\kappa}{16\pi(\kappa T)^2}$ for the ordinary form of entropy, where the obtained black hole entropy is consistent with the result obtained in ref. [36] when $\alpha=0$.

Also, the entropy can be rewritten in terms of the area of the horizon $A=4\pi r_S^2=4\ell_P^2\left(\frac{S_0}{\kappa}\right)$, where $r_S=\frac{2GM}{c^2}$ is the Schwarzschild radius of the black hole and ℓ_P is the Planck length and S_0 is the semi-classical Bekenstein-Hawking entropy for the Schwarzschild black hole.

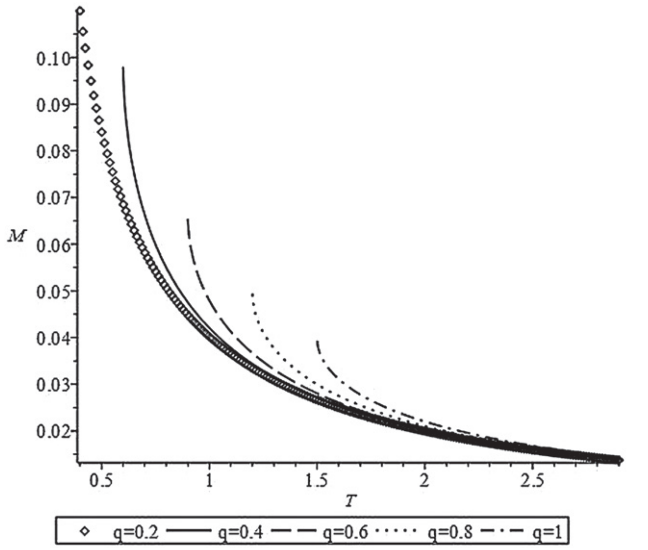

Fig. 1: The EUP-corrected mass *vs.* T for $m_p=0.5$, $c=\hbar=\kappa=1$.

From the mass-temperature relation for EUP black hole eq. (12), the temperature of a black hole can be generally calculated as

$$T=\frac{-c^5\hbar+4c^3G\hbar M\pi q-16cG^2\hbar M^2\pi^2q^2}{8\pi\kappa GM\left(-c^2+4GM\pi q\right)}, \tag{20}$$

where for $q=0$, the above equation reduces to $T=\frac{c^3\hbar}{8\pi\kappa GM}=\frac{(m_Pc)^2}{8\pi\kappa M}$, which is consistent with the previously obtained result in the usual form for the temperature of a Schwarzschild black hole in the absence of correction due to EUP [36]. Substituting eq. (20) into eq. (19) and by considering $A=4\pi r_S^2=\frac{16\pi G^2M^2}{c^4}$, the entropy in terms of the area of the horizon is given by

$$\begin{aligned} S &= \frac{m_P^2\kappa^2c^6\left(c^2-4GM\pi q\right)^2}{2\hbar^2\left(c^4-4c^2GM\pi q+16G^2M^2\pi^2q^2\right)^2}\\ &\quad\times\left[\frac{A}{2\kappa}-\frac{8\pi qAGM\left(c^2-4\pi qGM\right)}{3\kappa\left(c^4-4\pi qc^2GM+16\pi^2q^2G^2M^2\right)}\right.\\ &\quad\left.+\frac{12\pi^2q^2AG^2M^2\left(c^2-4\pi qGM\right)^2}{\kappa\left(c^4-4\pi qc^2GM+16\pi^2q^2G^2M^2\right)^2}+\cdots\right], \end{aligned}\tag{21}$$

with $q=0$ and considering the relationships giving the Planck length as $\ell_P^2=\frac{\hbar G}{c^3}$ and the Planck mass as $(m_Pc)^2=\frac{\hbar c^3}{G}$, the entropy relationship reduces to

$$S_0=\frac{\kappa^2m_P^2c^2A}{4\kappa\hbar^2}=\frac{\kappa A}{4\ell_P^2}. \tag{22}$$

We can see that, in the absence of correction due to EUP ($q=0$), the relationship obtained for entropy reduces to the semiclassical Bekenstein-Hawking entropy for the Schwarzschild black hole.

We have investigated the EUP-corrected black hole thermodynamics in figs. 1–3. We have investigated in fig. 1

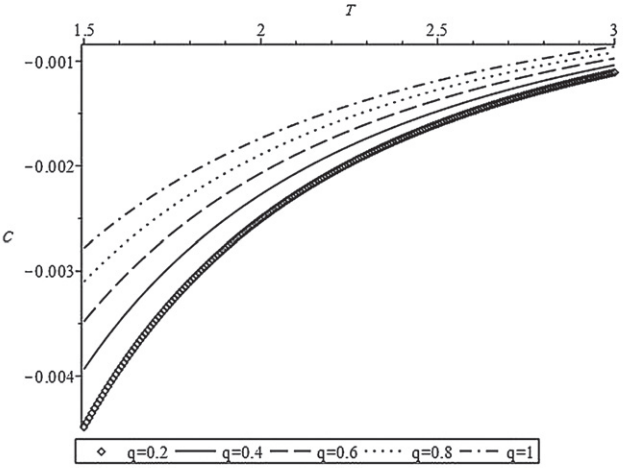

Fig. 2: The EUP-corrected heat capacity *vs.* T for $m_p = 0.5$, $c = \hbar = \kappa = 1$.

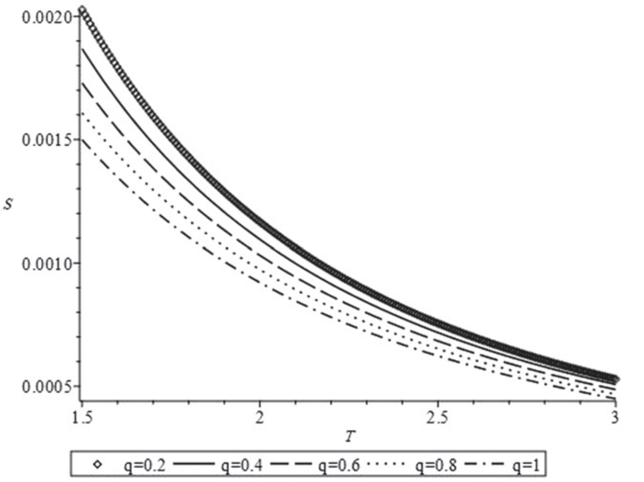

Fig. 3: The EUP-corrected entropy *vs.* T for $m_p = 0.5$, $c = \hbar = \kappa = 1$.

the EUP-corrected mass of AdS black hole *vs.* the black hole temperature T. Figure 2 is included to give a better insight of the EUP-corrected black hole heat capacity *vs.* T. Also, we have investigated the entropy of a black hole under the conditions of the EUP-corrected case for $q \neq 0$ *vs.* the black hole temperature in fig. 3. We can see from fig. 1 that although the EUP-corrected mass of AdS black hole increases as q increases, it decreases as T increases. In fig. 2, by taking a set of parameters $m_p = 0.5$, $c = \hbar = \kappa = 1$, we have investigated the effects of the q-parameter and the black hole temperature T on the black hole heat capacity under the conditions of the EUP correction and we can see that the black hole heat capacity increases with increasing q and T. Figure 3 shows the EUP-corrected entropy is always positive while the EUP-corrected heat capacity is always negative. The behavior of the black hole entropy *vs.* T is satisfied and we have investigated the effects of the q-parameter and T on the black hole entropy under the conditions of the EUP correction in fig. 3. We can see that the entropy decreases with increasing q and T. Our obtained results have been compared with the previous works of other authors in the literature and our obtained results for thermodynamic properties when $q = 0$ reduced to the work reported [36] in the absence of correction due to deformation of the ordinary Heisenberg uncertainty principle or when $\alpha = 0$.

Conclusions. – In this paper we have investigated the thermodynamic properties of the Schwarzschild black hole by using the EUP. We have obtained the mass-temperature relation for EUP black hole. From the minimal momentum of EUP we found that a lower bound for the EUP black hole temperature should exist. Also by using the EUP black hole corrected mass-temperature relation, we have obtained the heat cepacity and entropy of the EUP-corrected Schwarzschild black hole in terms of the black hole temperature. We have found that the heat capacity is always negative and increasing with the temperature. Moreover, the obtained results in this work have been compared with the previous ones already given in the literature.

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